Supplement 1*:*

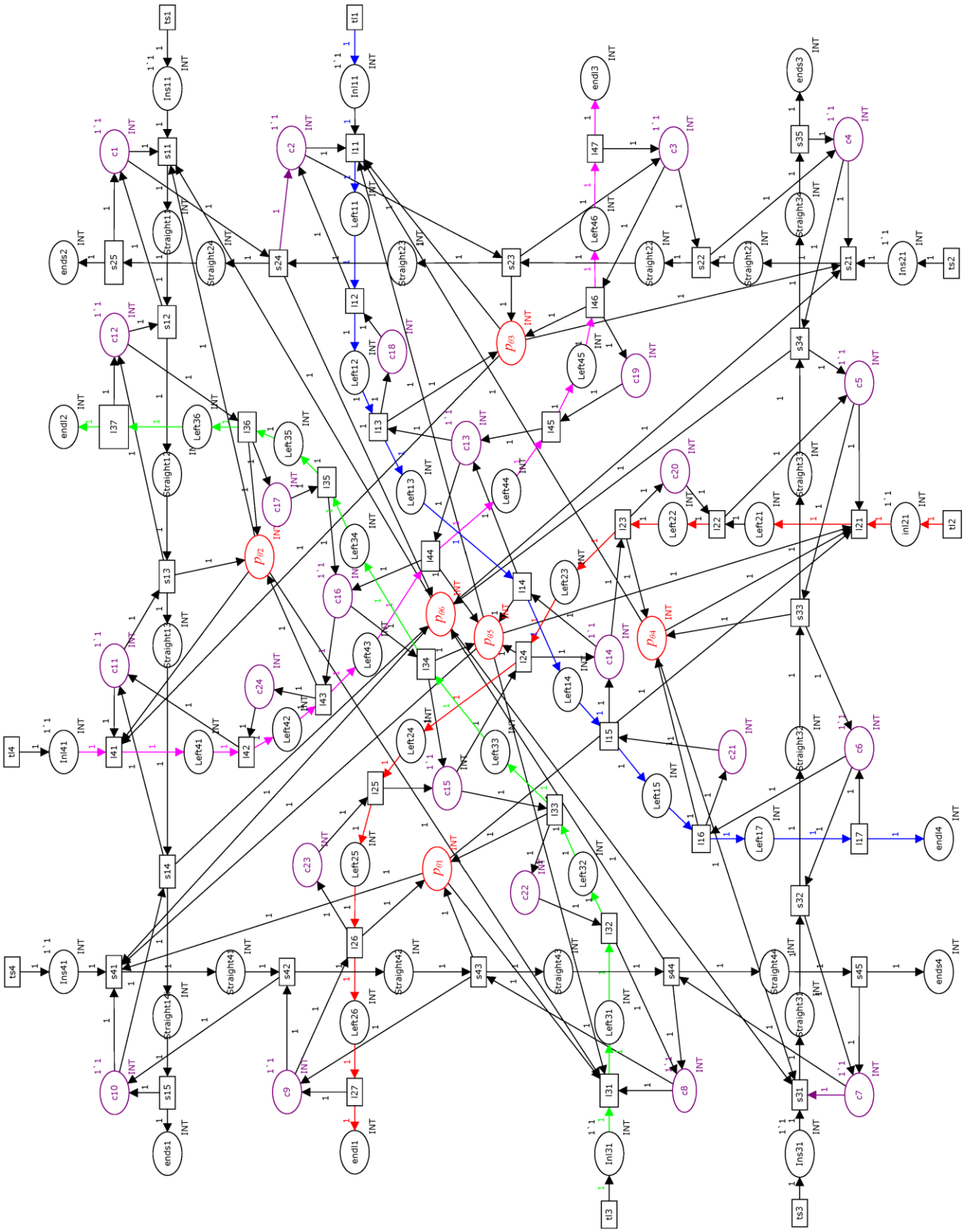

Fig.1. The controlled model of the entire intersection *CN*

Supplement 2:

Example 2: The potential vehicle flow deadlock situation shown in Fig. 10 (e) can be resolved by the added control place p_{θ_5}, as shown in Fig. 12. The figure shows the left-turn vehicle flows in four directions at an intersection.

