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Math for Machine Learning

Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

W2 Lesson 1

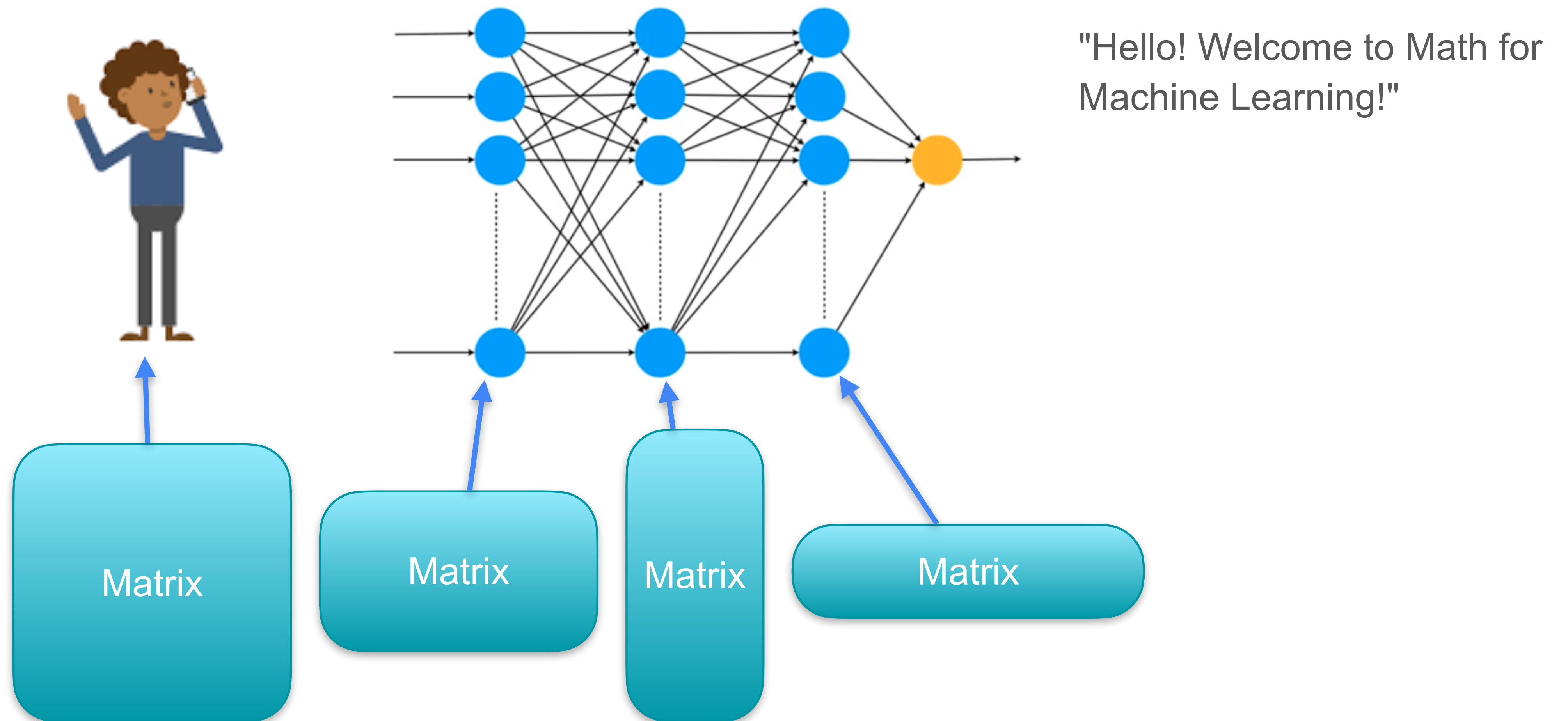


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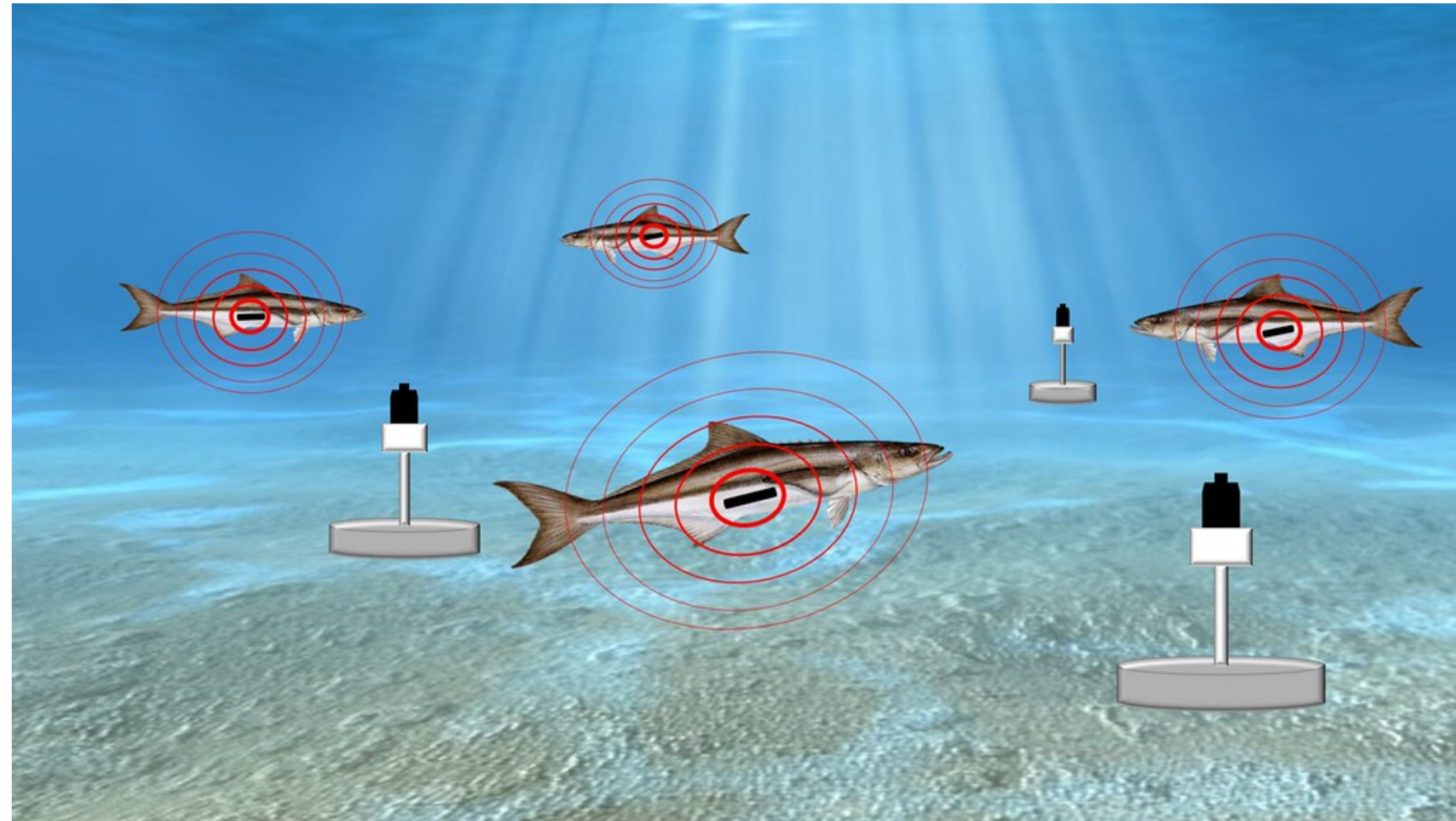
Solving System of Linear Equations

