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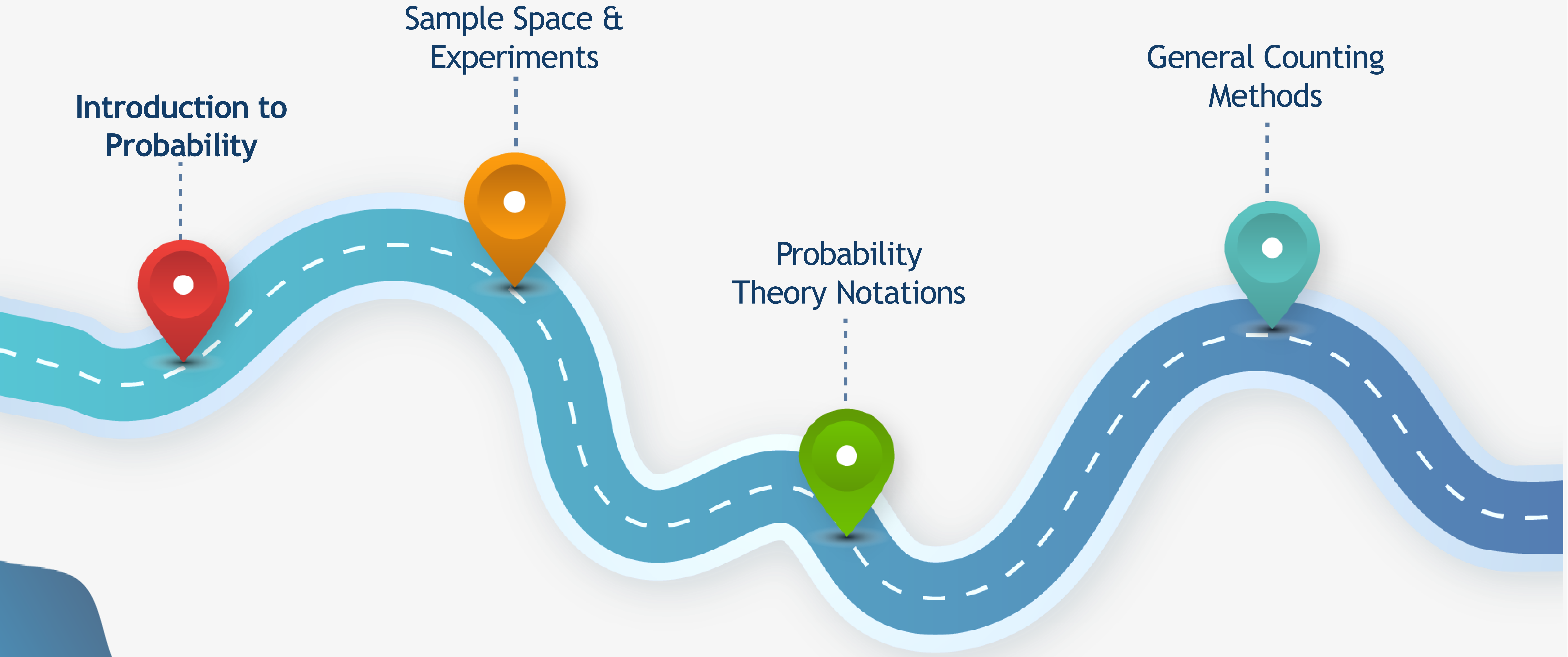
Data Science Fundamentals

Probability Theory 1


Introduction to Probability
Sample Space & Experiments
Probability Theory Notations
General Counting Methods

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Introduction to Probability

What is Probability?

- **Probability:** is a universal tool to measure the likelihood of an event occurring in the presence of incomplete information or uncertainty.
- Probability can range from 0 up to 1, where 0 means the event to be an impossible one and 1 indicates a certain event.
- Probability is based on past experiences to infer conclusions about future **events**.

What is Probability?

