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CSE160
HW9
10/31/2017

The problem says that typically only 5% of existing customers and 1% of potential customers make a purchase. And from the confusion matrices, we found that the sum of column for p is 5 for existing customers, and the sum of column for p is 1 for potential customers, which match what the problem says. Therefore, we can conclude the confusion matrices set positive for customers who will make a purchase, and negative for customers who won't make a purchase. After figuring out this, we can draw the **cost-benefit matrix** for both the existing customers and potential customers:

For existing customers:

	Actual	
Predicted	p	n
Y	30	-1
N	0	0

For potential customers:

	Actual	
Predicted	p	n
Y	20	-1
N	0	0

Traditional way of sending holiday catalogs:

$$4{,}000{,}000 * 5\% * \$30 + 1{,}000{,}000 * 1\% * \$20 - (4{,}000{,}000 * 95\% + 1{,}000{,}000 * 99\%) * (\$1) = \$1{,}410{,}000$$

Then we calculate the total money the company made for this season. From the confusion matrix, we could see that 4% of existing customers are predicted to buy and they buy, so the net sale is 30 per customer; 5% of existing customers are predicted to purchase but they don't buy, so the net sale is -1 per customer. And 1% of potential customers are predicted to buy and they buy, so the net sale is 20 per customer; 9% of potential customers are predicted to buy and they don't buy, so the net sale is -1 per customer.

For existing customers:

T = 100

P = 5 N = 95

p(p) = 0.05 p(n) = 0.95

tp rate = 4/5 = 0.8 fp rate = 5/95 = 0.05
fn rate = 1/5 = 0.2 tn rate = 90/95 = 0.95

$$\textbf{Expected profit} = p(p) * [p(Y|p) * b(Y,p) + p(N|p) * b(N,p)] + p(n) * [p(N|n) * b(N,n) + p(Y|n) * b(Y,n)] = 0.05 * [0.8 * b(Y,p) + 0.2 * b(N,p)] + 0.95 * [0.95 * b(N,n) + 0.05 * b(Y,n)] = 0.05 * [0.8 * 30 + 0.2 * 0] + 0.95 * [0.95 * 0 + 0.05 * (-1)] = \$1.15$$

We can expect to make an average of about $1.15 profit per consumer:
$1.15 * 4,000,000 = $4,600,000

For potential customers:
T = 100
P = 1 N = 99
p(p) = 0.01 p(n) = 0.99
tp rate = 1/1 = 1 fp rate = 9/99 = 0.09
fn rate = 0/1 = 0 tn rate = 90/99 = 0.91

$$\textbf{Expected profit} = p(p) * [p(Y|p) * b(Y,p) + p(N|p) * b(N,p)] + p(n) * [p(N|n) * b(N,n) + p(Y|n) * b(Y,n)] = 0.01 * [1 * b(Y,p) + 0 * b(N,p)] + 0.99 * [0.91 * b(N,n) + 0.09 * b(Y,n)] = 0.01 * [1 * 20] + 0.99 * [0.91 * 0 + 0.09 * (-1)] = \$0.11$$

We can expect to make an average of about $0.11 profit per consumer:
$0.11 * 1,000,000 = $110,000

Combine the profit from existing customers and potential customers together:
Total profit = 4,600,000 + 110,000 = $4,710,000

Since 4,710,000 > 1,410,000, the company would make more money this season by only sending catalogs to the prospects that are predicted to purchase.