Machine learning motivation

Neural networks - Matrix operations



Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.





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Solving System of Linear Equations



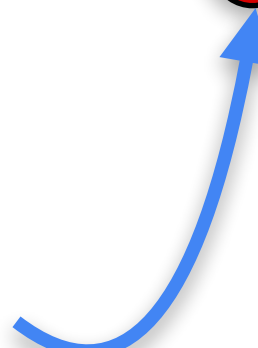

**Solving non-singular system
of linear equations**




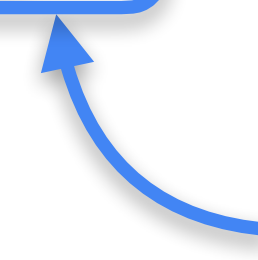
Solving systems of equations

System

- $a + b = 10$
 

- $a + 2b = 12$
 

 +  = \$10
\$8  \$2 

 +  +  = \$12
 \$2

Solving systems of equations

System

- $a + b = 10$



- $a + 2b = 12$



Some process

Manipulating equations

Swapping equations

Adding equations

Multiplying equations by a constant

Solved system

- $a = 8$



- $b = 2$



Solving systems of equations

System

- $a + b = 10$



- $a + 2b = 12$



Eliminate 'a' from this equation

Solved system

- $a = 8$



- $b = 2$



Manipulating equations

Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations


$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 22 \\ \hline 3a + 4b = 32 \end{array}$$

Systems of equations

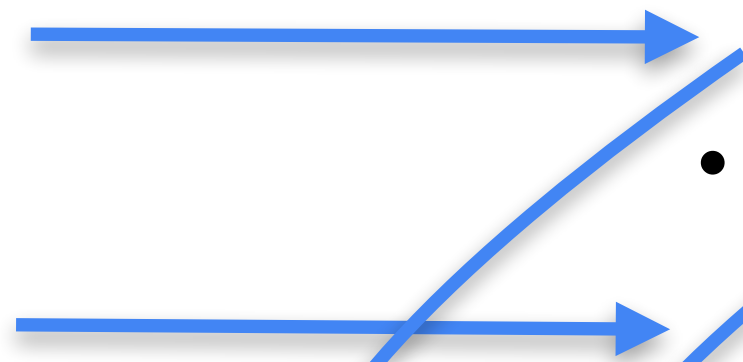
System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate 'a'
from this equation