- What is the probability of a coin flip being heads ? **0.5**
- What is the probability of a dice roll being 5 ? **1/6**
- What is the probability of having a baby boy ? **0.5**
- What is the probability of waking up tomorrow and finding people walking on their hands ? **0**

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Sample Space & Experiments

Sample Space

- **Sample Space:** in statistics is the set of all possible outcomes of a given statistic.
- In the rolling of six-sided die experiment, the possible outcomes are the numbers from one to six, the sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$.
- Each item within the sample space is termed a **Sample Point**.
- **Events:** are a set of sample points.

Experiments

- **Experiments:** is a physical occurrence that can be repeated infinitely.

- Tossing a coin n number of times.
- Choosing a random number between 1 and 50.
- Measuring someone heart rate.
- Tossing two dice
- Draw n balls from a box containing balls of various colours.

Examples

- Tossing a Coin

  Sample Space, S = { H, T } = { Head, Tail }

- Tossing Two Coins

  Sample Space, S = { (H1, H2), (H1, T2), (T1, H2), (T1, T2) }

Examples

- Tossing Three Coins

Therefore, the possible number of outcomes will be 2^3 = 8 outcomes

Sample space for tossing three coins is written as

Sample Space S = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

- In general, if you have "n" coins, then the possible number of outcomes will be $\mathbf{2^n}$

Examples

- Throwing a Dice

Sample Space, S = {1, 2, 3, 4, 5, 6}

- Throwing Two Dice

Sample Space, S = $$\left\{\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix}\right\}$$

Examples

- Throwing Three Dice

If three dice are thrown, the possible outcomes of 216 where n in the experiment is taken as 3, so it becomes $6^3 = 216$.

- In general, if you have "n" dices, then the possible number of outcomes will be 6^n

Examples

- In Two Dice rolling trail, write the sample space and find the probability that the sum is:

1. Equal to 1?
2. Equal to 4?
3. Less than 13?

Sample Space S= { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }

Examples

- Solution:

1. Let E be the event "sum equal to 1". Since, there are no outcomes a sum is equal to 1, hence, **$P(E) = n(E) / n(S) = 0 / 36 = 0$**.

2. Let A be the event of getting the sum of numbers on dice equal to 4. Three possible outcomes give a sum equal to 4 they are:

   $A = \{(1,3),(2,2),(3,1)\}$, $n(A) = 3$

   Hence, $P(A) = n(A) / n(S) = 3 / 36 = 1 / 12$

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Examples

3. Let B be the event of getting the sum of numbers on dice is less than 13.

From the sample space, we can see all possible outcomes for event B,

which gives a sum less than B. Like: (1,1) or (1,6) or (2,6) or (6,6).

So you can see the limit of an event to occur is when both dices have the number 6, i.e. (6,6).

Thus, $n(B) = 36$ Hence,

$P(B) = n(B) / n(S) = 36 / 36 = 1$

Examples

- A box contains 15 red, 3 Blue, and 7 Green balls. A Ball is drawn at random. Find the probability of this ball is a:

