Corporate finance theory 2.1: Moral Hazard and Asymmetric Information

A Connected Study Structure with Core Equations

1 One Unifying Lens: *Pledgeable Income* and Financing Feasibility

The system you should keep in your head

Across both topics, the logic is a single feasibility check:

$$\underbrace{\text{Maximum cash flow outsiders can be promised}}_{\text{“pledgeable income”}} \geq \underbrace{\text{External funding need}}_{\text{investment minus internal funds}}.$$

The friction (moral hazard or asymmetric information) forces the borrower/issuer to keep some cash-flow *inside* the firm (as rents, incentives, or to avoid mispricing), which lowers what can be pledged to outsiders. That is why positive-NPV projects may still fail to raise funds.

We now study two frictions with the same structure:

$$\begin{aligned}&\textbf{Friction} \Rightarrow \textbf{constraint(s)} \Rightarrow \textbf{reduced pledgeable income}\\ &\Rightarrow \textbf{credit rationing / security choice distortions}.\end{aligned}$$

2 Moral Hazard: Hidden Action / Effort

2.1 Setup → Timing → Why contracts cannot pledge everything

Assumptions (discipline the algebra)

Risk neutral, no time preference (investors require zero expected return), limited liability, competitive lending (zero profit).

Basic fixed-investment project

Entrepreneur must invest I but has only $A < I$ (internal cash), so external funding need is $I - A$. The project pays verifiable revenue $R > 0$ in success and 0 in failure. Hidden effort:

$$\text{Behave: } p_H, \text{ no private benefit;} \qquad \text{Misbehave: } p_L < p_H, \text{ private benefit } B > 0.$$

Two benchmark inequalities (social viability + inefficient shirking):

$$p_H R - I > 0, \tag{1}$$

$$p_L R - I + B < 0. \tag{2}$$

Timing (the connection)

$$\text{Contract} \rightarrow \text{Investment} \rightarrow \text{Effort choice (hidden)} \rightarrow \text{Outcome}.$$

Because effort is unobservable, contracts cannot directly enforce effort. They can only *indirectly* shape effort by how they split the success payoff.

2.2 Contract form → IC → PC → Pledgeable income

Profit sharing in success

Let entrepreneur receive R_b and lenders receive R_ℓ in success:

$$R_b + R_\ell = R, \qquad R_b \geq 0,\ R_\ell \geq 0, \tag{3}$$

and both get 0 in failure (limited liability).

(1) Investor participation / break-even (PC)

If the entrepreneur behaves, lenders' expected payoff is $p_H R_\ell$. Competitive lenders require:

$$p_H R_\ell \geq I - A \quad \Longleftrightarrow \quad R_\ell \geq \frac{I - A}{p_H}. \tag{4}$$

(2) Incentive compatibility (IC)

The entrepreneur must prefer behave to misbehave:

$$\text{Behave payoff} = p_H R_b, \tag{5}$$

$$\text{Misbehave payoff} = p_L R_b + B, \tag{6}$$

so

$$p_H R_b \geq p_L R_b + B \quad \Longleftrightarrow \quad (\Delta p)\, R_b \geq B, \qquad \Delta p \equiv p_H - p_L. \tag{7}$$

Hence the entrepreneur must keep at least

$$R_b \geq \frac{B}{\Delta p}. \tag{8}$$

Pledgeable income (IC ⇒ an upper bound on what lenders can get)

Since $R_\ell = R - R_b$, IC implies

$$R_\ell \leq R - \frac{B}{\Delta p}. \tag{9}$$

In expected terms under honest behavior, *maximum pledgeable income* is

$$\text{Pledgeable Income} = p_H R_\ell \leq p_H \Big(R - \frac{B}{\Delta p} \Big). \tag{10}$$

2.3 Feasibility → Net-worth threshold → Credit rationing

Financing is feasible only if maximum pledgeable income covers the external funding need:

$$p_H \Big(R - \frac{B}{\Delta p} \Big) \geq I - A. \tag{11}$$

Equivalently,

$$A \geq \bar{A} \equiv p_H \frac{B}{\Delta p} - (p_H R - I) \tag{12}$$

$$= I - p_H \Big(R - \frac{B}{\Delta p} \Big). \tag{13}$$

Connection: Positive NPV ($p_H R - I > 0$) is *not enough*; IC forces a minimum rent $B/\Delta p$, which reduces pledgeable income, which then creates the threshold $\bar{A}$. If $A < \bar{A}$, credit rationing occurs even when the project is socially valuable.

