COMPX310-20A
Assignment 3

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Part I: BFS Fours

Implementation

Fours implement the built-in FIFO queue as the basis for its algorithm. Each state object also stores its own depth level, and this is to allow the algorithm to find multiple solutions that are of the same shallowest depth possible, instead of just stopping at the first solution found - once the first solution is found, the algorithm will continue to run until a deeper state is dequeued.

The program will also accept a depth limit as an optional parameter. In such case, calling
`java Fours 35 10`
will look for statements that evaluate to the answer n = 35, but only up to the depth d = 10 (where "4" is depth 0). If there is no depth limit given as a parameter, the program will simply run until the stack overflows (implemented internally as d = 2^{32}; this should practically serve the same purpose as not keeping track of the depth, unless you literally have exabytes of RAM to give the JVM).

Grammar and Parsing

The parser used in this program was created with the purpose of parsing expressions. Originally, the java built in ScriptEngine was used to evaluate the expressions. This class is going to be depreciated , this led to a warning message being printed every time the ScriptEngine was used to evaluate an expression. While this wasn't ideal it still evaluated most of the expressions. By most it wasn't until fully implemented and tested that we noticed that ScriptEngine didn't evaluate expontials. As this was a built in class changing the underlying code was too difficult so a new parser was written. This parser was designed specifically to account for "^" symbol as well as evaluate the rest of the expression.

Results for n in [1..100]

```
Looking for expressions matching 1
Shortest solutions:
4/4 = 1

Looking for expressions matching 2
Shortest solutions:
(4+4)/4 = 2
4/4+4/4 = 2
```

```
Looking for expressions matching 3
Shortest solutions:
4-4/4 = 3

Looking for expressions matching 4
Shortest solutions:
4 = 4

Looking for expressions matching 5
```

Shortest solutions:
4+4/4 = 5
4/4+4 = 5

Looking for expressions matching 6
Shortest solutions:
4+4/4+4/4 = 6
(4+4)/4+4 = 6
4/4+4+4/4 = 6
4/4+4/4+4 = 6
44/4.4-4 = 6

Looking for expressions matching 7
Shortest solutions:
4+4-4/4 = 7
4-4/4+4 = 7
44/4-4 = 7

Looking for expressions matching 8
Shortest solutions:
4+4 = 8

Looking for expressions matching 9
Shortest solutions:
4+4+4/4 = 9
4+4/4+4 = 9
4/4+4+4 = 9

Looking for expressions matching 10
Shortest solutions:
44/4.4 = 10

Looking for expressions matching 11
Shortest solutions:
44/4 = 11

Looking for expressions matching 12
Shortest solutions:
4+4+4 = 12
4*4-4 = 12

Looking for expressions matching 13
Shortest solutions:
4+4+4+4/4 = 13
4+4+4/4+4 = 13
4+4/4+4+4 = 13
4*4+4/4-4 = 13
4*4-4+4/4 = 13
4/4+4+4+4 = 13
4/4+4*4-4 = 13

4/4-4+4*4 = 13

Looking for expressions matching 14
Shortest solutions:
4+44/4.4 = 14
44/4.4+4 = 14

Looking for expressions matching 15
Shortest solutions:
4+44/4 = 15
4*4-4/4 = 15
44/4+4 = 15

Looking for expressions matching 16
Shortest solutions:
4*4 = 16

Looking for expressions matching 17
Shortest solutions:
4*4+4/4 = 17
4/4+4*4 = 17

Looking for expressions matching 18
Shortest solutions:
4+4+44/4.4 = 18
4+44/4.4+4 = 18
(4+4)/4+4*4 = 18
4*4+4/4+4/4 = 18
4*4.4+4.4-4 = 18
4*4.4-4+4.4 = 18
4/4+4*4+4/4 = 18
4/4+4/4+4*4 = 18
4.4+4*4.4-4 = 18
4.4+4.4*4-4 = 18
4.4-4+4*4.4 = 18
4.4-4+4.4*4 = 18
4.4*4+4.4-4 = 18
4.4*4-4+4.4 = 18
44/4.4+4+4 = 18

Looking for expressions matching 19
Shortest solutions:
4+4+44/4 = 19
4+4*4-4/4 = 19
4+44/4+4 = 19
4-4/4+4*4 = 19
4*4+4-4/4 = 19
4*4-4/4+4 = 19
44/4+4+4 = 19

Looking for expressions matching 20
Shortest solutions:
4+4*4 = 20
4*4+4 = 20

Looking for expressions matching 21
Shortest solutions:
4+4*4+4/4 = 21
4+4/4+4*4 = 21
4*4+4+4/4 = 21
4*4+4/4+4 = 21
4/4+4+4*4 = 21
4/4+4*4+4 = 21

Looking for expressions matching 22
Shortest solutions:
4*4.4+4.4 = 22
4.4+4*4.4 = 22
4.4+4.4*4 = 22
4.4*4+4.4 = 22

Looking for expressions matching 23
Shortest solutions:
4+4+4+44/4 = 23
4+4+4*4-4/4 = 23
4+4+44/4+4 = 23
4+4-4/4+4*4 = 23
4+4*4+4-4/4 = 23
4+4*4-4/4+4 = 23
4+44/4+4+4 = 23
4-4/4+4+4*4 = 23
4-4/4+4*4+4 = 23
4*4+4+4-4/4 = 23
4*4+4-4/4+4 = 23
4*4+44/4-4 = 23
4*4-4+44/4 = 23
4*4-4/4+4+4 = 23
44/4+4+4+4 = 23
44/4+4*4-4 = 23
44/4-4+4*4 = 23

Looking for expressions matching 24
Shortest solutions:
4+4+4*4 = 24
4+4*4+4 = 24
4*4+4+4 = 24

Looking for expressions matching 25
Shortest solutions:
4+4+4*4+4/4 = 25

4+4+4/4+4*4 = 25
4+4*4+4+4/4 = 25
4+4*4+4/4+4 = 25
4+4/4+4+4*4 = 25
4+4/4+4*4+4 = 25
4*4+4+4+4/4 = 25
4*4+4+4/4+4 = 25
4*4+4/4+4+4 = 25
4/4+4+4+4*4 = 25
4/4+4+4*4+4 = 25
4/4+4*4+4+4 = 25

Looking for expressions matching 26
Shortest solutions:
4+4*4.4+4.4 = 26
4+4.4+4*4.4 = 26
4+4.4+4.4*4 = 26
4+4.4*4+4.4 = 26
4*4+44/4.4 = 26
4*4.4+4+4.4 = 26
4*4.4+4.4+4 = 26
4.4+4+4*4.4 = 26
4.4+4+4.4*4 = 26
4.4+4*4.4+4 = 26
4.4+4.4*4+4 = 26
4.4*4+4+4.4 = 26
4.4*4+4.4+4 = 26
44/4.4+4*4 = 26

