Demand Estimation in Empirical IO (EIOBook-2)

A fast, coherent, and intuitive review guide

Core mental model

Demand estimation is the task of recovering **consumer valuation** and **substitution patterns** from observed data on *prices*, *product characteristics*, and *market shares*. Once demand is identified and estimated, you can compute objects that matter for IO counterfactuals:

- marginal revenue and implied markups (market power),
- merger substitution/diversion and competitive effects,
- welfare impacts of taxes, subsidies, quality changes, and new products,
- (in some settings) cost-of-living and price-index implications.

Notation preference. Throughout, write α as a and β as b.

A 60-minute study plan (high-ROI order)

Pass 1 (5–10 min): What are we trying to learn?

Across demand models, the empirical targets are stable:

- **Price sensitivity** (a): how utility changes with price.
- **Taste for characteristics** (b): how utility loads on observed product attributes.
- **Substitution patterns**: who steals share from whom when a price changes.
- **Unobserved quality** (ξ_j): the latent component of product appeal that is observed by consumers/firms but not fully by the econometrician.

These objects are exactly what you need for welfare and merger counterfactuals.

Pass 2 (10–15 min): Two modeling traditions

It helps to organize the chapter into two "worlds".

A. Product-space demand systems (classic)

These treat consumption as **continuous quantities** across many goods (e.g., LES/CES/AIDS). They are analytically convenient, but face practical limitations in differentiated-product IO:

1. **No zero purchases:** many such systems imply $q_j > 0$ for all goods, which is unrealistic for differentiated varieties (e.g., cars, brands).
2. **Representative-consumer limitation:** heterogeneity in income/tastes is hard to incorporate transparently.
3. **Dimensionality:** unrestricted substitution patterns require too many parameters as the number of products J grows (elasticities scale roughly with J^2).

Example intuition (CES). CES often implies a single "substitution strength" shared across all product pairs, which can be too restrictive for differentiated markets.

Common fix: multi-stage budgeting. A nesting/budgeting structure (category $\rightarrow$ group $\rightarrow$ product) reduces dimensionality by restricting how substitution operates across levels.

B. Characteristics-space discrete choice (modern workhorse)

Here each product is a **bundle of characteristics**, and each consumer typically chooses **at most one** option in a market (including an outside option). This framework naturally generates:

- realistic **zero purchases**,
- interpretable **heterogeneity** (via observed or random tastes),
- tractable substitution patterns with many products.

Pass 3 (15–20 min): The discrete-choice engine

Let products be $j = 1, \ldots, J$ and outside option be 0.

1) Logit (baseline)

The core object is **mean utility**:

$$\delta_j = -ap_j + X_j b + \xi_j,$$

where p_j is price, X_j are observed characteristics, and ξ_j is unobserved quality.

With i.i.d. Type-I extreme value idiosyncratic shocks, market shares take the familiar logit form:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}.$$

A useful implication is the log-odds representation:

$$\log\left(\frac{s_j}{s_0}\right) = \delta_j,$$

where s_0 is the outside option share.

2) What fails in simple logit? IIA

Simple logit implies **Independence of Irrelevant Alternatives (IIA)**: the relative odds of choosing j over k do not depend on the presence or characteristics of other alternatives. In practice, IIA often generates implausible cross-price elasticities (e.g., a price increase for a compact car shifts demand proportionally to SUVs and to similar compacts).

3) Beyond logit: Nested logit and Random Coefficients (BLP)

Two standard paths relax IIA and improve realism:

- **Nested logit:** allows stronger substitution within groups ("nests") of similar products.
- **Random coefficients logit (BLP):** allows heterogeneous tastes, producing flexible substitution patterns.