2.4 Comparative statics → Why "raise interest" may fail

Agency cost increases with B or with $1/\Delta p$: larger B or smaller Δp raises the required rent R_b and lowers pledgeable income. A useful informativeness statistic is the likelihood ratio:

$$\frac{\Delta p}{p_H} = \frac{p_H - p_L}{p_H}, \tag{14}$$

measuring how informative/verifiable performance is about effort.

Connection: Raising required repayment reduces R_b (entrepreneur's stake), making IC harder to satisfy. Hence quantities (credit rationing) adjust rather than prices.

2.5 Reputation capital as "virtual collateral"

A track record can reduce the *effective* private benefit from misbehaving, replacing B by $b < B$. The threshold becomes

$$A(B) = p_H \frac{B}{\Delta p} - (p_H R - I), \qquad \text{so} \qquad A(b) < A(B). \tag{15}$$

Connection: Better reputation ⇒ smaller effective agency wedge ⇒ higher pledgeable income ⇒ easier external finance.

3 Moral Hazard II: Debt Overhang and Diversification

3.1 Debt overhang: old senior debt blocks new positive-NPV investment

General setup with legacy debt

Collateral/net worth A, legacy senior debt (face value) D, new project outlay I, success payoff R with probability p_H under effort, 0 under shirking, private benefit B. IC is unchanged:

$$R_b \geq \frac{B}{\Delta p}, \qquad \Delta p = p_H - p_L, \tag{16}$$

so pledgeable income without legacy debt is still

$$p_H\Big(R - \frac{B}{\Delta p}\Big). \tag{17}$$

Channel 1: Net-worth erosion

Because legacy creditors are senior, effective net worth is reduced to $A - D$. Even if $A \geq \bar{A}$, the project becomes unfundable when

$$A - D < \bar{A}. \tag{18}$$

Channel 2: Renegotiation breakdown ("static debt overhang")

If new investors enter while legacy debt remains senior, the most that can be pledged to newcomers in success is

$$R - \frac{B}{\Delta p} - D. \tag{19}$$

New investors fund only if

$$p_H\Big(R - \frac{B}{\Delta p} - D\Big) \geq I, \quad \text{i.e.} \quad \bar{A} + p_H D \leq 0. \tag{20}$$

Connection: Even when total surplus exists, the *distribution* created by seniority plus coordination frictions can prevent the haircut that would unlock financing.

If collective action were possible (a haircut restores efficiency)

Suppose old creditors reduce face value from D to $d < D$ such that

$$A + p_H d = 0. \tag{21}$$

Then newcomers can be promised $R - \frac{B}{\Delta p} - d$ and just break even:

$$p_H\Big(R - \frac{B}{\Delta p} - d\Big) = I. \tag{22}$$

Old creditors obtain $p_H d = -A > 0$, while the borrower receives the usual rent $p_H B/\Delta p$.

3.2 Diversification and cross-pledging: pledgeable income rises when outcomes are not perfectly aligned

Two independent projects

Two projects, each costs I, each yields R with probability p_H under work and p_L under shirking, private benefit B. Per-project net worth A. Single-project feasibility (recall) is

$$A \geq \bar{A} \equiv I - p_H\Big(R - \frac{B}{\Delta p}\Big). \tag{23}$$

Cross-pledging (core connection)

If lenders can seize income from one project as collateral for the other, reward design changes. Let R_i be the borrower's reward when the number of successful projects is i. Risk-neutrality implies concentrating rewards is optimal:

$$(R_2, R_1, R_0) = (R_2, 0, 0).$$

Then IC for working on both projects becomes

$$p_H^2 R_2 - 2B \geq p_L^2 R_2 \quad \Longleftrightarrow \quad (p_H + p_L)\, R_2 \geq \frac{2B}{\Delta p}. \tag{24}$$