Looking for expressions matching 27
Shortest solutions:
4*4+44/4 = 27
44/4+4*4 = 27

Looking for expressions matching 28
Shortest solutions:
44-4*4 = 28

Looking for expressions matching 29
Shortest solutions:
4/4+44-4*4 = 29
4/4-4*4+44 = 29
44+4/4-4*4 = 29
44-4-44/4 = 29
44-4*4+4/4 = 29
44-44/4-4 = 29

Looking for expressions matching 30
Shortest solutions:
4+4+4*4.4+4.4 = 30

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4+4+4.4+4*4.4 = 30
4+4+4.4+4.4*4 = 30
4+4+4.4*4+4.4 = 30
4+4*4+44/4.4 = 30
4+4*4.4+4+4.4 = 30
4+4*4.4+4.4+4 = 30
4+4.4+4+4*4.4 = 30
4+4.4+4+4.4*4 = 30
4+4.4+4*4.4+4 = 30
4+4.4+4.4*4+4 = 30
4+4.4*4+4+4.4 = 30
4+4.4*4+4.4+4 = 30
4+44/4.4+4*4 = 30
4*4+4+44/4.4 = 30
4*4+44/4.4+4 = 30
4*4.4+4+4+4.4 = 30
4*4.4+4+4.4+4 = 30
4*4.4+4.4+4+4 = 30
4.4+4+4+4*4.4 = 30
4.4+4+4+4.4*4 = 30
4.4+4+4*4.4+4 = 30
4.4+4+4.4*4+4 = 30
4.4+4*4.4+4+4 = 30
4.4+4.4*4+4+4 = 30
4.4*4+4+4+4.4 = 30
4.4*4+4+4.4+4 = 30
4.4*4+4.4+4+4 = 30
44-4-44/4.4 = 30
44-44/4.4-4 = 30
44/4.4+4+4*4 = 30
44/4.4+4*4+4 = 30

Looking for expressions matching 31
Shortest solutions:
4+4*4+44/4 = 31
4+44/4+4*4 = 31
(4+4)*4-4/4 = 31
4*4+4+44/4 = 31
4*4+4*4-4/4 = 31
4*4+44/4+4 = 31
4*4-4/4+4*4 = 31
44/4+4+4*4 = 31
44/4+4*4+4 = 31

Looking for expressions matching 32
Shortest solutions:
(4+4)*4 = 32
4*4+4*4 = 32

Looking for expressions matching 33
```

```
Shortest solutions:
44-44/4 = 33

Looking for expressions matching 34
Shortest solutions:
44-44/4.4 = 34

Looking for expressions matching 35
Shortest solutions:
44-4-4-4/4 = 35
44-4-4/4-4 = 35
44-4/4-4-4 = 35

Looking for expressions matching 36
Shortest solutions:
44-4-4 = 36

Looking for expressions matching 37
Shortest solutions:
4+44-44/4 = 37
4-44/4+44 = 37
4/4+44-4-4 = 37
4/4-4+44-4 = 37
4/4-4-4+44 = 37
44+4-44/4 = 37
44+4/4-4-4 = 37
44-4+4/4-4 = 37
44-4-4+4/4 = 37
44-44/4+4 = 37

Looking for expressions matching 38
Shortest solutions:
(4+4.4)*4+4.4 = 38
4+44-44/4.4 = 38
4-44/4.4+44 = 38
4*4+4*4.4+4.4 = 38
4*4+4.4+4*4.4 = 38
4*4+4.4+4.4*4 = 38
4*4+4.4*4+4.4 = 38
4*4.4+4*4+4.4 = 38
4*4.4+4.4+4*4 = 38
4.4+4*4+4*4.4 = 38
4.4+4*4+4.4*4 = 38
4.4+4*4.4+4*4 = 38
4.4+4.4*4+4*4 = 38
(4.4+4)*4+4.4 = 38
4.4*4+4*4+4.4 = 38
4.4*4+4.4+4*4 = 38
44+4-44/4.4 = 38
44-4-4/4-4/4 = 38
```

```
44-4/4-4-4/4 = 38
44-4/4-4/4-4 = 38
44-44/4.4+4 = 38


Looking for expressions matching 39
Shortest solutions:
44-4-4/4 = 39
44-4/4-4 = 39


Looking for expressions matching 40
Shortest solutions:
44-4 = 40


Looking for expressions matching 41
Shortest solutions:
4/4+44-4 = 41
4/4-4+44 = 41
44+4/4-4 = 41
44-4+4/4 = 41


Looking for expressions matching 42
Shortest solutions:
44-4/4-4/4 = 42


Looking for expressions matching 43
Shortest solutions:
44-4/4 = 43


Looking for expressions matching 44
Shortest solutions:
44 = 44


Looking for expressions matching 45
Shortest solutions:
4/4+44 = 45
44+4/4 = 45


Looking for expressions matching 46
Shortest solutions:
(4+4)/4+44 = 46
4/4+4/4+44 = 46
4/4+44+4/4 = 46
44+4/4+4/4 = 46


Looking for expressions matching 47
Shortest solutions:
4+44-4/4 = 47
4-4/4+44 = 47
44+4-4/4 = 47
44-4/4+4 = 47
```

```
Looking for expressions matching 48
Shortest solutions:
4+44 = 48
44+4 = 48


Looking for expressions matching 49
Shortest solutions:
4+4/4+44 = 49
4+44+4/4 = 49
4/4+4+44 = 49
4/4+44+4 = 49
44+4+4/4 = 49
44+4/4+4 = 49


Looking for expressions matching 50
Shortest solutions:
4+4/4+4/4+44 = 50
4+4/4+44+4/4 = 50
4+44+4/4+4/4 = 50
(4+4)/4+4+44 = 50
(4+4)/4+44+4 = 50
4/4+4+4/4+44 = 50
4/4+4+44+4/4 = 50
4/4+4/4+4+44 = 50
4/4+4/4+44+4 = 50
4/4+44+4+4/4 = 50
4/4+44+4/4+4 = 50
44+4+4/4+4/4 = 50
44+4/4+4+4/4 = 50
44+4/4+4/4+4 = 50
44+44/4.4-4 = 50
44-4+44/4.4 = 50
44/4.4+44-4 = 50
44/4.4-4+44 = 50


Looking for expressions matching 51
Shortest solutions:
4+4+44-4/4 = 51
4+4-4/4+44 = 51
4+44+4-4/4 = 51
4+44-4/4+4 = 51
4-4/4+4+44 = 51
4-4/4+44+4 = 51
44+4+4-4/4 = 51
44+4-4/4+4 = 51
44+44/4-4 = 51
44-4+44/4 = 51
44-4/4+4+4 = 51
44/4+44-4 = 51
```

```
44/4-4+44 = 51

Looking for expressions matching 52
Shortest solutions:
4+4+44 = 52
4+44+4 = 52
44+4+4 = 52

Looking for expressions matching 53
Shortest solutions:
4+4+4/4+44 = 53
4+4+44+4/4 = 53
4+4/4+4+44 = 53
4+4/4+44+4 = 53
4+44+4+4/4 = 53
4+44+4/4+4 = 53
4*4*4-44/4 = 53
4/4+4+4+44 = 53
4/4+4+44+4 = 53
4/4+44+4+4 = 53
(4^4-44)/4 = 53
4^4/4-44/4 = 53
44+4+4+4/4 = 53
44+4+4/4+4 = 53
44+4/4+4+4 = 53

Looking for expressions matching 54
Shortest solutions:
44+44/4.4 = 54
44/4.4+44 = 54

Looking for expressions matching 55
Shortest solutions:
44+44/4 = 55
44/4+44 = 55

Looking for expressions matching 56
Shortest solutions:
4+4+4+44 = 56
4+4+44+4 = 56
4+44+4+4 = 56
4*4+44-4 = 56
4*4-4+44 = 56
4*4*4-4-4 = 56
4^4/4-4-4 = 56
44+4+4+4 = 56
44+4*4-4 = 56
44-4+4*4 = 56

Looking for expressions matching 57
```

```
Shortest solutions:
4+4+4+4/4+44 = 57
4+4+4+44+4/4 = 57
4+4+4/4+4+44 = 57
4+4+4/4+44+4 = 57
4+4+44+4+4/4 = 57
4+4+44+4/4+4 = 57
4+4*4*4-44/4 = 57
4+4/4+4+4+44 = 57
4+4/4+4+44+4 = 57
4+4/4+44+4+4 = 57
4+4^4/4-44/4 = 57
(4+4^4)/4-4-4 = 57
4+44+4+4+4/4 = 57
4+44+4+4/4+4 = 57
4+44+4/4+4+4 = 57
4-44/4+4*4*4 = 57
4-44/4+4^4/4 = 57
4*4+4/4+44-4 = 57
4*4+4/4-4+44 = 57
4*4+44+4/4-4 = 57
4*4+44-4+4/4 = 57
4*4-4+4/4+44 = 57
4*4-4+44+4/4 = 57
4*4*4+4-44/4 = 57
4*4*4+4/4-4-4 = 57
4*4*4-4+4/4-4 = 57
4*4*4-4-4+4/4 = 57
4*4*4-44/4+4 = 57
4/4+4+4+4+44 = 57
4/4+4+4+44+4 = 57
4/4+4+44+4+4 = 57
4/4+4*4+44-4 = 57
4/4+4*4-4+44 = 57
4/4+4*4*4-4-4 = 57
4/4+4^4/4-4-4 = 57
4/4+44+4+4+4 = 57
4/4+44+4*4-4 = 57
4/4+44-4+4*4 = 57
4/4-4+4*4+44 = 57
4/4-4+4*4*4-4 = 57
4/4-4+4^4/4-4 = 57
4/4-4+44+4*4 = 57
4/4-4-4+4*4*4 = 57
4/4-4-4+4^4/4 = 57
(4^4+4)/4-4-4 = 57
(4^4-44)/4+4 = 57
4^4/4+4-44/4 = 57
4^4/4+4/4-4-4 = 57
4^4/4-4+4/4-4 = 57
```

