The Learning Hub

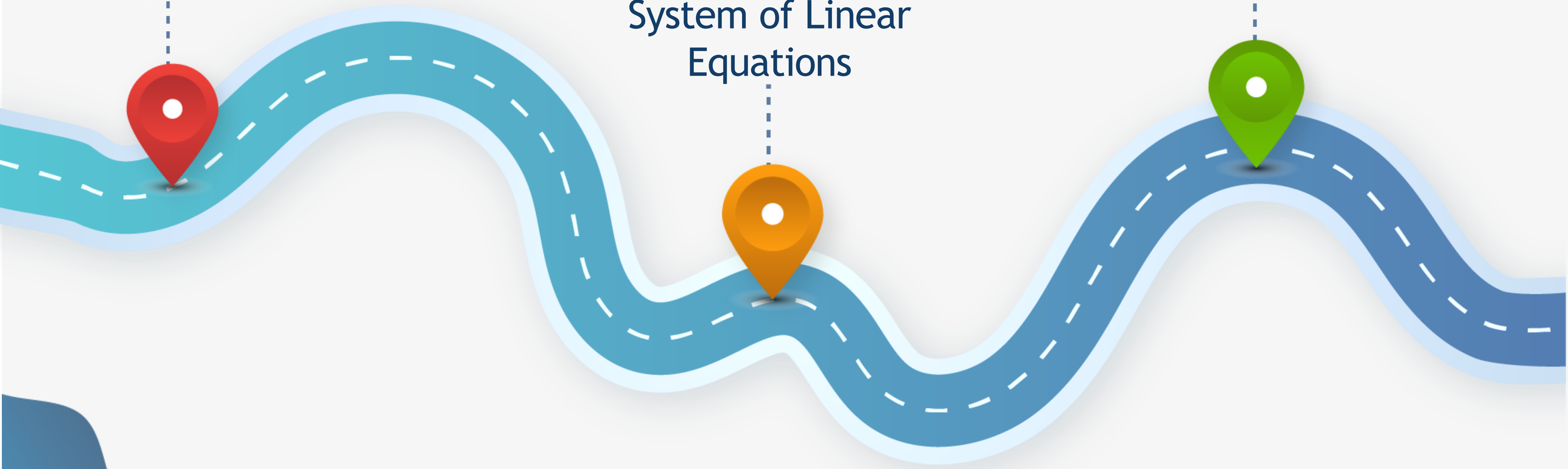
Data Science Fundamentals

Introduction to Linear Algebra

What Is Linear Algebra

System of Linear Equations

Solving Linear Equations



What is Linear Algebra?

What Is Linear Algebra?

- Linear Algebra is a branch of mathematics that is foundational in data science and machine learning.
- Linear algebra is the most important math skill in machine learning. It deals with linear equations and, more generally, their representations in the vector space using matrices and linear transforms.

What Is Linear Algebra?

- Linear algebra is used in data preprocessing, data transformation, and model evaluation.
- Every Machine Learning Model is also based on Linear Algebra. Where linear algebra is a key basis to the field of machine learning, from notations used to express the operation of algorithms to the execution of algorithms in code.

Agenda

1. Linear Equations
2. Matrices
3. Matrices Operations
4. Elementary matrices
5. Determinant of a matrix
6. Vectors
7. Vector Spaces
8. Dimension
9. Dot product
10. Eigenvalues
11. Eigenvectors

Import necessary libraries for linear algebra

```
import numpy as np
import pandas as pd
import pylab
import matplotlib.pyplot as plt
import seaborn as sns
```

System of Linear Equations

Linear Equations

A linear equation is an equation that has the standard form $a_1x_1 + a_2x_2 + \ldots + a_nx_n$.

It is the fundamental component of linear algebra.