1. Red ball?
2. Green ball ?
3. Not a blue ball ?

P(Red ball)=15/25

P(Green ball)=7/25

P(Not a blue ball)=22/25

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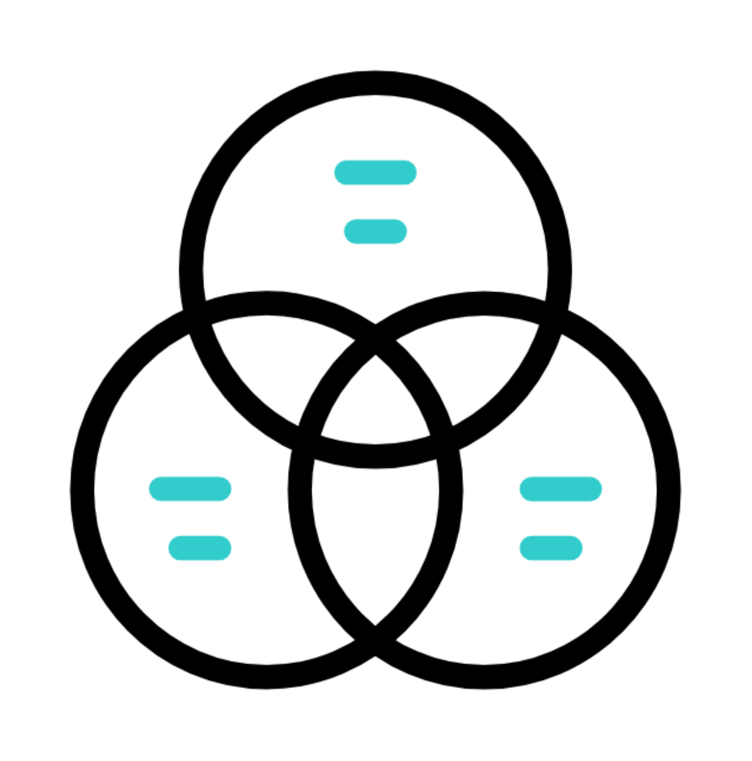
Probability Theory Notations

Operations Rules

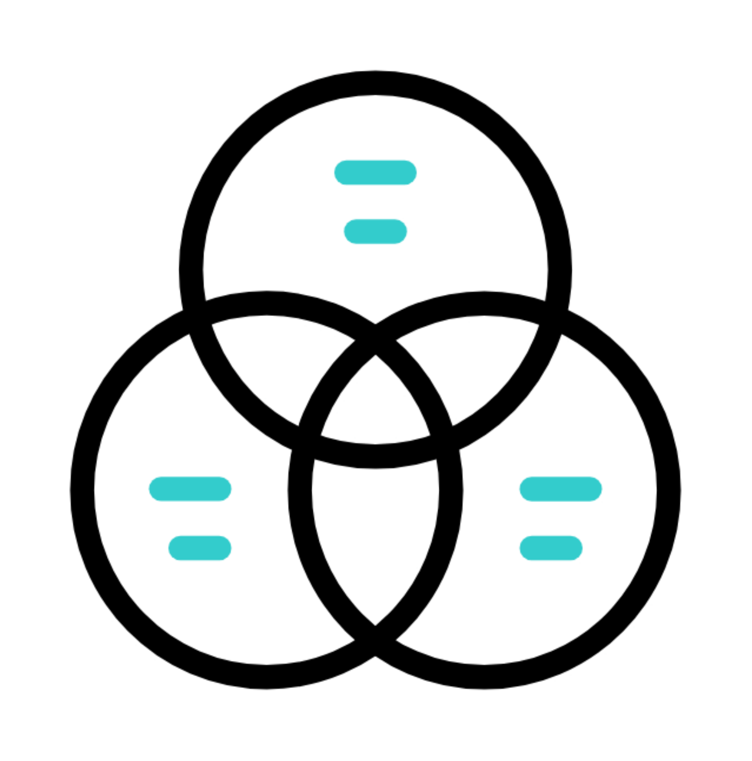
- Ac = set of points of Ω not in A
- A $\cap$ B = set of points of Ω that are in both A and B
- A $\cup$ B = set of points of Ω that are in at least one of A and B
- A - B = A−(A $\cap$ B) = set of points of Ω that are in A but not in B