According to Table III, initially, here are three tokens in p_{θ_5}. Therefore, only vehicles from three directions are allowed to enter the intersection at the same time. We assume that vehicles turning left from south to west, turning left from west to north, and turning left from east to south enter the intersection first, while vehicles turning left from north to east wait there. That is, transitions t_{l1}, t_{l2}, t_{l3} are enabled.

At this point, the initial state is:

$$M_0(p)=\begin{cases} 1`car & if\ p=Inl_{i1}, i=1,2,3 \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,9,11\cdots 24 \\ 3`token & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (1)$$

At M_0, the transitions l_{11}, l_{21}, l_{31} are enabled. The vehicles in each direction enter the next right-of-way points. At this time, the marking of CN_2 is:

$$M_1(p)\begin{cases} 1`car & if\ p=Left_{i1}, i=1,2,3 \\ 1`resource & if\ p=c_i, i=3,6,9,11\cdots 24 \\ \varnothing & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (2)$$

At M_1, the transitions l_{12}, l_{22}, l_{32} are triggered. The number of tokens in the control place p_{θ_5} reduced to 0, which means that no new vehicles are allowed to enter the intersection at this time. Therefore, the marking of CN_2 becomes:

$$M_2(p)=\begin{cases} 1`car & if\ p=Left_{i2}, i=1,2,3 \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,9,11 \\ & \cdots 17,19,21,23,24 \\ \varnothing & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (3)$$

At this state, the transitions l_{13}, l_{23}, l_{33} may be triggered.

$$M_3(p)=\begin{cases} 1`car & if\ p=Left_{i3}, i=1,2,3 \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,9, \\ & 11,12,16\cdots 24 \\ \varnothing & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (4)$$

At M_3, to prevent vehicle flow deadlock, only transition l_{34} is enabled, meaning that vehicles turning left from west to north can continue to enter into the next "right-of-way" point $Left_{34}$, while vehicles turning left from east to south and left from south to west stay at their current right-of-way point and wait. The CN_2 reaches the marking M_4:

$$M_4(p)=\begin{cases} 1`car & if\ p=Left_{i3}, i=1,2 \\ 1`car & if\ p=Left_{34} \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,9,11, \\ & 12,15,17\cdots 24 \\ 1`token & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (5)$$

At M_4, transitions l_{24} and l_{35} are triggered, and vehicles turning left from east to south continue their waiting. At the same time, one token is released into place p_{θ_5}, which means that a new vehicle can enter the interior of the intersection. Assume that a new vehicle turning left from north to east into the interior area of the intersection, meaning that the transition l_{41} is triggered. In this case, the marking of CN_2 becomes:

$$M_5(p)=\begin{cases} 1`car & if\ p=Left_{24}, Left_{13}, \\ & Left_{35}, Left_{41} \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,9,12, \\ & 14,16,18\cdots 24 \\ 1`token & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (6)$$

At M_5, the transitions l_{36}, l_{25}, l_{14}, l_{42} are triggered.

$$M_6(p)=\begin{cases} 1`car & if\ p=Left_{14}, Left_{25}, \\ & Left_{42}, Left_{36} \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,9, \\ & 11,13,15,\cdots,22 \\ 2`token & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (7)$$

Now, the number of tokens in the control place p_{θ_5} is 2, which means that new vehicles in the other three directions can enter the region. Assume that one from west to north and another from east to south do so, respectively. That is, transition l_{31}, l_{11} are triggered and M_7 is reached.

$$M_7(p)=\begin{cases} 1`car & if\ p=Left_{14}, Left_{25}, Left_{42}, \\ & Left_{31}, Left_{36} \\ 1`resource & if\ p=c_i, i=3,5,6,9,11, \\ & 13,15,\cdots,22 \\ \varnothing & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (8)$$

