OFDM Communication over Time-Varying Frequency-Selective Fading Channels

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***Abstract*—To contemplate the wireless channel effect on a signal, two different models for fading channels are studied during different circumstances. The main objective is to do a parameter analysis, and the results are presented through plots for different scenarios.**

I. Introduction

When a wireless channel is frequency selective, a signal sent through it is considered broadband. The sample time is then smaller than the delay spread, such that the channel cannot be considered constant in within the spectrum of the signal. This means different frequency components will be affected differently by the channel. A common way to circumvent the consequences of a frequency selective channel is to divide it into *sub-channels*. The technique presented in this report is the orthogonal frequency division multiplexing (OFDM). Generally, the data is then split up in the sub-channels to locally lower the rate (and requirements) and sent on top of each other as an OFDM symbol. Orthogonality in this context means that all sub-carriers are multiples of a fundamental frequency such that they can be separated at the receiver.

Given in the assignment is the relative velocity $v = 15$ m/s and the carrier frequency $f_c = 2$ GHz. This defines the coherence time as

$$T_{coh} = \frac{1}{f_D} = \frac{\lambda}{v} = 0.01s, \tag{1}$$

where f_D is the Doppler frequency. In words, the coherence time means that the channel can be considered stationary within an interval of 0.01 s. The signal bandwidth is provided as $B = 1$ MHz, giving a sample time of $T_s = 1\mu$s. Theoretically, the maximum number of samples that can be sent within one coherence time is thus T_{coh}/T_s. Although, transmission over the full coherence time is typically not implemented, since this is not a static metric. We choose the requirement that we only transmit over a tenth of the coherence time to have a more robust system. Since OFDM system implementation requires Fourier transforms, the number of samples are chosen as an integer power of two:

$$N = \text{pow2}(\frac{T_{coh}}{10T_s}) = 512. \tag{2}$$

A cyclic prefix is required when using OFDM, to null the ISI. The length of that must be decided to know what actual data can be sent in one OFDM symbol. The delay spread is provided in the task as $\tau_D = 5.4\mu$s, which sets the requirement of the number of taps L, which is also the minimum required length of the cyclic prefix. The number of taps is calculated as

$$L = \text{ceil}(\tau_D/T_s) = 6, \tag{3}$$

where the operator "ceil()" denotes rounding up to the closest integer. This way we make sure the number of taps is not too small.

The number of sub-channels is a design parameter that should be chosen wisely for the best performance. We want to choose the sub-carriers such that the effective data rate is as large as possible. Generally, the relative number of samples that are dedicated as cyclic prefix should be as small as possible to minimize the loss in data rate. One OFDM symbol is encoded such that the length in samples in the symbol is the number of sub-carriers. For this reason, the number of sub-carriers are equal to N.

In the task given, for $\tau_D = 5.4\mu$s and bandwidth $B = 1$ MHz ($T_s = 1\mu$s), a single carrier system would suffer from ISI. The solution is that one OFDM symbol will have the duration of $T_{\text{OFDM}} = NT_s$. In this way we make the symbol time $T_{\text{OFDM}} >> \tau_D$. At the same time, the channel must be constant for each OFDM symbol, such that $T_{\text{OFDM}} < T_{coh}$. Choosing N is a trade-off within these limits. We choose N as the closest power of 2 such that $T_{\text{OFDM}} < T_{coh}/10 = 1$ ms.

II. Simulation Task

Assume that the tap gains $c_l(nTs)$ are i.i.d. Rayleigh fading with Clarke's spectrum and a flat power delay profile (so $E[|c_l(nTs)|^2] = \frac{1}{L}\ \forall l$). The communication system is operating at $f_c = 2$ GHz carrier frequency with a bandwidth of 1 MHz. The noise spectral density receiver is $N_0/2 = 2.07{\times}10^{-14}\ \mu M/Hz$. The wireless link is experiencing a path loss of $101dB$ (we assume the shadow fading is negligible). The speed of receiver is $15m/s$ and the delay spread is 5.4 μs.

A. Choose appropriate values for N and N_{cp}. Verify through calculation and simulation that the conditions are satisfied. Relate (though a mathematical expression) E to the average transmit power P (if the average transmit power P is 0.1 W, what is the value of E?)

