Corporate Finance Theory 3: Full Derivations of the Super-Basic Equations

Monitoring and Control Rights

Common Setup (Used Throughout)

- A project requires investment I. The entrepreneur has internal funds A, so external financing need is $I - A$.
- Payoff: in success the project generates verifiable cash flow $R > 0$; in failure it generates 0.
- Hidden action (moral hazard): the entrepreneur chooses effort H ("work") or L ("shirk").
- Success probabilities:

$$\Pr(\text{success} \mid H) = p_H, \qquad \Pr(\text{success} \mid L) = p_L, \qquad p_H > p_L.$$

- Shirking yields a private (non-pledgeable) benefit $B > 0$.
- Limited liability: the entrepreneur cannot be forced to make payments in failure beyond available cash flow.

1 Monitoring: Super-Basic Equations and Derivations

1.1 (1) Success gap from effort (informativeness)

Definition.

$$\Delta p \equiv p_H - p_L.$$

Why $\Delta p > 0$. By assumption, effort increases the probability of success: $p_H > p_L$, hence $\Delta p > 0$.

1.2 (2) Incentive compatibility (IC) $\Rightarrow$ minimum rent

Contract primitive. Let the entrepreneur receive $R_b \geq 0$ in success and 0 in failure.

Entrepreneur's expected payoff.

$$U(H) = p_H R_b,$$
$$U(L) = p_L R_b + B.$$

Incentive compatibility (choose H rather than L).

$$U(H) \geq U(L) \iff p_H R_b \geq p_L R_b + B \iff (p_H - p_L) R_b \geq B.$$

Using $\Delta p \equiv p_H - p_L$, we obtain the core IC constraint:

$$(\Delta p) R_b \geq B.$$

Solving for R_b gives the minimum success-state reward (the "minimum rent"):

$$R_b \geq \frac{B}{\Delta p}.$$

1.3 (3) Pledgeable income (benchmark)

Outsiders' success-state claim. Let outsiders receive x in success. Since total verifiable cash flow in success is R,

$$x = R - R_b.$$

Expected (pledgeable) repayment to outsiders. If the entrepreneur works, expected repayment is

$$\Pi \equiv p_H x = p_H (R - R_b).$$

Maximizing pledgeable income subject to IC. To maximize what can be pledged to outsiders, choose the smallest R_b that still satisfies IC:

$$R_b^\star = \frac{B}{\Delta p}.$$

Substitute into $\Pi = p_H(R - R_b)$:

$$\Pi_0 = p_H \left(R - \frac{B}{\Delta p} \right).$$

This is the benchmark pledgeable income.

1.4 (4) With an early verifiable signal (clean monitoring)

Signal technology. An early verifiable signal $j \in \{H, L\}$ is realized after effort choice. Let

$$\sigma_{HH} = \Pr(j = H \mid \text{effort } H), \qquad \sigma_{LH} = \Pr(j = H \mid \text{effort } L).$$

Assume the signal is informative: $\sigma_{HH} > \sigma_{LH}$.

Signal-based compensation. Consider a contract that pays the entrepreneur a bonus $R_b \geq 0$ *only if* the signal is high ($j = H$). (Otherwise the entrepreneur receives 0.)

Expected payoff under each effort.

$$U(H) = \sigma_{HH} R_b,$$
$$U(L) = \sigma_{LH} R_b + B.$$

IC with the signal.

$$U(H) \geq U(L) \iff \sigma_{HH} R_b \geq \sigma_{LH} R_b + B \iff (\sigma_{HH} - \sigma_{LH}) R_b \geq B.$$

Hence the minimum incentive-compatible bonus is

$$R_b^\star = \frac{B}{\sigma_{HH} - \sigma_{LH}}.$$

Pledgeable income with the signal. Expected verifiable output under effort H is still $p_H R$. But the entrepreneur is paid R_b only when $j = H$, which occurs with probability σ_{HH}. Thus outsiders' expected pledgeable repayment is

$$\Pi_1 = p_H R - \sigma_{HH} R_b.$$

Substitute the minimal bonus $R_b^\star = \frac{B}{\sigma_{HH} - \sigma_{LH}}$:

$$\Pi_1 = p_H R - \sigma_{HH} \cdot \frac{B}{\sigma_{HH} - \sigma_{LH}} = p_H R - \frac{\sigma_{HH}}{\sigma_{HH} - \sigma_{LH}} B.$$

This shows precisely how a cleaner signal (larger $\sigma_{HH} - \sigma_{LH}$) reduces the required incentive payment and increases pledgeable income.

2 Control Rights: Super-Basic Equations and Derivations

2.1 (1) Financing feasibility

External funding need. The entrepreneur contributes A, so outsiders must provide $I - A$.

Competitive break-even condition (risk-neutral investors, zero required profit). Outsiders participate only if the expected repayment they can be promised (pledgeable income) covers their outlay:

$$\Pi \geq I - A.$$

This is the basic financing feasibility constraint.

2.2 (2) Control as collateral (enforced action raises success probability by τ)

Baseline pledgeable transfer per success. From the moral-hazard benchmark above, the smallest incentive-compatible reward is $R_b^\star = \frac{B}{\Delta p}$, so the maximum success-state payment to outsiders is

$$x^\star = R - R_b^\star = R - \frac{B}{\Delta p}.$$

Control right: an enforceable intervention. Suppose that if investors hold control rights, they can enforce an action that increases success probability by $\tau > 0$.

- Under **investor control**, success probability becomes $p_H + \tau$.
- Under **insider control**, assume the action is not taken (e.g., due to insiders' private cost), so success probability remains p_H.

Pledgeable income under the two control regimes. Since outsiders can receive at most $x^\star$ in success:

$$\Pi_I = (p_H + \tau)x^\star = (p_H + \tau)\left(R - \frac{B}{\Delta p}\right), \qquad \Pi_E = p_H x^\star = p_H\left(R - \frac{B}{\Delta p}\right).$$

Value of control as collateral.

$$\Pi_I - \Pi_E = \Big[(p_H + \tau) - p_H\Big]\left(R - \frac{B}{\Delta p}\right) = \tau\left(R - \frac{B}{\Delta p}\right).$$

One-per-topic (Minimal)

Monitoring (IC): $(\Delta p)R_b \geq B$. **Control rights (feasibility):** $\Pi \geq I - A$.