

COVID-19 Mathematical Model

April 4, 2020

Model Derivation

Susceptible (S) – latent (E) – infectious (I) – quarantined (Q) – recovered (R) model for spread virus. Numerical methods are employed to solve the obtained system of differential equations.

In the SEIQRS model for infections that do not confer immunity, the dynamic behavior described as:

- susceptible nodes first goes through a latent period (and is said to become exposed)
- after infection before becoming infectious, thereafter some infected nodes stay in the I class while they are infectious and then move to the removed class R upon temporary recovery
- Other infected nodes are transferred into the quarantine class Q while they are infectious and then move to the R class.
- Since in the acquired immunity is not permanent, the recovered nodes return back to the susceptible class.

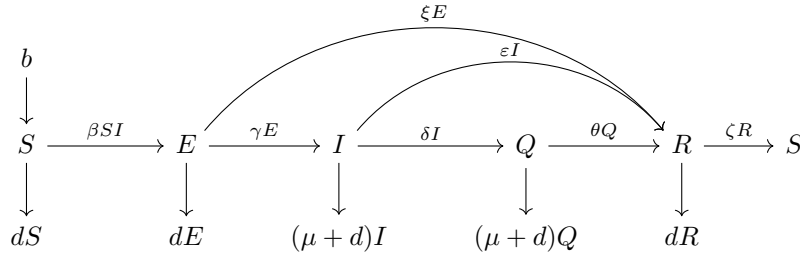


Figure 1: Schematic diagram for COVID19-SEIQRS model.

Mathematical Model hypothesis:

- We assume the population has a homogeneous spatial distribution and the mixing of hosts follow the law of mass action. More specifically, we assume that the local density of the total population is a constant through the total population size $N(t) = S(t) + E(t) + I(t) + Q(t) + R(t)$ may vary with time. Here $S(t), E(t), I(t), Q(t), R(t)$ denote the sizes of S, E, I, Q, R classes at any time t , respectively.
- The per capita contact rate β , which is the average number of effective contacts with other nodes per unit time is assumed to be a constant. The total number of new infections at a time t is given by βSI , with β as a mass action coefficient and is used for incidence.
- The constant b is the recruitment rate of susceptible nodes to the population. d is the per capita natural mortality rate, γ is the rate constant for nodes leaving the exposed class E for infective compartment, δ is the rate constant for nodes leaving the infective compartment I for quarantine compartment, μ is the disease related death rate constant in the compartments I and Q . ϵ, θ are the rates at which nodes recover temporarily after recovered(negative test result) and return to recovered class R from compartments I and Q respectively, ζ is the loss of immunity rate constant.
- In this $SEIQRS$ model, the flow is from the S class to the E class, E class to the I class, and then directly to the R class or to the Q class and then to the R class and as the recovery is not permanent in the world, it again returns back to the S class.

Mathematical Model

$$\begin{aligned}
 \frac{dS}{dt} &= b - \beta SI + \zeta R - dS, \\
 \frac{dE}{dt} &= \beta SI - (\gamma + \xi)E - dE, \\
 \frac{dI}{dt} &= \gamma E - (\epsilon + \delta)I - (\mu + d)I, \\
 \frac{dQ}{dt} &= \delta I - \theta Q - (\mu + d)Q, \\
 \frac{dR}{dt} &= \epsilon I + \theta Q + \xi E - \zeta R - dR,
 \end{aligned} \tag{1}$$

MATLAB CODE

```

1 function [ ] = COVID19b( b,beta,gamma,xi,epsilon , mu, delta ,theta ,zeta ,d,ti ,tf ,S0,E0 ,I0 ,
   Q0, R0)
2
3 %COVID19 virus spread model
4
5 f=@(t,x) [b-beta*x(1)*(x(2))+zeta*x(5)-d*x(1);
6           beta*x(1)*(x(2))-(gamma+xi)*x(2)-d*x(2);
7           gamma*x(2)-(epsilon+delta)*x(3)-(mu+d)*x(3);
8           delta*x(3)-theta*x(4)-(mu+d)*x(4);
9           epsilon*x(3)+theta*x(4)+xi*x(2)-zeta*x(5)-d*x(5)];
10
11 [t,xa]=ode45(f,[ti tf],[S0 E0 I0 Q0 R0]);
12
13 plot(t,xa(:,1))
14 hold on
15
16 for i=2:5
17     plot(t,xa(:,i))
18 end
19 legend('S','E','I','Q','R')
20 end

```

Experiment

$COVID19b(0, .9, 0.5, .0, .200, 0.00, .00, .00, .00, .00, 0, 30, 1000, 1, 0, 0, 0)$

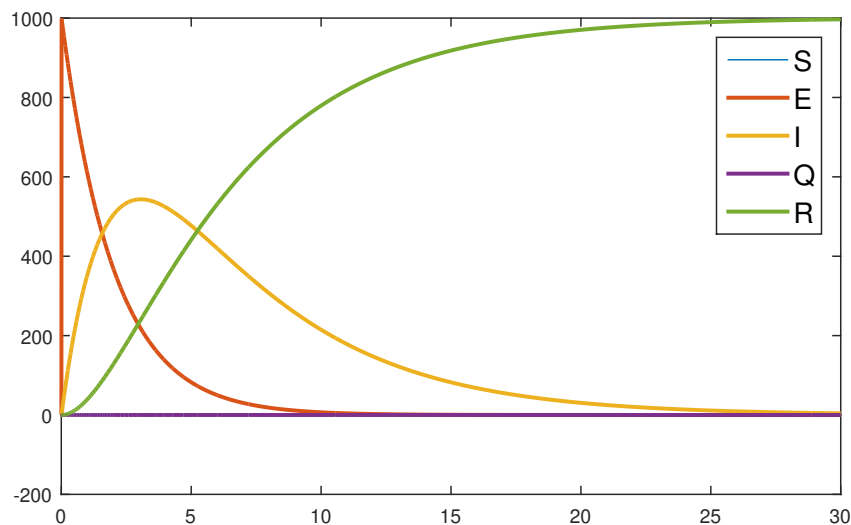


Figure 2: