



Investing in misallocation[☆]

Mete Kılıç^a, Şelale Tüzel^{a,*}

^a Marshall School of Business, University of Southern California, 3670 Trousdale Pkwy, Los Angeles, CA 90089, USA

ARTICLE INFO

Dataset link: [Kılıç & Tüzel - Investing in Misallocation \(Final\) \(Reference data\)](#)

JEL classification:

D21
D22
D24
D25
E22
G31
O30
O47

Keywords:

Misallocation
MPK
Endogenous growth
Investment
Jumps

ABSTRACT

We document that 20% of Compustat firms exhibit above-median investment rates despite having below-median marginal product of capital (MPK), seemingly “misallocating” resources. These firms are typically younger and more likely to experience substantial upwards jumps in sales and MPK in subsequent years. They contribute significantly to innovation, and their investments predict future aggregate productivity, creating value beyond their current MPK. We propose and estimate a simple endogenous firm growth model that captures key cross-sectional features and enables counterfactual analysis. Ignoring the potential for future jumps in hypothetical investment policies reduces MPK and investment dispersion but also lowers aggregate productivity.

1. Introduction

Recent empirical studies have uncovered significant heterogeneity in productivity levels across firms, even within industries. This finding is often interpreted as evidence of an inefficient capital allocation among firms, stemming from various distortions, and is referred to as “misallocation” (Hsieh and Klenow, 2009). The literature has focused on understanding the role of specific factors, such as adjustment costs, financing frictions, and firm risk in hindering the equalization of productivity across firms. A key insight emerging from this body of research is that reallocating capital from firms with lower marginal productivity to those with higher marginal productivity, such as by lifting financial constraints, can result in improvement in overall output and efficiency while reducing cross-sectional dispersion in productivity.

From the perspective of misallocation, which is usually measured by the dispersion in firms’ value added to input ratios, firms that make large investments despite having low output can be seen as recipients of

misallocated capital. However, many well-known firms, such as Tesla or Amgen, adopted a strategy of making substantial investments for several years after going public, even though their output levels were initially low. This approach ultimately led to a significant increase in their sales and productivity. This observation challenges the notion of the initial “misallocation” of capital, as it suggests that such investments were, in fact, successful strategies. Reallocating capital away from these firms, based solely on their lower productivity at the time, could potentially undermine long-run efficiency and hinder overall productivity growth.

In this paper, our focus is on firms that appear to be accumulating too much capital relative to their output, i.e. “investing in misallocation”, relative to other firms in the cross-section. We study the properties of these firms and explore their role for the dispersion across firms and aggregate efficiency. Empirically, we document that 20% of firms in the Compustat data set simultaneously have below-median

[☆] Toni Whited was the editor for this article. We thank the Editor and an anonymous referee for their valuable comments and suggestions. We are also grateful to Andrea Caggese, Yuchen Chen (discussant), Jaewon Choi, Jason Donaldson, Fotis Grigoris (discussant), Jerry Hoburg, Lars Kuehn (discussant), Adriano Rampini, Lukas Schmid, Martin Souchier (discussant), Amir Sufi, Neng Wang, Yufeng Wu, David Zeke, Miao Ben Zhang, Zhao Zhang, and participants at the Holden Conference in Finance at Indiana University, Frontiers in Finance Conference in Banff, SFS Cavalcade, FIRS, and seminars at University of Miami, CKGSB, Bilkent University, Frankfurt School of Finance and Management, and USC Marshall for helpful comments. Link to Research Data: <https://doi.org/10.17632/vmhdhtzxg.1>.

* Corresponding author.

E-mail addresses: mkilic@marshall.usc.edu (M. Kılıç), tuzel@usc.edu (Ş. Tüzel).

marginal product of capital (MPK) and above-median investment rates. These firms are typically younger and approximately 4% of them experience a large upward jump in their sales and MPK in the subsequent period, while the remaining firms have a much lower probability of experiencing such jumps. Importantly, these firms contribute significantly to the innovative activity within the economy. They exhibit a patent issuance rate that is more than twice as high as other firms, and their rates of breakthrough patent issuance and patent citation are more than three times as large. They are also more likely to be in the innovation stage along their product life cycle.

We explore whether the presence of infrequent but significant growth spurts, characterized as jumps, can explain the existence and characteristics of high investment-low MPK firms. To explore this, we develop and estimate a simple model that incorporates firm heterogeneity, capturing key empirical patterns. In this model, firms are classified as either high-type or low-type, reflecting their differing potential for productivity growth. Unlike low-type firms, high-type firms have the potential to experience a substantial positive productivity jump, with the probability of such a jump increasing as they invest more. Our model considers a single type of capital, encompassing both physical capital (e.g., equipment) and intangible capital, such as innovative capacity, organizational capital, brand capital, and customer base. Within this framework, high-type firms optimally invest at high levels, not merely for immediate production gains, but to maximize their chances of a jump occurring and to be prepared for its arrival. The presence of these firms disrupts the tight correlation between MPK and investment in the cross-section, resulting in a realistic proportion of firms that exhibit high investment despite having low MPK levels.

We also examine firm dynamics in the years before and after a jump realization. Both in the data and the model, firms maintain persistently high investment rates and low MPK levels relative to peers in the years leading up to a jump. When a jump occurs, investment increases further as firms capitalize on the surge in productivity. After this initial surge, investment tapers off and stabilizes at levels similar to peer firms. Similarly, MPK remains elevated post-jump and does not revert to pre-jump levels. Empirically, we also find that firms engage more actively in innovation in the years preceding and around the jump, with this activity gradually declining over the post-jump decade—suggesting a lasting shift in firm type.

We conduct a counterfactual experiment where firms' investment policies ignore the potential of future jumps even though jumps are still present in the data-generating process. That is, high-type firms do not engage in “investing in misallocation” and just invest the same amount as a low-type firm with the same productivity and capital. In the counterfactual model, investment and MPK become closely aligned in the cross-section, resulting in the absence of firms in the high investment-low MPK portfolio. This contradicts the empirical evidence where MPK dispersion among high investment firms is as high as it is among all firms. Furthermore, the overall MPK and investment dispersion becomes lower suggesting a decline in misallocation. However, this removal of “investing in misallocation” also leads to lower productivity in aggregate demonstrating that eliminating this source of cross-sectional MPK dispersion does not result in higher efficiency. Consistent with the lower aggregate productivity in the model counterfactual, our empirical findings reveal that the median investment rate among firms in the high investment-low MPK portfolio serves as a robust predictor of future aggregate growth in total factor productivity (TFP). Specifically, a one standard deviation increase in the investment rate corresponds to a one standard deviation increase in 5-year TFP growth. In sum, while traditional metrics may classify these firms as misallocating resources, they are creating value in ways that are not captured by their current MPK levels.

The influential works of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) introduced heterogeneous distortions to input and

output prices faced by firms, but did not explicitly identify the sources of misallocation. Subsequent research has identified various types of distortions as possible sources of misallocation, including adjustment costs and volatility (Asker et al., 2014), information frictions (David et al., 2016), excess investor demand Choi et al. (2025), and tradeoff between information disclosure and investment efficiency (Terry et al., 2023). The common thread in this literature is the exploration of the factors underlying the deviations of firm-level capital from an efficient allocation across firms, leading to disparities in MPK and productivity losses.

Another prominent strand of literature has proposed financial frictions as a significant factor contributing to misallocation (Midrigan and Xu, 2014; Moll, 2014; Whited and Zhao, 2021; Bau and Matray, 2023). These studies emphasize the importance of financial constraints that impede firms, especially those with high MPK and low investment, from accessing an optimal level of capital. Notably, financial frictions tend to have a particularly pronounced impact on innovative firms. Li (2011) asserts that financial constraints are more likely to curtail the investment opportunities of these firms, which face additional financial constraints due to information asymmetry and agency problems (Hall and Lerner, 2010) as well as low asset tangibility (Almeida and Campello, 2007). Drawing on the insights derived from our model and the empirical evidence, we posit that within an environment characterized by financial frictions, efficient resource allocation following the elimination of these constraints may require a higher allocation of resources to low MPK but innovative firms that exhibit the potential for rapid future growth, thereby further increasing the dispersion in MPKs.

To distinguish and quantify the role of various channels on misallocation, David and Venkateswaran (2019) employ a framework that accounts for adjustment costs, information frictions, and firm-specific factors to capture all remaining drivers of investment decisions. Their findings indicate a significant influence of highly persistent firm-specific factors on investment decisions. Subsequently, David et al. (2022) demonstrate that differences in firm risk premia can create a persistent firm-specific factor that drives a wedge in firms' investment decisions, potentially explaining part of the observed misallocation. Our model generates a similarly persistent wedge in investment behavior, but for a different reason—heterogeneous growth prospects lead firms to sustain persistently different MPKs. However, unlike the risk premium channel, which amplifies MPK dispersion while lowering aggregate productivity, our mechanism shows that “investing in misallocation” increases both MPK dispersion and productivity.

Recently, Sraer and Thesmar (2023) proposed a novel method to quantify the effect of quasi-experiments that reduce frictions for firms, such as tax reforms or banking deregulation, on allocative efficiency. Their method involves a simple transformation of the differences in MPK dispersion between treated and control firms and represents a variant of the sufficient statistic approach by Hsieh and Klenow (2009). This method is valid for a wide range of commonly used assumptions about the production function and frictions which lead to deviations of MPK from the user cost of capital. However, firm heterogeneity, where certain firms have the capability to generate endogenous growth through investment, is not within the scope of models considered by Sraer and Thesmar (2023), thus altering the interpretation of outcomes from natural experiments on misallocation. For example, an increase in MPK dispersion among treated firms following a weakening of financial constraints could imply improved allocative efficiency, provided that capital flows to firms with endogenous growth opportunities.

Our study challenges the conventional notion that high productivity dispersion always indicates inefficient outcomes. We find that investments made in anticipation of potential rapid growth, whether in equipment, software, R&D, or other types of intangibles, play a crucial role in fueling economic growth, despite the initial appearance

of misallocated capital.¹ Therefore, our results align with the findings of Haltiwanger (2016), Haltiwanger et al. (2018), who emphasize that conventional measures of misallocation may not only identify spurious inefficiencies but are also more likely to do so among firms that possess unique strengths, such as future demand and customer base. Our work provides compelling evidence that high investment-low MPK firms exhibit superior future productivity prospects, whether driven by technological advancements and innovation or by strong demand dynamics.

While the elimination of most mechanisms for misallocation generally leads to increased efficiency by reducing dispersion (such as easing financial constraints, minimizing risk premium dispersion, or lowering adjustment costs), Kehrig and Vincent (2020) present an exception to this pattern. They demonstrate that within-firm dispersion, specifically MPK dispersion across establishments, can enhance efficiency. Their model focuses on firms with multiple establishments subject to fixed adjustment costs and financial frictions, highlighting the benefits of this “good” dispersion. In our research, we explore a different source of “good” dispersion, namely the investment choices made by firms based on their growth prospects. Our findings indicate that allowing firms to make such investment decisions can contribute to a higher level of efficiency, even in the presence of dispersion.

Our paper is informed by the extensive literature on endogenous firm growth, which investigates how innovation can enhance productivity and competitiveness, leading to growth, increased profits, market share, and overall technological advancement. We connect to this vast and evolving body of work in various ways. In particular, we draw heavily from (Acemoğlu et al., 2018), incorporating exogenous heterogeneity in productivity growth prospects along the life cycle of innovative firms, and from (Klette and Kortum, 2004), who develop an endogenous growth model in which firm investment and capital accumulation increase the likelihood of output-enhancing booms. While this literature focuses exclusively on R&D and innovation as drivers of future growth, our empirical findings suggest that both intangible investments (e.g., R&D and innovation) and physical investments (e.g., equipment and software) are associated with a higher likelihood of jumps. Consequently, we adopt a reduced-form approach, modeling jumps with a single type of capital while remaining agnostic about the specific micro-level drivers of jumps within firm investment.

Furthermore, our work shares a connection with (Acemoğlu et al., 2018) in that they expand on the concept of misallocation beyond conventional productive inputs to include R&D inputs. Their research illustrates that reallocating innovative resources, such as skilled R&D workers, from less innovative established firms to younger, innovative firms can lead to significant welfare gains, which is in line with our results. This finding underscores the importance of adopting a more comprehensive perspective on misallocation, rather than solely focusing on realized MPK.

Our paper is organized as follows: In Section 2, we present the empirical evidence on the interplay between firm jumps, investment, MPK, and innovative activities. Section 3 presents our model, explicitly characterizing the endogenous growth mechanism involving heterogeneous firms. Section 4 discusses the model's fit to the data and explores various quantitative aspects of model counterfactuals. Section 5 provides concluding remarks.

¹ While the seminal work by Hsieh and Klenow (2009) examines the MPK dispersion as an indicator of misallocation, contrasting MPK dispersion in emerging economies like India and China with that in the United States, subsequent international comparisons reveal significant differences in the life cycle growth patterns of firms across countries. Hsieh and Klenow (2014) compares firm life cycles in the US with those in India and Mexico, finding that surviving US firms experience much higher growth rates, which they attribute to their investments in organizational capital. Additionally, Eslava et al. (2022) highlight the stronger ‘up-or-out’ dynamics and highly skewed growth rates prevalent in the US as contributing factors. Consequently, utilizing MPK dispersion as a measure of misallocation presents additional challenges when applied to the US context.

2. Empirical evidence

This section presents empirical evidence that serves as the foundation for our model and quantitative analysis. We hypothesize that firms investing with the aim of unlocking the potential for rare but substantial productivity growth spurts in the future may exhibit a greater disparity between their current MPK and optimal investment levels. These firms might appear to hold excess capital when evaluated solely based on their current MPK. However, this accumulation may be strategically motivated, reflecting an increased probability of future rapid growth, as documented in studies such as (Klette and Kortum, 2004) and Acemoğlu et al. (2018).