A generic random-coefficients utility can be written as:

$$u_{ij} = \delta_j + \mu_{ij}(\theta) + \varepsilon_{ij},$$

where $\mu_{ij}(\theta)$ captures consumer heterogeneity (random slopes on characteristics and/or price). Market shares become integrals over heterogeneity:

$$s_j(\theta) = \int \frac{\exp(\delta_j + \mu_{ij}(\theta))}{1 + \sum_{k=1}^{J} \exp(\delta_k + \mu_{ik}(\theta))} \, dF(i),$$

so shares generally have no closed form and must be computed numerically.

Pass 4 (10–15 min): The "magic trick" — Berry inversion and IV/GMM

This is the key computational and econometric insight.

Berry inversion (concept). Under regularity conditions, the mapping from mean utilities to market shares is invertible:

$$\delta = \sigma^{-1}(s \mid \theta).$$

Given observed shares s and a candidate parameter vector θ, you can recover the implied δ.

Why this matters. Once δ is recovered, the unobserved quality enters **additively**:

$$\xi_j = \delta_j + ap_j - X_j b,$$

which makes it natural to use **IV/GMM** to address price endogeneity.

One-line estimation loop (BLP intuition).

1. Guess parameters (including heterogeneity parameters) θ.
2. Invert shares to obtain $\delta = \sigma^{-1}(s \mid \theta)$.
3. Form implied unobservables ξ from $\xi_j = \delta_j + ap_j - X_j b$.
4. Choose θ to satisfy moment conditions using instruments Z:

$$\mathbb{E}[Z'\xi] = 0.$$

Instrument quality. In practice, demand instruments can be weak depending on market structure and product menus. A professional checklist is:

- justify exclusion restrictions carefully (why Z shifts price/cost but not ξ),
- diagnose weak instruments (first-stage strength, sensitivity),
- consider alternative sources of variation when standard BLP-style instruments underperform (e.g., cross-market price variation under strong assumptions, or dynamic-panel style strategies when appropriate).

Pass 5 (10 min): Welfare and new products

With estimated demand, you can quantify:

- welfare gains from **new varieties**,
- welfare effects of **quality changes** and **policy shocks** (tax/subsidy),
- counterfactual outcomes that depend on substitution (e.g., merger welfare).

A caution: standard logit-style errors can make welfare gains from variety grow too quickly as J increases; careful modeling/normalizations may be needed in applications with expanding product sets.

Pass 6 (5 min): Complementarity via bundles

Standard single-product discrete choice struggles with complementarity ("buying A makes B more valuable"). A bundle-choice extension lets consumers choose among:

$$\{0,\ A,\ B,\ AB\},$$

with a complementarity parameter Γ:

$$u_{AB} = u_A + u_B + \Gamma.$$

Interpretation: $\Gamma > 0$ implies complementarity; increasing the price of B can reduce demand for A because consumers value the joint bundle.

Six equations worth memorizing

1. Logit shares:
$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}.$$
2. Mean utility decomposition:
$$\delta_j = -ap_j + X_j b + \xi_j.$$
3. Log-odds form:
$$\log\left(\frac{s_j}{s_0}\right) = \delta_j.$$
4. Random-coefficients shares:
$$s_j(\theta) = \int \frac{\exp(\delta_j + \mu_{ij}(\theta))}{1 + \sum_{k=1}^{J} \exp(\delta_k + \mu_{ik}(\theta))}\, dF(i).$$
5. Berry inversion and additivity of ξ:
$$\delta = \sigma^{-1}(s \mid \theta), \qquad \xi_j = \delta_j + ap_j - X_j b.$$
6. Bundle complementarity:
$$u_{AB} = u_A + u_B + \Gamma.$$

Five-minute self-test

If you can answer these cleanly, you understand the chapter at a practical level:

1. Why do product-space demand systems struggle in differentiated-product settings?
2. What is IIA, and why does it generate unrealistic substitution patterns?
3. In logit, what are δ_j and ξ_j conceptually?
4. What does Berry inversion buy you for estimation and identification?
5. In the bundle model, what is the economic meaning of $\Gamma > 0$?