Linear Equations

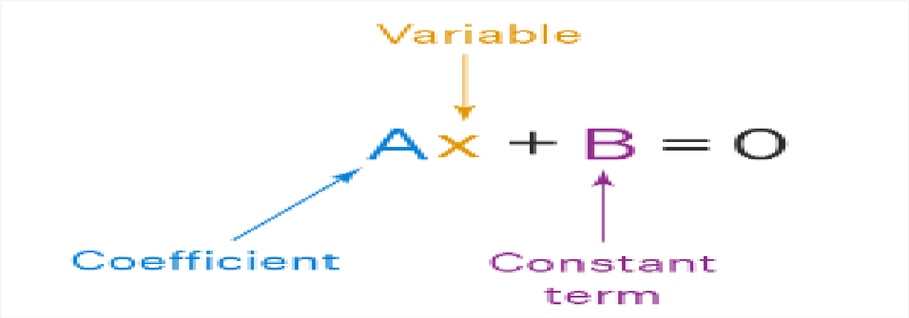
The linear equation in **n** variables $x_1, x_2, \ldots\ldots, x_n$ has the form :

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$$

The coefficients $a_1, a_2, \ldots\ldots, a_n$ are real numbers, and a constant term **b** is a real number. The number a_1 is the leading coefficient and is the x_1 leading variable.

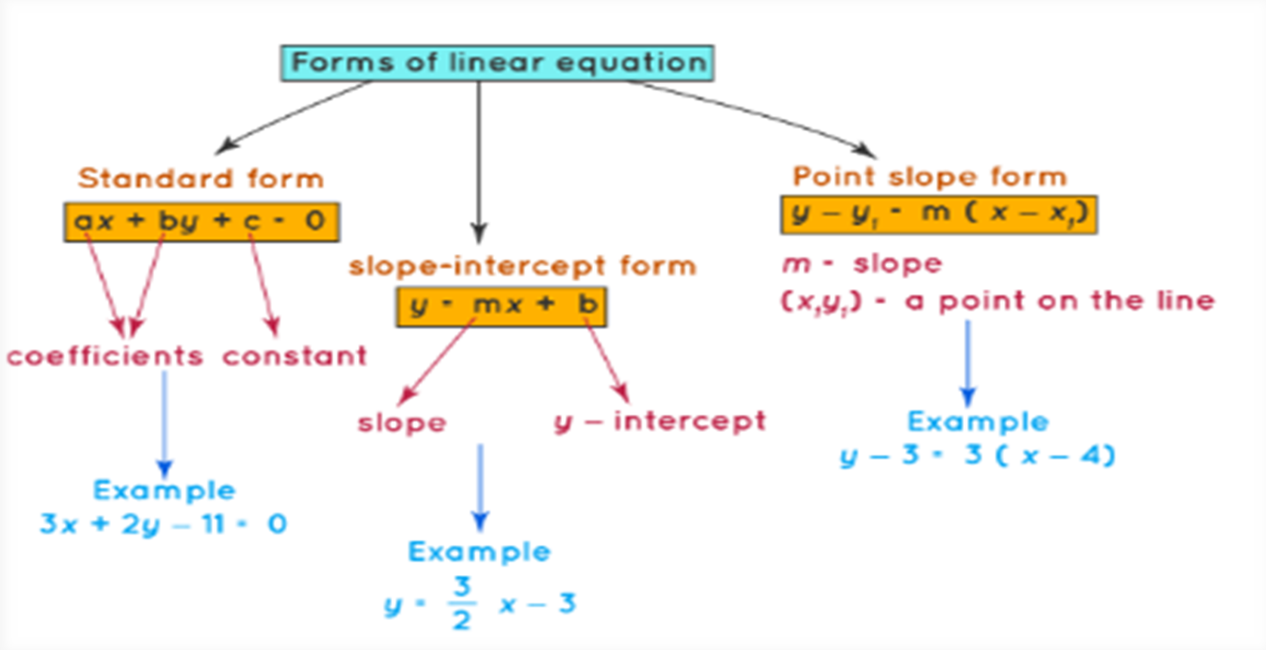
Linear Equation in One Variable

- The variable degree in linear equations must be equal to one.
- The linear equation graph in one variable is a straight line, maybe horizontal or vertical.
- The linear equation solution in one variable is unchanged if you add, subtract, multiply, or divide the number into both sides of the equation.



Linear Equations in Two Variables

- The standard form of a two-variable linear equation is **ax+ by+ c= 0** where x and y are the two variables.
- This is called a linear equation in two variables x and y.



Examples

- Similarly, the equation of a line in three-dimensional space is : $a_1 X + a_2 Y + a_3 Z = b$

  This is called a linear equation in three variables: x, y, and z.