2.1. Jumps in the data

We begin our discussion by establishing a connection between the probability of firm jumps (large increases in output and marginal product of capital), investments, and the current marginal product of capital. “Jumps” are defined as instances where a company's sales more than double, while its marginal product of capital increases by at least 50%. From 1975 to 2019, the unconditional annual probability of a firm experiencing a jump was 1.62%.² Although jumps were infrequent on an annual basis, many Compustat firms encountered a performance jump at some point during their existence. Among firms that joined Compustat after 1975 and remained for at least 5 years, 17% experienced at least one jump until 2019. We observe that jumps are not confined to just a few industries within the economy; rather, firms across many industries experience jumps. In Internet Appendix Table IA.III, we present the frequency of jumps across industries. Although there is some heterogeneity in jump frequency, firms from a wide range of industries exhibit higher propensity for jumps, including chemicals (particularly pharmaceuticals), metal mining, oil and gas extraction, and professional and scientific instruments.

Next, we explore the relationship between firm characteristics and the probability of jumps according to our defined criteria. The purpose of this empirical analysis is to examine whether the distribution of capital across firms is linked to the occurrence of large booms at the individual firm level. Specifically, we investigate whether the likelihood of jumps is influenced by the marginal product of capital, and whether firms allocate more capital to current investments in anticipation of future jumps.

Table 1 presents the parameter estimates obtained from a linear probability model that investigates the relationship between experiencing a jump and several variables, including lagged investment rates (I/K) in physical and intangible capital, lagged marginal product of capital (MPK), and firm age. The regressions incorporate year \times industry fixed effects, ensuring that all comparisons are made across firms operating in the same industry-year pair.

In Column 1, we find that firms with initially low MPK can experience a significant increase in both MPK and sales. This suggests that MPK is not a fixed characteristic of a firm and can vary over time. Furthermore, Column 2 shows that investment in both physical and intangible capital predicts a higher likelihood of a jump occurring.

² Internet Appendix I.1 provides detailed information about the data and variable construction. To minimize the impact of noise, we measure jumps over a four-year period where the thresholds are applied to the growth of sales and MPK from the first two years to the last two years. Our timing convention treats growth from years $t-1$ and t to years $t+1$ and $t+2$ as events in year t . For instance, jumps recorded in 2019 are based on growth rates from 2018–2019 to 2020–2021. Lowering the jump cutoff thresholds to a sales increase of at least 50% and an MPK rise of 30 log points results in a higher classification of jumps, leading to a 3.2% annual jump probability. Conversely, raising the cutoff thresholds to a sales increase of at least 150% and an MPK rise of 50 log points results in a 1% annual jump probability.

Table 1
Determinants of firm jumps.

	(1)	(2)	(3)	(4)	(5)
Physical I/K		0.020*** (11.20)	0.020*** (12.29)		
Intangible I/K		0.023*** (7.13)	0.031*** (9.97)		
Total I/K				0.050*** (15.24)	0.039*** (11.42)
Log MPK	-0.025*** (-18.49)		-0.027*** (-21.44)	-0.027*** (-21.15)	-0.027*** (-21.55)
Log age					-0.011*** (-18.09)
Ind × Year FE	x	x	x	x	x
R ²	0.043	0.031	0.055	0.053	0.057
N	203,253	203,054	203,054	203,253	203,253

This table presents coefficient estimates obtained from linear probability models analyzing realized jumps. The dependent variable is the jump dummy, which takes a value of 1 when a jump occurs from the current period to the next period. Jumps are defined as cases where a company's sales double, accompanied by a minimum 50% increase in its MPK. For a comprehensive explanation and the definition of explanatory variables, please refer to Internet Appendix I. The regressions incorporate 2-digit SIC industry-year fixed effects. The corresponding *t*-statistics are presented in parentheses, and standard errors are clustered at the firm-year level. Statistical significance levels are indicated by one, two, and three stars, denoting significance at the 10%, 5%, and 1% levels, respectively. The variable *N* denotes the count of firm-year observations, while *R*² represents the adjusted R-squared value.

In Column 3, which includes both MPK and investment measures, all variables demonstrate strong predictive power. Specifically, given a certain level of current MPK, a higher investment rate is linked to a greater probability of experiencing a jump. Consequently, firms that continue to invest in spite of having low MPK are more likely to encounter rapid growth.

Columns 2 and 3 of Table 1 highlight the importance of both physical and intangible investments as relevant predictors of jumps, with comparable magnitudes. As per (Peters and Taylor, 2017), we combine physical and intangible investments to derive the firm's total investment and total capital. The results in Column 4 indicate that total investment is able to capture the explanatory power of its individual physical and intangible components in predicting jumps. In Column 5, we introduce firm age, demonstrating that younger firms are more likely to experience jumps.

Internet Appendix Table IA.IV reproduces the same regressions for the lower and higher jump thresholds mentioned above. Although the coefficient estimates vary slightly due to the identification of more or fewer jumps, the statistical significance of the predictive relationships persists.

The implications of the results presented in Table 1 become clearer when we examine two firms with identical and high investment rates but differing levels of current MPK. Consider a firm with high MPK, indicating that it already has high output relative to its capital. In this case, the firm's high investment rate can be justified by the potential benefits of having more capital to further enhance its output. On the other hand, we have a low MPK firm that seemingly has excess capital compared to its output, yet it still falls within the high investment group. This combination of characteristics, namely high investment and low MPK, poses a challenge when analyzing the firm solely based on its current observable productivity characteristics. Nevertheless, they are more likely to experience rapid growth in the future, suggesting that their investment decisions are influenced by factors beyond current MPK. This indicates the presence of alternative channels or factors driving their investment behavior, possibly related to their expectations of future growth opportunities, and empirically challenges the prevailing notion that a lower MPK among firms in the cross-section represents misallocated capital that could be more efficiently utilized by high MPK firms.

2.2. Firm characteristics across investment and MPK portfolios

We next adopt a portfolio approach and analyze the characteristics of firms that are sorted based on both their total I/K and MPK. This

approach allows us to investigate the significance and implications of MPK variation among firms with similar investment, and helps us understand the potential economic channels behind the high I/K-low MPK phenomenon. Accordingly, we sort firms into four portfolios based on their placement in below or above median groups for both I/K and MPK. Based on the evidence presented in Table 1, we have defined total capital as the sum of physical and intangible capital. Our I/K and MPK are also consistent with this definition.

To ensure that our findings are not driven by variations across industries, we conduct the sorting within each 2-digit SIC industry, allowing us to focus on within-industry comparisons. In a frictionless economy where firm productivity is static or completely exogenous, we would expect investment rate and MPK to be perfectly correlated. Consequently, all firms would fall into either the low I/K-low MPK ($I/K_1, MPK_1$) portfolio or the high I/K-high MPK ($I/K_2, MPK_2$) portfolio. However, Panel A of Table 2 reveals that 42% of Compustat firms are placed in off-diagonal portfolios, indicating a more complex relationship between investment and MPK than standard models might suggest.

We observe significant persistence in firms' allocations to I/K and MPK sorted portfolios, consistent with previous findings in the literature. Internet Appendix Table IA.V presents the 1-year transition matrix of firms across these portfolios. The likelihood of a firm remaining in the same portfolio varies from 60% to 73%. This is especially notable for the ($I/K_2, MPK_1$) portfolio, where firms continue to invest heavily despite having low marginal products of capital.

In line with the results of Table 1, we observe that firms in high I/K-low MPK ($I/K_2, MPK_1$) portfolio have a significantly higher probability of experiencing a jump than firms in other portfolios. Specifically, these firms have an annual jump probability of 4.09%, which is more than twice as high as the average jump probability of the entire sample. These firms are also younger, which is associated with a higher likelihood of experiencing a jump, as shown in Table 1. While 21% of all firms belong to the ($I/K_2, MPK_1$) portfolio, this proportion increases to 27% among young firms, defined as those firms that have been included in the Compustat sample for 10 years or less.

To better understand the nature of jumps across portfolios, we examine the distribution of sales and MPK growth, comparing firms in the ($I/K_2, MPK_1$) portfolio to the broader population. Specifically, we compute the unconditional percentiles of the two-year sales and MPK growth distributions and measure the share of firms falling between the 50th and Xth percentiles of both growth rates, where X ranges from 50 to 100, for both all firms and firms in the ($I/K_2, MPK_1$) portfolio.

Panel A of Fig. 1 presents these distributions. We observe no difference in the share of firms between the 50th and 90th percentiles

Table 2

Descriptive statistics for total I/K and MPK sorted portfolios.

	($I/K_1, MPK_1$)	($I/K_1, MPK_2$)	($I/K_2, MPK_1$)	($I/K_2, MPK_2$)	($I/K_2, MPK_1$) – All	
					Difference	t-stat
Panel A: Portfolio properties						
N	1428.3	993.8	997.2	1393.9		
Total I/K (median, ind. adj.)	-0.058	-0.048	0.085	0.097		
Log MPK (median, ind. adj.)	-0.40	0.35	-0.36	0.44		
Portfolio share	0.30	0.21	0.21	0.29		
Portfolio share among young firms (≤ 10 years)	0.22	0.14	0.27	0.37		
Age (median)	15.1	16.2	9.20	9.42	-2.96***	-4.65
Jump probability (%)	1.46	0.56	4.09	0.93	2.46***	8.25
Panel B: Innovative activity and product development						
Patents/K (mean)	9.24	6.67	26.5	17.0	11.9***	5.29
Patent value/K (mean)	24.9	22.8	90.1	93.0	32.4**	2.58
Patent Citations/K (mean)	293.9	142.1	1157.3	594.5	627.9***	4.04
Top 10% patents/K - 5 yr (mean)	1.17	0.47	4.40	2.20	2.40***	3.92
Top 10% patents/K - 10 yr (mean)	1.31	0.48	4.69	2.26	2.58***	3.62
Exposure to Life1 stage (median)	0.22	0.19	0.29	0.25	0.061***	13.2
Panel C: Productivity, returns, financial constraints						
Log TFP (median, ind. adj.)	-0.056	-0.033	0.020	0.074	0.020***	4.09
Log TFP (90th pctl, ind. adj.)	0.36	0.34	0.50	0.59	0.048**	2.33
Log future TFP (5yr later, median, ind. adj.)	-0.021	-0.019	0.012	0.010	0.015***	3.09
Log future TFP (5yr later, 90th pctl, ind. adj.)	0.41	0.35	0.55	0.50	0.10***	4.53
Excess future stock returns (VW mean, annual, %)	8.46	8.88	9.67	9.85	0.77	0.19
Total q (median)	0.43	0.44	0.86	0.87	0.27***	4.97
SA index (median)	-0.12	-0.26	0.21	0.14	0.21***	11.8
LW equity index (median)	0.026	-0.16	0.27	-0.027	0.27***	15.2

To construct the four portfolios, all firms are annually sorted into below and above median I/K and MPK groups. The resulting portfolio statistics are presented in columns 1 to 4, while columns 5 and 6 display the differences between firms in the ($I/K_2, MPK_1$) group and the entire sample and their t -statistics. For each variable, the statistics are initially computed for all firms within each portfolio and then averaged across years. Internet Appendix I provides a detailed explanation of variable definitions and sources. To account for industry differences, total I/K, log MPK, and log TFP are normalized by subtracting the median values of their respective 2-digit SIC industries, and the median SA and LW index are normalized to be 0 each year to increase readability. Excess future stock returns ($r - r^f$) are measured from July of year $t + 1$ to June of year $t + 2$. Most variables are reported as portfolio medians, except for TFP where the 90th percentile is also presented. Excess returns ($r - r^f$) are calculated as value-weighted averages, while patent-based variables are reported as means due to the highly skewed nature of patenting activity.

of growth across the two groups. However, beyond the 90th percentile, the distributions begin to diverge, with a higher fraction of ($I/K_2, MPK_1$) firms experiencing extreme growth. Notably, the point at which this divergence emerges aligns with our baseline jump definitions, where the cutoff points are marked with a dotted line. This pattern suggests that while ($I/K_2, MPK_1$) firms resemble other firms in normal growth scenarios, they are disproportionately represented in extreme growth episodes, which we classify as jumps.

Panel B further illustrates this distinction by plotting the difference in probability distributions between the two groups. The probability gap remains near zero at lower growth levels but begins to diverge precisely around the jump cutoffs, reinforcing the idea that firms in the ($I/K_2, MPK_1$) portfolio are uniquely positioned for extreme-growth episodes.

In Internet Appendix Figure IA.1, we plot sales and MPK growth across percentiles within both all firms and those in the ($I/K_2, MPK_1$) portfolio. We find that the growth trajectories of high investment-low MPK firms closely track those of all firms up to approximately the 80th percentile. However, beyond this point, the two distributions diverge, with high I/K-low MPK firms exhibiting significantly higher growth rates compared to the full sample. Internet Appendix Figure IA.2 extends this analysis to 10-year growth rates, revealing a strikingly similar pattern. This finding supports the idea that the extreme growth (jumps) observed in high I/K-low MPK firms does not reverse over the long run, reinforcing the persistence of their productivity gains.

While our primary focus is on endogenous growth and the occurrence of positive productivity jumps, we also examine the presence of negative productivity jumps, which we define as simultaneous, significant declines in both sales and MPK, characterized by logarithmic changes of opposite signs. Negative shocks have been widely used

in the disaster risk literature to model aggregate quantities such as consumption shocks ((Rietz, 1988), Barro (2006)). Additionally, some firm-level productivity models incorporate negative jumps to capture higher-order moments of cross-sectional firm growth (e.g., Ehouarn et al. (2017)). We find that the unconditional likelihood of negative jumps closely mirrors that of positive jumps. However, unlike positive jumps, the probabilities of observing negative jumps are nearly uniform across portfolios, ranging from 1.4% to 1.8%, with no statistically significant deviation from the overall unconditional probability of 1.7%. Importantly, negative jumps do not exhibit any systematic relationship with MPK or investment, suggesting that they are not driven by the same economic mechanisms as positive jumps. Internet Appendix Figures IA.1 and IA.2 support this conclusion, as they show no significant divergence in sales or MPK growth between high investment-low MPK firms and all firms at the lower percentiles. Given their lack of connection to investment dynamics, we do not focus on negative jumps in our analysis.

