

# Finding Surprising Patterns in a Time Series Database in Linear Time and Space

Eamonn Keogh  
eamonn@cs.ucr.edu

Stefano Lonardi  
stelo@cs.ucr.edu

Bill 'Yuan-chi' Chiu  
ychiu@cs.ucr.edu

Department of Computer Science and Engineering  
University of California  
Riverside, CA 92521

## ABSTRACT

The problem of finding a specified pattern in a time series database (i.e. query by content) has received much attention and is now a relatively mature field. In contrast, the important problem of enumerating all surprising or interesting patterns has received far less attention. This problem requires a meaningful definition of "surprise", and an efficient search technique. All previous attempts at finding surprising patterns in time series use a very limited notion of surprise, and/or do not scale to massive datasets. To overcome these limitations we introduce a novel technique that defines a pattern surprising if the frequency of its occurrence differs substantially from that expected by chance, given some previously seen data.

## Categories and Subject Descriptors

H.2.8 [Database Management]: Database Applications—Data Mining

## Keywords

Time series, Suffix Tree, Novelty Detection, Anomaly Detection, Markov Model, Feature Extraction.

## 1. INTRODUCTION

The problem of finding a specified pattern in a time series database (i.e. query by content) has received much attention and is now a relatively mature field [8, 18, 15, 17]. In contrast, the problem of enumerating all surprising or interesting patterns has received far less attention. The utility of such an algorithm is quite obvious. It would potentially allow a user to find surprising patterns in a massive database without having to specify in advance what a surprising pattern looks like.

Note that this problem should not be confused with the relatively simple problem of outlier detection. Hawkins' classic definition of an outlier is "... an observation that deviates so much from other observations as to arouse suspicion that it was generated from a different mechanism" [14]. However we are not interested in finding individually surprising datapoints, we are interested in finding surprising patterns, i.e., combinations of datapoints whose structure and frequency somehow defies our expectations. The problem is referred to under various names in the literature, including novelty detection [6] and anomaly detection [28].

The problem requires a meaningful definition of "surprise". The literature contains several such definitions for time series; however they are all too limited for a useful data-mining tool. Consider for example the notion introduced by Shahabi *et al.* [24]. They define *surprise* in time series as "...sudden changes in the original time series data, which are captured by local maximums of the absolute values of (wavelet detail coefficients)". However it is not difficult to think of very surprising patterns that defy this rule.

Several other definitions of surprise for time series exist, but all suffer from similar weaknesses [4, 28, 29, 6]. To overcome these limitations we introduce a novel definition that defines a pattern surprising if the frequency of its occurrence differs substantially from that expected by chance, given some previously seen data. This notion has the advantage of not requiring an explicit definition of surprise, which may in any case be impossible to elicit from a domain expert. Instead the user simply gives the algorithm a collection of previously observed data, which is considered normal. The measure of surprise of a newly observed pattern is considered relative to this data collection, and thus eliminates the need for a specific model of normal behavior.

Note that unlike all previous attempts to solve this problem, the measure of surprise of a pattern is not tied exclusively to its structure. Instead it depends on the departure of the frequency of the pattern from its expected frequency. This is the crucial distinction of our approach from all the others.

Our definition of surprise would be of little utility to the data mining community without a technique that allowed efficient determination of the expected frequency

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SIGKDD '02, July, 23-26 2002, Edmonton, Alberta, Canada.  
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Symbol	
$R$	the reference time series database (consisting of real numbers)
$X$	the time series database (to be mined for surprising patterns)
$r$	the discrete version of $R$
$x$	the discrete version of $X$
$l_1$	the feature window length
$l_2$	sliding window length
$a$	the alphabet size

**Table 1: A summary of the major notation use in the work. More complete definitions are given in the relevant sections**

of a pattern. We demonstrate how a suffix tree can be used to efficiently encode the frequency of all observed patterns. Since it is possible that a pattern observed in the new data was not observed in the training data, we demonstrate a technique based Markov models to calculate the expected frequency of previously unobserved patterns. Once the suffix tree has been constructed, the measure of surprise for all the patterns in a new database can be determined in time linear in the size of the database.

## 2. DISCRETIZING TIME SERIES

For concreteness we more formally define our intuition of surprise as follows.