```
4^4/4-4-4+4/4 = 57                          Shortest solutions:
4^4/4-44/4+4 = 57                           4*4+44 = 60
44+4+4+4+4/4 = 57                           4*4*4-4 = 60
44+4+4+4/4+4 = 57                           4^4/4-4 = 60
44+4+4/4+4+4 = 57                           44+4*4 = 60
44+4*4+4/4-4 = 57
44+4*4-4+4/4 = 57                           Looking for expressions matching 61
44+4/4+4+4+4 = 57                           Shortest solutions:
44+4/4+4*4-4 = 57                           (4+4^4)/4-4 = 61
44+4/4-4+4*4 = 57                           4*4+4/4+44 = 61
44-4+4*4+4/4 = 57                           4*4+44+4/4 = 61
44-4+4/4+4*4 = 57                           4*4*4+4/4-4 = 61
                                            4*4*4-4+4/4 = 61
Looking for expressions matching 58         4/4+4*4+44 = 61
Shortest solutions:                         4/4+4*4*4-4 = 61
4+44+44/4.4 = 58                            4/4+4^4/4-4 = 61
4+44/4.4+44 = 58                            4/4+44+4*4 = 61
4*4.4+44.4-4 = 58                          4/4-4+4*4*4 = 61
4*4.4-4+44.4 = 58                          4/4-4+4^4/4 = 61
(4^4-4-4)/4-4 = 58                          (4^4+4)/4-4 = 61
4.4*4+44.4-4 = 58                          4^4/4+4/4-4 = 61
4.4*4-4+44.4 = 58                          4^4/4-4+4/4 = 61
44+4+44/4.4 = 58                            44+4*4+4/4 = 61
44+44/4.4+4 = 58                            44+4/4+4*4 = 61
44/4.4+4+44 = 58
44/4.4+44+4 = 58                            Looking for expressions matching 62
44.4+4*4.4-4 = 58                           Shortest solutions:
44.4+4.4*4-4 = 58                           4*4.4+44.4 = 62
44.4-4+4*4.4 = 58                          (4^4-4-4)/4 = 62
44.4-4+4.4*4 = 58                          4.4*4+44.4 = 62
                                            44.4+4*4.4 = 62
Looking for expressions matching 59         44.4+4.4*4 = 62
Shortest solutions:
4+44+44/4 = 59                              Looking for expressions matching 63
4+44/4+44 = 59                              Shortest solutions:
4*4+44-4/4 = 59                             4*4*4-4/4 = 63
4*4-4/4+44 = 59                             (4^4-4)/4 = 63
4*4*4-4-4/4 = 59                            4^4/4-4/4 = 63
4*4*4-4/4-4 = 59
(4^4-4)/4-4 = 59                            Looking for expressions matching 64
4^4/4-4-4/4 = 59                            Shortest solutions:
4^4/4-4/4-4 = 59                            4*4*4 = 64
44+4+44/4 = 59                              4^4/4 = 64
44+4*4-4/4 = 59
44+44/4+4 = 59                              Looking for expressions matching 65
44-4/4+4*4 = 59                             Shortest solutions:
44/4+4+44 = 59                              (4+4^4)/4 = 65
44/4+44+4 = 59                              4*4*4+4/4 = 65
                                            4/4+4*4*4 = 65
Looking for expressions matching 60         4/4+4^4/4 = 65
```

(4^4+4)/4 = 65
4^4/4+4/4 = 65

Looking for expressions matching 66
Shortest solutions:
(4+4+4^4)/4 = 66
(4+4^4+4)/4 = 66
4*4*4.4-4.4 = 66
4*4.4*4-4.4 = 66
(4^4+4+4)/4 = 66
4.4*4*4-4.4 = 66

Looking for expressions matching 67
Shortest solutions:
4+4*4*4-4/4 = 67
4+4^4/4-4/4 = 67
4-4/4+4*4*4 = 67
4-4/4+4^4/4 = 67
4*4*4+4-4/4 = 67
4*4*4-4/4+4 = 67
(4^4-4)/4+4 = 67
4^4/4+4-4/4 = 67
4^4/4-4/4+4 = 67
444/4-44 = 67

Looking for expressions matching 68
Shortest solutions:
4+4*4*4 = 68
4+4^4/4 = 68
4*4*4+4 = 68
4^4/4+4 = 68

Looking for expressions matching 69
Shortest solutions:
4+4*4*4+4/4 = 69
4+4/4+4*4*4 = 69
4+4/4+4^4/4 = 69
4+4^4/4+4/4 = 69
(4+4^4)/4+4 = 69
4*4*4+4+4/4 = 69
4*4*4+4/4+4 = 69
4/4+4+4*4*4 = 69
4/4+4+4^4/4 = 69
4/4+4*4*4+4 = 69
4/4+4^4/4+4 = 69
(4^4+4)/4+4 = 69
4^4/4+4+4/4 = 69
4^4/4+4/4+4 = 69

Looking for expressions matching 70

Shortest solutions:
(4+4+4^4)/4+4 = 70
4+4*4*4.4-4.4 = 70
4+4*4.4*4-4.4 = 70