At M_7, transitions l_{12}, l_{15}, l_{26}, l_{37}, l_{32}, l_{43} are triggered and CN_2 reaches M_8:

$$M_8(p)=\begin{cases} 1`car & if\ p=Left_{12}, Left_{15}, Left_{26}, \\ & Left_{32}, Left_{43}, \\ 1`resource & if\ p=c_i, i=2,3,5,6,8,11\cdots \\ & 15,17,19,20,23,24 \\ \varnothing & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (9)$$

At M_8, transitions l_{16}, l_{27}, l_{33}, l_{44} are fired. Therefore, the marking of CN_2 becomes:

$$M_9(p)=\begin{cases} 1`car & if\ p=Left_{12}, Left_{16}, Left_{33}, Left_{44} \\ 1`resource & if\ p=c_i, i=2,3,5,8,9,11,12, \\ & 14,16,19\cdots 24 \\ 1`token & if\ p=p_{\theta_5} \\ \varnothing & otherwise \end{cases} \quad (10)$$

Now, the number of tokens in the control place p_{θ_5} is 1. Assume that a new left-turn vehicle from south to west enters the intersection, i.e., transition l_{21} is triggered. Then, the transitions l_{17}, l_{34}, l_{45}, l_{22} are triggered. The CN_2 reaches

marking M_{10}:

$$M_{10}(p)=\begin{cases} 1`car & if\ p=Left_{12},Left_{22},\\ & \quad Left_{34},Left_{45},\\ 1`resource & if\ p=c_i,i=2,3,5,6,8,9,11\cdots14,\\ & \quad 15,17,21\cdots24\\ 1`token & if\ p=p_{\theta_5}\\ \varnothing & otherwise \end{cases} \tag{11}$$

Thus, the possible vehicle flow deadlock as shown in Fig. 10 (e) has been effectively prevented. After that, this type of deadlock can be prevented by continuing evolution of the model according to the controller.