**DEFINITION 2.1.** *A time series pattern  $P$ , extracted from database  $X$  is surprising relative to a database  $R$ , if the frequency of its occurrence is greatly different to that expected by chance, assuming that  $R$  and  $X$  are created by the same underlying process.*

In order to compute this measure, we must calculate the probability of occurrence for the pattern of interest. Here we encounter the familiar paradox that the probability of a particular real number being chosen from any distribution is zero [11]. Since a time series is an ordered list of real numbers the paradox clearly applies. The obvious solution to this problem is to discretize the time series into some finite alphabet  $\Sigma$ . Using a finite alphabet allows us to avail of Markov models to estimate the expected probability of occurrence of a previously unseen pattern.

The problem of discretizing time series into a finite alphabet has received much attention in diverse fields, including astronomy, medicine, chemistry, etc. (See [7] for an exhaustive overview). The representation has also captured the attention of the data mining community who use discretized time series to support similarity search [15] and to enable change point detection [12].

Below we give a generic algorithm to discretize a time series dataset such that each symbol is equiprobable. A table of notation used in this, and subsequent algorithms is given in Table 1.

The inputs are a reference time series database  $R$ , the feature window length and the size of the desired

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string DISCRETIZE_TIME_SERIES (time_series  $X$ ,
                                int  $l_1$ , int  $a$ )
  for  $i = 1, |X| - l_1 + 1$ 
    let  $\text{features}_{\{i\}} = \text{EXTRACT\_FEATURE}(X_{\{i, i+l_1\}})$ 
    let  $\text{sorted\_features} = \text{SORT}(\text{features})$ 
    for  $j = 1, a$ 
      let  $\text{pointer} = j \lfloor \text{features} \rfloor / a$ 
      let  $\text{boundaries}_{\{j\}} = \text{sorted\_features}[\text{pointer}]$ 
    for  $i = 1, |\text{features}|$ 
      let  $x_{\{i\}} =$ 
        MAP_REAL_TO_INT( $\text{boundaries}$ ,  $\text{features}_{\{i\}}$ )
    return  $x$ 

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**Table 2: Outline of the algorithm for the discretization of the time series:  $t$  is the time series data,  $l_1$  is the feature window length,  $a$  is the alphabet size**

alphabet. The feature window length is the length of a sliding window that is moved across the time series. At each time step, the portion of data falling within the window is examined, and a single real number, describing some feature of the data is extracted. After the features have been extracted, they are sorted so the boundaries that contain an equal number of extracted features can be determined. At this point the unsorted features are scanned, each feature is tested to see which range it maps to, and matching symbol is assigned. An outline of the algorithm is shown in Table 2.

Note that the one element of the algorithm we did not specify is the EXTRACT\_FEATURE subroutine. Here we have been deliberately vague. The best feature extraction technique may be domain dependent. Possible features include the mean of the data [17], the slope of the best-fitting line [12, 18], the second wavelet coefficient, the second real Fourier coefficient [8], etc. For simplicity we will consider only the slope of the best-fitting line for the rest of this paper.

We also have not stated how the two parameters, the feature window length and the size of the desired alphabet, are chosen. As emphasized in [5] and elsewhere, data mining is an iterative activity, and “discovery algorithms should be run several times with different parameter settings”. Alternatively, techniques that use maximum entropy based methods can be used to decide reasonable parameters to discretize time series [22].

The time complexity for the above algorithm is dominated by the need to sort the features to allow determination of the feature boundaries. However these feature boundaries are very stable, and can be reliably estimated from a subsample of the data [5]. For large databases we can determine the feature boundaries from a subsample of size  $s = \sqrt{|R|}$  [5]. Since  $s \log(s) < |R|$ , the feature extraction algorithm is  $O(|R|)$ .

## 3. BACKGROUND ON STRING PROCESSING

We use  $\Sigma$  to denote a nonempty alphabet of symbols.

A *string* over  $\Sigma$  is an ordered sequence of symbols from the alphabet. Given a string  $x$ , the number of symbols in  $x$  defines the *length*  $|x|$  of  $x$ . Henceforth, we assume  $|x| = n$ . The empty string has length zero, and is denoted by  $\epsilon$ .

Let us decompose a text  $x$  in  $uvw$ , i.e.,  $x = uvw$  where  $u, v$  and  $w$  are strings over  $\Sigma$ . Strings  $u, v$  and  $w$  are called *substrings*, or *words*, of  $x$ . Moreover,  $u$  is called a *prefix* of  $x$ , and  $w$  is called a *suffix* of  $x$ .