Divide by coefficient of a

- $a + 0.2b = 3.4$
 - $a - 0.75b = 1.5$
- 


Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$


$$\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline \end{array}$$


$$0a - 0.95b = -1.9$$

$$-0.95b = -1.9$$

$$b = 2$$


Solved system

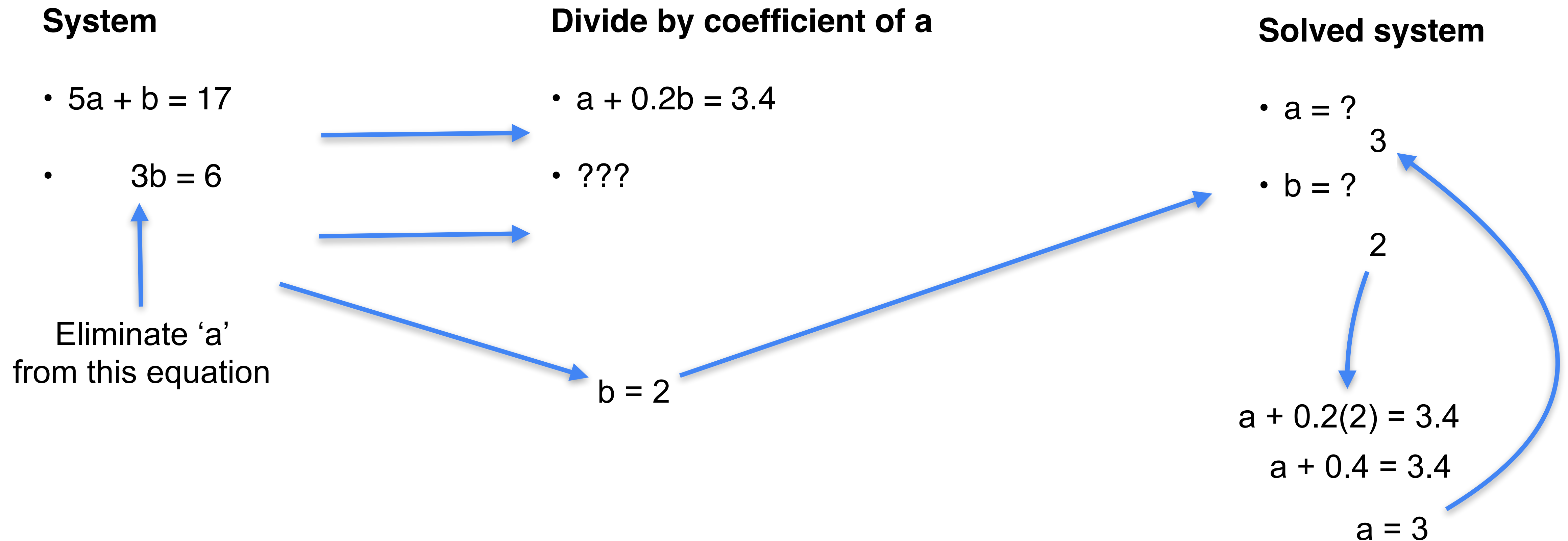
- $a = ?$ 3
 - $b = ?$ 2
- 

$$a + 0.2(2) = 3.4$$

$$a + 0.4 = 3.4$$

$$a = 3$$

What if one of the coefficients of a is zero?



Quiz

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- $a = 3$
- $b = 8$



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Solving System of Linear Equations

**Solving singular system of
linear equations**

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate 'a'
from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$$a + b = 10$$

$$- \quad a + b = 10$$

$$\hline 0 = 0$$

Solved system

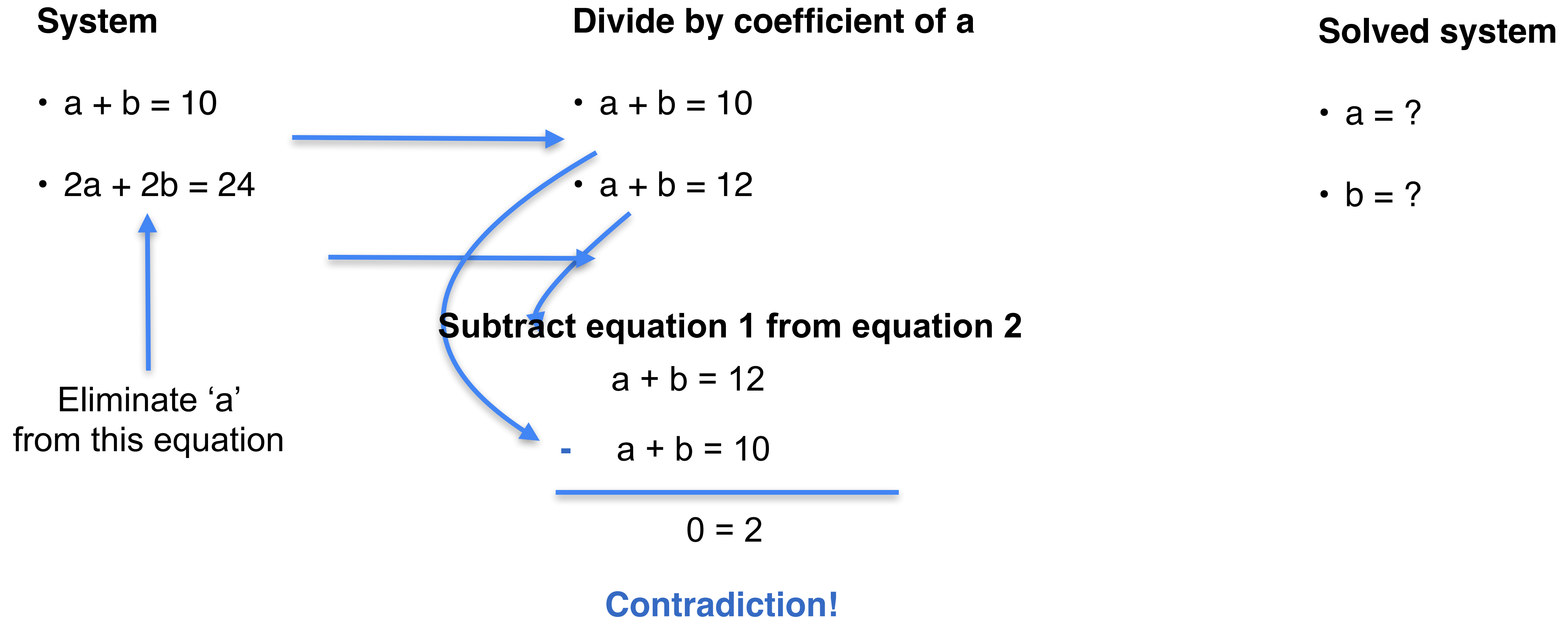
- $a = ?$
 - $b = ?$
- x
 $10 - x$

Degree of
freedom x

Solved system

- $a + b = 10$
- no other equation

What if the system is singular (contradictory)?



Quiz

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution: If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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Solving System of Linear Equations

**Solving system of equations
with more variables**

Elimination method

System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by
itself

Divide each row
by the
coefficient of 'a'

- $a + b + 2c = 12$

- $a - b - \frac{1}{3}c = 1$

- $a - \frac{b}{2} + 3c = 12$

Use the first
equation to
remove 'a' from
the others

- $a + b + 2c = 12$

- $-2b - \frac{7}{3}c = -11$

- $-\frac{3}{2}b + c = 0$

Isolated 'a'

Solve this new
system of 2
equations

Elimination method

System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $-11/6 c = -11/2$

Isolated 'b'

$c = 3$

Elimination method

System

- $a + b + 2c = 12$ 
- $b + \frac{7}{6}c = \frac{11}{2}$ 
- $c = 3$

$a + 2 + 6 = 12$
 $a = 4$

$b + \frac{7}{2} = \frac{11}{2}$
 $b = 2$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

The solution is
 $a = 4$
 $b = 2$
 $c = 3$



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Solving System of Linear Equations