From the description in the part of introduction, N should satisfy the equation

$$N \ll \frac{T_{coh}}{T_s}, \tag{4}$$

for which we choose

$$N < \frac{T_{coh}}{10T_s} = \frac{0.01}{10^{-6} \times 10} = 1000 \Rightarrow N = 512 \tag{5}$$

For the choice of the length of cyclic prefix N_{cp}, it depends on the delay spread. At the receiver, we get a sequence of OFDM symbols (an OFDM symbol as an OFDM block). If the N_{cp} is not larger than the delay spread τ_D, some data samples from the previous OFDM block may still be delayed to the next block across the cyclic prefix. This may cause ISI, which means that the cyclic prefix can not prevent ISI in this case. To be ISI-free, the length of the cyclic prefix should be larger than the delay spread:

$$N_{cp} > \frac{\tau_D}{T_s} = \frac{5.4 \times 10^{-6}}{10^{-6}} = 5.4 \Rightarrow N_{cp} = 6. \tag{6}$$

If N_{cp} is large compared to N, the communication system design is poor since all samples serving as cyclic prefix are discarded at the receiver. For the chosen parameters, the loss rate is $N_{cp}/(N + N_cp) = 1.16\%$, which is considered acceptable. The value of N and N_{cp} satisfy the conditions that

$$(N+N_{cp})f_D T_s = (512+6)\times 100 \times 10^{-6} = 0.0518 \ll 1 \tag{7}$$

$$N_{cp} > L - 1 = 5, \tag{8}$$

Where L is the number of channel taps.

The expected value of the symbol energy $E[|s_k^{(m)}|^2] = E$, where $s_k^{(m)}$ is the kth sample of the mth OFDM symbol. For the average transmit power P and sample time T_s, the relation between symbol energy E and average transmit power P is

$$E = P \cdot T_s \tag{9}$$

For $P = 0.1$ W, $E = 1 \times 10^{-7}$ J.

B. Generate independent QPSK data over all N sub-carriers for multiple OFDM symbols and implement the transmitter and receiver. To test the system, run the transmitter and receiver over an AWGN channel with $c_0(nTs)$ $\forall n$ and $c_{l\neq 0}(nTs) = 0$ $\forall n$. Make scatter plots of the received signal on a few sub-carriers. Explain how you implemented the different stages of the transmitter and receiver.

To simulate the transmission of OFDM over the AWGN channel, we generate a vector of data and map them to a QPSK constellation. Since we want to be able to control the SNR, the symbol energy is calculated for a specific transmit power. By varying the power we can then study the system performance. By the inverse fourier transform (IFFT), the symbols are converted to time domain according to

$$z_n^{(m)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k^{(m)} \exp(j2\pi \frac{nk}{N}), n = 0, 1, ..., N-1. \tag{10}$$

Now we set the ISI guards between each OFDM symbol by adding the cyclic prefix of length N_{cp}. We precede the data by a copy of the last N_{cp} number of samples in each OFDM symbol. At this stage we have a matrix of size $O \times (N + N_{cp})$, where O is the number of OFDM symbols. The signal is transmitted through an AWGN channel by multiplying with the path loss and adding complex noise of variance $N0$. By setting the number of taps L to 1, we obey that $c_0(nTs) = 1$, $\forall n$ and $c_{l\neq 0}(nTs) = 0$, $\forall n$.

In the receiver, we have the received symbols $y_n^{(m)}$. We first discard the cyclic prefix, and then transform back to frequency domain by using the FFT. In the case of AWGN channel, there is simply noise added and no equalizer is required.

Symbol detection is performed through the maximum likelihood method, i.e. making the decision from the minimum euclidean distance between the received symbols and the original constellation points from the QPSK map.

The scatter plots of the received symbols for 10 sub-carriers are generated. From figures 1, 2 and 3 we can conclude that the constellation will be much clearer when using higher averaged transmitted power P. When $P = 10$ dBm, the received SNR is lower compared to when $P = 30$ dBm, since the noise power is constant. From the constellation we can see that the noise is spread out similarly for all the sub-carriers (scatter points in one color denotes one sub-carrier data) because we introduce Gaussian noise in our simulation. When the spread around each original QPSK symbol in the scatter plot is smaller, the SER will be lower, which is beacause of higher SNR. For the AWGN channel, we emphasize that for the different sub-carriers, the SNR (consequently the spread in the scatter plot, and the SER) is very similar. Since the statistical distribution is equal for all sub-carriers, the SER will be equal for all of them when the number of transmitted symbol grows larger.