(4+4^4+4)/4+4 = 70
4+4.4*4*4-4.4 = 70
4-4.4+4*4*4.4 = 70
4-4.4+4*4.4*4 = 70
4-4.4+4.4*4*4 = 70
4*4*4.4+4-4.4 = 70
4*4*4.4-4.4+4 = 70
4*4.4*4+4-4.4 = 70
4*4.4*4-4.4+4 = 70
(4^4+4+4)/4+4 = 70
4.4*4*4+4-4.4 = 70
4.4*4*4-4.4+4 = 70

Looking for expressions matching 71
Shortest solutions:
4+4+4*4*4-4/4 = 71
4+4+4^4/4-4/4 = 71
4+4-4/4+4*4*4 = 71
4+4-4/4+4^4/4 = 71
4+4*4*4+4-4/4 = 71
4+4*4*4-4/4+4 = 71
4+4^4/4+4-4/4 = 71
4+4^4/4-4/4+4 = 71
4+444/4-44 = 71
4-4/4+4+4*4*4 = 71
4-4/4+4+4^4/4 = 71
4-4/4+4*4*4+4 = 71
4-4/4+4^4/4+4 = 71
4-44+444/4 = 71
4*4+44+44/4 = 71
4*4+44/4+44 = 71
4*4*4+4+4-4/4 = 71
4*4*4+4-4/4+4 = 71
4*4*4+44/4-4 = 71
4*4*4-4+44/4 = 71
4*4*4-4/4+4+4 = 71
(4^4+44)/4-4 = 71
(4^4-4)/4+4+4 = 71
4^4/4+4+4-4/4 = 71
4^4/4+4-4/4+4 = 71
4^4/4+44/4-4 = 71
4^4/4-4+44/4 = 71
4^4/4-4/4+4+4 = 71
44+4*4+44/4 = 71
(44+4^4)/4-4 = 71
44+44/4+4*4 = 71

44/4+4*4+44 = 71
44/4+4*4*4-4 = 71
44/4+4^4/4-4 = 71
44/4+44+4*4 = 71
44/4-4+4*4*4 = 71
44/4-4+4^4/4 = 71
444/4+4-44 = 71
444/4-44+4 = 71

Looking for expressions matching 72
Shortest solutions:
4+4+4*4*4 = 72
4+4+4^4/4 = 72
4+4*4*4+4 = 72
4+4^4/4+4 = 72
4*4*4+4+4 = 72
4^4/4+4+4 = 72

Looking for expressions matching 73
Shortest solutions:
4+4+4*4*4+4/4 = 73
4+4+4/4+4*4*4 = 73
4+4+4/4+4^4/4 = 73
4+4+4^4/4+4/4 = 73
4+4*4*4+4+4/4 = 73
4+4*4*4+4/4+4 = 73
4+4/4+4+4*4*4 = 73
4+4/4+4+4^4/4 = 73
4+4/4+4*4*4+4 = 73
4+4/4+4^4/4+4 = 73
4+4^4/4+4+4/4 = 73
4+4^4/4+4/4+4 = 73
(4+4^4)/4+4+4 = 73
(4-4/4)^4-4-4 = 73
4*4*4+4+4+4/4 = 73
4*4*4+4+4/4+4 = 73
4*4*4+4/4+4+4 = 73
4/4+4+4+4*4*4 = 73
4/4+4+4+4^4/4 = 73
4/4+4+4*4*4+4 = 73
4/4+4+4^4/4+4 = 73
4/4+4*4*4+4+4 = 73
4/4+4^4/4+4+4 = 73
(4/4-4)^4-4-4 = 73
(4^4+4)/4+4+4 = 73
4^4/4+4+4+4/4 = 73
4^4/4+4+4/4+4 = 73
4^4/4+4/4+4+4 = 73

Looking for expressions matching 74

Shortest solutions:
4*4*4+44/4.4 = 74
(4^4+44-4)/4 = 74
(4^4-4+44)/4 = 74
4^4/4+44/4.4 = 74
(44+4^4-4)/4 = 74
(44-4+4^4)/4 = 74
44/4.4+4*4*4 = 74
44/4.4+4^4/4 = 74

Looking for expressions matching 75
Shortest solutions:
4*4*4+44/4 = 75
(4^4+44)/4 = 75
4^4/4+44/4 = 75
(44+4^4)/4 = 75
44/4+4*4*4 = 75
44/4+4^4/4 = 75

Looking for expressions matching 76
Shortest solutions:
4+4+4+4*4*4 = 76
4+4+4+4^4/4 = 76
4+4+4*4*4+4 = 76
4+4+4^4/4+4 = 76
4+4*4*4+4+4 = 76
(4+4*4)*4-4 = 76
4+4^4/4+4+4 = 76
(4+4)*4+44 = 76
4*4+4*4+44 = 76
4*4+4*4*4-4 = 76
4*4+4^4/4-4 = 76
4*4+44+4*4 = 76
(4*4+4)*4-4 = 76
4*4-4+4*4*4 = 76
4*4-4+4^4/4 = 76
4*4*4+4+4+4 = 76
4*4*4+4*4-4 = 76
4*4*4-4+4*4 = 76
4^4-4-4*44 = 76
4^4-4-44*4 = 76
4^4-4*44-4 = 76
4^4-44*4-4 = 76
4^4/4+4+4+4 = 76
4^4/4+4*4-4 = 76
4^4/4-4+4*4 = 76
44+4*4+4*4 = 76

Looking for expressions matching 77
Shortest solutions:

(4-4/4)^4-4 = 77
(4/4-4)^4-4 = 77

Looking for expressions matching 78
Shortest solutions:
4+4*4*4+44/4.4 = 78
4+4^4/4+44/4.4 = 78
(4+4.4)*4+44.4 = 78
4+44/4.4+4*4*4 = 78
4+44/4.4+4^4/4 = 78
(4-4/4)^4+4/4-4 = 78
(4-4/4)^4-4+4/4 = 78
4*4+4*4.4+44.4 = 78
4*4+4.4*4+44.4 = 78
4*4+44.4+4*4.4 = 78
4*4+44.4+4.4*4 = 78
4*4*4+4+44/4.4 = 78
4*4*4+44/4.4+4 = 78
4*4.4+4*4+44.4 = 78
4*4.4+44.4+4*4 = 78
(4/4-4)^4+4/4-4 = 78
(4/4-4)^4-4+4/4 = 78
(4^4+44-4)/4+4 = 78
(4^4-4+44)/4+4 = 78
(4^4-4-4)/4+4*4 = 78
4^4/4+4+44/4.4 = 78
4^4/4+44/4.4+4 = 78
(4.4+4)*4+44.4 = 78
4.4*4+4*4+44.4 = 78
4.4*4+44.4+4*4 = 78
(44+4^4-4)/4+4 = 78
44+44-44/4.4 = 78
(44-4+4^4)/4+4 = 78
44-44/4.4+44 = 78
44/4.4+4+4*4*4 = 78
44/4.4+4+4^4/4 = 78
44/4.4+4*4*4+4 = 78
44/4.4+4^4/4+4 = 78
44.4+4*4+4*4.4 = 78
44.4+4*4+4.4*4 = 78
44.4+4*4.4+4*4 = 78
44.4+4.4*4+4*4 = 78

Looking for expressions matching 79
Shortest solutions:
4+4*4*4+44/4 = 79
(4+4*4)*4-4/4 = 79
4+4^4/4+44/4 = 79
4+44/4+4*4*4 = 79
4+44/4+4^4/4 = 79

4*4+4*4*4-4/4 = 79
4*4+4^4/4-4/4 = 79
(4*4+4)*4-4/4 = 79
4*4-4/4+4*4*4 = 79
4*4-4/4+4^4/4 = 79
4*4*4+4+44/4 = 79
4*4*4+4*4-4/4 = 79
4*4*4+44/4+4 = 79
4*4*4-4/4+4*4 = 79
(4^4+44)/4+4 = 79
4^4-4*44-4/4 = 79