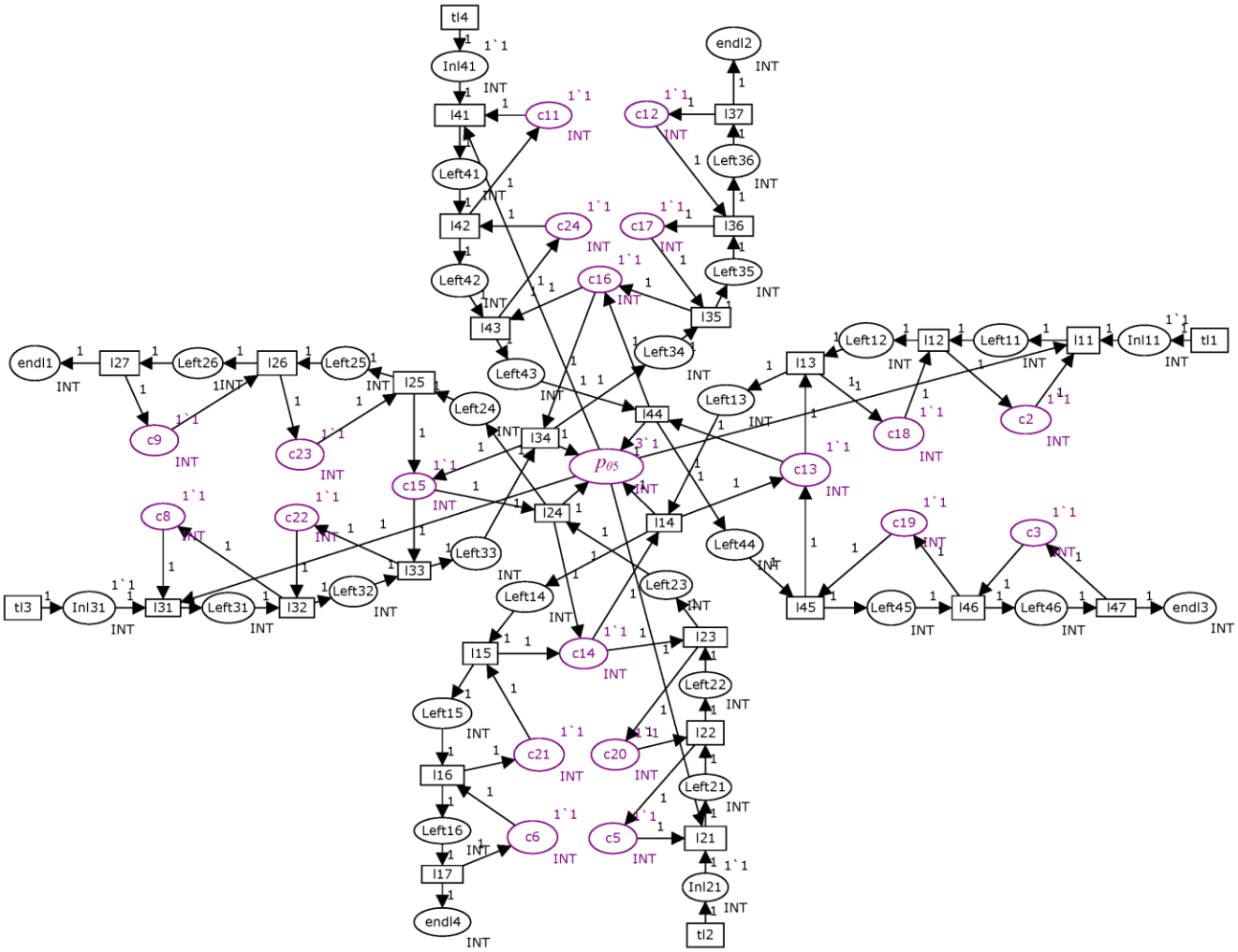

Fig. 12. Part of the controlled Petri net model CN_2

Supplement 3*:*

Example 3: The potential vehicle flow deadlock situation shown in Fig. 10 (a) can be resolved by the added control place p_{θ_1}, as shown in Fig. 13. Similarly, the vehicle flow deadlock situations shown in Fig. 10 (b), (c) and (d) can be resolved by the added control places p_{θ_2}, p_{θ_3} and p_{θ_4}, respectively. In these four cases, the control principle of the controllers is the same and we discuss one of the situations. Fig.13 shows the go-straight vehicle flow trajectories from south to north, left turn from south to west, and left turn from west to north.

According to Table III, by the control design, we put four tokens in p_{θ_1}. Therefore, vehicles from all three directions are allowed to enter the intersection at the same time at the initial state.

When the first car from each of the three directions enters the intersection internal area at the same time, there is only one token in each of the places Ins_{21}, Ins_{31}, Ins_{41} in CN_3, and the initial marking of CN_3 is:

$$M_0(p)=\begin{cases} 1`car & if\ p=Ins_{41} \\ 1`car & if\ p=Inl_{i1}, i=2,3 \\ 1`resource & if\ p=c_i, i=5,7,8,9,10,12, \\ & \quad 14\cdots 17,20,22,23 \\ 4`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{12}$$

At M_0, transitions l_{21}, l_{31}, and s_{41} are enabled, and the vehicles in each direction enter the next "right-of-way" points. At this time, the marking of CN_3 is:

$$M_1(p)=\begin{cases} 1`car & if\ p=Left_{i1}, i=2,3 \\ 1`car & if\ p=Straight_{41} \\ 1`resource & if\ p=c_i, i=7,9,12,14\cdots \\ & \quad 17,20,22,23 \\ 1`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{13}$$

At M_1, the transitions l_{22}, l_{32}, s_{42} are enabled, Therefore, the marking of CN_3 becomes:

$$M_2(p)=\begin{cases} 1`car & if\ p=Straight42 \\ 1`car & if\ p=Left_{i2}, j=2,3 \\ 1`resource & if\ p=c_i, i=5,7,8,10,12,14\cdots 17,23 \\ 1`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{14}$$