We write  $x_{[i]}$ ,  $1 \leq i \leq |x|$  to indicate the  $i$ -th symbol in  $x$ . We use  $x_{[i,j]}$  as shorthand for the substring  $x_{[i]}x_{[i+1]} \dots x_{[j]}$  where  $1 \leq i \leq j \leq n$ , with the convention that  $x_{[i,i]} = x_{[i]}$ . Substrings in the form  $x_{[1,j]}$  corresponds to the prefixes of  $x$ , and substrings in the form  $x_{[i,n]}$  to the suffixes of  $x$ .

We say that a string  $y$  has an *occurrence* at position  $i$  of a text  $x$  if  $y_{[1]} = x_{[i]}$ ,  $y_{[2]} = x_{[i+1]}$ ,  $\dots$ ,  $y_{[m]} = x_{[i+m-1]}$ , where  $m = |y|$ . For any substring  $y$  of  $x$ , we denote by  $f_x(y)$  the number of occurrences of  $y$  in  $x$ .

Throughout this document, variables  $y$  and  $w$  usually indicate substrings of the text  $x$ . Unless otherwise specified, we assume the generic term  $m$  as the length of any of these words.

### 3.1 Markov models

We consider a string generated by a stationary Markov chain of order  $M \geq 1$  on the finite alphabet  $\Sigma$ . Let  $x = x_{[1]}x_{[2]} \dots x_{[n]}$  be an observation of the random process and  $y = y_{[1]}y_{[2]} \dots y_{[m]}$  an arbitrary but fixed pattern over  $\Sigma$  with  $m < n$ .

The stationary Markov chain is completely determined by its *transition matrix*  $\Pi = (\pi(y_{[1,M]}, c))_{y_{[1]}, \dots, y_{[M]}, c \in \Sigma}$  where

$$\pi(y_{[1,M]}, c) = \mathbf{P}(X_{i+1} = c | X_{[i-M+1, i]} = y_{[1,M]})$$

are called *transition probabilities*, with  $y_{[1]}, \dots, y_{[M]}, c \in \Sigma$  and  $M \leq i \leq n-1$ . The vector of the *stationary probabilities*  $\mu$  of a stationary Markov chain with transition matrix  $\Pi$  is defined as the solution of  $\mu = \mu\Pi$ .

We now introduce the random variable which describes the occurrences of the word  $y$ . We define  $Z_i$ ,  $1 \leq i \leq n - m + 1$  to be 1 if  $y$  occurs in  $x$  starting at position  $i$ , 0 otherwise. We set  $Z_y = \sum_{i=1}^{n-m+1} Z_{i,y}$  so that  $Z_y$  is the random variable for the total number of occurrences  $f_x(y)$ .

In the stationary  $M$ -th order Markovian model the expectation of  $Z_i$ , which represents the probability that  $y$  occurs at a given position  $i$ , is given by

$$E(Z_i) = \mu(y_{[1,M]}) \prod_{i=1}^{m-M} \pi(y_{[i, i+M-1]}, y_{[i+M]}).$$

The expected count of the occurrences  $y$  under the Markov model is therefore

$$\begin{aligned} E(Z_y) &= (n - m + 1)E(Z_i) \\ &= (n - m + 1)\mu(y_{[1,M]}) \prod_{i=1}^{m-M} \pi(y_{[i, i+M-1]}, y_{[i+M]}) \end{aligned} \quad (1)$$

because the distribution of the  $Z_i$ 's does not depend on  $i$ .

When the true model is *unknown*, the transition and stationary probabilities have to be estimated from the observed sequence  $x$ . Let  $y$  be a substring of  $x$ , where  $m = |y| \geq M + 2$ . The transition probability can be estimated by the *maximum likelihood estimator* [23]

$$\hat{\pi}(y_{[1,M]}, c) = \frac{f_x(y_{[1,M]}c)}{f_x(y_{[1,M]})} \quad (2)$$

and the *stationary probability* by the maximum likelihood estimator

$$\hat{\mu}(y_{[1,M]}) = \frac{f_x(y_{[1,M]})}{n - M + 1}. \quad (3)$$

Substituting in equation (1) for the estimators (2) and (3) we obtain an estimator of the expected count of  $y$

$$\hat{E}(Z_y) = \frac{\prod_{i=1}^{m-M} f_x(y_{[i, i+M]})}{\prod_{i=2}^{m-M} f_x(y_{[i, i+M-1]})}.$$

A precise relationship between the expectation of  $y$  and the expectation of its prefix and suffix is established in the following fact.