Matrix row reduction

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form

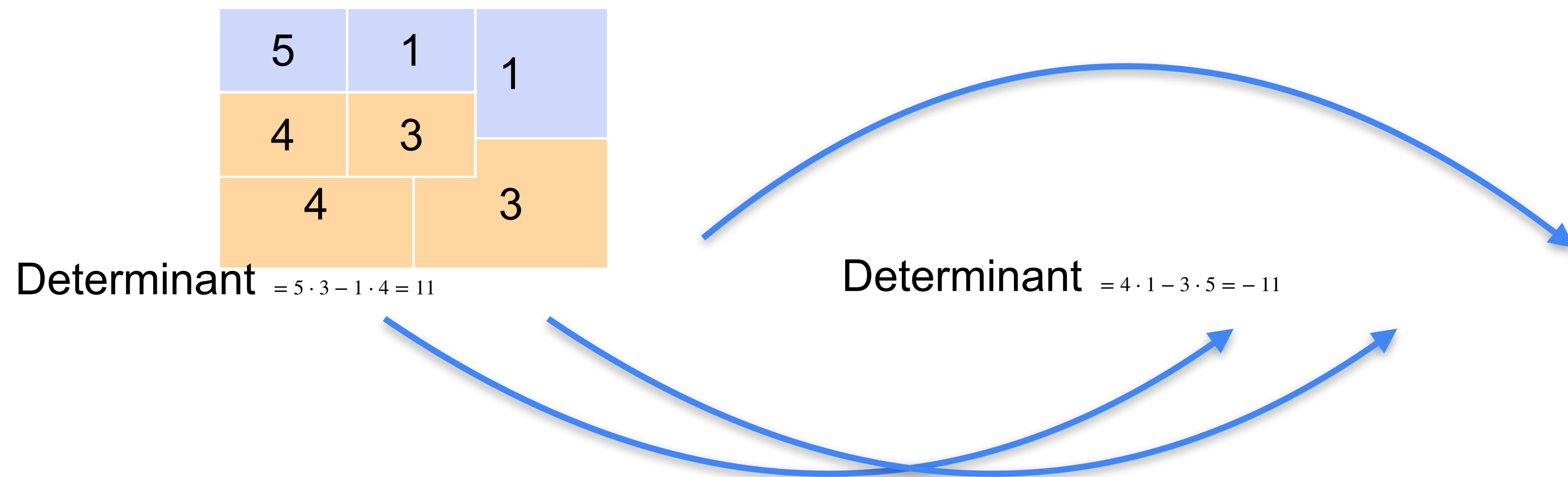


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Solving System of Linear Equations

**Row operations that preserve
singularity**

Switching rows



Multiplying a row by a (non-zero) scalar

5	1	1
4	3	
4		-3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

$\times 10 =$

50	10
----	----

Determinant $= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$

$= 10 \cdot 11$

Adding a row to another row



A 3x3 matrix is shown with blue and orange blocks. The top row has three blue blocks with values 5, 1, and 1. The middle row has two orange blocks with values 4 and 3, followed by a blue block with value 1. The bottom row has two orange blocks with values 4 and 3.

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

+



A 2x2 matrix is shown with teal blocks. The top row has two teal blocks with values 9 and 4.

Determinant $= 9 \cdot 3 - 4 \cdot 4$

$= 11$

W2 Lesson 2



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Solving System of Linear Equations

Rank of a matrix

Compressing Images - Reducing rank

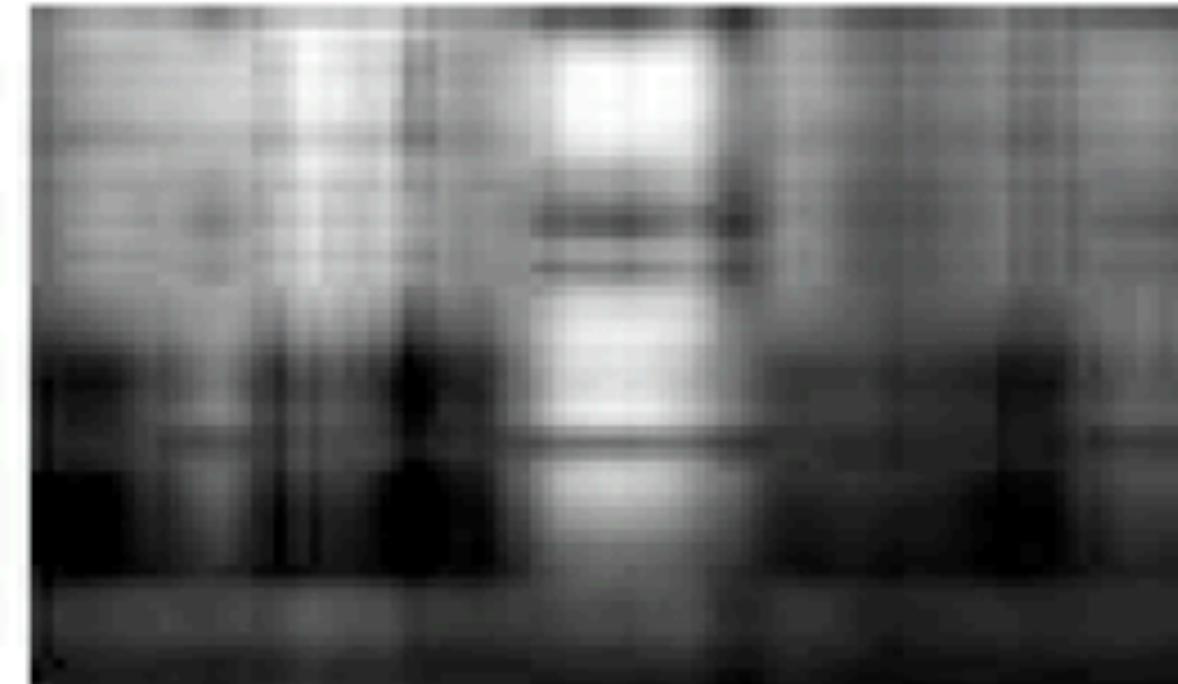
Original (Rank 200)



Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



Systems of information

System 1



 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2


 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

 The dog
 The dog

Two sentences

Zero pieces of information

Rank = 0

Systems of equations

System 1

$a + b = 0$
 
 $a + 2b = 0$
 

	
1	1
1	2





Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$
 
 $2a + 2b = 0$
 

	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$
 $0a + 0b = 0$

	
0	0
0	0

Rank = 0

Two equations

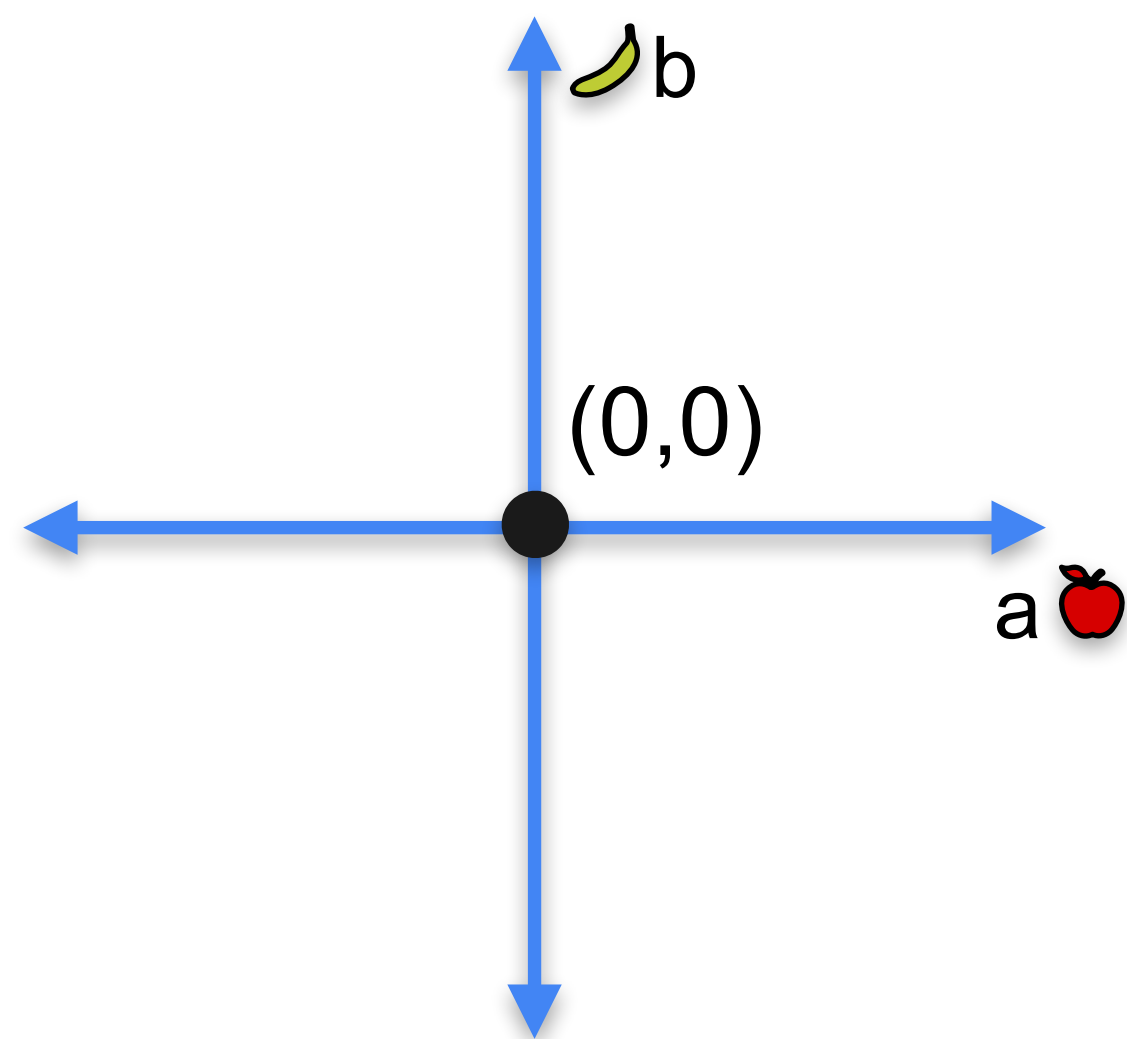
Zero pieces of information



Rank = 0

Rank and solutions to the system

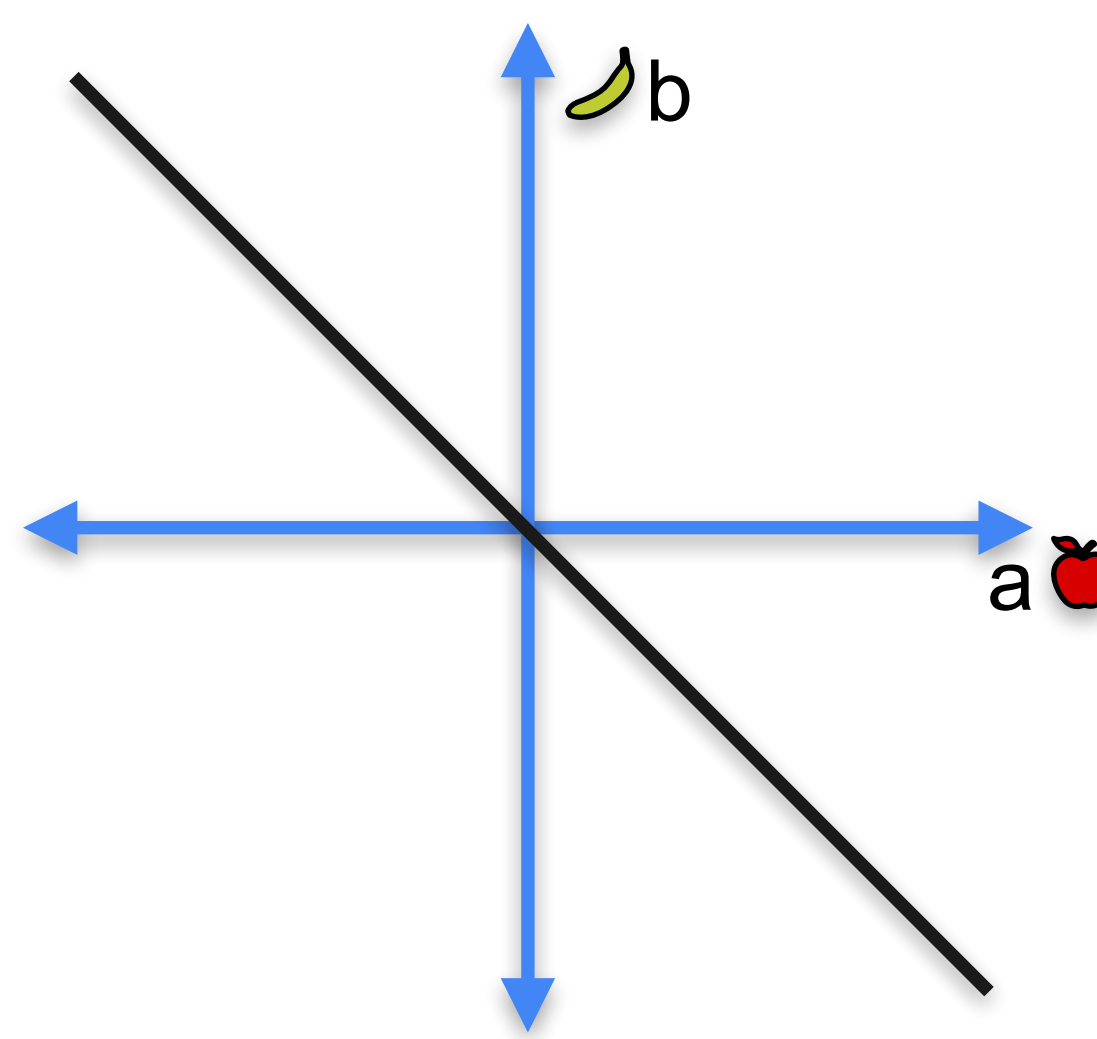
	
1	1
¹ Rank = 2	2