Examples of linear equations

$$3x + 2y = 73x + 2y = 7$$
$$x1 - 2x2 + 10x3 + x4 = 0$$

Examples of non-linear equations

$$x * y + z = 2$$
$$Sin(x1) + 2(x2) - 3(x3) = 0$$
$$\frac{1}{x} + \frac{1}{y} = 5$$

Solving Linear Equations

Solving Linear Equations

- Solutions of linear equations in n variables mean finding the value of the variable(s) given in the linear equations where is a sequence of n real numbers s1, s2,..sn that satisfy the equation when they replace the variables in the equation.

The set of all solutions of a linear equation is called its **solution set**

- **Solve the linear equation: x+2y=4**

  Solve for x in terms of y: x=4−2y

  We can get multiple solutions by assigning different values for y

Solving Linear Equations

A system of m linear equations in n variables is a set of equations, each of which is linear in the same variables, Like the singular linear equation, the solution is a sequence of n real numbers s1, s2,..sn that satisfy the equation when they replace the variables in all the equations.

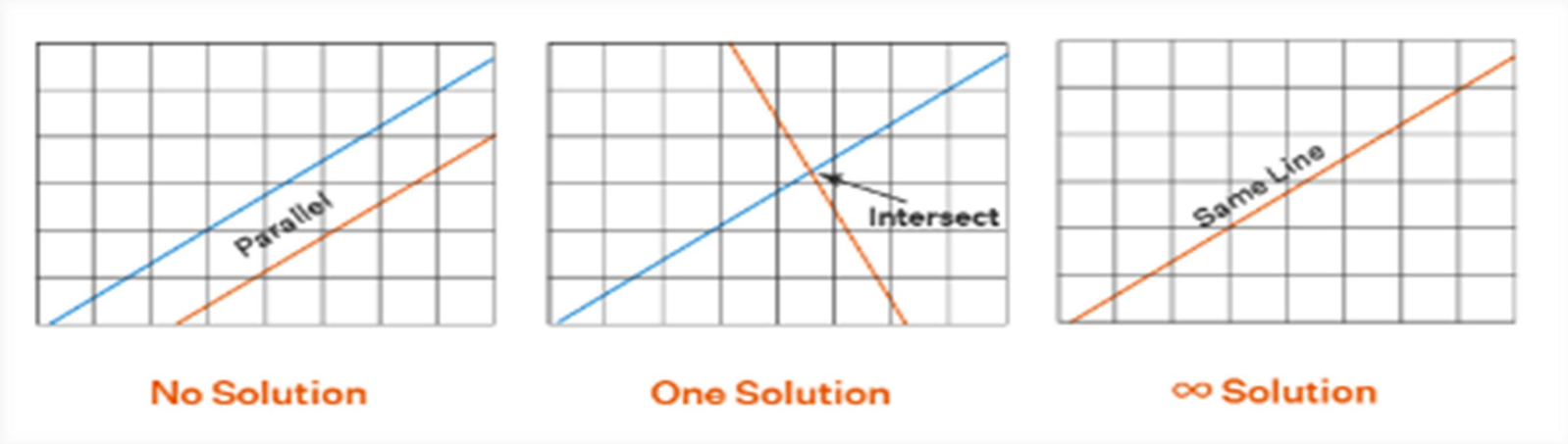
$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m.
\end{aligned}
$$

Solving Linear Equations

- maybe a system of linear equations has just one solution, an infinite number of solutions, or no solution.
- where the system of linear equations is called consistent if it has at least one solution and is inconsistent if it has no solution.

Number of Solutions of a Linear Equations System

- For linear equations in n variables, exactly one of the following is true:
  1. The system of linear equations has just one solution
  2. The system of linear equations has an infinite number of solutions.
  3. The system of linear equations has no solution



Solutions for Linear Equations in One Variable

Consider the equation: 3x + 6 = 9

- To find the value of x, first, we remove 6 from L.H.S, so we subtract 6 from both sides of the equation. 3x +(6 - 6)= (9 – 6)

Simply Now we get, 3x = 3

- Now we have to remove 3 from L.H.S in order to get x, therefore we divide the equation by 3. 3x/3 = 3/3, x=1

Hence, the solution of equation 3x + 6 = 9 is x=1.