4^4-4/4-4*44 = 79
4^4-4/4-44*4 = 79
4^4-44*4-4/4 = 79
(4^4-4)/4+4*4 = 79
4^4/4+4+44/4 = 79
4^4/4+4*4-4/4 = 79
4^4/4+44/4+4 = 79
4^4/4-4/4+4*4 = 79
(44+4^4)/4+4 = 79
44/4+4+4*4*4 = 79
44/4+4+4^4/4 = 79
44/4+4*4*4+4 = 79
44/4+4^4/4+4 = 79

Looking for expressions matching 80
Shortest solutions:
(4+4*4)*4 = 80
4*4+4*4*4 = 80
4*4+4^4/4 = 80
(4*4+4)*4 = 80
4*4*4+4*4 = 80
4^4-4*44 = 80
4^4-44*4 = 80
4^4/4+4*4 = 80

Looking for expressions matching 81
Shortest solutions:
(4-4/4)^4 = 81
(4/4-4)^4 = 81

Looking for expressions matching 82
Shortest solutions:
(4-4/4)^4+4/4 = 82
(4/4-4)^4+4/4 = 82

Looking for expressions matching 83
Shortest solutions:
44+44-4-4/4 = 83
44+44-4/4-4 = 83

44-4+44-4/4 = 83
44-4-4/4+44 = 83
44-4/4+44-4 = 83
44-4/4-4+44 = 83

Looking for expressions matching 84
Shortest solutions:
44+44-4 = 84
44-4+44 = 84

Looking for expressions matching 85
Shortest solutions:
(4-4/4)^4+4 = 85
(4/4-4)^4+4 = 85

Looking for expressions matching 86
Shortest solutions:
(4-4/4)^4+4+4/4 = 86
(4-4/4)^4+4/4+4 = 86
(4*4+4.4)*4+4.4 = 86
4*4*4+4*4.4+4.4 = 86
4*4*4+4.4+4*4.4 = 86
4*4*4+4.4+4.4*4 = 86
4*4*4+4.4*4+4.4 = 86
4*4.4+4*4*4+4.4 = 86
4*4.4+4^4/4+4.4 = 86
4*4.4+4.4+4*4*4 = 86
4*4.4+4.4+4^4/4 = 86
(4/4-4)^4+4+4/4 = 86
(4/4-4)^4+4/4+4 = 86
(4^4+44+44)/4 = 86
4^4/4+4*4.4+4.4 = 86
4^4/4+4.4+4*4.4 = 86
4^4/4+4.4+4.4*4 = 86
4^4/4+4.4*4+4.4 = 86
4.4+4*4*4+4*4.4 = 86
4.4+4*4*4+4.4*4 = 86
4.4+4*4.4+4*4*4 = 86
4.4+4*4.4+4^4/4 = 86
(4.4+4*4)*4+4.4 = 86
4.4+4^4/4+4*4.4 = 86
4.4+4^4/4+4.4*4 = 86
4.4+4.4*4+4*4*4 = 86
4.4+4.4*4+4^4/4 = 86
4.4*4+4*4*4+4.4 = 86
4.4*4+4^4/4+4.4 = 86
4.4*4+4.4+4*4*4 = 86
4.4*4+4.4+4^4/4 = 86
(44+4^4+44)/4 = 86
(44+44+4^4)/4 = 86

44+44-4/4-4/4 = 86
44-4/4+44-4/4 = 86
44-4/4-4/4+44 = 86

Looking for expressions matching 87
Shortest solutions:
44+44-4/4 = 87
44-4/4+44 = 87

Looking for expressions matching 88
Shortest solutions:
44+44 = 88

Looking for expressions matching 89
Shortest solutions:
4/4+44+44 = 89
44+4/4+44 = 89
44+44+4/4 = 89

Looking for expressions matching 90
Shortest solutions:
(4+4)/4+44+44 = 90
4/4+4/4+44+44 = 90
4/4+44+4/4+44 = 90
4/4+44+44+4/4 = 90
44+4/4+4/4+44 = 90
44+4/4+44+4/4 = 90
44+44+4/4+4/4 = 90

Looking for expressions matching 91
Shortest solutions:
4+44+44-4/4 = 91
4+44-4/4+44 = 91
4-4/4+44+44 = 91
44+4+44-4/4 = 91
44+4-4/4+44 = 91
44+44+4-4/4 = 91
44+44-4/4+4 = 91
44-4/4+4+44 = 91
44-4/4+44+4 = 91
444/4-4-4*4 = 91
444/4-4*4-4 = 91

Looking for expressions matching 92
Shortest solutions:
4+44+44 = 92
44+4+44 = 92
44+44+4 = 92

Looking for expressions matching 93

Shortest solutions:
4+4/4+44+44 = 93
4+44+4/4+44 = 93
4+44+44+4/4 = 93
4/4+4+44+44 = 93
4/4+44+4+44 = 93
4/4+44+44+4 = 93
44+4+4/4+44 = 93
44+4+44+4/4 = 93
44+4/4+4+44 = 93
44+4/4+44+4 = 93
44+44+4+4/4 = 93
44+44+4/4+4 = 93

Looking for expressions matching 94
Shortest solutions:
(444-4)/4-4*4 = 94
444/4-4*4-4/4 = 94
444/4-4/4-4*4 = 94

Looking for expressions matching 95
Shortest solutions:
444/4-4*4 = 95

Looking for expressions matching 96
Shortest solutions:
(4+4+4*4)*4 = 96
4+4+44+44 = 96
(4+4*4+4)*4 = 96
4+44+4+44 = 96
4+44+44+4 = 96
(4*4+4+4)*4 = 96
44+4+4+44 = 96
44+4+44+4 = 96
44+44+4+4 = 96

Looking for expressions matching 97
Shortest solutions:
(4-4/4)^4+4*4 = 97
(4/4-4)^4+4*4 = 97
4444/44-4 = 97

Looking for expressions matching 98
Shortest solutions:
44+44+44/4.4 = 98
44+44/4.4+44 = 98
44/4.4+44+44 = 98

Looking for expressions matching 99

Shortest solutions:
4+444/4-4*4 = 99
4-4*4+444/4 = 99
44+44+44/4 = 99
44+44/4+44 = 99
44/4+44+44 = 99
444/4+4-4*4 = 99
444/4-4-4-4 = 99
444/4-4*4+4 = 99

Looking for expressions matching 100
Shortest solutions:
4+4+4+44+44 = 100
(4+4+4*4)*4+4 = 100
4+4+44+4+44 = 100
4+4+44+44+4 = 100
(4+4*4+4)*4+4 = 100
4+44+4+4+44 = 100
4+44+4+44+4 = 100
4+44+44+4+4 = 100
(4*4+4+4)*4+4 = 100
4*4+44+44-4 = 100
4*4+44-4+44 = 100