Dimension of solution space = 0



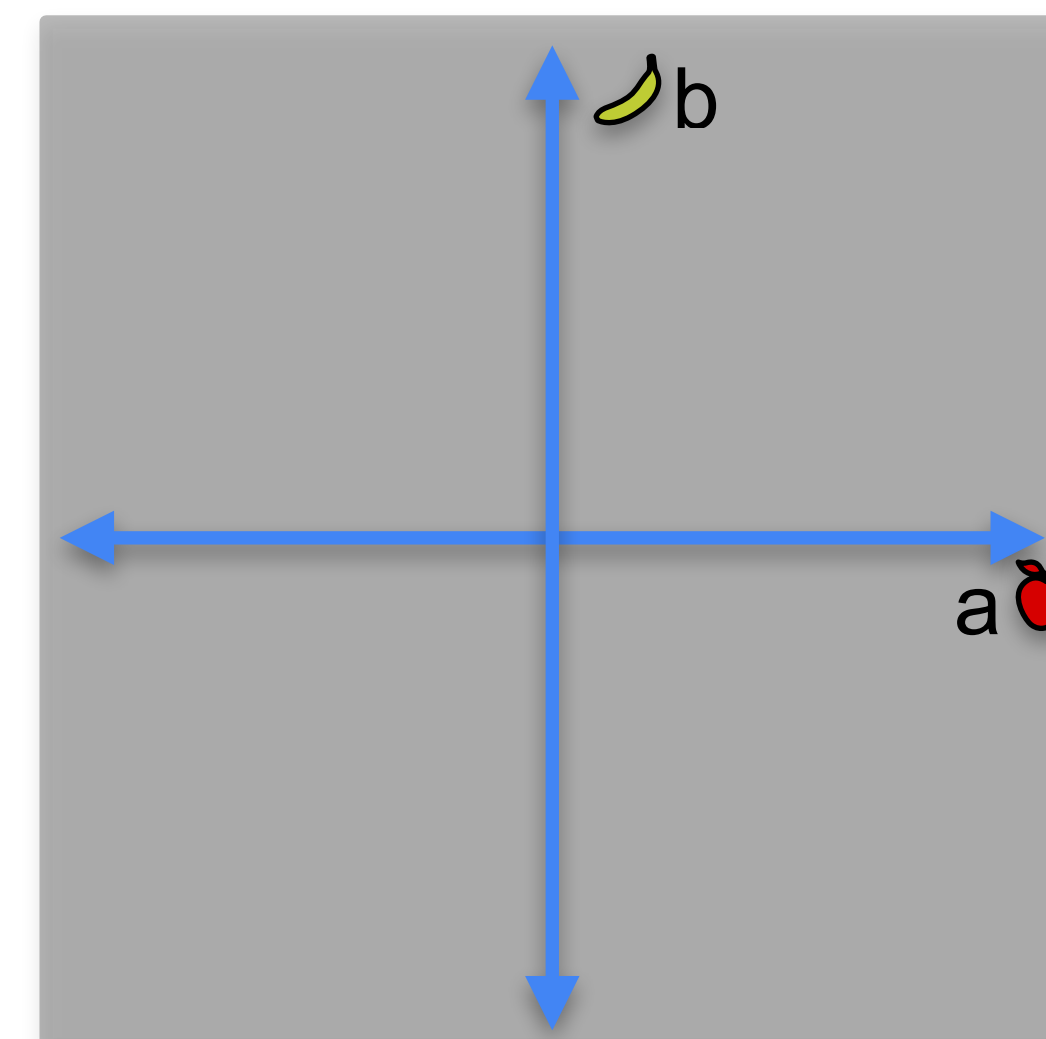
	
1	1
² Rank = 1	2

Dimension of solution space = 1



	
0	0
⁰ Rank = 0	0

Dimension of solution space = 2



Rank of a matrix

	
1	1
¹ Rank = 2	2

Dimension of solution space = 0

	
1	1
² Rank = 1	2

Dimension of solution space = 1

	
0	0
⁰ Rank = 0	0



Dimension of solution space = 2

$$\text{Rank} = 2 - (\text{Dimension of solution space})$$

Rank and singularity

	
1	1
¹ Rank = 2	2

Non-singular

	
1	1
² Rank = 1	2

Singular

	
0	0
⁰ Rank = 0	0

Singular

Quiz: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Solutions: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1: Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