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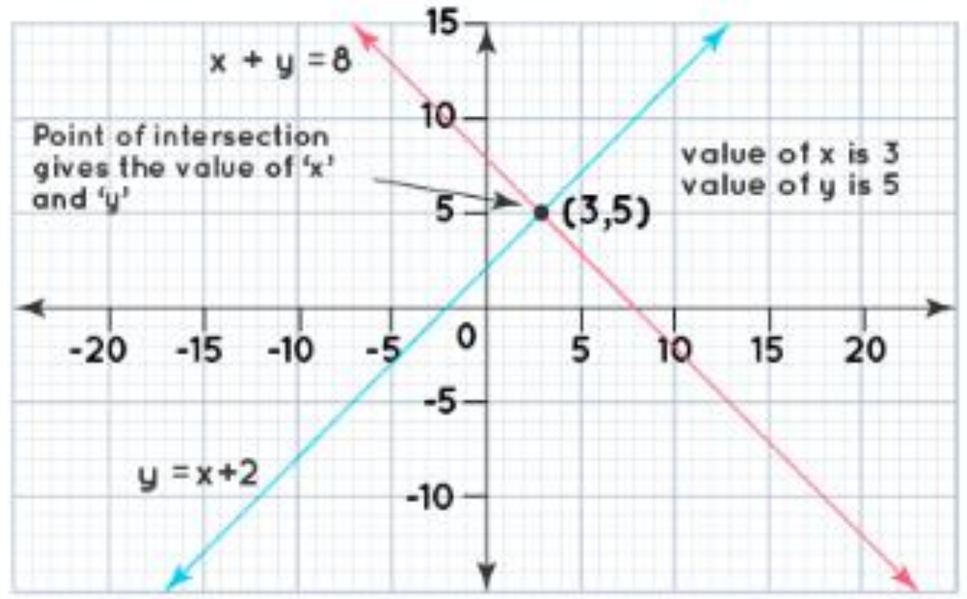
Solutions for Linear Equations of Two Variables

➢ We have different methods to solve linear equations of two variables:

1. Graphical method
2. Substitution method
3. Elimination method
4. Cross-multiplication method

Graphical Method [1]



Solutions for Linear Equations of Two Variables

Elimination method[1]

$$2x + y = 11$$

(-) (-) (-)

$$2x + 6y = 36$$

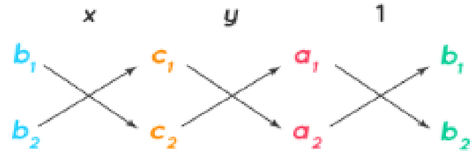
$$-5y = -25$$

$$y = \frac{-25}{-5}$$

$$\therefore \boxed{y = 5}$$

Cross-multiplication Method [1]

Coefficient



Formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

values of 'x' and 'y' are

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

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Solving Linear Equations

Example: Solve the following linear equations by the substitution method.

$2x + y = 14$ --------- (1)

$3x + 2y = 20$ -------- (2)

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Solving Linear Equations

Solution: using the substitution method, take the first equation and find the value of 'y' and substitute it in the second equation.

➔From equation (1), $y = 14-2x$.

➔ Now, substitute the value of 'y' in equation (2):

$3x + 2(14 - 2x) = 20$

$3x + 28 - 4x = 20$ ➔ $-x + 28 = 20$ ➔ $-x = 20- 28$ ➔ $-x = -8$ ➔ $x = -8/-1$ ➔ $x = 8$

➔ Now, substitute the value of 'x = 4' in equation (1) and find the value of 'y'.

$2x + y = 14$ ------- (1)

$2(8) + y = 14$ ➔ $16+ y = 14$ ➔ $y = 14 – 16$ ➔ $y = -2$

by the substitution method, the value of x is 8 and y is -2

Exercises

Example 1: Solve the following linear equations by the substitution method.

$x + 5y = 10$ --------- (1)

$6x + 10y = 20$ -------- (2)

Solution:

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Exercises

Example 2: Using the elimination method of solving linear equations find the values of 'x' and 'y'.

$7x + y = 21$ ------ (1)

$4x + 6y = 30$ -------- (2)

Solution:

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References

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THANK YOU