LEMMA 3.1. *Let  $y$  be a substring of  $x$  and  $w_1 = y_{[2,m]}$ ,  $w_2 = y_{[1,m-1]}$ . Then*

$$\hat{E}(Z_y) = \frac{f(y_{[1,M+1]})}{f(y_{[2,M+1]})} \hat{E}(Z_{w_1}) = \hat{E}(Z_{w_2}) \frac{f(y_{[m-M,m]})}{f(y_{[m-M,m-1]})}$$

### 3.2 Suffix Trees

A simple method to count the number of occurrences of each substring in a sequence is to create a look-up table. The table has an entry for each word. Given a word  $w$ , a one-to-one hash function returns the index in the table. The hash table is a convenient data structure as long as  $m$  is bounded by a relatively small constant. If we allow  $m$  to grow as a function of  $n$ , for example  $m \propto \log(n)$ , then the time and space required to build the hash table would be *exponential* in the size of the input.

A more space-efficient data structure to organize a dictionary of words is to use a *suffix tree* (see, e.g., [13] and references therein). The suffix tree is a type of digital search tree that represents a set of strings over a finite alphabet  $\Sigma$ . It has  $n$  leaves, numbered 1 to  $n$ . Each internal node, other than the root, has at least two children and each edge is labeled with a nonempty substring of  $x$ . No two edges outgoing from a node can have labels beginning with the same character. The tree has the property that for any leaf  $i$ , the concatenation of the labels on the path from the root to the leaf  $i$  spells out exactly the suffix of  $x$  that starts at position  $i$ , that is  $x_{[i,n]}$ . The substrings of  $x$  can be obtained by spelling out the words from the root to any internal node of the tree or to any position in the middle of an edge.

In order to achieve overall linear-space allocation, the labels on the edges are described implicitly: for each word, it suffices to save an ordered pair of integers indexing one of the occurrences of the label in the text. Each edge label requires thus constant space, which, in



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suffix_tree PREPROCESS (string  $r$ , string  $x$ )
  let  $T_r = \text{SUFFIX\_TREE}(r)$ 
  let  $T_x = \text{SUFFIX\_TREE}(x)$ 
  let  $\alpha = \frac{|x| - m + 1}{|r| - m + 1}$ 
  ANNOTATE- $f(w)(T_r)$ 
  ANNOTATE- $f(w)(T_x)$ 
  visit  $T_x$  in breadth-first traversal, for each node  $u$  do
    let  $w = L(u), m = |w|$ 
    if  $w$  occurs in  $T_r$  then
      let  $\hat{E}(w) = \alpha f_r(w)$ 
    else
      find the largest  $1 < l < m - 1$  such that
         $\prod_{j=1}^{m-l} f_r(w_{[j, j+l]}) > 0$ 
        using the suffix tree  $T_r$ 
      if such  $l$  exists then
        let  $\hat{E}(w) = \alpha \frac{\prod_{j=1}^{m-l} f_r(w_{[j, j+l]})}{\prod_{j=2}^{m-l} f_r(w_{[j, j+l-1]})}$ 
      else
        let  $\hat{E}(w) = (|x| - m + 1) \prod_{i=1}^m w_{y[i]}$ 
    let  $z(w) = f_x(w) - \hat{E}(w)$ 
    store  $z(w)$  in the node  $u$ 
  return  $T_x$ 

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**Table 3: Outline of the preprocessing algorithm for the computation of the scores obtained comparing the trees of a reference string  $r$  against the string under analysis  $x$**

The inputs are the reference database  $R$ , the database to be examined  $X$ , and the three parameters which control the feature extraction and representation. The algorithm begins by discretizing the data to the desired granularity. The two resultant strings are passed to the PREPROCESS algorithm which constructs the annotated suffix tree  $T_x$ . After this has been accomplished, the surprise of each substring found in  $x$  can be determined. Those substrings which have surprising ratings exceeding a certain user defined threshold (as defined by the absolute value of  $z(w)$ ) can be returned and examined by the user.