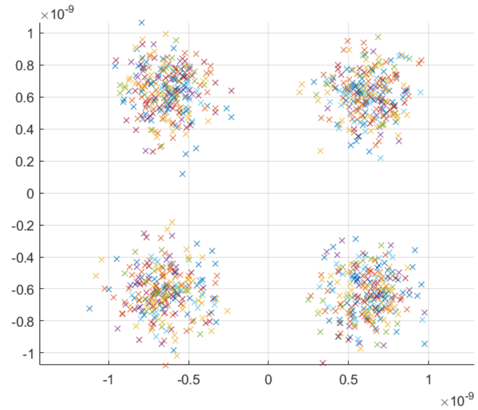

Fig. 1. Scatter plot of the received symbols for ten sub-carriers (different colors for different sub-carriers). The channel is AWGN and Pt = 10dBm.

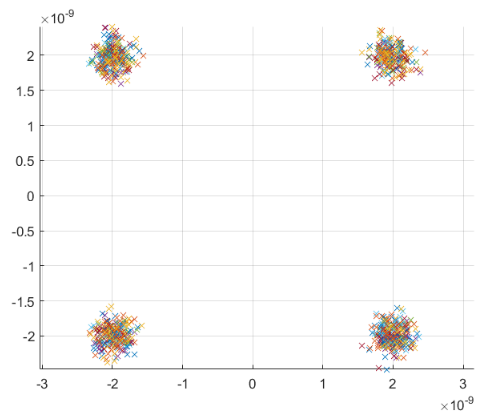

Fig. 2. Scatter plot of the received symbols for ten sub-carriers (different colors for different sub-carriers). The channel is AWGN and Pt = 20dBm.

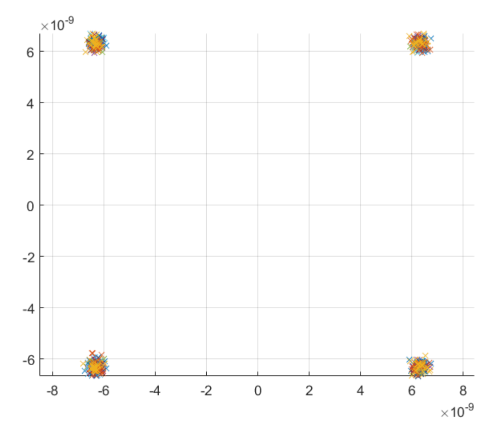

Fig. 3. Scatterplot of the received symbols AWGN when P = 30dBm.

To get an overview of the behavior due to the AWGN channel, the SER over SNR for the theoretical case and the estimated data is displayed in the figure below. The difference between the theoretical and estimated data comes from estimations in the theoretical model, using the Q-function.

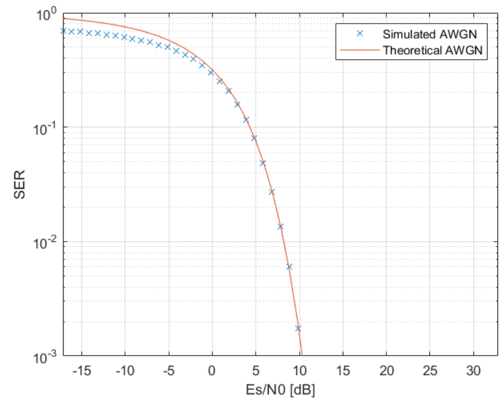

Fig. 4. Symbol error rate versus E_s/N_0 (SNR) for the AWGN channel.

C. Now include the time-varying frequency selective channel $c_l(nTs)$ between the transmitter and the receiver. Make scatter plots of the received signal on a few sub-carriers. Explain how you implemented the different stages of the transmitter and receiver. Derive and implement maximum likelihood detection for each sub-carrier. Predict the SNR per sub-carrier theoretically and compare with simulations. Explain your reasoning. By varying E, plot the average symbol error rate as a function of $E/N0$ (choose a meaningful range of symbol error rates, e.g., between 0.5 and 0.001) and compare with theoretical symbol error rate. What is the effective data rate?

The transmitter is implemented in the same way as for the AWGN channel case. The data sequence is first mapped into the QPSK symbols, then they are encoded into OFDM symbols through dividing the whole sequence of symbols into sub-carriers and performing an IFFT to convert to time domain. The cyclic prefix works as a guard interval for ISI, and is enabling transmission over a large bandwidth but degrades the effective data rate.

We now include the frequency varying channel, using the provided file ”Fading_Channel.p”. The input required is the power delay profile, the delay in samples, the normalized Doppler frequency and the input signal. The output is the signal passed through the channel, and the channel samples.

We add the path loss and noise to the received signal, and discard the trailing samples from the signal delay. The channel samples in frequency domain are retrieved by an N-point FFT of the channel samples in delay domain. By discarding the cyclic prefix from each OFDM symbol, the ISI-infected samples are removed. The received samples are converted to frequency domain, and are equalized by dividing with the channel samples also in frequency domain, which removes the affect of the fading channel on the received signal. The detection is made through a maximum likelihood decision through the minimum euclidean distance to the symbol map used at encoding.

The SNR per sub-carrier in a fading channel is generally calculated by the power on each sub-carrier and and the noise power within each sub-band as

$$\text{SNR}_k = \frac{\alpha_k^2 \cdot P_k}{N_0 \cdot B_k}, \tag{11}$$

for sub-carrier k and $\alpha_k = |H_k|$. To theoretically calculate the SNR it is thus required to know the channel gains for each sub-carrier. Since no power adaptation is implemented, the transmitted power per sub-carrier is $P_k = P/N$, where P is the average transmit power and N is the number of sub-carriers. Depending on the fading variations in the channel, α_k will vary. For this reason, the SNR for each channel can be very different.For a certain transmit power, figure 8 displays the simulation of the SNR for a flat i.e. AWGN channel and a frequency selective channel i.e. Rayleigh fading channel in our case. We can see that for AWGN channel, the SNR is constant while for Rayleigh fading channel the SNR will change with different channels, which is similar with the theoretical analysis.

For two different values of the transmit power, the scatter plots of the received symbols per sub-carrier are provided. Compared with the AWGN case, we can now see that some of the sub-channels have a rather large spread in the constellation relative to others. This is due to that the channels for the different sub-channels are different and vary in channel gain (i.e. SNR). The result from the different channel effects on the sub-channels is that the SER will be degraded compared to the AWGN case.

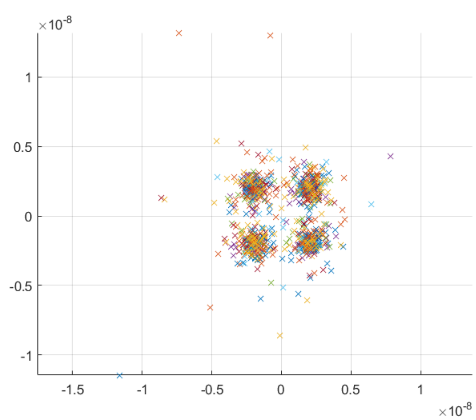

Fig. 5. Scatter plot of the received symbols for ten sub-channels (different color for different sub-carriers). The channel is a fading channel and Pt = 50 dBW.

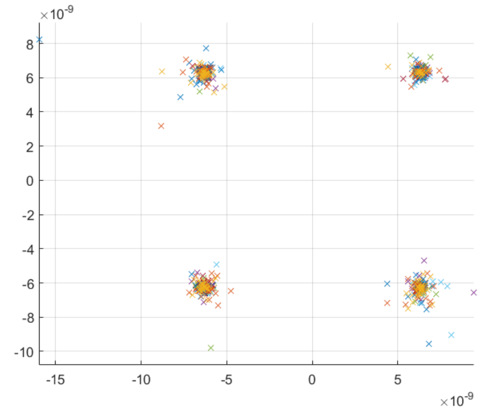

Fig. 6. Scatter plot of the received symbols for ten sub-channels (different color for different sub-carriers). The channel is a fading channel and Pt = 60 dBW.

The performance is again estimated through the SER to emphasize the effects of using a number of sub-channels. Figure 7 displays the degradation of the performance compared with the frequency selective channel and AWGN channel. Due to the introducing of the cyclic prefix, the pass loss and Rayleigh fading, to achieve the same SER, the frequency selective channel will required much transmitted power(i.e. SNR).