Define

$$d_2 \equiv \frac{p_L}{p_H + p_L} \in \Big(0, \frac{1}{2}\Big). \tag{25}$$

Pledgeable income increases (relative to separate contracting) and leads to the financing condition

$$p_H\Big[R - (1 - d_2)\frac{B}{\Delta p}\Big] \geq I - A. \tag{4.3}$$

Equivalently,

$$A \geq \hat{A} \equiv I - p_H\Big[R - (1 - d_2)\frac{B}{\Delta p}\Big] < \bar{A}. \tag{26}$$

Connection: Cross-pledging effectively reduces the private benefit term from B to $(1 - d_2)B$, so less "rent" must be left to sustain incentives, increasing pledgeable income and lowering required net worth.

4 Asymmetric Information: Hidden Type / Quality

4.1 Core: privately-known prospects → pooling vs breakdown

Setup

Entrepreneur has $A = 0$ and needs investment I. Two types:

$$\text{Good: success prob } p, \qquad \text{Bad: success prob } q < p.$$

Project payoff is R on success, 0 otherwise. Prior:

$$\alpha \equiv \Pr(\text{Good}), \qquad m = \alpha p + (1-\alpha)q. \tag{27}$$

Competitive, risk-neutral investors require break-even in expectation.

Benchmark: symmetric information

Good type financing under full information:

$$p(R - R_b^G) = I \quad \Longrightarrow \quad R_b^G = R - \frac{I}{p}. \tag{28}$$

Bad type is financed iff $qR \geq I$.

Pooling under asymmetric information

Restrict to contracts paying borrower $R_b \geq 0$ only on success. Expected investor profit:

$$\Pi = m(R - R_b) - I. \tag{29}$$

Market breakdown. No lending occurs if

$$mR < I. \tag{30}$$

Solving for the prior gives the breakdown threshold

$$\alpha < \alpha^{\equiv \frac{I-qR}{(p-q)R},}(31)$$

equivalently

$$\alpha^{(pR-I)+(1-\alpha^{)(qR-I)=0.}}(32)$$

Connection: With hidden types, investors price off the average m. If average pledgeable income mR cannot cover I, trade collapses even if the good type alone would be fundable.

Pooling contract and cross-subsidy. Break-even on the pooled distribution implies

$$m(R - R_b) = I \quad \Longrightarrow \quad R_b^{\text{pool}} = R - \frac{I}{m}. \tag{33}$$

Then investor payoff differs by type:

$$p(R - R_b^{\text{pool}}) > I, \qquad q(R - R_b^{\text{pool}}) < I,$$

so good borrowers "overpay" to subsidize bad borrowers.

Adverse-selection index and the lemons "tax"

Define

$$\chi \equiv (1-\alpha)\frac{p-q}{p} \in [0,1). \tag{34}$$

Since

$$m = p(1-\chi), \tag{35}$$

feasibility can be rewritten as

$$mR \geq I \quad \Longleftrightarrow \quad (1-\chi)\, pR \geq I. \tag{36}$$

Effective pledgeable income for the good borrower under pooling:

$$pR_b^{\text{pool}} = pR - \frac{pI}{m} = pR - \frac{I}{1-\chi} \tag{37}$$

$$= (pR - I) - \frac{\chi}{1-\chi}\, I, \tag{38}$$

where $\frac{\chi}{1-\chi} I$ is the lemons "tax".

4.2 Why rates may not clear: backward-bending "supply of capital"

Suppose entrepreneurs differ by success probability $p \sim F$ on $[0,1]$. Lenders post debt with repayment $D = 1+r$ in success, 0 in failure. Let borrower have a non-pledgeable control benefit $b > 0$ independent of success. Then borrower payoff is

$$U(p;r) = b - pD, \tag{39}$$

so participation is

$$U(p;r) \geq 0 \quad \Longleftrightarrow \quad p \leq \Lambda(r) \equiv \frac{b}{1+r}. \tag{40}$$

As r rises, $\Lambda(r)$ falls, so high-quality (high-p) borrowers exit first. Expected investor return can be written as

$$\mathbb{E}[\text{return}(r)] = p_{\text{avg}}(r) \cdot r, \tag{41}$$

and beyond a threshold $\bar{r}$, $p_{\text{avg}}(r)$ falls faster than r rises, so expected return declines (backward bend).