When examining the characteristics of the ($I/K_2, MPK_1$) portfolio over time, several intriguing patterns emerge. As shown in Panel A of Fig. 2, the share of firms in this portfolio remains relatively stable throughout the sample period with minor fluctuations. In contrast, the annual jump probability depicted in Panel B varies over time, reaching its first peak in the late 1990s, and its second peak in the last few years. Interestingly, late 1990s are also the years when firms in this group experienced their highest investment rate and had the lowest MPK relative to the median firms in their industry (Panel C). Moreover, the investment rate and MPK move in opposite directions, and jumps comove positively with the investment rate and negatively with MPK. These time series patterns are consistent with the cross-sectional results presented in Table 1: not only are firms with high investment and low

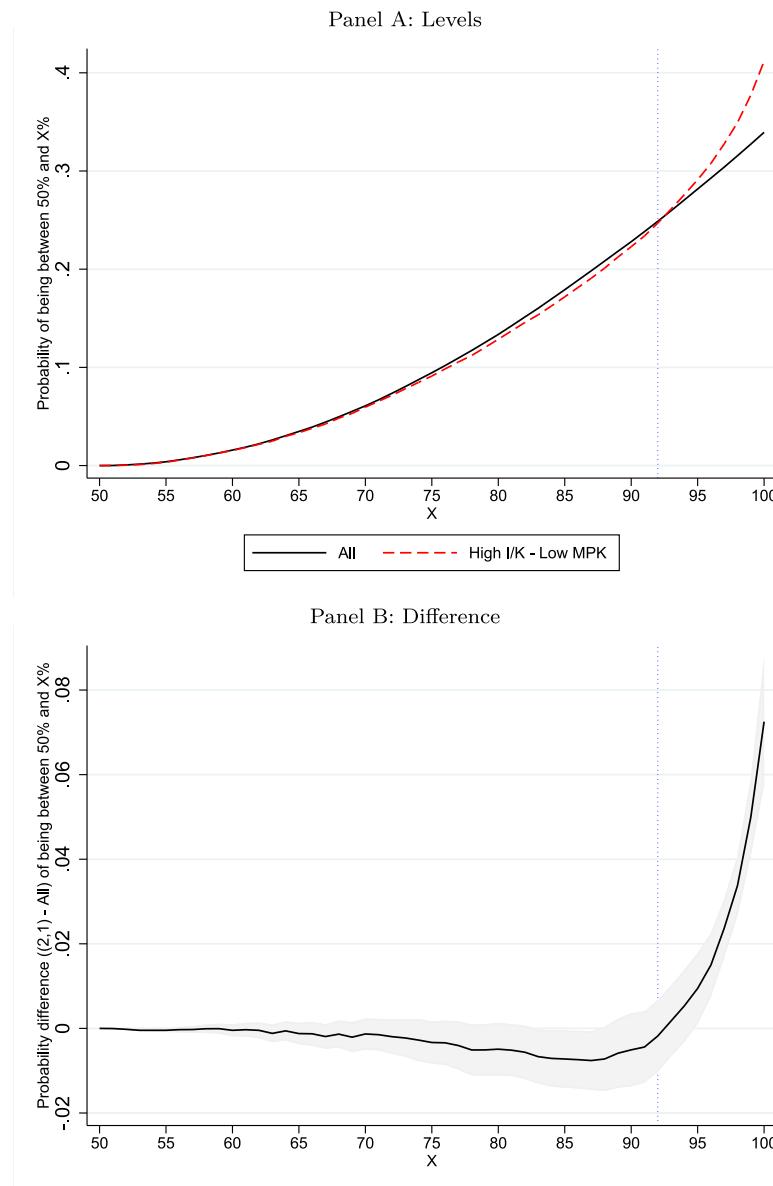


Fig. 1. Distribution of above-median growth across firms.

This figure presents the 50th to 100th percentiles of the unconditional two-year sales growth and MPK growth distributions in the panel data. In Panel A, each year, we compute the share of firms among all firms and $(I/K_2, MPK_1)$ firms that fall between the 50th percentile and X^{th} percentile of the unconditional growth distribution, where X ranges from 50 to 100. Each plotted point represents the average across years. Panel B displays the average difference in shares between all firms and $(I/K_2, MPK_1)$ firms, with 95% confidence intervals. Portfolio assignment occurs at time 0, while growth is measured from $t = -1, 0$ to $t = 1, 2$, following our standard convention.

MPK more likely to jump in the cross-section, but a larger fraction of these firms experience a jump when median investment is high and median MPK is low in the time series.

Jumps in firm performance can stem from various factors fueled by investment. These jumps are characterized by rapid growth, and they are not randomly distributed among firms but strongly associated with both investment and MPK. Examples of such factors are innovative activity and new product development, which often require sustained investment over a considerable period, with the anticipation of potential but uncertain future benefits.

To gain insights into the differences in firm jump propensity, we examine outcomes that capture innovative activity and new product development, as proposed in recent literature. To evaluate innovative quantity and quality, we employ multiple patent-based measures, such as traditional patent and citation counts, as well as patent values

derived from stock market reactions to patent news developed by [Kogan et al. \(2017\)](#) and breakthrough patent measures that identify the most innovative and influential patents using textual analysis proposed by [Kelly et al. \(2021\)](#). To ensure comparability of innovative intensity across firms of varying sizes, we scale all patent measures by the firm's total capital.

The average innovation metrics for the portfolios are presented in Panel B of [Table 2](#). We find that firms in the $(I/K_2, MPK_1)$ portfolio exhibit higher patent issuance, with these patents being more valuable and receiving more citations. Notably, this portfolio stands out in terms of top (breakthrough) patent issuance and patent citation measures, surpassing the performance of all other portfolios. This finding is consistent with a higher jump propensity in performance. To measure new product development activity, we use firm exposure to Life1 (product innovation) stage, which is identified by textual analysis of 10-K files by [Hoberg and Maksimovic \(2022\)](#). As shown in Panel B of [Table](#)

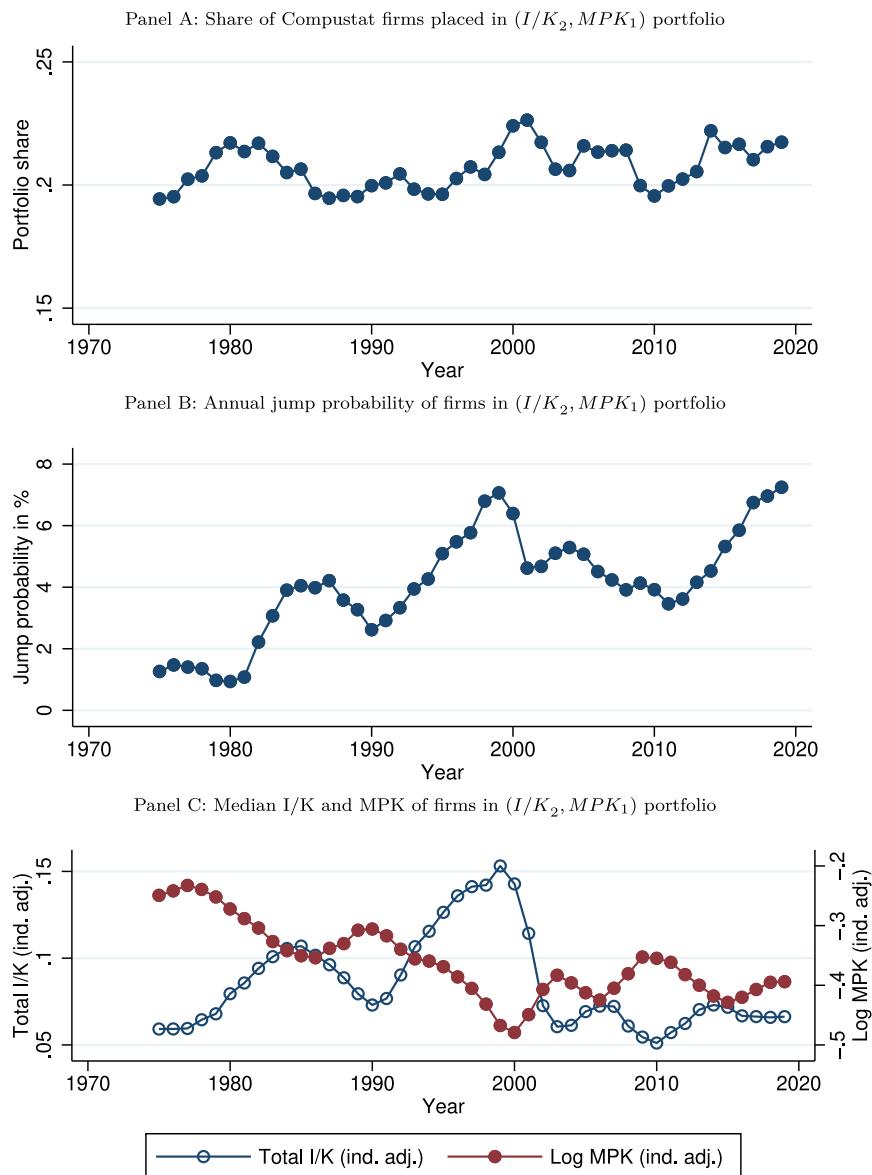


Fig. 2. $(I/K_2, MPK_1)$ Portfolio share and jump probability.

The figure illustrates the portfolio share, jump probability, and median investment rate and MPK for firms in the $(I/K_2, MPK_1)$ portfolio. Panel A presents the portfolio shares, Panel B displays the jump probabilities, and Panel C shows the industry-adjusted median investment rate and log MPK. All variables are presented as 3-year moving averages.

2, firms in the $(I/K_2, MPK_1)$ portfolio exhibit significantly higher exposure to this stage of product development compared to the other portfolios. The Life1 stage is characterized as risky since firms need to acquire capacity before knowing the outcome of product development, which aligns with the notion that investment predicts future jumps.

We also compare current and future firm productivity (total factor productivity, TFP) across portfolios using TFP estimates from (İmrohoroglu and Tüzel, 2014). Panel C of Table 2 shows that firms in high I/K portfolios exhibit higher productivity levels than firms in low I/K portfolios. Additionally, among the two high investment portfolios, firms in the high MPK portfolio have higher median and 90th percentile productivity levels compared to firms in the $(I/K_2, MPK_1)$ portfolio. However, the pattern shifts in the subsequent years, as TFP of firms in the $(I/K_2, MPK_1)$ portfolio exceeds other portfolios five years later. More strikingly, this improvement in TFP for firms in the $(I/K_2, MPK_1)$ portfolio is particularly prominent among top performing firms in each portfolio (such as firms in the 90th percentile),

indicating that currently high I/K -low MPK firms are likely to be in the right tail of TFP distribution in five years, experiencing substantial jumps in performance.

A large body of research has highlighted the importance of risk premia in explaining the cross-sectional differences in investment behavior. As noted by David et al. (2022), it is the appropriately discounted marginal product of capital (MPK) that should be equalized across firms when capital is efficiently allocated, rather than the MPK itself. Therefore, it is essential to account for any differences in risk premia across portfolios. Panel C of Table 2 reports the value-weighted average excess stock returns for the portfolios in the year following the portfolio formation. We find that the realized risk premia were remarkably similar across the portfolios in our sample period. This suggests that the dispersion in MPK and I/K were not driven by variations in the discount rates of the portfolios.

At first glance, it may seem surprising that the returns of the high I/K -low MPK portfolio, which consists of firms that tend to be relatively

young and focused on investment and growth, are not significantly different from the average returns of other portfolios. Empirical asset pricing studies suggest that growth firms and firms with high investment and low profitability — which are likely to overlap with the high investment–low MPK portfolio — tend to have lower expected equity returns (Hou et al., 2015; Fama and French, 2015). However, our findings indicate that the high investment–low MPK portfolio used in our empirical analysis and estimation does not correspond to the typical high investment–low profitability firms that exhibit low expected returns.

To explore the factors driving average portfolio returns, we estimate linear factor models, with results presented in Internet Appendix Table IA.1 and discussed in Section 2. In particular, the q^5 model proposed by Hou et al. (2021), based on the investment CAPM framework, provides valuable insights into the returns of the high investment–low MPK portfolio. The portfolio exhibits negative loadings on both the investment and profitability factors, but this is offset by a positive and statistically significant loading on the “expected growth” factor, which has the highest average returns among all q^5 factors. This finding sheds light on the elevated returns observed in the high investment–low MPK portfolio.

Another key insight is that, controlling for investment, Tobin’s q does not differentiate between low and high MPK portfolios. Within both low and high investment portfolios, those with low and high MPKs exhibit virtually identical total q values, as shown in Panel C Table 2. Specifically, while the $(I/K_2, MPK_1)$ portfolio and the $(I/K_2, MPK_2)$ portfolio fundamentally differ in terms of their current MPK and growth prospects, their q values are indistinguishable from one another.

Recent literature has highlighted financial frictions as a potential source of misallocation (Midrigan and Xu, 2014; Moll, 2014; Whited and Zhao, 2021; Bau and Matray, 2023). To examine whether such frictions may explain the low investment despite high MPK and high investment despite low MPK that we observe in off-diagonal portfolios, we employ various metrics from the existing literature. The first metric we utilize is the SA index introduced by Hadlock and Pierce (2010), which identifies firm age and size as key indicators of financial constraints. The second metric is a recent development by Lim and Weagley (2023), the LW equity index. This index broadens the application of text-based constraint measures by employing a random forest model trained on financial constraint metrics derived from textual analysis, originally formulated by Hoberg and Maksimovic (2014).