Matrix 2: Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3



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Solving System of Linear Equations

**Rank of a matrix:
General case**

Rank for matrices

System 1

$a + b + c = 0$ ✓
 $a + 2b + c = 0$ ✓
 $a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
 $a + b + 2c = 0$ ✗
 $a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
 $2a + 2b + 2c = 0$ ✗
 $3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
 $0a + 0b + 0c = 0$ ✗
 $0a + 0b + 0c = 0$ ✗

3 Equations
0 Pieces of information

Rank 0

0	0	0
0	0	0
0	0	0

Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



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Solving System of Linear Equations

Row echelon form

Row echelon form of a matrix

Original matrix

5	1
4	-3
5	1
10	2
0	0
0	0

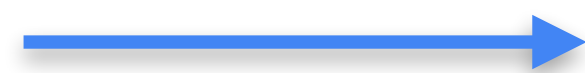
Row echelon form

1	0.2
0	1
1	1
0	0
0	0
0	0

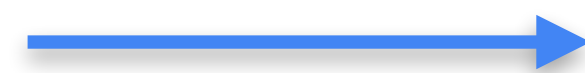
Row echelon form

Original matrix

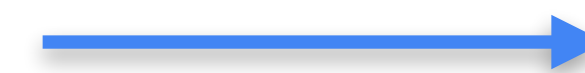
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Row echelon form

1	0.2
0	1

Divide each row by
the leftmost coefficient

-	1	-0.75
	1	0.2
<hr/>		
	0	-0.95

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by
the leftmost coefficient

1	0.2
1	0.2

Row echelon form

1	0.2
0	0

Divide the second row by
the leftmost non-zero coefficient

1	0.2
?	?

	1	0.2
-	1	0.2
<hr/>		
	0	0

Row echelon form for singular matrices

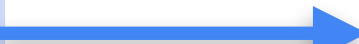
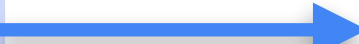
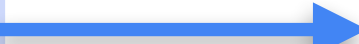
Row echelon form

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form, singularity, and rank

Non-singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>-3</td></tr></table> 	5	1	4	-3	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>1</td></tr></table>	1	0.2	0	1	Rank 2 2 ones in the diagonal
5	1										
4	-3										
1	0.2										
0	1										
Singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>10</td><td>2</td></tr></table> 	5	1	10	2	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>0</td></tr></table>	1	0.2	0	0	Rank 1 1 one in the diagonal
5	1										
10	2										
1	0.2										
0	0										
Singular matrix	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table> 	0	0	0	0	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table>	0	0	0	0	Rank 0 0 ones in the diagonal
0	0										
0	0										
0	0										
0	0										



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Solving System of Linear Equations

**Row echelon form:
General case**

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Matrix

1	1	2
3	-3	-1
2	-1	6



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Row echelon form matrix

1	1	2
0	-6	7
0	0	6

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

Rank 5

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

Row echelon form

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

$\div 3$

$\div 1$

$\div (-4)$

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Note:

- In general, pivots different than 1 are allowed
- For this class, pivots are 1. This makes no mathematical difference.

Another example

Matrix

1	1	1
1	2	1
1	1	2

Row echelon form

1	1	1
0	1	0
0	0	1

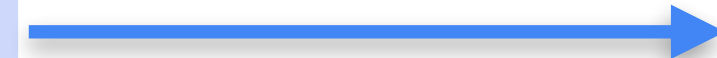
Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

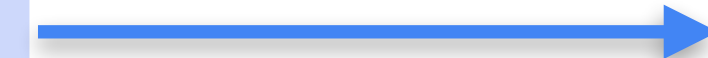
1	1	1
1	1	2
1	1	3

Subtract the first row
from the second and
the third ones



1	1	1
0	0	1
0	0	2

Subtract twice the
second row from the
third one



Row echelon form

1	1	1
0	0	1
0	0	0

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

Subtract twice the first
row from the second
row



1	1	1
0	0	0
3	3	3

Subtract three times
the first row from the
third row



Row echelon form

1	1	1
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0



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Solving System of Linear Equations

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Reduced row echelon form

Row echelon form

1	0.2
0	1

0	1
---	---

x 0.2

0	0.2
---	-----

Reduced row echelon form

1	0.2
0	1

1	0.2
0	0.2

-

1	0
---	---

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Rank 5

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the
value of the pivot

Reduced row
echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a
pivot to 0

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one

1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one

1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one

Reduced row echelon form

1	0	0
0	1	0
0	0	1



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Solving System of Linear Equations

The Gaussian Elimination Algorithm

Augmented matrix

$$\begin{array}{rcl} 2a - b + c & = & 0 \\ 2a + 2b + 4c & = & 0 \\ 4a + b & = & 0 \end{array}$$



Augmented matrix

2	-1	1
2	2	4
4	1	0

Proceed with the elimination method

Augmented matrix

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

 R_1

2

-1

1

1

 R_2

2

2

4

-2

 R_3

4

1

0

-1



Pivoting

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$



R_1	2	-1	1		1
R_2	2	2	4		-2
R_3	4	1	0		-1

$$R_1 \leftarrow \frac{1}{2}R_1$$

$$R_1 \leftarrow \frac{1}{2}$$

=	1	-1/2	1/2		1/2
---	---	------	-----	--	-----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

R_1	1	-1/2	1/2	1/2
R_2	2	2	4	-2
R_3	4	1	0	-1

$$R_2 \leftarrow R_2 - 2R_1$$

	2	2	4	-2
-2	1	-1/2	1/2	1/2
<hr/>				
$R_2 \leftarrow$	0	3	3	-3

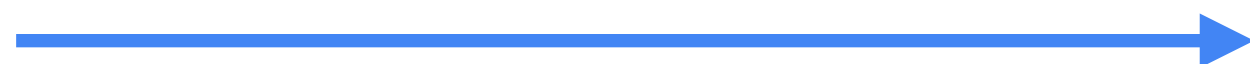
$$R_3 \leftarrow R_3 - 4R_1$$

	4	1	0	-1
-4	1	-1/2	1/2	1/2
<hr/>				
$R_3 \leftarrow$	0	3	-2	2

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$
$$3b + 3c = -3$$

$$3b - 2c = 2$$



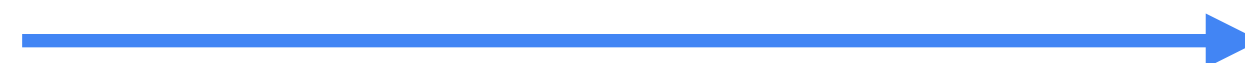
R_1	1	-1/2	1/2	1/2
R_2	0	3	3	-3
R_3	0	3	-2	2