At this state, the number of tokens in p_{θ_1} is 1, which means that a vehicle can enter the intersection area. According to the principle of go-straight priority, we assume that a new vehicle enters the go-straight lane from north to south. The transition s_{41} is triggered. At the same time, transitions s_{43}, l_{33}, and l_{23} are triggered. we can get a new marking M_3:

$$M_3(p)=\begin{cases} 1`car & if\ p=Straight_{4j}, j=1,3 \\ 1`car & if\ p=Left_{i3}, i=2,3 \\ 1`resource & if\ p=c_i, i=5,7,9,12,16, \\ & \quad 17,20,22,23 \\ 2`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{15}$$

At M_3, the number of tokens in p_{θ_1} is 2, transitions l_{21}, s_{44}, s_{42}, l_{34} are triggered such that CN_3 reaches marking M_4:

$$M_4(p)=\begin{cases} 1`car & if\ p=\text{Straight}_{4j}, j=2,4 \\ 1`car & if\ p=Left_{2j}, i=1,3 \\ 1`car & if\ p=Left_{34} \\ 1`resource & if\ p=c_i, i=8,10,12,15 \\ & \quad 17,20,22,23 \\ 1`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{16}$$

At M_4, the number of tokens in p_{θ_1} is 1. We assume that a new vehicle enters the left-turn lane from west to north. That is, the transition l_{31} is triggered. Then, transitions s_{45}, l_{24}, l_{22}, l_{35} are triggered such that CN_3 reaches M_5:

$$M_5(p)=\begin{cases} 1`car & if\ p=Straight_{42} \\ 1`car & if\ p=Left_{3j}, j=1,5 \\ 1`car & if\ p=Left_{2j}, j=2,4 \\ 1`resource & if\ p=c_i, i=5,7,10,12,14,16,22,23 \\ \varnothing & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{17}$$

At this point, the first vehicle entering the north-south go-straight lane has left the intersection. Then, transitions l_{36}, l_{32}, l_{25}, l_{23} are triggered such that CN_3 reaches marking M_6:

$$M_6(p)=\begin{cases} 1`car & if\ p=Straight_{42} \\ 1`car & if\ p=Left_{3j}, j=2,6 \\ 1`car & if\ p=Left_{2j}, j=3,5 \\ 1`resource & if\ p=c_i, i=5,7,8,10,15, \\ & \quad 16,17,20 \\ \varnothing & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{18}$$

At M_6, transitions s_{43}, l_{37}, l_{33} are triggered such that CN_3 reaches marking M_7:

$$M_7(p)=\begin{cases} 1`car & if\ p=Straight_{43} \\ 1`car & if\ p=Left_{33} \\ 1`car & if\ p=Left_{2j}, j=3,5 \\ 1`resource & if\ p=c_i, i=5,7,9,10,12,16, \\ & \quad 17,20,22 \\ 2`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \tag{19}$$

At M_7, the number of tokens in p_{θ_1} is 2. We assume that a new vehicle enters the go-straight lane from north to south, i.e., transitions l_{21} and s_{41} are triggered. Then, the transitions s_{44}, l_{34}, l_{26}, l_{22}, l_{24} are triggered such that CN_3 reaches marking M_8:

$$M_8(p)=\begin{cases} 1`car & if\ p=Straight_{4j},\ j=1,4 \\ 1`car & if\ p=Left_{34} \\ 1`car & if\ p=Left_{2j},\ j=2,4,6 \\ 1`resource & if\ p=c_i,\ i=5,8,12,14,17,22,23 \\ 1`token & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \quad (20)$$

At M_8, the number of tokens in p_{θ_1} is 1. We assume that transition l_{31} is triggered. Then, transitions s_{45}, l_{35}, l_{27}, l_{25}, l_{23} are triggered such that CN_3 reaches marking M_9:

$$M_9(p)=\begin{cases} 1`car & if\ p=Straight_{41} \\ 1`car & if\ p=Left_{3j},\ j=1,5 \\ 1`car & if\ p=Left_{2j},\ j=3,5 \\ 1`resource & if\ p=c_i,\ i=5,7,9,12,15,16,20,22 \\ \varnothing & if\ p=p_{\theta_1} \\ \varnothing & otherwise \end{cases} \quad (21)$$

At this point, the first vehicle in the straight lane from north to south, the left-turn lane from west to north, and the left-turn lane from south to west have safely passed through the intersection, confirming that deadlock is successfully prevented.