Our findings show that $(I/K_2, MPK_1)$ has the highest SA index and LW equity index among the four portfolios, indicating that it has the most financially constrained firms. In contrast, $(I/K_1, MPK_2)$, the portfolio of firms with low investment despite having high MPK, has the least financially constrained firms. Therefore, if financial constraints are indeed impeding investment, we would expect $(I/K_2, MPK_1)$ firms to invest even more in the absence of such constraints, leading to an even wider gap between investment and MPK. Conversely, the low investment of $(I/K_1, MPK_2)$ firms is unlikely to be due to financial constraints. Hence, the observed “misallocation” in the off-diagonal portfolios cannot be attributed to the financial constraints faced by these firms. In summary, our findings suggest that removing financial constraints would not necessarily redirect more capital exclusively to high MPK firms, but also potentially to firms in the $(I/K_2, MPK_1)$ portfolio.

We conducted two additional portfolio sorts using alternative capital definitions to test the robustness of our findings. The first approach applies a more restrictive capital definition, where I/K and MPK are based solely on physical capital. While the results, presented in Internet Appendix Table IA.VI, are largely consistent with those in Table 2, we observe that the distinctions between the $(I/K_2, MPK_1)$ portfolio and the other portfolios become less pronounced, both economically and statistically. The second approach adopts a broader capital definition, incorporating inventories and leased capital alongside the physical and

intangible capital used in our baseline case. The results, shown in Internet Appendix Table IA.VII, closely mirror the baseline findings, both in overall patterns and in the distinct behavior of the $(I/K_2, MPK_1)$ portfolio relative to the other groups. To further examine the consistency of these patterns over time, Internet Appendix Figure IA.3 replicates Fig. 2 using only physical capital, while Internet Appendix Figure IA.4 recreates it with the comprehensive capital definition (including physical, intangible, inventories, and leased capital). These figures demonstrate that the time-series dynamics of the $(I/K_2, MPK_1)$ portfolio’s share, jump probability, investment, and MPK remain largely unchanged, reinforcing the robustness of our findings.

2.3. Firm dynamics before, during, after a jump

After examining the characteristics of firms that invest heavily despite having low MPK — which are disproportionately represented among those experiencing large jumps in sales and MPK — we now shift our focus to the firms that actually undergo these jumps. Our goal is to analyze the dynamics surrounding these events and understand how firms evolve before, during, and after a jump.

Table 3 presents trends in investment, MPK, innovation, and new product development, tracking firms from five years before a jump event (-5) to ten years after ($+10$). Investment and MPK are measured relative to industry peers. We observe that firms’ I/K ratio is consistently 3 to 4 percentage points higher than their peers prior to the jump event. Beginning in the jump period, the investment rate rises further and remains elevated for several periods. This reflects the firm’s response to a surge in productivity, which initiates an investment boom to capitalize on the increase. Eventually, investment levels taper off, converging to those of industry peers approximately five years after the jump.

Conversely, firms’ MPK is significantly below industry peers five years before the jump, declining even further as they approach the event. This pattern is consistent with firms increasing investment and expanding their capital stock in anticipation of a jump, but not yet experiencing the productivity gain. When the jump occurs, MPK rises sharply and remains elevated for a brief period before returning to industry levels, driven by higher post-jump investment.

Product and technology innovation activity is high in the pre-jump period but declines significantly in the post-jump years. Firms’ engagement in product innovation and patenting activity — measured by the number of patents, their value, citations, and representation in breakthrough patent categories — peaks around the time of the jump at very high levels. However, innovation activity declines substantially in the following years, gradually converging toward the levels observed in average firms by the end of the decade following the jump.

2.4. Interpreting the evidence

The evidence showing that firms that invest heavily despite having low MPK have the potential to make substantial leaps in innovation, productivity, and sales challenges the standard neoclassical firm model, which bases capital allocation primarily on marginal product of capital (MPK). Instead, this pattern aligns more closely with endogenous growth models, which suggest that firms invest not just in response to current productivity, but to innovate, enhance production processes, and introduce new products to drive future growth.

A key implication of this empirical evidence is that current productivity does not fully capture the marginal value of investing today. Some models, such as (Kogan and Papanikolaou, 2013), feature a decoupling between firm productivity and growth opportunities, where the arrival rate of such opportunities is exogenous and independent of current productivity in the cross-section of firms. By contrast, our model incorporates endogenous growth, maintaining a connection between investment and productivity akin to the neoclassical framework, but also recognizing that for some firms, growth opportunities emerge

Table 3
Jump event study.

	-5	-4	-3	-2	-1	0	1	2	3	4	6	8	10
Total I/K (median, ind. adj.)	0.029	0.035	0.035	0.032	0.037	0.073	0.076	0.077	0.064	0.044	0.015	0.006	-0.003
Log MPK (median, ind. adj.)	-0.514	-0.602	-0.638	-0.712	-0.931	-0.666	-0.112	0.105	0.035	-0.013	-0.058	-0.054	-0.097
Exposure to Life1 stage (median)	0.364	0.360	0.371	0.355	0.354	0.349	0.344	0.338	0.327	0.325	0.317	0.311	0.293
Patents/K (mean)	48.0	39.4	36.5	39.6	41.6	34.9	31.1	22.5	21.4	20.6	16.6	14.3	12.8
Patent value/K (mean)	164.0	166.7	148.8	176.7	177.0	233.4	227.6	184.3	160.9	165.1	184.4	161.2	167.7
Patent Citations/K (mean)	2735.9	2213.0	2123.3	2162.5	2829.5	2518.5	1795.7	1248.1	1132.6	957.7	684.3	453.1	388.7
Top 10% patents/K - 5 yr (mean)	6.7	8.0	8.4	8.4	10.8	10.2	7.4	5.5	4.7	4.8	3.1	1.8	1.7
Top 10% patents/K - 10 yr (mean)	8.3	9.5	8.5	10.2	13.1	11.5	8.2	6.7	5.1	4.9	3.0	2.5	2.1
Observations	1266	1420	1651	1949	2273	3234	3146	3070	2729	2405	1947	1574	1279

This table summarizes firm dynamics related to investment, MPK, and innovation before, during, and after a jump event. The top row indicates years relative to the jump, where 0 represents the period in which the jump occurs (i.e., the transition from the current period to the next). Negative values denote pre-jump periods, while positive values indicate post-jump periods. Jumps are defined as instances where a firm's sales double, accompanied by a minimum 50% increase in its MPK. A detailed explanation of variable definitions and data sources is provided in Internet Appendix I. To account for industry differences, total I/K and log MPK are normalized by subtracting the median values within their respective 2-digit SIC industries. The top panel reports medians for variables, while the bottom panel presents patent-based variables as means, reflecting the highly skewed distribution of patenting activity.

as a result of their investment decisions rather than being purely exogenous.

In the following section, we present a simple quantitative model that extends the neoclassical framework by introducing a jump-augmented productivity process, incorporating elements of endogenous growth in a reduced-form manner. This addition disrupts the near-perfect correlation between I/K and MPK, introducing a more nuanced and heterogeneous relationship between the two. When applied to Compustat firm data, the model generates significant MPK variation, enabling counterfactual analysis of different firm policies and their impact on MPK and productivity outcomes.

3. Model

In this section, we introduce a simple model that distinguishes between firms based on whether they have the potential for a significant positive jump in capital productivity. We then structurally estimate the model and derive its quantitative implications. A simplified version of the model with closed-form solutions is provided in Internet Appendix III.

Let i denote the index of firms. Each firm utilizes capital $K_{i,t}$ as its sole productive input at time t , with its law of motion is given by

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t}, \quad (1)$$

where δ is the depreciation rate and $I_{i,t}$ is investment. In line with our empirical analysis, capital in the model encompasses both physical capital and intangible capital such as R&D, branding, and new product development. Investment is subject to standard quadratic adjustment costs on net investment, and the total cost of investment is given by $I_{i,t} + \frac{1}{2}c\left(\frac{I_{i,t}}{K_{i,t}} - \delta\right)K_{i,t}$.

Firms differ in their potential for experiencing productivity jumps. Following (Acemoğlu et al., 2018), we assume that upon entering the economy, a firm's type θ_i , either high (h) or low (l), is randomly assigned. Each newly established firm has an identical probability p of being high-type:

$$\Pr(\theta_i = \theta^h) = p \text{ and } \Pr(\theta_i = \theta^l) = 1 - p, \quad (2)$$

where $p \in (0, 1)$.

Firm i produces output $Y_{i,t}$ based on the technology given by

$$Y_{i,t} = Z_{i,t}^\alpha K_{i,t}^{1-\alpha}, \quad (3)$$

where $Z_{i,t}$ denotes productivity. High-type and low-type firms differ in their stochastic productivity process. Log productivity $z_{i,t}$ of a high-type firm follows

$$z_{i,t+1} = z_{i,t} + \epsilon_{i,t+1} + J_{i,t+1}, \quad (4)$$

where ϵ_i is a Gaussian shock with mean zero and volatility σ , and J_i represents a jump shock. This specification follows (Andrei et al., 2019), incorporating both a Gaussian component and an innovation-driven jump component. In each period, jump probability is given by

$$\Pr(J_{i,t+1} = \zeta) = \lambda_{i,t} \text{ and } \Pr(J_{i,t+1} = 0) = 1 - \lambda_{i,t}, \quad (5)$$

where $\lambda_{i,t}$ is the probability of a jump and $\zeta > 0$ is the time-invariant jump size. Low-type firms' productivity does not feature jumps and follows $z_{i,t+1} = z_{i,t} + \epsilon_{i,t+1}$.

We model the probability of jumps similarly to innovation intensity in the endogenous growth model of Klette and Kortum (2004). In their model, the probability of upward jumps in firm sales is driven by how much "knowledge capital" the firm has accumulated in the past. This capital stock is inspired by Griliches (1979)'s concept of the stock of knowledge, defined as the cumulative sum of a firm's past innovative activities.

We parameterize the jump probability $\lambda_{i,t}$ as

$$\lambda_{i,t} = \lambda_0 \left(\frac{k_{i,t}}{k^{ss}} \right)^t, \quad (6)$$

where $k_{i,t} = K_{i,t}/Z_{i,t}$ and k^{ss} is the steady-state value of $k_{i,t}$. The parameter λ_0 controls the level of the jump probability and $t > 0$ determines its curvature with respect to the firm capital level. As in Klette and Kortum (2004), this reduced-form formulation of jump probability reflects the idea that past innovative investments lay the groundwork for firm's future expansion. The firm's capital stock, representing its revenue-generating capacity, serves as a key indicator of its potential for future jumps. Similarly, we assume decreasing returns to expansion efforts, with $0 < t < 1$.

In our model, firms can accumulate capital beyond what is needed for their current productivity to increase the likelihood of a jump, giving rise to a new determinant of capital investment. In other words, firms invest not only for current production opportunities, but also to increase their expected future production. For instance, to achieve potentially rapid growth, firms need to invest in building up brand capital, developing technology, as well as obtaining market share to become potential "superstar" firms, as defined by Autor et al. (2020) as the most productive firms within an industry. In that sense, our broad definition of capital encompasses factors that not only drive a firm's current production but also its future potential.

As in Acemoğlu et al. (2018), we assume that high-type firms transition to low-type at the exogenous flow rate μ , where the low-type state is absorbing. Additionally, each firm faces an exogenous destruction rate φ . In case of destruction, firm value declines to zero and the firm exits the economy.

Given the firm structure outlined above and assuming a discount rate of R , high-type firms solve the following value maximization problem:

$$\begin{aligned} V^h(K_{i,t}, Z_{i,t}) = \max_{I_{i,t}} & \left(Z_{i,t}^\alpha K_{i,t}^{1-\alpha} - I_{i,t} - \frac{1}{2}c \left(\frac{I_{i,t}}{K_{i,t}} - \delta \right)^2 K_{i,t} \right) \\ & + \frac{1}{R}(1-\varphi)((1-\mu)\mathbb{E}_t[V^h(K_{i,t+1}, Z_{i,t+1})] \\ & + \mu\mathbb{E}_t[V^l(K_{i,t+1}, Z_{i,t+1})]), \end{aligned} \quad (7)$$

and low-type firms solve

$$\begin{aligned} V^l(K_{i,t}, Z_{i,t}) = \max_{I_{i,t}} & \left(Z_{i,t}^\alpha K_{i,t}^{1-\alpha} - I_{i,t} - \frac{1}{2}c \left(\frac{I_{i,t}}{K_{i,t}} - \delta \right)^2 K_{i,t} \right) \\ & + \frac{1}{R}(1-\varphi)\mathbb{E}_t[V^l(K_{i,t+1}, Z_{i,t+1})]. \end{aligned} \quad (8)$$

To obtain optimal policies, we utilize value function iteration by normalizing the firm value functions V^h and V^l in Eqs. (7) and (8) with respect to productivity Z . In the quantitative implementation of the model described in Section 4, we make the assumption that all shocks $\epsilon_{i,t}$ and $J_{i,t}$ are independent across firms and time, and we focus solely on the cross-sectional implications by abstracting from any common variations in these shocks. Additionally, we consider a fixed number of firms, meaning that when a firm is hit by an exit shock (with probability φ), it is replaced by a new firm with a high-type probability of p .

4. Quantitative analysis

In this section, we discuss our estimation strategy and examine the quantitative implications of our model, considering both targeted and non-targeted moments. Additionally, we perform counterfactual exercises utilizing the estimated model to illustrate the role of jumps within the overall framework and the impact of expected jumps on investment policy, thereby influencing both the cross-sectional distribution of firms and aggregate productivity. Lastly, we provide empirical evidence that supports the predictions derived from comparing the baseline model to the counterfactual model.

4.1. Model estimation

We estimate the model parameters using data from Compustat firms. In order to streamline the computation process, we predetermined two parameters outside of the estimation. First, we set the discount rate $1/R$ to 0.91 for all firms, which corresponds to the (inverse of the) value-weighted average real stock return for the firms in our sample. Second, we determined the exit probability directly from the data and set φ to 3.1%. We observed that 7.1% of firms in the Compustat sample exited each year. Of these, 4% were due to mergers and acquisitions, a rate that was consistent across portfolios, leaving a failure rate of 3.1%. Assuming that, in the case of mergers, investors receive the current market value of the firm, mergers do not impact the firm's optimization problem. However, we account for the merger rate in our model simulations. Further details on Compustat exit classification are provided in Internet Appendix I.3.

