EIOBook-7: Dynamic Consumer Demand

Fast, intuitive, and highly coherent review notes

Coherence spine (read this first)

Dynamic consumer demand starts from a measurement problem: promotions and temporary price changes generate stockpiling and waiting, so observed purchase spikes reflect intertemporal timing rather than true substitution. That diagnosis immediately implies the missing object a correct model must include: an intertemporal state variable—**inventory**—and beliefs about future prices. Once inventory enters, demand becomes a dynamic decision problem, but the state is potentially enormous because it would include the full vector of brand prices and attributes. The inclusive value is introduced precisely to compress the many-brand choice environment into a low-dimensional index while preserving the relevant substitution incentives. Estimation then naturally splits into (i) within-period brand choice parameters (identified from brand choice variation conditional on purchase, with IV/GMM for price endogeneity) and (ii) quantity/inventory dynamics (identified from household purchase histories and timing between trips, with inventory treated as an unobserved endogenous state). In short:

$$\text{static bias under promotions} \Rightarrow \text{need an inventory state} \Rightarrow \text{DP becomes high-dimensional}$$

$$\text{inclusive value compresses the state} \Rightarrow \text{two-part estimation identifies primitives.}$$

The one mental picture

Dynamic consumer demand = brand choice + quantity choice + an inventory state. When prices are temporarily low (promotions), consumers buy more today, store, and buy less later. Ignoring this intertemporal substitution can bias elasticities and implied price-cost margins, especially in "high–low" pricing environments.

A 60–90 minute study plan

1) Motivation: why static demand fails under promotions

Connection tag: this section establishes the empirical bias and identifies the missing state variable.

- Temporary price cuts ⇒ stockpiling today ⇒ reduced purchases later.
- A static model reads the spike as substitution ⇒ overstates own-price elasticity.
- Overstated elasticity ⇒ understates implied market power / price-cost margins.

Implication: incorporate inventory (and expectations about future prices) to separate timing from true substitution.

2) Data: why household purchase histories are required

Connection tag: once the object is intertemporal substitution, market-level shares are insufficient; dynamics live in purchase paths.

- Inventory is not directly observed in aggregate data.
- Dynamics are revealed by sequences: quantities, timing between trips, and response to promotions over time.

Therefore, scanner-style household panels are a natural data environment.

3) Model structure: brand choice + quantity choice + inventory dynamics

Connection tag: the model mirrors the causal mechanism implied by the motivation: promotions affect both *which brand* and *how much*, while inventory links quantities across time.

- **State:** inventory (and possibly additional states such as household characteristics).
- **Actions:** (i) whether to purchase and how much (quantity/size), (ii) which brand to buy conditional on purchasing.

Key simplification: inventory is often modeled as *aggregate* rather than brand-specific.
Connection: this makes brand choice approximately static conditional on purchasing size x, while dynamics concentrate in the quantity/inventory problem.

4) Inclusive value: compress many brands into one index

Connection tag: a full dynamic model would need to track the entire vector of brand prices/attributes; inclusive value is introduced to reduce dimensionality while preserving substitution incentives.

Conditional on purchasing size x, brand choice can be written as a (micro) logit:

$$u_{hjxt} \; = \; b_h a_{jxt} - a_h p_{jxt} + \xi_{jxt} + \varepsilon_{hjxt},$$

(where α is written as a and β as b). The **inclusive value** for size x is:

$$\omega_h(x, p_t) \; = \; \ln \left(\sum_{j=1}^{J} \exp \left\{ b_h a_{jxt} - a_h p_{jxt} + \xi_{jxt} \right\} \right).$$

Interpretation: $\omega_h(x, p_t)$ summarizes the "best available brand value" for size x given the current price/attribute environment, allowing the dynamic problem to treat ω as a low-dimensional sufficient index rather than tracking all brands separately.

5) State-space reduction: the sufficiency/Markov assumption

Connection tag: forward-looking choice requires forecasting future environments; the chapter introduces a sufficient-statistic assumption to make expectations tractable.

Instead of modeling transitions for the full price vector p_t, assume inclusive values are sufficient for predicting future inclusive values:

$$\Pr\left(\omega_h(p_{t+1}) \mid p_t\right) \; = \; \Pr\left(\omega_h(p_{t+1}) \mid \omega_h(p_t)\right).$$

Connection: this turns an intractable forecasting problem (over all prices) into a manageable one (over inclusive values).

6) Estimation: why it splits into two components

Connection tag: the estimation strategy follows the model decomposition: within-period brand choice vs intertemporal quantity/inventory choice.

A) Brand-choice estimation (within-period)

- Identify price sensitivity and tastes for brand attributes using cross-sectional variation in prices and characteristics, conditional on purchase.
- Use IV/GMM because prices may correlate with unobserved quality ξ.

Connection: this component is "static-looking" because aggregate inventory is assumed not to enter the relative utility comparisons across brands conditional on buying.

B) Quantity/inventory dynamics (intertemporal)

- Inventory is an **unobserved endogenous state**: current purchases change tomorrow's inventory.
- Therefore dynamics are identified from purchase histories and timing between trips, not from a single-period demand equation.

7) Inventory is unobserved: proxies and identification (why $(x^{\textbf{last}}, T)$ matters)

Connection tag: because inventory is central but unobserved, the chapter introduces observable proxies that summarize inventory evolution.

A practical approach is to proxy inventory using observables such as last purchase size and time since last purchase:

$$i_{ht} \approx f_h(x_{ht}^{\text{last}}, T_{ht}),$$

where T_{ht} is weeks since last purchase.

- If current purchase probabilities depend systematically on (x^{last}, T), inventory is empirically relevant.
- Storage costs are identified through purchase frequency and stockpiling behavior (holding price paths fixed, higher frequency indicates higher storage costs).

Connection: this step closes the loop by linking the dynamic model's unobserved state to observable variation in household purchase paths.

One-page connection tags (memory anchors)

- **Motivation** $\rightarrow$ shows bias and implies the missing state (inventory).
- **Data** $\rightarrow$ dynamics live in purchase paths, requiring micro panels.
- **Model** $\rightarrow$ decomposes the problem into brand vs quantity/inventory.
- **Inclusive value** $\rightarrow$ compresses the many-brand environment.
- **Sufficiency/Markov** $\rightarrow$ makes expectations about the future computable.
- **Estimation split** $\rightarrow$ follows the model split (brand "static", quantity dynamic).
- **Inventory proxy** $\rightarrow$ converts unobserved state into estimable variation.

Five-minute self-test

1. Why does high–low pricing make static elasticity estimates too large?
2. What does the inclusive value $\omega_h(x, p_t)$ summarize, and what does it replace?
3. What does the sufficient-statistic transition assumption buy computationally?
4. Why is inventory an endogenous state rather than a standard error term?
5. How can (x^{last}, T) help identify storage costs?