Distributive Law

For any sets A, B, and C we have

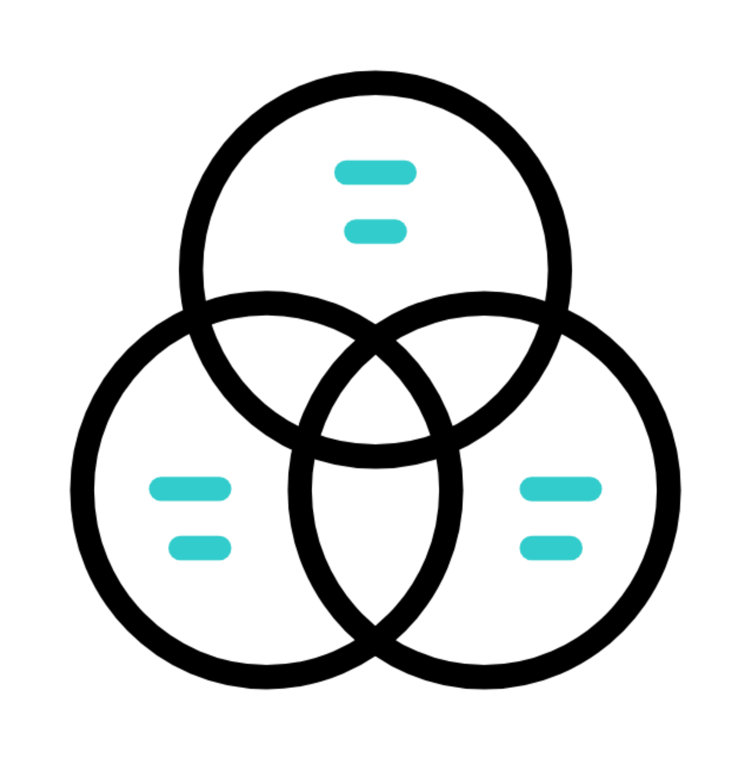
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$



De Morgan Law

For any sets A1, A2, ···, An, we have

- $(A \cup B)^C = A^C \cap B^C$
- $(A \cap B)^C = A^C \cup B^C$



Examples

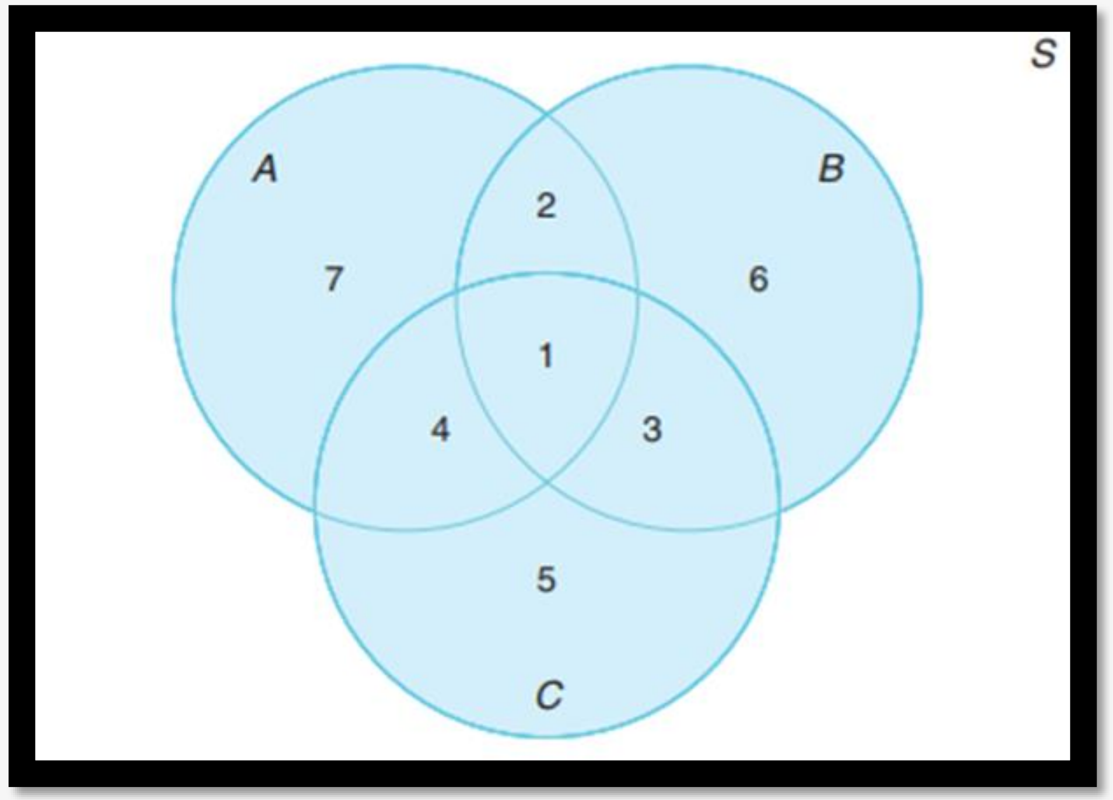
- If the universal set is given by S={1,2,3,4,5,6,7,8 }, and A={1,2,5,8}, B={2,4,5},C={1,5,6,7} are three sets, find the following sets:

1. $A \cup B$ **=1,2,4,5,8**
2. $A \cap B$ **=2,5**
3. A^C **=3,4,6,7**
4. B^C**= 1,3,6,7,8**

Examples

- In a Venn Diagram, Find the regions representing the following:

1. $A \cup C$ = **1, 2, 3, 4, 5,7**
2. $B' \cap A$ = **4 and 7**
3. $A \cap B \cap C$ = **1**
4. $(A \cup B) \cap C$ = **1,3,4**



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General Counting Methods

Propositions

(a) In the case of the **order of arrangement matters**, then the number of ways of linearly arranging n distinct objects=**n!**

(b) In case of the **order of selection is important**, then the number of ways of choosing r distinct objects from n distinct objects can be calculated by =

$$\left(\frac{n}{r}\right) = \frac{n!}{r!\,(n-r)!}$$

Examples

- How many alternative ways can a chair and a treasurer be elected from 22-member ?

Solution: there are 22 total possibilities For the chair position. While for each of these 22 possibilities, there are 21 possibilities to elect the treasurer. Using the multiplication rule, we obtain **$n1 \times n2 = 22 \times 21 = 462$ other ways**.

Examples

- Adam goes to assemble a computer by himself. He has a selection of chips from two brands, a hard drive from four, memory from three, and an adjunct bundle from five local stores. how many other ways can Sam order the parts?

Solution:

$n1 = 2$, $n2 = 4$, $n3 = 3$, and $n4 = 5$, number of ways to order the parts

$$n1 \times n2 \times n3 \times n4 = 2 \times 4 \times 3 \times 5 = 120$$

Examples

- Three awards (research, teaching, and development) will be given to a class of 25 graduate students in an Artificial Intelligence department. If the winning student can receive just one award, how many possible selections are there?

Solution:

The total number of sample points is $_{25}P_3 = 25!/(25-3)! = 25!/22! = (25)(24)(23) = 13,800$.

Examples

How many even four-digit numbers can be formed from the digits { 0, 1, 2, 5, 6, 9}, if each digit can be used only once?

Solution:

Since the number must be even, we have only $n1 = 3$ choices for the unit's position. However, for a four-digit number, the thousands position cannot be 0. Hence, we consider the unit's position in two parts, 0 or not 0. If the units position is 0 (i.e., $n1 = 1$), we have $n2 = 5$ choices for the thousands position, $n3 = 4$ for the hundreds position, and $n4 = 3$ for the tens position. Therefore, in this case, we have a total of

$n1n2n3n4 = (1)(5)(4)(3) = 60$

Examples

Even four-digit numbers. On the other hand, if the units position is not 0 (i.e., n1 = 2), we have n2 = 4 choices for the thousands position, n3 = 4 for the hundreds position, and n4 = 3 for the tens position. In this situation, there are a total of **n1n2n3n4 = (2)(4)(4)(3) = 96** even four-digit numbers. Since the above two cases are mutually exclusive, the total number of even four-digit numbers can be calculated as **60 + 96 = 156.**

References

1. Scribd. (n.d.). Chapter 2 | PDF | Experiment | Statistics.
2. Javed, O.A. (n.d.). Probability and Statistics for Engineers and Scientists - 9th Edition (by Walpole, Mayers, Ye)

THANK YOU