We estimate the remaining 9 parameters jointly using the simulated method of moments (SMM). Let $\theta = \{\alpha, \delta, c, \sigma, \lambda_0, \iota, \zeta, \mu, p\}$ denote the set of nine parameters to be estimated. The SMM approach minimizes the weighted quadratic distance between a vector of moments in the data Ψ^d and model-simulated data $\Psi^m(\theta)$:

$$\arg \min_{\theta} g(\theta)' W g(\theta), \quad (9)$$

where $g(\theta)$ is a column vector of moment conditions as a function of parameters. Details on model solution and simulation can be found in Internet Appendix IV. In order to ensure that our estimated structural parameters are economically meaningful, we carefully select moments from the data that are closely related to them. As a result, we identify

three sets of moment conditions: the first set consists of general moments that capture the volatility of investment and output outcomes as well as the level of investment rate and profitability. The second set includes moments that are specifically linked to the jump mechanism that we empirically illustrate in Section 2 and discuss in Section 3. Finally, the third set of moments pertains to the portfolio characterized by high investment and low MPK, which represents the primary focus of our paper.

Table 4 lists the target moments in our indirect inference procedure. Details on the empirical computations can be found in Internet Appendix I.2. We formulate the model at an annual frequency, consistent with our empirical work, and **Table 5** displays the calibrated and estimated parameter values.

Although all model parameters influence all moments, certain moments are particularly informative for identifying specific parameters. The volatility of Gaussian shocks (σ) increases dispersion in both output and investment. To measure this dispersion, we use the interquartile range (IQR) instead of the standard deviation, which is highly sensitive to outliers and the winsorization of firm-level data, as discussed in Internet Appendix V. Unlike the standard deviation, the IQR remains unaffected by extreme values, making it a more robust measure of dispersion. Conversely, adjustment costs constrain large capital adjustments, significantly dampening investment volatility while having a relatively smaller effect on sales volatility. By jointly targeting investment and output dispersion, we can more precisely estimate these parameters.

The model also captures distinct investment dynamics: high-type firms, primarily young, experience both Gaussian and jump shocks, whereas low-type firms, mostly older, are exposed only to Gaussian shocks. To account for these differences, we separately analyze investment dispersion for young and old firms in the targeted moments, ensuring accurate estimation of the parameters governing investment behavior. Additionally, profitability levels help identify the curvature of the production function, while the investment rate informs the estimation of the capital depreciation rate.

The core mechanism in our model revolves around the impact of endogenous rare jumps on firm investment. To capture this, we include two sets of moment conditions that discipline the same underlying parameters.

To account for realized jumps, we incorporate key moments that discipline the jump frequency and jump size parameters. The jump probability of young firms helps identify both the parameters governing jump frequency, λ_0 and ι , and the probability of being born as a high-type firm, p . The magnitude of realized jumps, conditional on the returns-to-scale parameter α , is primarily determined by the jump size parameter ζ . To discipline ζ , we include the measured jump sizes for sales and log MPK in the moment conditions, as shown in Panel B of **Table 4**, using the same measurement procedure in simulations as in the data, outlined in Section 2. Additionally, the median jump age serves as a key moment that helps discipline both the level of jump probability, λ_0 , and the type-switching probability, μ , given that jumps do not occur after firms transition to low-type.

Our model also highlights the role of rare jumps in shaping the cross-sectional distribution of firms. High-type firms invest more than expected given their current productivity because they anticipate rapid growth in the future, which they can influence through investment. As a result, there is a substantial share of firms with high capital relative to sales (low MPK) but high investment rates. To estimate the parameters governing jump probability, λ_0 and ι , we leverage moment conditions that capture the presence of high investment-low MPK firms, presented in Panel C of **Table 4**. The fraction of these firms among all firms is shaped by changes in investment behavior in response to anticipated jumps, which in turn are dictated by jump probability.

Empirically, we observe that 27% of young firms, defined as those within ten years of establishment, fall into the high investment-low MPK category, compared to 21% across all firms. This pattern supports

Table 4
Moment conditions.

	Data	Baseline
Panel A: General moments		
IQR of I/K among young firms	0.252	0.226
IQR of I/K among mature firms	0.092	0.118
IQR of sales growth	0.243	0.213
Median I/K	0.164	0.164
Median profitability	0.370	0.389
Panel B: Moments related to jump realizations		
Jump probability of young firms	0.028	0.023
Median sales jump size	2.966	2.595
Median log MPK jump size	0.720	0.665
Median jump age	6.000	6.000
Panel C: Moments for the $(I/K_2, MPK_1)$ portfolio		
Portfolio share	0.207	0.187
Jump probability	0.041	0.050
I/K (ind. adj.)	0.086	0.097
Portfolio share among young firms	0.269	0.309

This table presents the target empirical moments and model-generated moments obtained from the estimation of our baseline and counterfactual models. The empirical moments, presented in column 1, are computed using Compustat data from the period 1975–2021. For more details, please refer to Internet Appendix I.2. The parameter estimates for the benchmark model can be found in [Table 5](#).

Table 5
Parameter estimates.

Panel A: Calibrated parameters	
Discount rate, $1/R$	0.91
Exit probability, φ	0.031
Panel B: Estimated parameters	
Production function curvature, α	0.361 (0.004)
Capital depreciation rate, δ	0.196 (0.001)
Gaussian shock volatility, σ	0.328 (0.003)
Adjustment cost parameter, c	2.914 (0.023)
Jump probability level, λ_0	0.031 (0.001)
Jump probability curvature, ι	0.484 (0.006)
Jump size, ζ	2.187 (0.017)
Type switching probability, μ	0.101 (0.002)
Probability of being born high-type, p	0.928 (0.020)

This table provides the parameters resulting from the estimation of the baseline model. Panel A displays the calibrated parameters, while Panel B presents the estimated parameters, with the target and model-generated moments shown in [Table 4](#). The standard errors for the parameters are presented in parentheses.

the idea that high-type firms transition to low-type over time, a process governed by the type-switching probability, μ . The type-switching probability plays a crucial role in determining the expected duration of high-type status and directly influences the present value of additional capital investment for these firms. Consequently, μ is central in shaping both the average jump age and the relative investment rates of high-versus low-type firms.

[Table 4](#) demonstrates that our baseline model provides a reasonably good fit to the data, despite targeting 13 moments with only 9 parameters, resulting in an overidentified system. In particular, the model successfully generates substantial dispersion in sales growth and investment rates, with higher investment dispersion among young firms, consistent with empirical observations. Importantly, our model also replicates large movements in sales and MPK that qualify as jumps, applying the same empirical jump identification method used in the data to the model simulations.

Panel B of [Table 5](#) reports the parameter estimates underlying the simulated moments in [Table 4](#). The estimated values of c and σ ensure

that the model matches the investment and sales growth dispersions well. Targeting the median investment rate and profitability identifies the production function curvature, α , and the depreciation rate, δ .

Moments related to jumps and the $(I/K_2, MPK_1)$ portfolio in Panels B and C of [Table 5](#) play a crucial role in shaping the estimation of other parameters. Specifically, the estimation selects a jump size of 2.19 to match the observed jumps characterized by a doubling of sales and a 50% increase in MPK over a two-year period. Additionally, the jump probabilities of young firms and the $(I/K_2, MPK_1)$ portfolio aid in identifying both the level and the curvature of the jump probability.

The estimation also reveals a 10.1% probability of transitioning from high-type to low-type status, along with a high fraction of new firms (92.8%) being born as high-type. While this proportion may seem high in the general population, it reflects the fact that in our sample, firm birth corresponds to completing an IPO and going public. Given this initial selection process, it is expected that most firms enter as high-type. As conjectured, the moment conditions effectively pin down the estimated parameters, leading to low standard errors overall.

The construction of I/K and MPK-sorted portfolios using simulated data follows the methodology used in their empirical construction. In the model, 18.7% of simulated firms belong to the $(I/K_2, MPK_1)$ portfolio (compared to 20.7% in the data). These firms exhibit an average annual jump probability of 5.0% in the model (4.1% in the data). Notably, the jump probability in this portfolio significantly exceeds the probability among all firms, both in the model (1.32%) and in the data (1.62%), even though it is not a directly targeted moment in the estimation. Also consistent with the data, the proportion of the $(I/K_2, MPK_1)$ portfolio is higher among young firms. Moreover, the median investment rate within this portfolio surpasses the median among all firms by approximately 9.7 percentage points in the model (8.6pp in the data).

While the model estimation targets the jump frequency in the $(I/K_2, MPK_1)$ portfolio under the baseline jump definition, it also successfully replicates the observed jump propensity for this portfolio across alternative jump cutoffs, as defined in footnote 2.1, despite not explicitly targeting those statistics. For higher jump cutoffs, which result in fewer observed jumps, the model predicts a 2.8% jump probability, closely matching the empirical estimate of 2.9%. Conversely, for lower jump cutoffs, which increase the number of observed jumps, the model generates a 5.6% jump probability, compared to 6.2% in the data. These results indicate that the model captures a rich and realistic productivity process, despite its parsimonious structure with relatively few parameters.

Furthermore, although not specifically targeted, additional model-generated moments of investment resemble their empirical counterparts. Simulated firms display similar persistence in their investment and MPK to that observed in the data. The model-generated probabilities of remaining in the same portfolio for two consecutive years range from 66% to 82%, aligning closely with the empirical data presented in Internet Appendix Table IA.V. The model also produces right-skewed investment rates, with the time series average of cross-sectional skewness at 1.6. Empirically, the skewness measure is highly sensitive to the level of winsorization. Winsorizing the investment rate at the 0.5% level results in an average skewness of 4.3, while winsorizing at the 1% level reduces the average skewness to 3.2. These levels are consistent with the skewness reported by Bai et al. (2024) for 40 different definitions of tangible investment rates, which range from 1.5 to 4.5 based on 1% winsorization. Finally, the model predicts that 96% of simulated investment values are non-negative, closely aligning with the 98.9% observed in the data, suggesting that the model captures realistic investment dynamics.

Next, we examine the implications of the model for firm dynamics in the years leading up to and following realized jumps, which are not part of targeted moments in our estimation. To do so, we take the sample of firms that experience a jump in year $t = 0$, as measured using sales and MPK growth from $t = -1, 0$ to $t = 1, 2$. We then compute the share of these firms that fall into the high I/K portfolio and low MPK portfolio, for each year from $t = -5$ to $t = 10$, comparing these shares in both the data and model, conditional on firm presence in the real and simulated datasets, respectively.

Panel A of Fig. 3 shows that the investment dynamics in both the data and model exhibit a similar qualitative pattern around jumps, although the portfolio fluctuations are more amplified in the model. Prior to jumps, firms are likely to have above-median investment rates both in the data and the model. Beginning in the jump period, even more firms switch to the high I/K portfolio and remain there for a few years. Both in the data and in the model, the share of high I/K firms then drops below the pre-jump levels around 5 years after the jump and further falls by year 10 after the jump.

Similarly, Panel B of Fig. 3 shows that a large share of firms that jump in year 0 are in the low MPK portfolio in the prior five years, consistent with the notion that these firms accumulate more capital than predicted by their current productivity. Following a jump, most firms in the model transition to the high MPK portfolio, driven by a

sharp increase in productivity. While this shift is more pronounced in the model, it qualitatively aligns with the observed decline in the low MPK share, which drops from nearly 80% to below 50% in the data. In the 10 years after the jump, the low MPK share does not reach the elevated pre-jump levels anymore and converges towards 50% both in the data and in the model.

Finally, Panel C of Fig. 3 shows that in the years leading up to a jump, firms are significantly more likely to be in the high I/K-low MPK portfolio compared to the unconditional probability of 21% in the data and 19% in the model. Furthermore, a jump in year 0 is associated with an increasing probability of being in this portfolio in the prior years, again consistent with persistently high investment motivated by the possibility of a productivity jump and a concurrent decline in MPK due to high capital accumulation. The high I/K-low MPK share then sharply drops due to the productivity and investment boom upon jump realization, albeit this effect is more pronounced in the model compared to the data. Overall, firms appear to enter a “new regime” in the post-jump period where the high I/K-low MPK share does not reach the pre-jump levels again and converges to a lower level from around 5 years after the jump.

In sum, the model rationalizes the presence of firms with above-median investment rates despite below-median MPKs, which may initially appear to be misallocating productive capital. The characteristics of the $(I/K_2, MPK_1)$ portfolio guide our mechanism that generates this portfolio: the anticipation of large positive moves in their future productivity drives high investment rates despite their low current productivity levels. As a result, we argue that the high capital allocation to low MPK firms does not necessarily indicate a misallocation of resources.