4.3 Market timing twist (public outlook τ)

Let success probabilities shift with a publicly observed outlook parameter τ:

$$\text{Good: } p+\tau, \qquad \text{Bad: } q+\tau.$$

Then financing condition becomes

$$(m+\tau)R > I, \tag{42}$$

and the lemons index becomes

$$\chi(\tau) = (1-\alpha)\frac{p-q}{p+\tau}, \tag{43}$$

which decreases in τ. **Connection:** booms (τ high) reduce adverse selection severity, so equity is comparatively cheaper and issuance clusters in good times.

5 Asymmetric Information II: Equity issues, pecking order, and certification

5.1 Assets in place → seasoned equity offering (SOE) pooling condition

Existing asset pays R if it succeeds. Fundamental success probability $\theta \in \{p, q\}$, with $p > q$. Insider knows θ; outsiders know prior α, so pricing uses

$$m = \alpha p + (1 - \alpha) q. \tag{44}$$

A deepening investment of size I raises success probability by $\tau > 0$. If type were observable, deepening NPV would be $\tau R - I$.

If the firm issues equity giving new investors a claim R_ℓ on payoff R, then under pooling beliefs investors break even if

$$(m + \tau) R_\ell = I \quad \Longrightarrow \quad R_\ell = \frac{I}{m + \tau}, \qquad 0 < R_\ell < R. \tag{45}$$

If the good insider does not issue, her reservation payoff is pR. Good type participates iff

$$(p + \tau)(R - R_\ell) \geq pR \tag{46}$$

$$\Longleftrightarrow \qquad \tau R \geq \frac{p + \tau}{m + \tau} I. \tag{6.1}$$

Define adverse-selection severity at depth τ:

$$\chi_\tau = (1 - \alpha) \frac{p - q}{p + \tau} = \frac{p - m}{p + \tau}. \tag{47}$$

Then (6.1) is equivalent to

$$\tau R - I \geq \frac{\chi_\tau}{1 - \chi_\tau} I, \tag{48}$$

i.e., deepening NPV must exceed the lemons tax.

5.2 Pooling vs separation → announcement effects

Pooling (if (6.1) holds): both types issue; price impact at announcement is flat under pooling pricing.

Separation (if (6.1) fails): only bad types issue; investors demand

$$R_\ell^B = \frac{I}{q + \tau}. \tag{49}$$

Announcement return expressions in the slides:

$$V_0 = \alpha p R + (1 - \alpha) \big[(q + \tau) R - I \big], \tag{50}$$

$$V_1 = (q + \tau) R - I, \tag{51}$$

$$\Delta \equiv V_0 - V_1 = \alpha \big[pR - (q + \tau) R + I \big] = \alpha \big[I + (p - q - \tau) R \big]. \tag{52}$$

An equivalent price-drop form:

$$\Delta P = (q + \tau) R - I - mR < 0. \tag{53}$$

5.3 Pecking order via security design (salvage value flattens payoffs)

Project yields R^S in success and R^F in failure, with $R^S > R^F$. If funded, investors receive $(R^S - R_b^S)$ in success and $(R^F - R_b^F)$ in failure. Investor break-even (binding at optimum):

$$m(R^S - R_b^S) + (1-m)(R^F - R_b^F) = I. \tag{54}$$

Good borrower chooses (R_b^S, R_b^F) to maximize

$$\max_{R_b^S, R_b^F} \; pR_b^S + (1-p)R_b^F \quad \text{s.t. breakeven.} \tag{55}$$

Good borrower utility at the optimum:

$$U^G = pR_b^S + (1-p)R_b^F \tag{56}$$

$$= \underbrace{pR^S + (1-p)R^F - I}_{\text{NPV under symmetric info}} - \underbrace{(1-\alpha)(p-q)\Big[(R^S - R_b^S) - (R^F - R_b^F)\Big]}_{\text{adverse-selection discount}}. \tag{57}$$

Connection: adverse-selection losses rise with how *success-tilted* investors' claim is. Thus optimal structure pushes toward *information-insensitive* payoffs.