4*4-4+44+44 = 100
4*4*4+44-4-4 = 100
4*4*4-4+44-4 = 100
4*4*4-4-4+44 = 100
4^4/4+44-4-4 = 100
4^4/4-4+44-4 = 100
4^4/4-4-4+44 = 100
44+4+4+4+44 = 100
44+4+4+44+4 = 100
44+4+44+4+4 = 100
44+4*4+44-4 = 100
44+4*4-4+44 = 100
44+4*4*4-4-4 = 100
44+4^4/4-4-4 = 100
44+44+4+4+4 = 100
44+44+4*4-4 = 100
44+44-4+4*4 = 100
44-4+4*4+44 = 100
44-4+4*4*4-4 = 100
44-4+4^4/4-4 = 100
44-4+44+4*4 = 100
44-4-4+4*4*4 = 100
44-4-4+4^4/4 = 100
(444-44)/4 = 100
444/4-44/4 = 100

Part II: A* Pathfinding

Usage

As a GUI implementation was required, the program uses JavaFX to render a completed map.

Compiling:
```
javac --module-path "[File path to the jfx lib folder]" --add-modules
javafx.controls AStar.java
```

Running:
```
java --module-path "[File path to the jfx lib folder]" --add-modules
javafx.controls AStar map1.txt
```
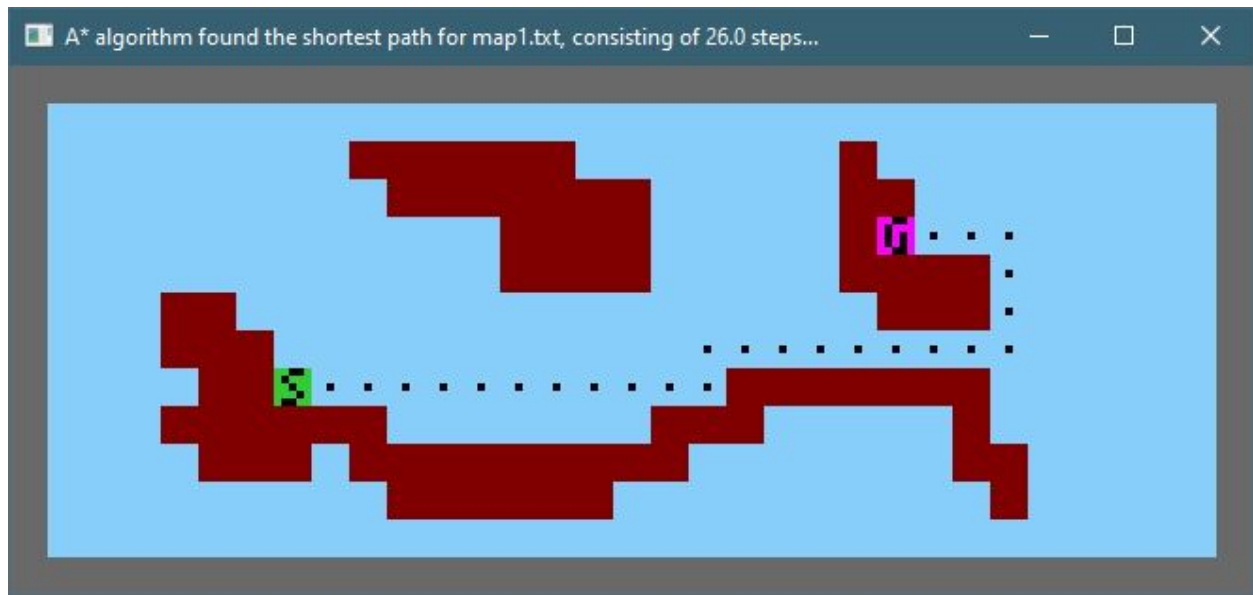
By default, the program only searches paths consisting of tiles that are up/down/left/right. The optional flag `-d` can be used to search diagonal paths:
```
java --module-path "[File path to the jfx lib folder]" --add-modules
javafx.controls AStar -d map1.txt
```

The output to console presents the path on the map in ASCII-art form, path length, and the time it took to run the search algorithm:

The GUI output presents the same information (except the run timer):

Performance Optimisation

When the first iteration of the program was completed, the algorithm by itself failed to provide satisfactory performance, both in terms of time and resource. Runs were taking excessively long, and failed to complete for all but the most basic maps, even with the heap size increased. Basic looping- and retracing-avoidance was in place, but these did little to improve the overall performance.

The exponential complexity of A* algorithm is one of its known drawbacks. The assignment brief mentioned *multiple path pruning*, where each time an item is added, all other more expensive paths to the same point are removed from the frontier. While this would have improved the performance of the program, it had two drawbacks:

1. Time complexity still increases along with the frontier size, although not as severely as the algorithm itself.
2. Requires a custom implementation of a priority queue beyond the standard java Collections API to search through the internal array.

For this application, the "shape" of the path was irrelevant, and only maximising the efficiency of the path length was to be considered, so I implemented an alternative method of reducing the time complexity: a replication matrix of the input map that stores a single state value in each cell. I don't know if there is a term for this, but I'll just call it *enqueue filtering*, and the difference to path pruning is that less efficient duplicate paths to the same point are never added to the queue in the first place.

The principle of enqueue filtering was that, between p_0 and p_2, for **an** optimal path $<p_0, p_1, p_2>$, the sub-path $<p_0, p_1>$ is also **an** optimal path between p_0 and p_1. Therefore, I could store the most optimal path to a given point, along with its cost. If a prospective state arrived at the same point, but incur a higher cost, then it would be impossible for this new state's subsequent children states to be more efficient than the children states of the existing recorded state. If this new state proved more efficient, on the other hand, it would replace the old state as the best state for that given point, and its subsequent children states added to the queue for processing. In other words, for each and every given point, only a single most efficient state was allowed to add subsequent states to the priority queue.

Without enqueue filtering (map1.txt):

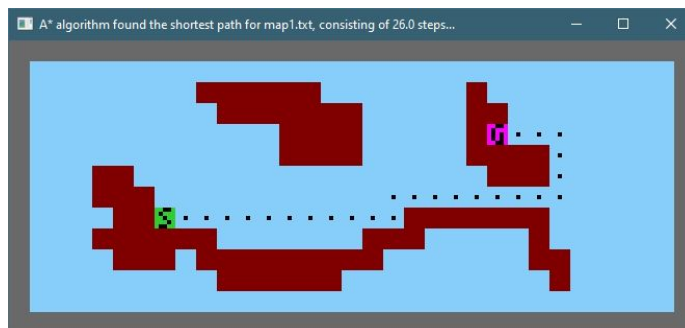```
Program took 84362 milliseconds to run.
```