4.2. Inspecting the mechanism

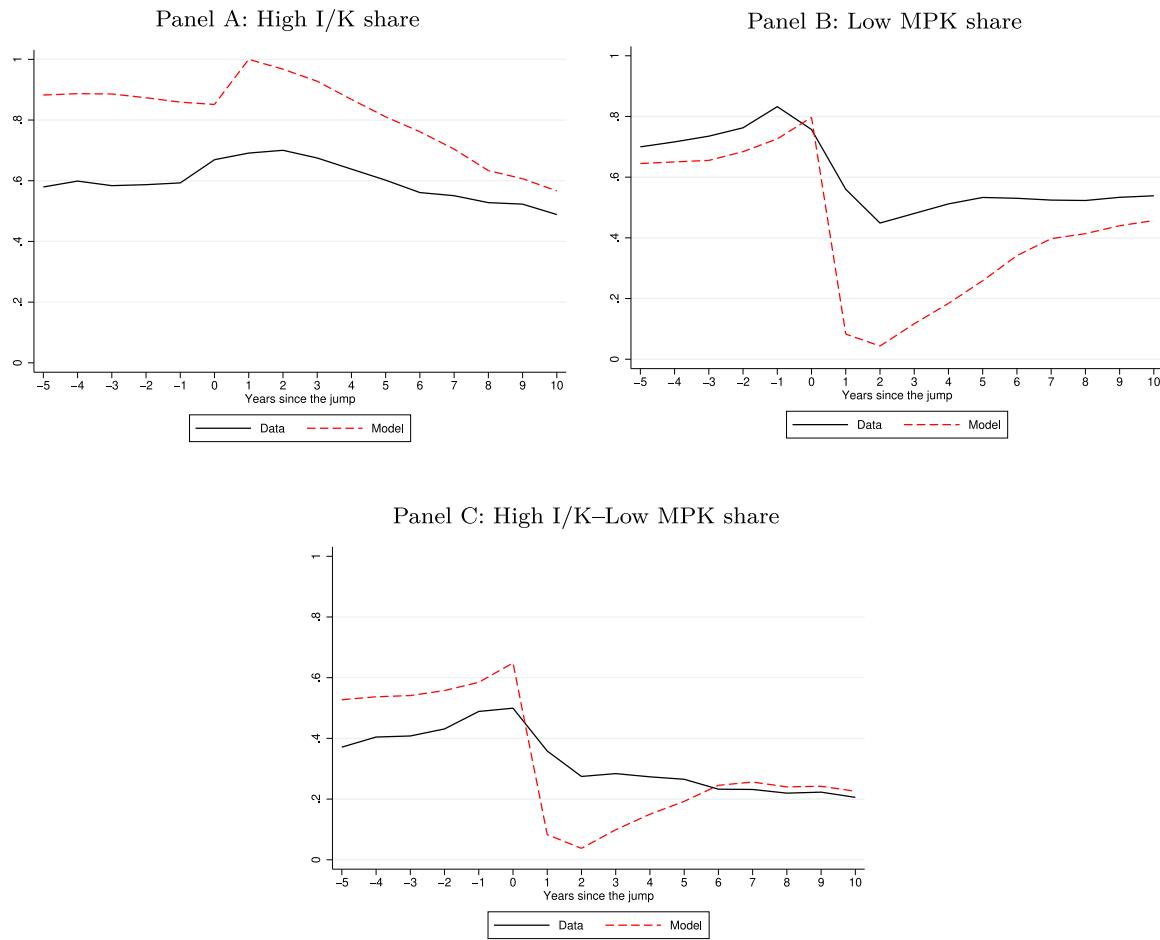
In this section, we explore our model’s mechanism in greater detail and address two fundamental questions. First, can a model with conventional Gaussian shocks match the targeted empirical facts regarding jumps without using Poisson shocks as in our baseline model? Second, what is the role of investment policy in response to anticipated jumps? Specifically, what can we infer from a counterfactual scenario where firms do not invest and grow in anticipation of productivity jumps?

By addressing these questions, we aim to gain a better understanding of the mechanisms underlying our model and shed light on the role and consequences of investment policies tied to the anticipation of jumps.

4.2.1. Model implications in the absence of jumps

To answer the first question, we perform two estimations using a modified version of the model that completely eliminates jumps and keeps all other features the same as in Section 3. This model is essentially a standard neoclassical model with homogeneous firms, random walk productivity, and quadratic adjustment costs. Consequently, the only parameters to estimate are α , δ , c and σ , as all other estimated parameters in the baseline model relate to jumps and firm types.

Table 6 presents the simulated moments obtained from the estimation of the no-jump model. First, we target the moments in Panel A and observe that the estimation selects higher values for c and σ compared to the baseline model (Table 7). The absence of jumps in this model requires higher volatility to match the dispersion of sales growth. However, this also necessitates higher adjustment costs to prevent investment dispersion from being counterfactually high. While this model successfully matches the investment rate dispersion among mature firms, it fails to generate additional investment dispersion among young firms due to the absence of young high-type firms. Panel B of Table 6 reveals that, unlike our baseline model, this model generates no jumps. Furthermore, in this model, the $(I/K_2, MPK_1)$ portfolio is nearly empty containing only 0.1% of firms compared to 20.7% in the data. This is because in the absence of jumps, there is a counterfactually tight

**Fig. 3. Investment and MPK Portfolios around Jumps.**

The figure plots the portfolio shares among firms that jump at year 0, based on sales and MPK growth from $t = -1, 0$ to $t = 1, 2$ following our standard convention, from 5 years prior to the jump to 10 years after the jump in the data and the model. Panel A plots the fraction of jump firms that are in the high I/K portfolio. Panel B plots the fraction in the low MPK portfolio. Panel C plots the fraction in the high I/K-low MPK portfolio.

Table 6
Moments in a model with no jumps.

	Data	Targeting Panel A	Targeting Panel A & B
Panel A: General moments			
IQR of I/K among young firms	0.252	0.090	0.139
IQR of I/K among mature firms	0.092	0.090	0.140
IQR of sales growth	0.243	0.165	0.265
Median I/K	0.164	0.156	0.138
Median profitability	0.370	0.425	0.419
Panel B: Moments related to jump realizations			
Jump probability of young firms	0.028	0.000	0.000
Median sales jump size	2.966	n/a	2.057
Median log MPK jump size	0.720	n/a	0.410
Median jump age	6.000	n/a	11.000
Panel C: Moments for the ($I/K_2, MPK_1$) portfolio			
Portfolio share	0.207	0.001	0.002
Jump probability	0.041	0.000	0.000
I/K (ind. adj.)	0.086	0.001	0.001
Portfolio share among young firms	0.269	0.001	0.002

This table displays the target empirical moments and model-generated moments obtained from the estimation of a model without jumps, as discussed in Section 4.2.1. The empirical moments, presented in column 1, are computed using Compustat data from the period 1975–2021. For more detailed information, please refer to Internet Appendix I.2. Column 2 presents the model-generated moments resulting from an estimation that focuses solely on matching the moments in Panel A. Panel B presents the moments when both Panels A and B are targeted. Parameter estimates for both estimations can be found in Table 7.

Table 7
Parameter estimates in a model with no jumps.

	Targeting Panel A	Targeting Panel A & B
Production function curvature, α	0.310 (0.003)	0.280 (0.002)
Capital depreciation rate, δ	0.224 (0.002)	0.318 (0.002)
Gaussian shock volatility, σ	0.359 (0.004)	0.611 (0.005)
Adjustment cost parameter, c	4.605 (0.100)	3.202 (0.032)

This table presents the parameters obtained from the estimation of the no-jump model, as discussed in Section 4.2.1. The corresponding target and model-generated moments can be found in Table 6. Column 1 displays the parameters resulting from an estimation that only targets the moments in Panel A of Table 6. Column 2 presents the parameters when both Panels A and B are targeted. The standard errors for the parameters are presented in parentheses.

positive relationship between investment and MPK in the cross-section of firms. Consequently, MPK resembles the endogenous state variable that determines investment for all firms, and low MPK directly implies low investment.

The inability of the no-jump model to produce large movements in sales and MPK, as observed in the data, motivates our next estimation. This approach targets both the general moments in Panel A and the jump-related moments in Panel B of Table 6. As a result, the estimation yields a significantly higher value for σ and a lower value for c , as shown in Table 7.

This outcome arises because increasing Gaussian volatility while reducing adjustment costs can generate occasional jumps and a higher dispersion in sales growth. However, the overall jump probability remains extremely low at just 0.002%, far from the data value of 1.62%. Similar to the case where jumps are not explicitly targeted, the $(I/K_2, MPK_1)$ portfolio remains nearly empty, and the observed jumps in Panel B stem solely from large random realizations of Gaussian shocks.

4.2.2. Model implications without “investing in misallocation”

To address the second question posed in Section 4.2, we conduct a counterfactual analysis using our baseline model. In this counterfactual scenario, jumps in productivity are still present in the data-generating process, but firms’ investment policies ignore the potential of future jumps. That is, high-type firms do not engage in “investing in misallocation”, meaning they do not invest more than a low-type firm with the same productivity and capital, even though they have a positive probability of experiencing a jump and additional capital could potentially increase their likelihood of jumping. The only distinction between low-type and high-type firms in this counterfactual scenario arises from the realized jumps, which are not anticipated. For this counterfactual exercise, we maintain the same model parameters as in the baseline model.

Table 8 presents a comparison of the simulated moments in the baseline model and the counterfactual model. The results show that the counterfactual model generates significantly lower investment dispersion for young firms compared to the baseline model and the data (Panel A), indicating that the difference in investment dispersion between young and mature firms can be attributed to high-type firms’ heightened investment in anticipation of jumps. A reduction in sales growth dispersion is also observed, although not to the same extent as the decline in young firms’ investment dispersion. Despite these differences, the counterfactual model simulations still exhibit jumps, and their magnitudes closely resemble those observed in the data, albeit the jump probability for young firms drops from 2.3% in the baseline model to 1.1% in the counterfactual (Panel B). Similarly, the unconditional probability of jumps is significantly lower, at 0.62% compared to 1.32% in the baseline model. This reduction arises from the jump specification in Eq. (6). In the counterfactual scenario, where firms ignore potential productivity jumps, they also overlook the heightened jump probability associated with capital accumulation. Consequently, firms do not invest

to increase their jump probability, leading to an overall decline in the average jump probability.

Panel C of Table 4 illustrates that in the counterfactual model, the joint distribution of investment and MPK leads to the $(I/K_2, MPK_1)$ portfolio share becoming zero. When firms ignore jumps in their investment policies, investment is solely determined by the current MPK, thereby eliminating the occurrence of high investment despite a low MPK. Consequently, our model predicts that this portfolio, which constitutes 21% of Compustat firms, would be absent when the mechanism in our model concerning the impact of anticipated jumps on investment is shut down, even though jump realizations still form part of the data-generating process.

We also examine the relationship between predicted jumps, investment, and MPK in both the baseline and alternative models. The results of this comparison can be found in Table 9. In the data, we observe that investment positively predicts jumps while controlling for MPK, indicating that higher levels of investment are associated with a greater likelihood of jumps. Conversely, MPK negatively predicts jumps while controlling for investment, suggesting that lower MPK values are associated with a higher probability of jumps (Table 1). The baseline model successfully replicates this pattern.

The core mechanism of the baseline model hinges on this relationship. For a given level of investment across firms, a lower MPK is associated with a higher probability of a jump. This is because high-type firms, despite experiencing a low current MPK, are incentivized to invest due to their higher expected growth prospects. However, this mechanism is absent in the counterfactual model. Consequently, the predicted coefficients on investment and MPK in the counterfactual model are statistically insignificant.

The last two columns of Table 9 present the regression results from the no-jump model. In the version estimated using Panel A moments of Table 6, there are no jump realizations. In the model targeting Panel A and B moments, neither investment nor MPK predict jumps, as their occurrence is purely driven by random realizations of large Gaussian shocks.

The cross-sectional dispersion in realized MPK is a commonly used metric for misallocation in the literature, where firms with low MPK are deemed to have too much capital relative to their more productive high MPK counterparts. We next investigate the extent to which cross-sectional dispersion in MPKs would be reduced in the counterfactual scenario where firms cease “investing in misallocation” compared to the baseline model. Fig. 4 illustrates that the interquartile range of MPK is 17.2% lower in the counterfactual model. This reduction indicates a convergence of MPK values among firms. Additionally, the investment rates of firms also exhibit convergence, leading to a lower cross-sectional dispersion, particularly among young firms.

The 17.2% reduction in overall MPK dispersion may appear modest, but this figure fails to capture the heterogeneous effects on the cross-section of firms. A more nuanced understanding of this reduction emerges when examining MPK dispersion within portfolios categorized as above- and below-median I/K, as illustrated in Table 10. In the data,

Table 8
Moment conditions: Baseline and Counterfactual.

	Data	Baseline	Counterfactual
Panel A: General moments			
IQR of I/K among young firms	0.252	0.226	0.136
IQR of I/K among mature firms	0.092	0.118	0.103
IQR of sales growth	0.243	0.213	0.186
Median I/K	0.164	0.164	0.160
Median profitability	0.370	0.389	0.488
Panel B: Moments related to jump realizations			
Jump probability of young firms	0.028	0.023	0.011
Median sales jump size	2.966	2.595	2.435
Median log MPK jump size	0.720	0.665	0.747
Median jump age	6.000	6.000	5.000
Panel C: Moments for the ($I/K_2, MPK_1$) portfolio			
Portfolio share	0.207	0.187	0.000
Jump probability	0.041	0.050	n/a
I/K (ind. adj.)	0.086	0.097	n/a
Portfolio share among young firms	0.269	0.309	0.000

This table presents the target empirical moments and model-generated moments obtained from the estimation of our baseline and counterfactual models. The empirical moments, presented in column 1, are computed using Compustat data from the period 1975–2021. For more details, please refer to Internet Appendix I.2. The parameter estimates for the benchmark model can be found in Table 5. The counterfactual model, as discussed in Section 4.2.2, adopts the same parameter estimates as the baseline model, except that firms' investment policy ignores the possibility of jumps.

Table 9
Predicting jumps in the model.

	Data	Baseline	Counterfactual	No Jump Targeting A	No Jump Targeting A & B
Total I/K	0.050	0.112 [0.085; 0.141]	-0.034 [-0.084; 0.029]	0.000 [0.000; 0.000]	0.002 [0.000; 0.021]
Log MPK	-0.027	-0.062 [-0.079; -0.046]	0.015 [-0.010; 0.036]	0.000 [0.000; 0.000]	-0.000 [-0.004; 0.000]

This table presents coefficient estimates obtained from linear probability models analyzing realized jumps using both empirical data and model-simulated data. The dependent variable is the jump dummy, which takes a value of 1 when a jump occurs from the current period to the next period. Jumps are defined as cases where a company's sales double, accompanied by a minimum 50% increase in its MPK. Column 1 reproduces Column 4 of Table 1, which contains coefficient estimates based on empirical data. Columns 2 to 5 correspond to coefficient estimates obtained from simulations of the Baseline model, the counterfactual model, and two versions of the no-jump model, respectively. Point estimates for the coefficients are simulation averages, while the confidence intervals (presented in parentheses) are constructed from the 2.5th and 97.5th percentiles of the simulated distribution of each coefficient.