Slides' optimal structure sets

$$R_b^F = 0, \tag{58}$$

so breakeven becomes

$$m(R^S - R_b^S) + (1-m)R^F = I \tag{59}$$

$$\iff \quad m(R^S - R_b^S) = I - R^F. \tag{60}$$

Define safe debt face value $D = R^F$ (commit the entire failure payoff to investors), then residual equity (success-only claim) covers the remaining financing gap:

$$m(R^S - R_b^S) = I - D. \tag{61}$$

Two canonical cases

Case A: default-free debt feasible ($I \le R^F$). Perfect flattening:

$$R^S - R_b^S = R^F - R_b^F = I. \tag{62}$$

Borrower retains

$$R_b^S = R^S - I, \qquad R_b^F = R^F - I.$$

Case B: pledge all salvage, top up in success ($R^F < I \le mR^S + (1-m)R^F$). Investors get all salvage:

$$R^F - R_b^F = R^F \quad \Rightarrow \quad R_b^F = 0, \tag{63}$$

and the success-state transfer is

$$R^S - R_b^S = R^F + \frac{I - R^F}{m}. \tag{64}$$

If $I > mR^S + (1-m)R^F$, pooling issuance is infeasible (insufficient pledgeable income).

Numerical example (from slides)

Parameters: $R^S = 1,\ I = 0.25,\ m = 0.64,\ p = 0.8.$

- Case A ($R^F = 0.35 \geq I$):

$$(R^S - R_b^S,\ R^F - R_b^F) = (I, I) = (0.25, 0.25), \quad (R_b^S, R_b^F) = (0.75, 0.10),$$
$$\mathbb{E}[U^G] = pR_b^S + (1-p)R_b^F = 0.62.$$

- Case B ($R^F = 0.15 < I$):

$$R_b^F = 0, \qquad R^S - R_b^S = R^F + \frac{I - R^F}{m} = 0.15 + \frac{0.10}{0.64} = 0.30625,$$
$$(R_b^S, R_b^F) = (0.69375, 0), \qquad \mathbb{E}[U^G] = pR_b^S = 0.555.$$

5.4 Costly certification (reduce lemons severity)

A certifier truthfully reveals θ at an upfront real cost c (paid out of proceeds). Without certification (pooling), equity issuance implies investor claim

$$R_\ell = \frac{I}{m + \tau}. \tag{65}$$

With certification, the good type raises $I + c$ at fair terms for type p:

$$p(R - R_\ell^G) = I + c \quad \Longrightarrow \quad R_\ell^G = R - \frac{I + c}{p}. \tag{66}$$

Certification incentive (good type certifies iff):

$$R_\ell^G > R - \frac{I}{m} \tag{67}$$
$$\iff \quad \frac{c}{I + c} < \chi, \qquad \chi \equiv \frac{p - m}{p} = (1 - \alpha)\frac{p - q}{p}. \tag{68}$$

Connection: higher χ (more severe lemons) makes certification more valuable.

6 Connecting the Two Topics (keep this map consistent)

Same pipeline, different wedge

- **Moral hazard (hidden action):**

$$\text{IC forces rent } \frac{B}{\Delta p} \ \Rightarrow\ \text{max pledgeable } p_H\Big(R - \frac{B}{\Delta p}\Big) \ \Rightarrow\ A \geq \bar{A}.$$

- **Adverse selection (hidden type):**

$$\text{Pooling prices on } m \ \Rightarrow\ \text{lemons tax } \frac{\chi}{1-\chi} I \ \Rightarrow\ \text{breakdown if } mR < I, \text{ and safer claims (debt) preferred.}$$

Mitigators line up by "what they improve"

- Moral hazard mitigators raise incentives / monitoring: higher A, reputation (lower B), cross-pledging (reduces effective B).
- Adverse selection mitigators raise information / screening: higher α, higher public outlook τ, safer contracts (flatten payoffs), certification (pay c to reveal type).