With enqueue filtering (map1.txt):

```
Program took 2 milliseconds to run.
```
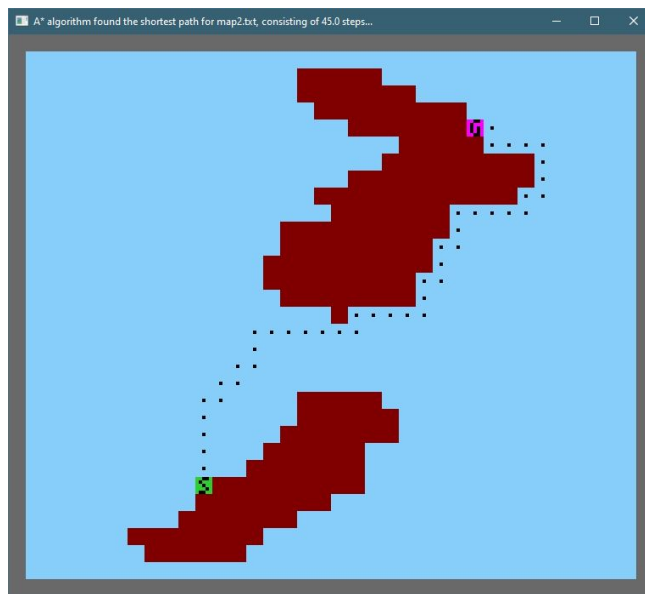
With enqueue filtering enabled, no example maps took longer than 10 milliseconds to run.
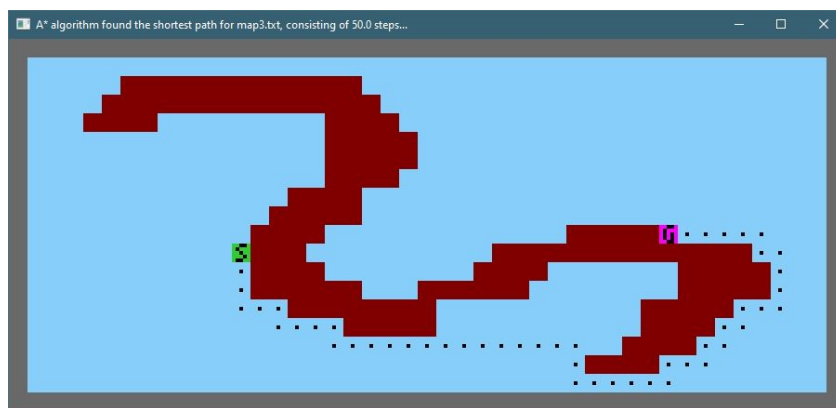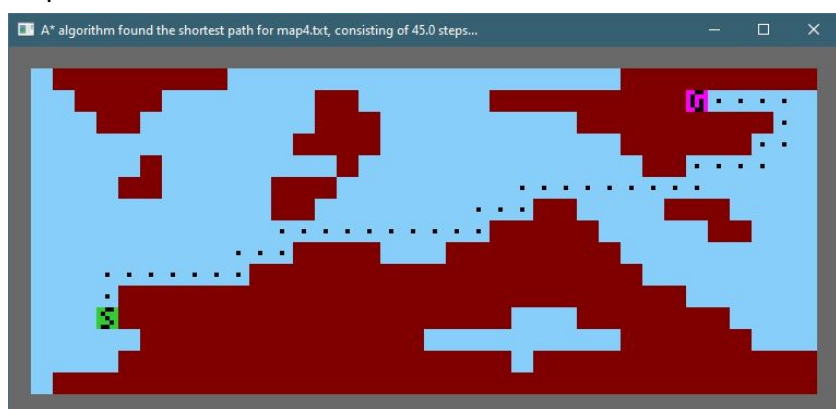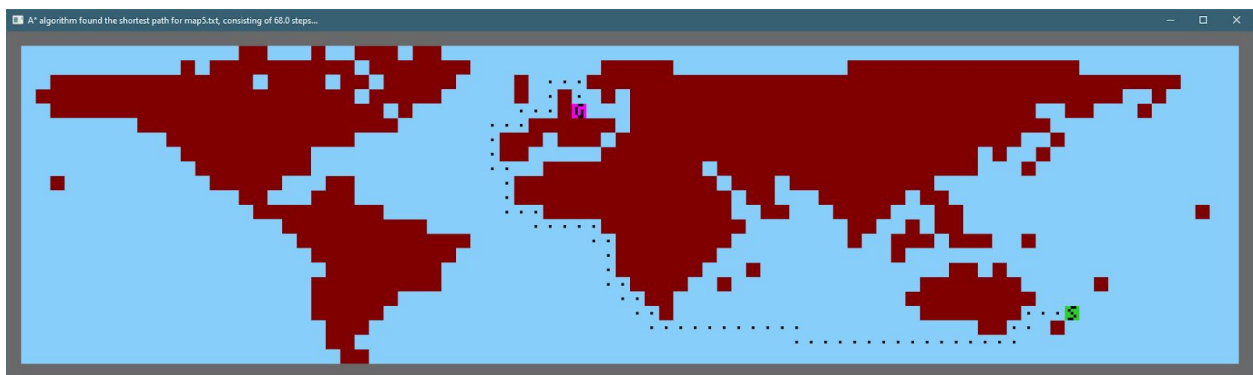
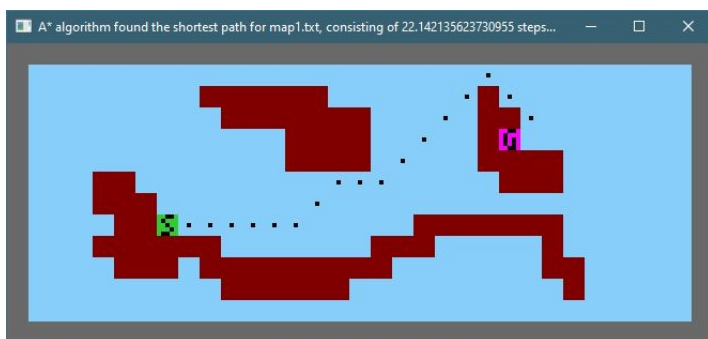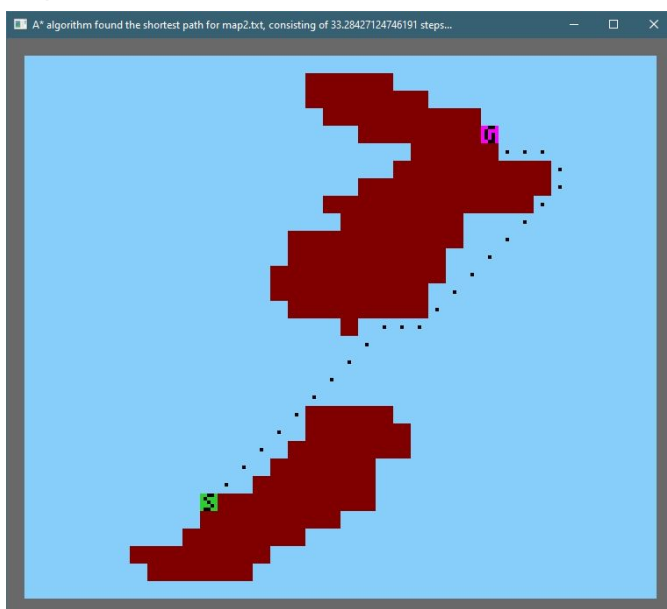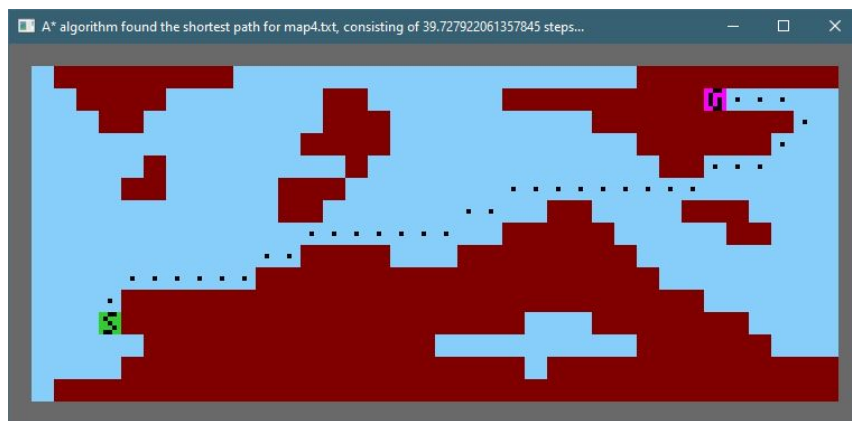## Results (No diagonal paths)

Map 1:

Map 2:



Map 3:



Map 4:

Map 5:



# Results (Diagonal paths)

Map 1:



Map 2:

Map 3:



A* algorithm found the shortest path for map3.txt, consisting of 42.97056274847713 steps...

Map 4:



A* algorithm found the shortest path for map4.txt, consisting of 39.727922061357845 steps...

Map 5:



A* algorithm found the shortest path for map5.txt, consisting of 57.45584412271567 steps...