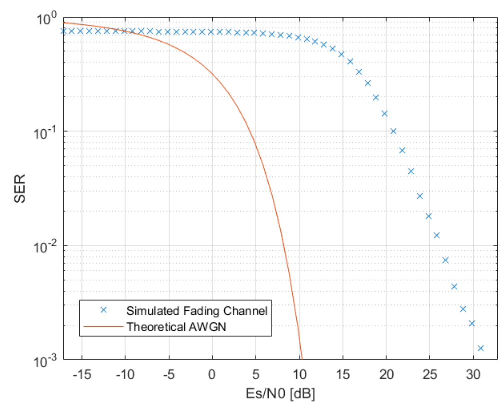

Fig. 7. Symbol error rate (SER) as a function of Es/N0 for all sub-carriers in a fading channel simulation.

The effective data rate, that is the result of using a cyclic prefix, is calculated by

$$R_b = \frac{N \log_2(M)}{(N + N_{cp})T_s} \approx 1.977\text{Mbps}. \tag{12}$$

The rate loss is thus in this case 1.16%.

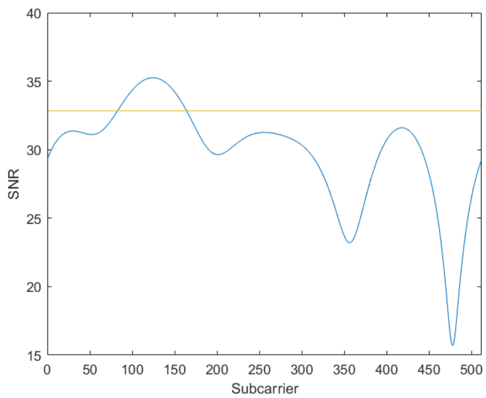

Fig. 8. SNR for an AWGN channel (yellow) and for the Fading channel (blue) for a certain transmit power.

D. *What happens with the scatter plots when you choose N_{cp} too small? What happens to the symbol error rate?*

When we change the length of N_{cp} to a small value in our code, it gives almost no change in the scatter plots and symbol error rate. This is because in our simulation we transmit only one OFDM symbol at a time, so we will not get interference between each OFDM symbol. But in theory, when the cyclic prefix is too short (shorter than the delay spread), the preceding OFDM symbol will leak into the adjacent OFDM symbol, resulting in ISI. And if the length of the cyclic prefix is very small compared with the length of the data sequence N, the rate loss will be small meaning that we use each sub channel effectively. However, this will reduce the robustness of the system, which may increase the symbol error rate.

III. Contribution from group members

Lise Aabel: I have spent approximately 16 hours on writing the report and 40 hours writing the MATLAB code presented for the entire project.

Yaxi Xie : I have spent approximately 27 hours on trying to write and modify the Matlab code. For the report, I have spent about 12 hours in answering the questions in the simulation part in a basic version, which has been modified by Lise Aabel later.