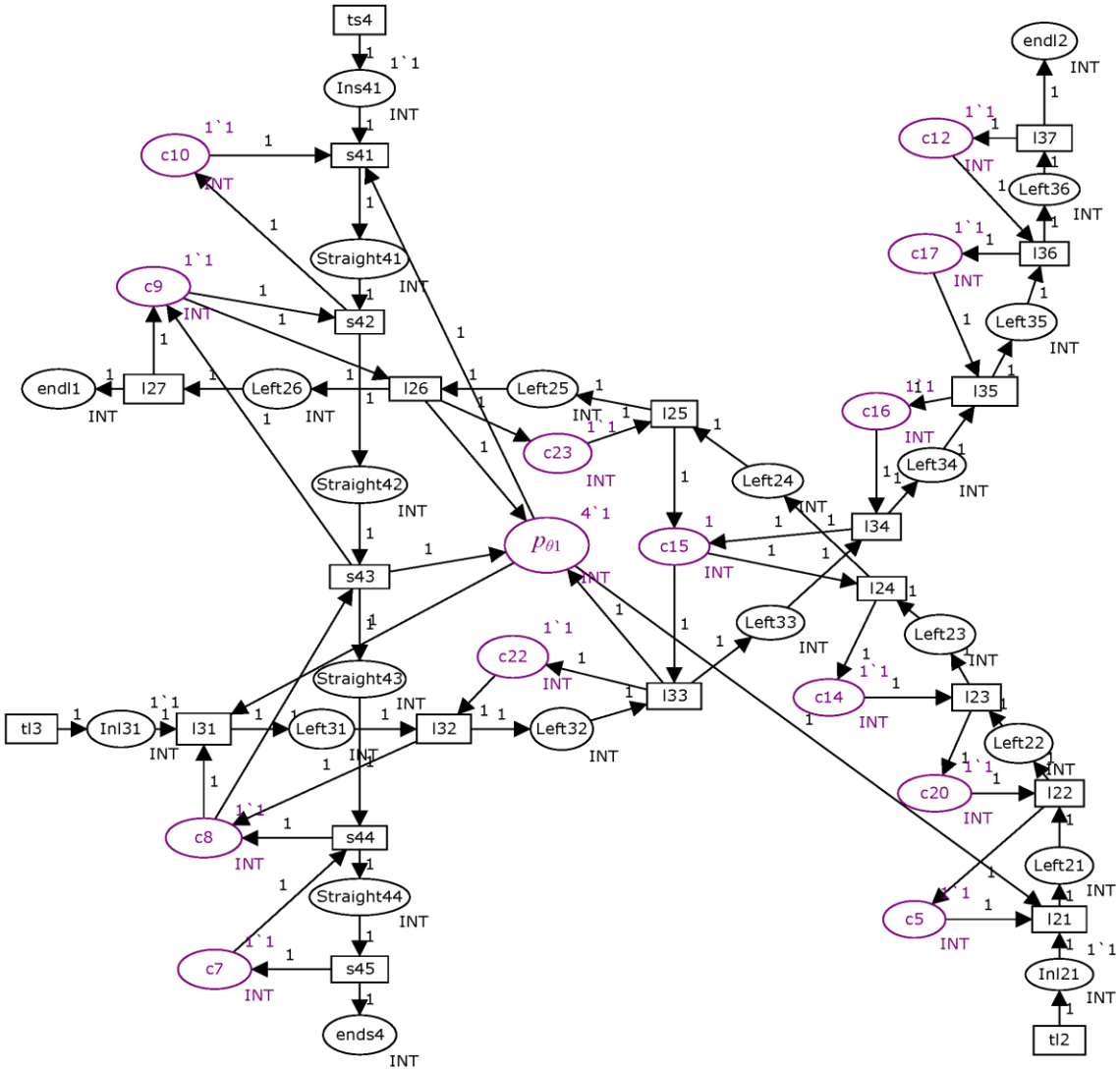

Fig. 13. Part of the controlled Petri net model CN_3