The length  $l_2$  of the sliding window is connected with the feature window length  $l_1$  and the alphabet size  $a$  (which have been discussed in Section 2). We suggest choosing  $l_2 < \log_{|\Sigma|} |x|$  because words longer than  $\log_{|\Sigma|} |x|$  have extremely small expectations and belong to a different probabilistic regime. In fact, scores  $z(w)$  are asymptotically Gaussian distributed when  $|w| < \log_{|\Sigma|} |x|$  and Poisson distributed for longer words [23]. The threshold  $c$  can be identified by gathering statistics about the distribution of the scores and/or assuming the distribution of the scores to be normal.

## 6. EXPERIMENTAL EVALUATION

that the heart of the algorithm relies on comparing two suffix trees, “tree to tree”. Tarzan (R) is a registered trademark owned by Edgar Rice Burroughs, Inc.

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void TARZAN (time_series  $R$ , time_series  $X$ ,
  int  $l_1$ , int  $a$ , int  $l_2$ , real  $c$ )
  let  $x = \text{DISCRETIZE\_TIME\_SERIES}(X, l_1, a)$ 
  let  $r = \text{DISCRETIZE\_TIME\_SERIES}(R, l_1, a)$ 
  let  $T_x = \text{PREPROCESS}(r, x)$ 
  for  $i = 1, |x| - l_2 + 1$ 
    let  $w = x_{[i, i+l_2-1]}$ 
    retrieve  $z(w)$  from  $T_x$ 
    if  $|z(w)| > c$  then print  $i, z(w)$ 

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**Table 4: Outline of the Tarzan algorithm:  $l_1$  is the feature window length,  $a$  is the alphabet size for the discretization,  $l_2$  is the scanning window length and  $c$  is the threshold**

We compare our approach with the TSA-tree Wavelet based approach of Shahabi *et al.* [24] and to the Immunology (IMM) inspired work of Dasgupta and Forrest [6], which are the only obvious candidates for comparison. More details about these approaches are contained in Section 7.

We begin with a very simple experiment as a reality check. We constructed a reference dataset by creating a sine wave with 800 datapoints and adding some Gaussian noise (each complete sine wave is 32 datapoints long). We then built a test dataset using the same parameters as the reference set, however we also inserted an artificial anomaly by halving the period of the time series in the region between the 400<sup>th</sup> and 432<sup>th</sup> datapoints. In other words, that small subsection of the test time series has two short sine waves instead of one. We compared all three approaches under consideration. The results are shown in Figure 2. We used a feature window of length  $l_1 = 12$  for TARZAN and IMM, and an alphabet of size  $a = 4$  for TARZAN.

The IMM approach was unable to find the anomaly, and it introduced some false alarms. The TSA approach also failed to find the anomaly. In contrast to the other techniques TARZAN shows a strong peak for the duration of the anomaly. Note that for consistency with the other techniques we flipped the results for TARZAN upside down, so the low expectation for the anomaly shows as a peak.

Testing the ability of the algorithms to find surprising patterns on real data is a greater challenge, since the results may be subjective. To address this problem we consider a dataset that contains the power demand for a Dutch research facility for the entire year of 1997 [26]. The data is sampled over 15 minute averages, and thus contains 35,040 points. The nice feature of this dataset is that although it contains great regularity, as shown in Figure 3, it also contains regions that could objectively be said to be surprising or anomalous. In particular, there are several weeks on which one or more days were national holidays, and thus the normal pattern of five weekday peaks, followed by a relatively flat weekend, is disturbed.

We used from Monday January 6<sup>th</sup> to Sunday March 23<sup>rd</sup> as reference data. This time period is devoid of

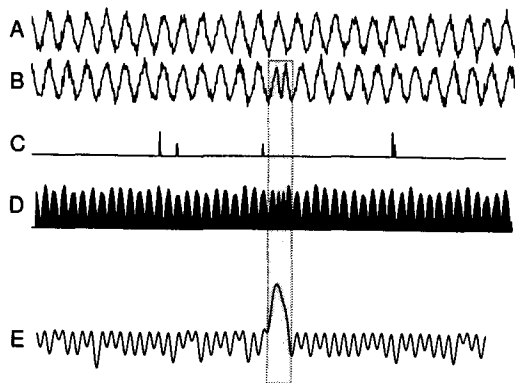


Figure 2: A comparison of three anomaly detection algorithms on the same task. A) The training data, a slightly noisy sine wave. B) A time series containing a synthetic “anomaly”, it is a noisy sine wave that was created with the same parameters as the training sequence. Then the period of the sine wave between the 400<sup>th</sup> and 432<sup>th</sup> points (denoted by the gray bar) was halved. C) The IMM anomaly detection algorithm failed to find the anomaly, and introduced some false alarms. D) The TSA-Tree approach is also unable to detect the anomaly. E) Tarzan shows a strong peak for the duration of the anomaly