APPENDIX A
MATLAB CODE

```
clear all; close all; clf

par.M = 4; % QPSK
par.B = 1e6; % Bandwidth (tot)
par.fc = 2e9; % Carrier freq
par.lambda = 3e8/par.fc;
par.N0 = 2*2.07e-20; % Noise power [W/Hz](-164 dBm)
par.v = 15; % Relative velocity [m/s]
par.fD = par.v/par.lambda; % Doppler freq
par.Tcoh = 1/par.fD; % Coherence time
par.PL = 10^-10.1; % Path loss in linear (-101 dB)
par.tau = 5.4e-6; % Delay spread
par.P = -50:1:0; % Avg TX power [dBW]
par.Bcoh = 1/par.tau; % Coherence BW
par.O = 100; % Number of OFDM symbols
par.bps = log2(par.M); % Spectral eff / bits per symbol
par.channel = 'Fading Channel'; % {'Fading Channel' or 'AWGN'}
par.Ts = 1/par.B; % Sample time
par.L = ceil(par.tau/par.Ts); % Minimum number of taps/Ncp
par.Ncp = par.L;

% Max number of samples, N + Ncp must be within Tcoh/10
par.N = floor(par.Tcoh/10/par.Ts); % Max no samples for a ~stationary channel
par.N = par.N - par.Ncp; % Actual data samples w/o CP
par.N = pow2(floor(log2(par.N))); % actual data samples in power of 2 (# subcarriers)
par.R = par.N*par.bps/((par.N+par.Ncp)*par.Ts); % Data rate


% Preallocate SER
SER = zeros(1,size(par.P,2));
SER_theory_AWGN = zeros(1,size(par.P,2));
SER_theory_Rayleigh = zeros(1,size(par.P,2));
E_N0 = zeros(1,size(par.P,2));
for p = 1:length(par.P)
    % Create and scale symbols
    clear r
    clear h

    map = [ -1-1i, -1+1i, 1-1i, 1+1i ]; % Get the sombol map
    map = map./sqrt(mean(abs(map).^2)); % Normalize symbol energy, Es = 1
    bits = de2bi(0:par.M-1,par.bps,'left-msb'); % Create the bit pattern matching map
    E = 10^(par.P(p)/10)*par.Ts;
    map = map.*sqrt(E); % Scale to TX power

    % Create random data and map them to symbols
    data = randi(2,par.N*par.O,par.bps)-1;
    [~,idx] = ismember(data,bits,'rows'); % Find the corresponding symbol index
    S = map(idx); % SxO vector of symbols

    % Serial to parallel
    S_p = reshape(S,[par.N par.O]); % Split up into subchannels

    % Convert to time domain
    s_p = sqrt(par.N)*ifft(S_p,par.N,1);

    % Add CP (precede s_p with end of itself)
    s = [s_p(par.N-par.Ncp+1:par.N,:); s_p];

    % Parallel to serial, signal to transmit
    x = s.';

    tau = (0:par.L-1);
    P = ones(1,par.L)*10^(par.P(p)/10)/par.L; % Power delay profile

    switch par.channel
        case 'Fading Channel'

            % [r,h] = Fading_Channel(s, tau, fdts, P);
            % s = channel input (One OFDM symbol)
```

```matlab
            % P = power delay profile [P/L]
            % tau = path delay in samples

            % Pass each OFDM symbol through the channel, r and h will be an OxS matrix
            for i = 1:par.O
                [r_temp,h_temp] = Fading_Channel(x(i,:), tau', par.fD*par.Ts, P);
                r(i,:) = r_temp;
                h(i,:) = h_temp(1,:);
            end

            % Add noise and path loss
            noise = sqrt(par.N0/2)*(randn(size(r))+1j*randn(size(r)));
            r = r*sqrt(par.PL) + noise;

            y = r.'; % Flip back
            h = h.'; % Flip back

            % discard trailing samples
            y = y(1:end-par.L+1,:);

            H = fft(h,par.N,1);

            SNR_k = 10*log10(abs(H(:,1)).^2*10^(par.P(p)/10)*par.PL/(par.N0*par.B));
            SNR_0 = 10*log10(10^(par.P(p)/10)*par.PL/(par.N0*par.B));

        case 'AWGN'

            noise = sqrt(par.N0/2)*(randn(size(x))+1j*randn(size(x)));
            r = x*sqrt(par.PL) + noise;
            y = r.';
    end

    %---------------------------------------------------------------------------

    % Discard CP (The ISI guard samples)
    y = y(par.Ncp+1:end,:);

    % Convert to freq domain
    Y = 1/sqrt(par.N)*fft(y,par.N,1);

    % Equalizer
    switch par.channel
        case 'Fading Channel'
            Y_hat = Y./H;
        case 'AWGN'
            Y_hat = Y;
    end

    % Parellel to serial
    Y_s = reshape(Y_hat,1,[]);

    [~,idx] = min(abs(Y_s.' - map),[],2); % Get the closest points in the constellation
    S_hat = map(idx); % Estimated symbols

    SER(p) = numel(find(S_hat~=S))/numel(S_hat); % Symbol error rate
    E_N0(p) = E*par.PL/(par.N0);
    SER_theory_AWGN(p) = 2*qfunc(sqrt(E_N0(p)));
end

Y_hat_sub = reshape(Y_hat,par.N,par.O);
for k = 1:10:100 % Plot scatter for 10 number of sub-carriers
    figure(1), hold on
    scatter(real(Y_hat_sub(k,:)),imag(Y_hat_sub(k,:)),'x'); % Scatterplot
    hold on
end
axis equal

grid on
figure(2), semilogy(10*log10(E_N0),SER,'x','DisplayName',['Simulated ' par.channel])
hold on, semilogy(10*log10(E_N0),SER_theory_AWGN,'DisplayName','Theoretical AWGN');
axis([10*log10(E_N0(1)) 10*log10(E_N0(end)) 0.001 1])
grid on
ylabel('SER')
xlabel('Es/N0 [dB]')
```

legend