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$3b + 3c = -3$$

$$3b - 2c = 2$$


 R_1

1	-1/2	1/2	1/2
---	------	-----	-----

 R_2

0	3	3	-3
---	---	---	----

 R_3

0	3	-2	2
---	---	----	---

$$R_2 \leftarrow \frac{1}{3}R_2$$

$$R_2 \leftarrow \frac{1}{3}$$

0	3	3	-3
---	---	---	----

 $=$

0	1	1	-1
---	---	---	----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$

 R_1

1	-1/2	1/2	1/2
---	------	-----	-----

 R_2

0	1	1	-1
---	---	---	----

 R_3

0	3	-2	2
---	---	----	---

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$



R_1	1	-1/2	1/2		1/2
R_2	0	1	1		-1
R_3	0	3	-2		2

$$R_3 \leftarrow R_3 - 3R_2$$

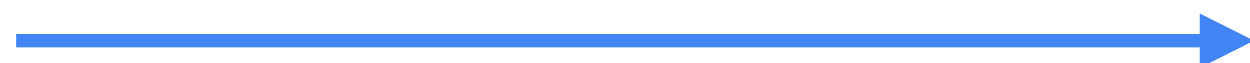
	0	3	-2		2
-3	0	1	1		-1
<hr/>					
$R_3 \leftarrow$	0	0	-5		5

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$-5c = 5$$

 R_1

1	-1/2	1/2		1/2
---	------	-----	--	-----

 R_2

0	1	1		-1
---	---	---	--	----

 R_3

0	0	-5		5
---	---	----	--	---

$$R_3 \leftarrow -\frac{1}{5}R_3$$

$$R_3 \leftarrow -\frac{1}{5}$$

0	0	-5		5
---	---	----	--	---

$$=$$

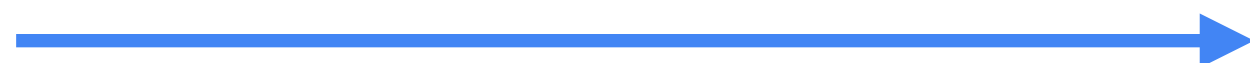
0	0	1		-1
---	---	---	--	----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 R_1

1	-1/2	1/2	1/2
---	------	-----	-----

 R_2

0	1	1	-1
---	---	---	----

 R_3

0	0	1	-1
---	---	---	----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$



R_1

R_2

R_3

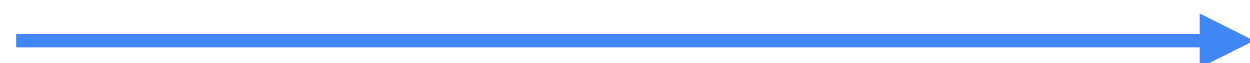
1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 R_1

1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

 R_2 R_3

Back substitution

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

R_1	1	-1/2	1/2		1/2
R_2	0	1	1		-1
R_3	0	0	1		-1

$$R_2 \leftarrow R_2 - R_3$$

	0	1	1		-1
-	0	0	1		-1

$$R_2 \leftarrow$$

0	1	0		0
---	---	---	--	---

$$R_1 \leftarrow R_1 - \frac{1}{2}R_3$$

$$-\frac{1}{2}$$

$$R_1 \leftarrow$$

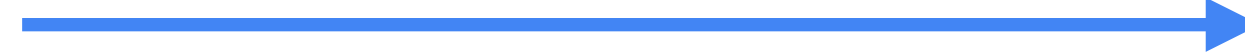
0	0	1		-1
1	-1/2	1/2		1/2
1	-1/2	0		1

Back substitution

$$a - \frac{1}{2}b = 1$$

$$b = 0 \quad -1$$

$$c = -1$$



R_1	1	-1/2	0		1
R_2	0	1	0		0
R_3	0	0	1		-1

$$R_1 \leftarrow R_1 + \frac{1}{2}R_2$$

$$+\frac{1}{2}$$

$$R_1 \leftarrow$$

1	0	0		1
---	---	---	--	---

The result

$a = 1$
 $b = 0$
 $c = -1$



R_1

1	0	0	1
---	---	---	---

R_2

0	1	0	0
---	---	---	---

R_3

0	0	1	-1
---	---	---	----

Solution to the system

Identity matrix

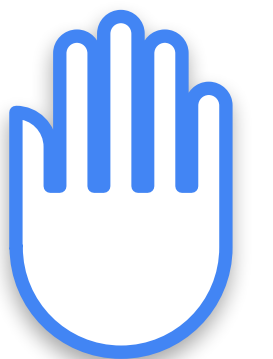
What if the system is singular?

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



1	2	-1	5
0	0	-7	9
0	0	0	0



There is no need to worry!

Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0



$$0a + 0b + 0c = 0$$

Infinitely many
solutions

Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	10

After row reduction...



Look at the column of constants

1	2	-1	5	
0	0	-7	9	
0	0	0	0	
				4

$$0a + 0b + 0c = 4$$

The system has
no solutions

Checking if it has infinitely many or no solutions

- Row full of zeroes in row echelon form
- Constant in that row is zero
- **Infinitely many solutions**

1	2	-1	5
0	0	-7	9
0	0	0	0

- Row full of zeroes in row echelon form
- Constant in that row is not zero
- **No solutions**

1	2	-1	5
0	0	-7	9
0	0	0	4

Gaussian Elimination - Summary

1. Create the augmented matrix
2. Get the matrix into reduced row echelon form
3. Complete back substitution
4. Stop if you encounter a row of 0s



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Solving System of Linear Equations

Conclusion