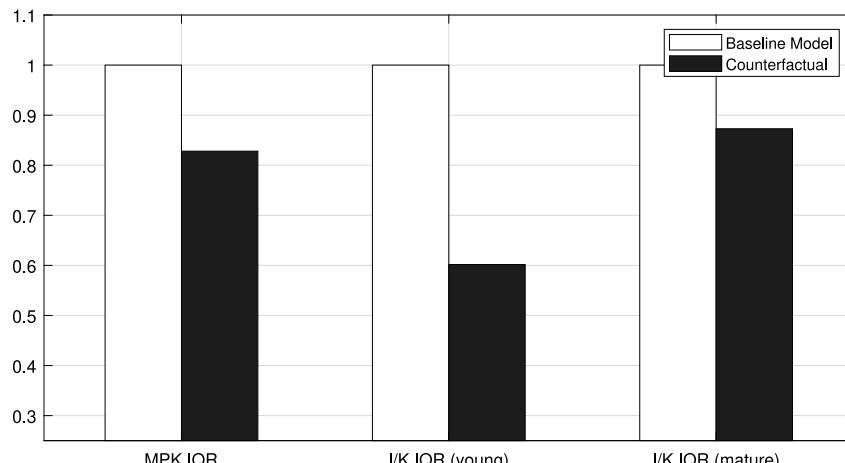


Fig. 4. Dispersion in the Model.

The figure displays the interquartile range (IQR) for MPK and the investment rate IQR separately for young and mature firms. In order to facilitate presentation and comparisons, the values in the baseline model are normalized to 1.

we observe significant MPK variation across portfolios independently sorted based on MPK within high and low investment groups. Notably, the baseline model generates approximately half of the dispersion in MPK observed in the data. Interestingly, this variation is entirely eliminated in the counterfactual model due to the near-perfect correlation between investment and MPK. Therefore, MPK dispersion is not reduced randomly when jumps are ignored in investment policy.

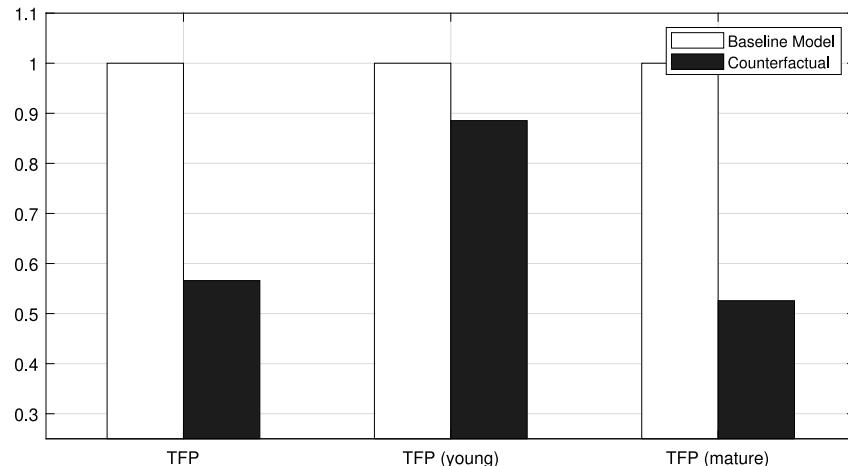
The unique feature of our model implies that it is the MPK dispersion between diagonal and off-diagonal portfolios sorted on investment and MPK in the cross-section, that is eliminated in the counterfactual case. Similarly, while the no-jump model generates substantial overall MPK dispersion, as presented in the last two columns of Table 10, it also completely misses the mark in the contribution of off-diagonal firms to the MPK dispersion within high and low investment firms.

Table 10

Model-generated MPK dispersion across portfolios.

	Data	Baseline	Counterfactual	No Jump Targeting A	No Jump Targeting A & B
$(I/K_1, MPK_1)$	-0.40	-0.22	-0.18	-0.20	-0.29
$(I/K_1, MPK_2)$	0.35	0.13	n/a	0.02	-0.00
Difference	0.75	0.35	n/a	0.22	0.29
$(I/K_2, MPK_1)$	-0.36	-0.24	n/a	0.01	-0.01
$(I/K_2, MPK_2)$	0.44	0.34	0.19	0.19	0.27
Difference	0.80	0.58	n/a	0.19	0.28

This table compares the median MPK for I/K and MPK Sorted Portfolios using both empirical data and model-simulated data. Column 1 displays the median industry-adjusted MPK values for the portfolios, along with the MPK dispersion for the low and high investment groups. Columns 2 to 5 present the same median-adjusted statistics obtained from simulations of different models. Column 2 represents the Baseline model, Column 3 represents the counterfactual model, and Columns 4 and 5 represent two versions of the no-jump model.

**Fig. 5. Aggregate Productivity in the Model.**

The figure displays the aggregate productivity level for all firms as well as for young and mature firms separately. The logarithm of aggregate productivity level is computed as a Solow residual, obtained by subtracting $(1 - \alpha)$ times the log total capital from the log total output. In Panel B, the investment rate IQR is displayed separately for young and mature firms. In Panel C, productivity obtained from the Solow residual is displayed separately for young and mature firms obtained using the total output and capital for each group. In order to facilitate presentation and comparisons, the values in the baseline model are normalized to 1.

What does our model reveal about the aggregate impact of eliminating our mechanism? Specifically, does the reduction in MPK dispersion in the counterfactual scenario have a positive or negative impact on overall efficiency? To address this question, we compute the aggregate Total Factor Productivity (TFP) in both the baseline and counterfactual models. In particular, we compute the aggregate log TFP in the model as a Solow residual, obtained by subtracting $(1 - \alpha)$ times log total capital from the log total output produced by all firms. Fig. 5 illustrates a striking 43% reduction in aggregate TFP when firms cease “investing in misallocation” and the associated reductions in MPK and investment dispersions take place. While the differences are largely driven by the investment policies of young firms, which are more likely to be high-type, Fig. 5 shows that TFP losses in the counterfactual model are more pronounced for mature firms. This occurs because productivity is path-dependent, reflecting the cumulative impact of jumps since a firm’s inception. As a result, mature firms experience greater productivity losses, having foregone investment in their upside potential during their early years.

In the baseline model, the infrequent realization of anticipated jumps leads to substantial increases in productivity when they do occur, ultimately contributing to an overall rise in TFP. According to our model, higher investment driven by jump anticipation is justified by the expected gains from jumps, therefore such investment is expected to enhance overall productivity in the economy by allocating more capital to high-type firms. In this sense, our mechanism generates favorable cross-sectional dispersion in MPK and investment, aligning with our interpretation of jumps as sources of innovation or other

activities that foster future growth, despite their limited immediate impact on output. Therefore, a reduction in this beneficial dispersion implies diminished productivity in the economy, despite being driven by the avoidance of investments by currently low-productivity firms, whose investments may superficially resemble misallocation. In Section 4.3, we present empirical evidence that supports the notion that “investments in misallocation” have a net positive effect on aggregate productivity.

It is worth noting that our mechanism is distinct from the conventional view in the literature, which attributes misallocation primarily to distortions such as adjustment costs or financial constraints (Hsieh and Klenow, 2009). In this literature, financially constrained firms cannot invest as much as they efficiently should, leading to persistent under-investment and high MPK. Removing financial constraints reallocates capital to high MPK firms, reduces cross-sectional dispersion in MPK, increases output by directing capital toward more productive firms, while generally leaving individual firm productivity unchanged (Bau and Matray, 2023). However, financial constraints — though absent from our model — are difficult to reconcile with high-investment, low-MPK firms and would not explain why firms with abundant capital and low MPK still maintain high investment rates relative to their industry peers. Similarly, investment frictions generally lead to sluggish capital adjustment, and eliminating these frictions leads to greater capital allocation to high MPK firms (David and Venkateswaran, 2019). We illustrate this point within our model by eliminating capital adjustment costs in the counterfactual model (Internet Appendix Figure IA.5). As expected, eliminating adjustment costs significantly reduces MPK

Table 11
Effect of investment on firm and competitor growth.

	(1) $\log \frac{Sale_{t+5}}{Sale_t}$	(2) $\log \frac{K_{t+5}}{K_t}$	(3) $\log \frac{Profit_{t+5}}{Profit_t}$	(4) $\log TFP_{t+5}$
I/K_{firm}	0.418*** (10.36)	0.639*** (13.98)	0.373*** (7.11)	0.060*** (4.06)
$I/K_{comp} \in (I/K_1, MPK_1)$	-0.265 (-0.30)	1.333 (1.44)	0.083 (0.08)	0.083 (0.46)
$I/K_{comp} \in (I/K_1, MPK_2)$	0.571* (2.01)	1.412*** (5.70)	0.230 (0.72)	0.373*** (3.57)
$I/K_{comp} \in (I/K_2, MPK_1)$	-0.339** (-2.48)	-0.414*** (-3.40)	-0.301** (-2.19)	-0.054*** (-3.99)
$I/K_{comp} \in (I/K_2, MPK_2)$	0.416* (1.70)	0.437 (1.63)	0.335 (1.19)	0.086 (1.24)
Firm FE	x	x	x	x
R^2	0.015	0.088	0.011	0.004
N	134,860	134,860	126,492	90,533

This table presents the point estimates of Eq. (10) for firm sales, total capital, gross profits, and TFP. The analysis relates firm growth and productivity both to the firm's own investment and the median investment rate of firms in the same SIC2 industry within each portfolio. The regressions include firm fixed effects and the corresponding *t*-statistics are reported in parentheses. The standard errors are clustered by firm and year, and corrected for serial correlation using the Newey-West correction with 8 lags. Statistical significance is denoted by one, two, or three stars, indicating significance at the 10%, 5%, and 1% levels, respectively. The variable *N* represents the count of firm-year observations, while R^2 indicates the adjusted R-squared value.

dispersion, as firms can rapidly adjust capital based on their current productivity. This improvement in allocative efficiency enhances aggregate efficiency, ultimately increasing TFP in the economy.

Our mechanism, however, challenges the notion that allocating more resources to low MPK firms is inherently inefficient. Instead, it highlights that such allocations can be optimal when considering firms' future prospects in terms of productivity, innovation, and growth. Consequently, our results call for caution when interpreting increased capital allocation to high MPK firms as the only efficient outcome, such as in cases where financial constraints are lifted. Indeed, our framework suggests that directing capital toward high-type firms enhances overall productivity while allowing for the coexistence of low MPK firms. In summary, the patterns identified in our analysis caution against automatically labeling capital investment in low MPK firms as misallocation relative to high MPK firms.

4.3. Implications for firm growth and aggregate productivity

The analysis presented in Figs. 4 and 5 highlights the potential for lower aggregate productivity, despite reduced MPK dispersion, if firms cease investing in anticipation of jumps. However, our simple framework assumes independence among firms and thus overlooks potential spillover effects. In reality, firms' investments and outcomes can generate both positive and negative spillovers, influencing other firms in various ways. As a result, the overall impact of our proposed channel on aggregate outcomes remains uncertain. For instance, investments in innovation can push the technology frontier, generating benefits for all firms. Conversely, investments aimed at expanding market share might adversely affect the growth prospects of other firms, or technologies developed by one firm could render the technologies of rival firms obsolete.

To empirically investigate this question, we explore the predictive relationship between a firm's future growth and both its own current investment and that of its competitors. Specifically, we estimate the following regression model:

$$\log \frac{Y_{i,t+5}}{Y_{i,t}} = \alpha_0 + \alpha_1 I/K_{i,t} + \sum_{p=1,2} \sum_{q=1,2} \alpha_{pq} I/K_{comp} \in (I/K_p, MPK_q), t + \eta_i + \epsilon_{i,t+5}, \quad (10)$$

where $\log \frac{Y_{i,t+5}}{Y_{i,t}}$ represents the 5-year log growth rate in sales, capital,

gross profits, or total factor productivity (TFP)³ of firm *i* from year *t* to *t* + 5, $I/K_{i,t}$ denotes the investment rate of the firm *i* in year *t*, $I/K_{comp} \in (I/K_p, MPK_q), t$ refers to the median investment rate of competitor firms operating in the same industry as firm *i* and belonging to the portfolio $(I/K_p, MPK_q)$, and η_i represents a firm fixed effect, capturing unobserved heterogeneity specific to each firm. Hence, the regression model employs time-series variation to estimate the effect of a firm's own investment (α_1), as well as its competitors' investments — grouped by their positions in the joint distribution of investment and marginal product of capital (MPK) — (α_{pq}), on its future growth. The specification follows (Kogan et al., 2017), who analyze the role of firms' own and competitors' innovation activities in shaping firm growth.

Table 11 reveals significant positive effects of firms' own investment on sales growth, capital growth, profit growth, and a higher ranking in the future total factor productivity (TFP) distribution. A particularly striking finding is the impact of competitors' investment. While investments from other portfolios do not consistently predict firm growth, a higher median investment by competitor firms in the $(I/K_2, MPK_1)$ portfolio predicts lower firm growth and a decline in future TFP ranking. These findings align with the research conducted by Kogan et al. (2017) and the empirical findings presented in Section 2, which indicate that firms in the $(I/K_2, MPK_1)$ portfolio tend to engage in a higher degree of innovative activities and make significant contributions, as evidenced by patent values and citations. Interestingly, the use of a broad measure of investment, rather than precise indicators of innovation, proves sufficient to capture the effect of competitors on firm growth. This further supports the notion that firms in the $(I/K_2, MPK_1)$ portfolio hold a distinct role in the economy compared to other firms. Their observed high investment rates are motivated by future prospects, encompassing both their own growth prospects and the potential displacement of competitors.

The negative effect of $(I/K_2, MPK_1)$ portfolio investment on competitors can be attributed to the dominance of competition and creative destruction channels, which outweigh the potential benefits of technology spillovers, that are not accounted for in our model. Consequently, the TFP gains derived from our model's perspective can be considered an upper bound in quantitative terms. However, a crucial question

³ While sales, capital and profits reflect growth rates, TFP represents the level of TFP projected 5 years ahead, rather than its growth rate. The firm level TFP estimates used in this analysis are obtained from (Imrohoroglu and Tüzel, 2014). These estimates are cross-sectional and should not be directly compared across different years.

Table 12
Effect of investment on aggregate productivity.

	(1)	(2)	(3)	(4)	(5)
Panel A: Business sector TFP					
$I/K_{(I/K_1, MPK_1)}$	-0.013 (-0.07)				
$I/K_{(I/K_1, MPK_2)}$		0.091 (0.58)			
$I/K_{(I/K_2, MPK_1)}$			0.091*** (8.95)		0.128*** (4.28)
$I/K_{(I/K_2, MPK_2)}$				0.092*** (5.08)	-0.047 (-0.96)
Panel B: Utilization-adjusted TFP					
$I/K_{(I/K_1, MPK_1)}$	0.151 (0.63)				
$I/K_{(I/K_1, MPK_2)}$		0.263 (1.18)			
$I/K_{(I/K_2, MPK_1)}$			0.116*** (7.21)		0.141*** (3.36)
$I/K_{(I/K_2, MPK_2)}$				0.124*** (5.26)	-0.032 (-0.54)

This table presents the point estimates of Eq. (11), which examines the relationship between future 5-year aggregate productivity growth and the median investment rate in L/K and MPK-sorted portfolios. Panel A measures aggregate productivity using business sector TFP, while Panel B utilizes utilization-adjusted TFP. The regressions include controls for 3-lags of log TFP to account for the influence of past productivity levels. The standard errors are corrected for serial correlation using the Newey-West correction with 8 lags. The corresponding *t*-statistics are reported in parentheses, and statistical significance is indicated by one, two, or three stars, representing significance at the 10%, 5%, and 1% levels, respectively.

remains as to whether the negative effect on competitors is significant enough to entirely offset the advantages of increased investment stemming from the expectation of substantial improvements in a firm's own productivity.

To address this question, we analyze the relationship between firms' investment rates in portfolios sorted by I/K and MPK and future aggregate TFP growth. We utilize business sector and utilization-adjusted TFP measures developed by Fernald (2014) and estimate the following regression model:

$$\log \frac{TFP_{t+5}}{TFP_t} = \alpha_0 + \sum_{p=1,2} \sum_{q=1,2} \alpha_{pq} I/K_{(I/K_p, MPK_q)} + \sum_{l=0}^2 c_l \log TFP_{t-l} + \epsilon_{i,t+5}. \quad (11)$$

Here, $\log \frac{TFP_{t+5}}{TFP_t}$ represents the 5-year log growth rate in aggregate TFP, and $I/K_{(I/K_p, MPK_q)}$ represents the median investment rate of firms belonging to the $(I/K_p, MPK_q)$ portfolio. Additionally, we incorporate controls for the three lagged values of $\log TFP$, similar to the specification used by Kogan et al. (2017), who examine the impact of economy-wide innovation measures on aggregate growth.

Table 12 presents the results of univariate regressions, showing that median investment rates in low-investment portfolios ($I/K_1, MPK_1$) and ($I/K_1, MPK_2$) do not predict future TFP growth. However, the investment rates of above-median investment portfolios ($I/K_2, MPK_1$) and ($I/K_2, MPK_2$) positively predict future aggregate growth using both TFP measures. Given the common element in investment rates across high-investment portfolios, we perform a bivariate predictive regression using the median investment rates of ($I/K_2, MPK_1$) and ($I/K_2, MPK_2$). The investment rate of the ($I/K_2, MPK_1$) portfolio dominates that of ($I/K_2, MPK_2$) and remains a significant and positive predictor of aggregate TFP. This finding suggests that the high investments made by low MPK firms in the ($I/K_2, MPK_1$) portfolio contribute to overall productivity growth in the economy, and the observed effect is economically significant: A one-standard deviation increase in the median investment rate of firms in the ($I/K_2, MPK_1$) portfolio corresponds to an additional approximately one standard deviation increase in 5-year TFP growth.

The positive predictability of aggregate TFP by the investments of the ($I/K_2, MPK_1$) portfolio indicates that the negative effect on

competitors, as documented in Table 11, is outweighed by the positive effect on firms' own growth within the ($I/K_2, MPK_1$) portfolio. In summary, while our previous sections focused on the impact of high investment-low MPK firms' investment on their own growth, the positive relationship between these firms' investments and aggregate TFP provides qualitative support for our counterfactual calculation in Fig. 5, which suggests a reduction in aggregate TFP in the absence of "investing in misallocation".

5. Conclusion

This paper presents a novel empirical finding within Compustat firms: around 20% of these firms demonstrate above-median investment rates despite having below-median marginal product of capital. Within the conventional neoclassical framework, which views differences in MPK as indicators of misallocation to be avoided, these firms can be identified as misallocating resources. We argue that such "investing in misallocation" is not necessarily inefficient and is geared towards future growth. In particular, these firms possess distinct characteristics, such as being relatively young, and a small proportion (around 4%) experience significant jumps in their sales and MPK in the subsequent year, a rarity among other firms.

To capture the dynamics of heterogeneous firms, we propose and estimate a simple endogenous firm growth model. This model incorporates the idea that high-type firms' investments increase the likelihood of large upward moves in productivity. It aligns well with the observed data, explaining the high investment levels exhibited by firms despite their low MPK. Importantly, this model allows us to conduct counterfactual analyses by simulating a scenario in which firms are unable to invest in anticipation of such jumps. In this counterfactual scenario, we observe a better alignment between investment and MPK, resulting in a reduction in "misallocation". However, this adjustment also leads to a decrease in aggregate productivity.

Our contribution deviates from previous literature on misallocation in several aspects. While prior studies primarily focus on examining distortions and frictions such as adjustment costs, information asymmetries, and financial constraints that lead to misallocation, we take a distinct approach by considering the role of firms' investments in shaping their future productivity and show that some of the measured

misallocation may be attributed to an overlooked aspect of optimal investment policy.

Furthermore, our analysis employs the Compustat sample, which consists of publicly traded firms with access to public capital markets. This distinguishes our study from earlier research that often concentrates on smaller private firms, frequently from emerging economies, (e.g., [Midrigan and Xu, 2014](#); [Bau and Matray, 2023](#)). By examining publicly traded firms, which tend to be larger in scale, we mitigate the potential impact of financial constraints that can be more pronounced among smaller private firms. In fact, [Farre-Mensa and Ljungqvist \(2016\)](#) have documented that publicly traded firms typically face minimal financial constraints.

In summary, our paper offers a fresh perspective on the interpretation of MPK dispersion as an indicator of misallocation. Through our empirical analysis and the utilization of the Compustat sample, we provide new insights into the investment behavior and productivity dynamics of firms, contributing to a deeper understanding of resource allocation in the economy.

CRediT authorship contribution statement

Mete Kılıç: Writing – original draft, Visualization, Methodology, Investigation, Formal analysis, Conceptualization. **Selale Tüzel:** Writing – original draft, Visualization, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jfineco.2025.104208>.

Data availability

Kılıç & Tüzel - Investing in Misallocation (Final) (Reference data)
(Mendeley Data)

References

- Acemoğlu, D., Akçigit, U., Alp, H., Bloom, N., Kerr, W., 2018. Innovation, reallocation, and growth. *Am. Econ. Rev.* 108 (11), 3450–3491.
- Almeida, H., Campello, M., 2007. Financial constraints, asset tangibility, and corporate investment. *Rev. Financ. Stud.* 20 (5), 1429–1460.
- Andrei, D., Mann, W., Moyen, N., 2019. Why did the q theory of investment start working? *J. Financ. Econ.* 133 (2), 251–272.
- Asker, J., Collard-Wexler, A., De Loecker, J., 2014. Dynamic inputs and resource (mis)allocation. *J. Political Econ.* 122 (5), 1013–1063.
- Autor, D., Dorn, D., Katz, L.F., Patterson, C., Van Reenen, J., 2020. The fall of the labor share and the rise of superstar firms. *Q. J. Econ.* 135 (2), 645–709.
- Bai, H., Li, E.X.N., Xue, C., Zhang, L., 2024. Asymmetric investment rates. Working Paper.
- Barro, R.J., 2006. Rare disasters and asset markets in the twentieth century. *Q. J. Econ.* 121 (3), 823–866.
- Bau, N., Matray, A., 2023. Misallocation and capital market integration: Evidence from India. *Econometrica* 91 (1), 67–106.
- Choi, J., Kargar, M., Tian, X., Wu, Y., 2025. Investor demand, firm investment, and capital misallocation. *J. Financ. Econ.* 168 (104039).
- David, J., Hopenhayn, H., Venkateswaran, V., 2016. Information, misallocation, and aggregate productivity. *Q. J. Econ.* 131 (2), 943–1005.
- David, J.M., Schmid, L., Zeke, D., 2022. Risk-adjusted capital allocation and misallocation. *J. Financ. Econ.* 145 (3), 684–705.
- David, J., Venkateswaran, V., 2019. The sources of capital misallocation. *Am. Econ. Rev.* 109 (7), 2531–2567.
- Ehouarne, C., Kuehn, L.-A., Schreindorfer, D., 2017. Misallocation cycles. SSRN Working Paper.
- Eslava, M., Haltiwanger, J., Pinzón, Á., 2022. Job creation in Colombia versus the USA: ‘up-or-out dynamics’ meet ‘The life cycle of plants’. *Economica* 89, 511–539.
- Fama, E.F., French, K.R., 2015. A five-factor asset pricing model. *J. Financ. Econ.* 116 (1), 1–22.
- Farre-Mensa, J., Ljungqvist, A., 2016. Do measures of financial constraints measure financial constraints? *Rev. Financ. Stud.* 29 (2), 271–308.
- Fernald, J., 2014. A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Federal Reserve Bank of San Francisco.
- Griliches, Z., 1979. Issues in assessing the contribution of research and development to productivity growth. *Bell J. Econ.* 92–116.
- Hadlock, C.J., Pierce, J.R., 2010. New evidence on measuring financial constraints: Moving beyond the KZ index. *Rev. Financ. Stud.* 23 (5), 1909–1940.
- Hall, B.H., Lerner, J., 2010. The financing of R&D and innovation. In: Hall, B.H., Rosenberg, N. (Eds.), *Handbook of the Economics of Innovation*. Elsevier-North Holland.
- Haltiwanger, J., 2016. Firm dynamics and productivity: tfpq, tfpr, and demand-side factors. *Economía* 17 (1), 3–26.
- Haltiwanger, J., Kulick, R., Syverson, C., 2018. Misallocation Measures: The Distortion that Ate the Residual. Tech. Rep., National Bureau of Economic Research.
- Hoberg, G., Maksimovic, V., 2014. Redefining financial constraints: A text-based analysis. *Rev. Financ. Stud.* 28 (5), 1312–1352.
- Hoberg, G., Maksimovic, V., 2022. Product life cycles in corporate finance. *Rev. Financ. Stud.* 35 (9), 4249–4299.
- Hou, K., Mo, H., Xue, C., Zhang, L., 2021. An augmented q-factor model with expected growth. *Rev. Financ.* 25 (1), 1–41.
- Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. *Rev. Financ. Stud.* 28 (3), 650–705.
- Hsieh, C.-T., Klenow, P., 2009. Misallocation and manufacturing TFP in China and India. *Q. J. Econ.* 124, 1403–1448.
- Hsieh, C.-T., Klenow, P.J., 2014. The life cycle of plants in India and Mexico. *Q. J. Econ.* 129 (3).
- İmrohoroglu, A., Tüzel, c., 2014. Firm level productivity, risk, and return. *Manag. Sci.* 60 (8), 2073–2090.
- Kehrig, M., Vincent, N., 2020. Good dispersion, bad dispersion. Working Paper.
- Kelly, B., Papanikolaou, D., Seru, A., Taddy, M., 2021. Measuring technological innovation over the long run. *Am. Econ. Rev.: Insights* 3 (3), 303–320.
- Klette, T., Kortum, S., 2004. Innovating firms and aggregate innovation. *J. Political Econ.* 112 (5), 986–1018.
- Kogan, L., Papanikolaou, D., 2013. Firm characteristics and stock returns: The role of investment-specific shocks. *Rev. Financ. Stud.* 26 (11), 2718–2759.
- Kogan, L., Papanikolaou, D., Seru, A., Stoffman, N., 2017. Technological innovation, resource allocation and growth. *Q. J. Econ.* 132 (2), 665–712.
- Li, D., 2011. Financial constraints, R&D investment, and stock returns. *Rev. Financ. Stud.* 24 (9), 2974–3007.
- Linn, M., Weagley, D., 2023. Uncovering financial constraints. *J. Financ. Quant. Anal.* 1–36.
- Midrigan, V., Xu, D., 2014. Finance and misallocation: Evidence from plant-level data. *Am. Econ. Rev.* 104 (2), 422–458.
- Moll, B., 2014. Productivity losses from financial frictions: can self-financing undo capital misallocation? *Am. Econ. Rev.* 104 (10), 3186–3221.
- Peters, R.H., Taylor, L.A., 2017. Intangible capital and the investment-q theory relation. *J. Financ. Econ.* 123, 251–272.
- Restuccia, D., Rogerson, R., 2008. Policy distortions and aggregate productivity with heterogeneous establishments. *Rev. Econ. Dyn.* 11 (4), 707–720.
- Rietz, T.A., 1988. The equity risk premium a solution. *J. Monet. Econ.* 22 (1), 117–131.
- Sraer, D., Thesmar, D., 2023. How to use natural experiments to estimate misallocation. *Am. Econ. Rev.* 113 (4), 906–938.
- Terry, S.J., Whited, T., Zakolyukina, A., 2023. Information versus investment. *Rev. Financ. Stud.* 36, 1148–1191.
- Whited, T.M., Zhao, J., 2021. The misallocation of finance. *J. Financ.* 76 (5), 2359–2407.