Course's Name: Calculus 2
Course's Number: 15010102
Exam's period: 1.30 Hour
Total mark : 40
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Palestine Technical University-Kadoorie

Midterm Exam

Summer Semester 2017/2018

Instructor's Name : .....
Student's Name : .....
Student's Number:.....
Section's Number:.....
Form : A

Question 1) Choose the correct answer : **(15p)** (6)

The answer choice **NOTA** means none of the above answers is correct.

1) The volume of the solid generated by revolving the region enclosed by $y = \sqrt{x}$, $x - axis$, $x = 4$, about $x - axis$ is :

a) 2π
b) 4π
c) 6π
d) 8π
e) NOTA

2) $\frac{\ln 9}{\ln 3} =$

a) 3
b) $\ln 9 - \ln 3$
c) $\ln 3$
d) 2
e) NOTA

3) Let $f(x) = x^2 - 4x - 5$, $x > 2$, the value of $\frac{df^{-1}}{dx}$ at $x = 0$ is:

a) -5
b) $\frac{1}{6}$
c) 6
d) $-\frac{1}{6}$
e) NOTA

4) $\cosh(3x) + \sinh(3x) =$

a) e^{3x}
b) $2e^{3x}$
c) $2e^{-3x}$
d) e^{-3x}
e) NOTA

5) If $\ln(x - 3) = 0$, then

a) $x = 2$
b) $x = 3$
c) $x = 4$
d) $2 < x < 3$
e) NOTA

6) If $y = \tan^{-1}(x^4)$, then $\dot{y} =$

a) $\frac{x^4}{1+x^4}$
b) $\frac{3x^2}{1+x^6}$
c) $\frac{4x^3}{1+x^8}$
d) $-\frac{4x^3}{1+x^6}$
e) NOTA

7) $3^{2\log_3 x} = 36$, then $x =$

a) 6 c) -6 e) NOTA

b) 18 d) $6, -6$

8) Range of $cot^{-1}x$

a) $(-\infty, \infty)$ c) $(0, \pi)$ e) NOTA

b) $[0, \pi]$ d) $\mathcal{R}/\{0\}$

9) The form of the partial fraction decomposition of the function

$$\frac{1}{(x^2-4)(x-2)} \quad is:$$

a) $\frac{A}{x-2} + \frac{B}{x+2}$

b) $\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

c) $\frac{A}{x-2} + \frac{Bx+C}{x^2-4}$

d) $\frac{A}{x-2} + \frac{Bx+E}{x^2-2} + \frac{Cx+D}{x^2+2}$

e) NOTA

10) $\int \ln x \, dx =$

a) $x(\ln x - 1)$ c) $x(\ln x + 1)$ e) NOTA

b) $x(1 - \ln x)$ d) $x \ln x$

Question 2) Evaluate : (6) (6p)

$$\int \frac{dx}{(x+1)(x^2+1)}$$

$$\frac{dx}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$= A(x^2+1) + (Bx+C)(x+1)$$

$$= Ax^2 + A + Bx^2 + Bx + Cx + C$$

x^2: $A + B = 0$

x: $B + C = 0$

const: $A + C = 1$

$A + C = 1$

$C = 1 - A$

$A + B = 1$

$B + 1 - A = 0$

$2B + 1 = 0$

$2B = -1$

$B = -\frac{1}{2}$

$A + B = 0$

$A - \frac{1}{2} = 0$

$A = \frac{1}{2}$

$A + C = 1$

$\frac{1}{2} + C = 1$

$C = \frac{1}{2}$

Question 3) Evaluate (4p) (2)

1) $\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\left(\frac{x}{2}\right)\Big|_0^1 = \sin^{-1}\left(\frac{1}{2}\right) - \sin(0)$

$= \frac{\pi}{3}$ $\frac{\pi}{6} = 0$

$= \frac{\pi}{3}$

2) $\int \sin 5x \cos 3x \, dx$

$\sin(A-B) + \sin(A+B)$

$= \frac{1}{2}\int \sin(5-3)x + \sin(5+3)x \, dx$

$= \frac{1}{2}\int \sin 2x + \sin 8x \, dx$ (1.5)

$\frac{1}{2}\left(-\frac{\cos 2x}{2} - \frac{\cos 8x}{8} + C\right)$

Question 4) Find $\frac{dy}{dx}$ **(4points)** 4

$y = x^{x^2}$

$\ln y = \ln x^{x^2}$

$\ln y = x^2 \ln x$

$\frac{y'}{y} = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$y' = y(x + 2x \ln x)$

$y' = x^{x^2}(x + 2x \ln x)$

Question 5) Find the length of the curve **(6 points)** 3

$x = \frac{t^2}{2}$ $\quad y = \frac{1}{3}(2t+1)^{\frac{3}{2}}$ $\quad 0 \le t \le 4$

$\frac{dx}{dt} = \frac{2 \cdot 2t - t^2 \times 0}{4}$

$= \frac{4t}{4}$

$\frac{dx}{dt} = t$

$\frac{dx^2}{dt} = t^2$

$L = \int_0^4 \sqrt{t^2 + 2t^3 + 3t^2}\, dt$

$L = \int_0^4 \sqrt{2t^2 + 2t^3}\, dt$

$L = \int_0^4 \sqrt{2t^2(1+t)}\, dt$

$L = \sqrt{2} \int_0^4 \sqrt{t^2(1+t)}\, dt$

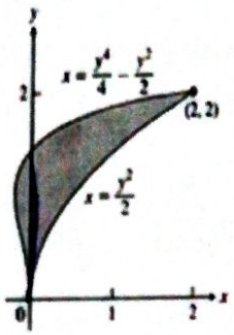
$\frac{dy}{dt} = \frac{1}{3} \cdot \frac{3}{2}(2t+1)^{1/2} \cdot 2t + (2t+1)^{3/2} \times 0$

$= t\sqrt{2t+1}$ $\quad \frac{dy^2}{dt} = t^2(2t+1)$

$\frac{dy^2}{dt} = t^2 + 2t\sqrt{2t+1} + 2t + 1$

Question 6) Find the volume of the solid generated by revolving the shaded region ***about x − axis .***

(5 points)



$X=\frac{y^4}{4}-\frac{y^2}{2}$ / $X=\frac{y^2}{2}$ $\quad y^2-\left(\frac{y^4}{4}-\frac{y^2}{2}\right)$

$$V=2\pi\int_0^2 (y)\left(\frac{y^4}{4}-\frac{y^2}{2}-\frac{y^2}{2}\right)dy$$

$$V=2\pi\int_0^2 (y)\left(-\frac{y^4}{4}+2\frac{y^2}{2}\right)dy$$

$$V=2\pi\int_0^2 [y)\left(-\frac{y^4}{4}+y^2\right)dy$$

$$V=2\pi\int_0^2 \left(-\frac{y^5}{4}+y^3\right)dy$$

$$V=\pi\int_0^2 (y^5-y^3)dy$$

$$V=\pi\left\{\frac{y^6}{6}+\frac{y^4}{4}\right\}_0^2$$

$$V=\pi\left(\left[\frac{(2)^6}{6}-\frac{(2)^4}{4}\right]-\left[\frac{(0)^6}{6}-\frac{(0)^4}{4}\right]\right)$$

$$V=\pi\left[\frac{64}{6}-\frac{16}{4}\right]$$

$$V=\pi\left[\frac{64}{6}-24\right]$$

$$V=\pi\left[\frac{64-24}{6}\right)$$

$$V=\frac{40\pi}{6}$$

Good Luck