Figure 3: The first three weeks of the power demand dataset. Note the repeating pattern of a strong peak for each of the five weekdays, followed by relatively quiet weekends

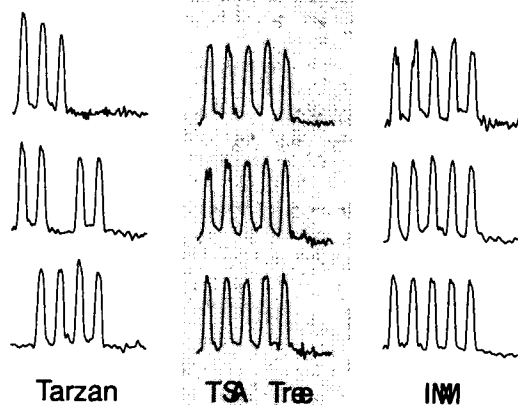


Figure 4: The three most surprising weeks in the power demand dataset, as determined by Tarzan, TSA-Tree and IMM

national holidays. We processed the remainder of the year with TARZAN, with a window size equivalent to 4 hours ( $l_1 = 16$  datapoints), and an alphabet of size  $a = 4$ . Because of the size of the dataset we will just show the three most surprising sequences found by each algorithm. For each of the three approaches we show the entire week (beginning Monday) in which the three largest values of surprise fell. The results are shown in Figure 4.

Both TSA-tree and IMM returned sequences that appear to be normal workweeks, however TARZAN returned three sequences that correspond to the weeks that contain national holidays in the Netherlands. These results present strong visual evidence that TARZAN is able to find surprising patterns in time series.

## 7. RELATED WORK

The task of finding surprising patterns in data has been an area of active research, which has long attracted the attention of researchers in biology, physics, astronomy and statistics, in addition to the more recent work by the data mining community. The problem, and closely related tasks are variously referred to as the detection of “Aberrant Behavior” [19], “Novelties” [6], “Faults” [29], “Surprises” [24, 4], “Deviants” [16], “Temporal Change” [3, 10], and “Outliers” [14].

Jagadish *et al.* [16] introduced a technique for mining deviants in time series, however deviants are simply “... points with values that differ greatly from that of surrounding points”, and thus this work may be considered more of a generalization of classic outlier detection [14].

In [24] and several follow up papers, Shahabi *et al.* suggest a method to find both trends and “surprises” in large time series datasets. The authors achieve this using a wavelet-based tree structure (TSA-Tree) that can represent the data at different scales, e.g., the weather trend in last month vs. last decade. However the definition of surprise used seems limited to dramatic shifts in the signal. In particular, this approach is not suitable for detecting unusual data patterns that hide inside the normal signal range. For example, the system would not be able to detect if we give it an EEG time series that we had flipped upside down, since the wavelet-based “surprise” features are invariant to this transformation of the data.

The immunological based approach of Dasgupta and Forrest [6], is inspired by the negative selection mechanism of the immune system, which discriminates between *self* and *non-self*. In this case *self* is the model of the time series learned from the reference dataset, and *non-self* are any observed patterns in the new dataset that do not conform to the model within some tolerance. A major limitation of the approach is that it is only defined when the space of *self* is not exhaustive. However, if you examine enough random walk data (or financial data, which is closely modeled by random walk [8]), *self* rapidly becomes saturated with every possible pattern, and thus *non-self* is the null set, and nothing encountered thereafter is considered surprising.

## 8. CONCLUSIONS

In this paper we introduced TARZAN, an algorithm that detects surprising patterns in a time series database in linear space and time. Our definition of surprising is general and domain independent, describing a pattern as surprising if the frequency with which we encounter it differs greatly from that expected given previous experience. We compared it to two other algorithms on both real and synthetic data, and found it to have much higher sensitivity and selectivity.

## 9. ACKNOWLEDGMENTS

We thank the anonymous referees for very useful comments on the paper.

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