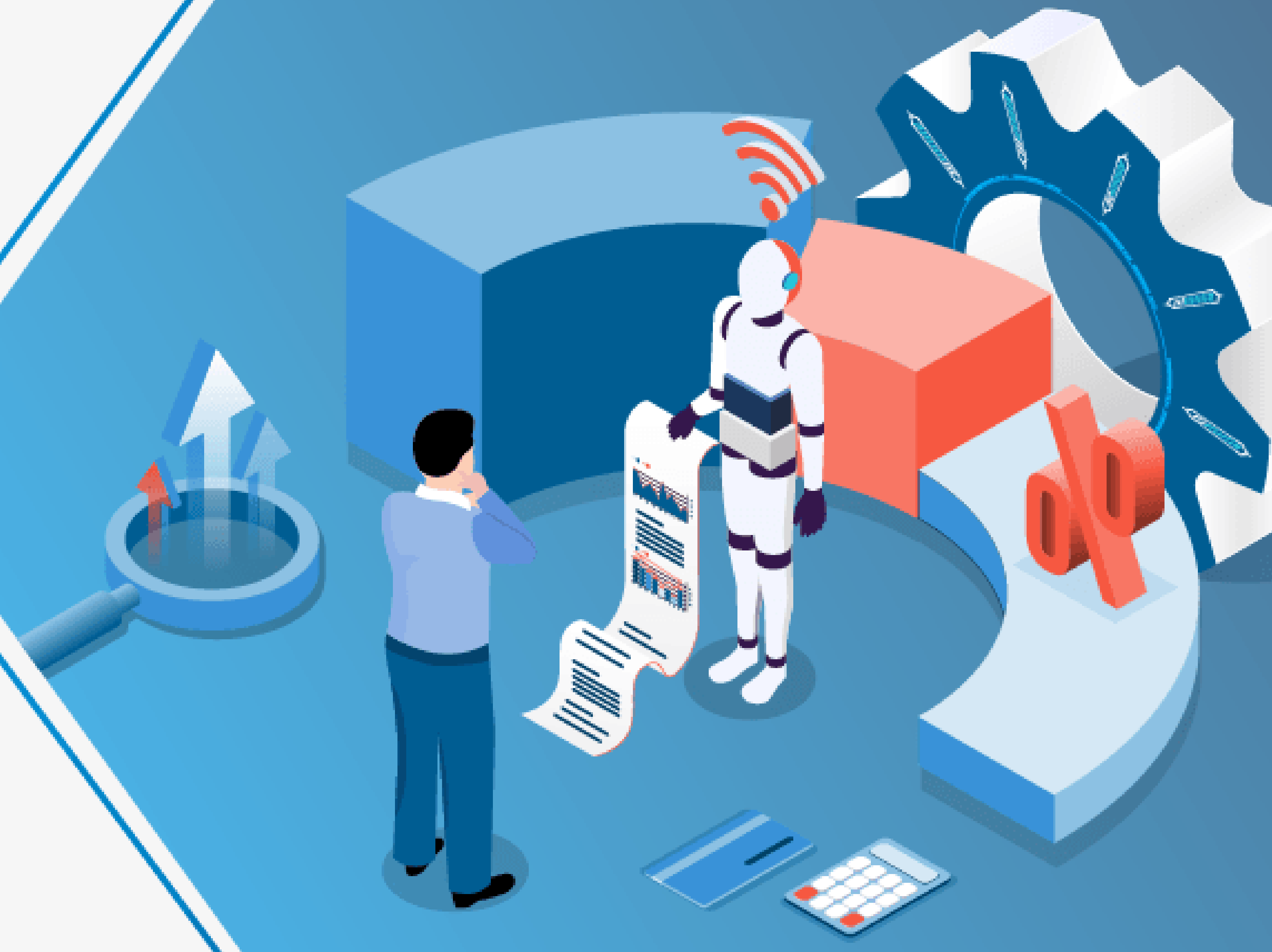
Data Science Fundamentals

Matrices 1



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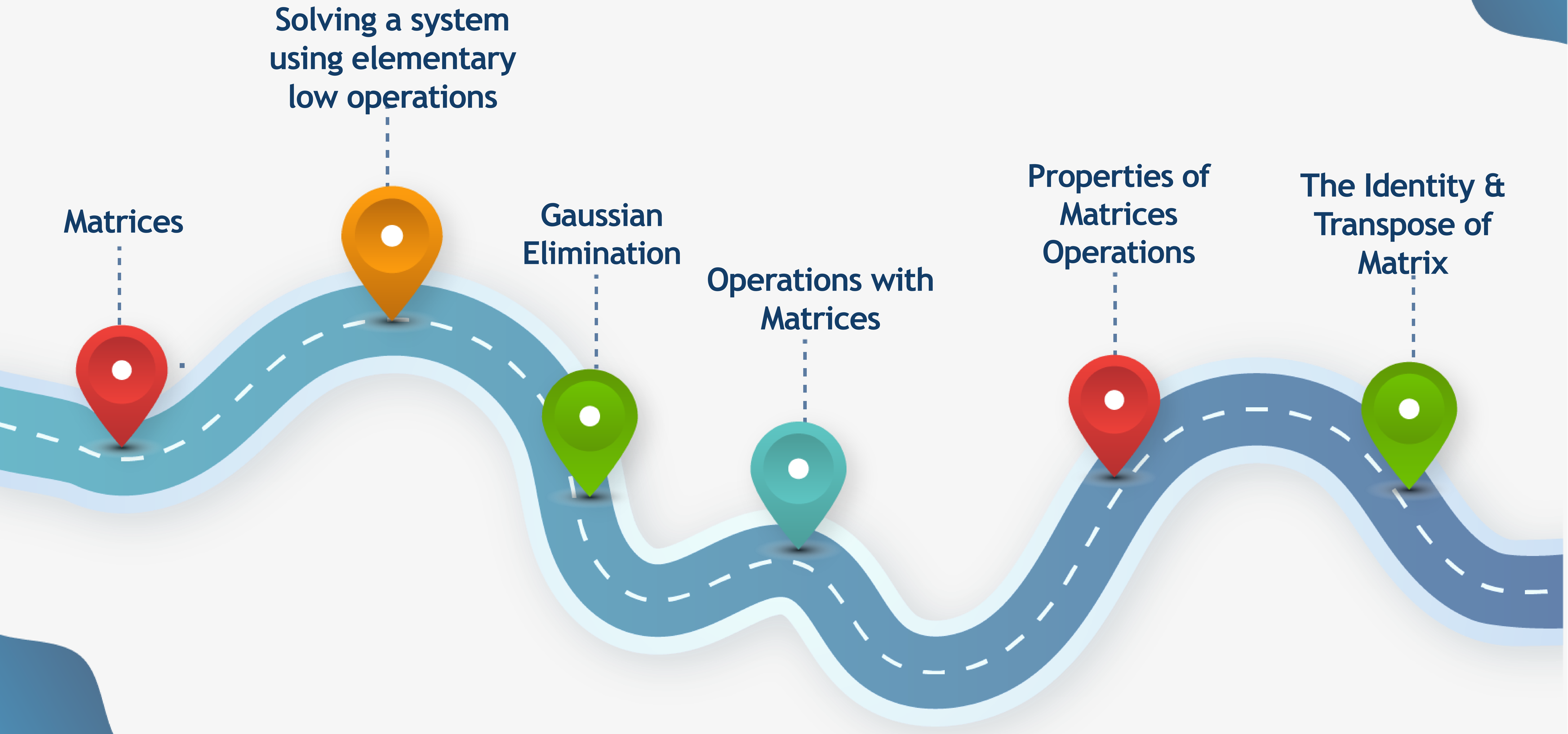

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Matrices

Matrices

- Matrices are used to store and represent the data on the machine.

- A matrix is a rectangular structure with rows and columns to organize data. It can be described as A m*n, where m represents the number of rows and n denotes the number of columns in the matrix. A matrix in linear algebra is used to express linear equations more compactly.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3}$$

Matrices

A matrix is a rectangular array if **m** and **n** are positive integers

$$\underbrace{\left.\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}\right\} m \text{ rows}}_{n \text{ columns}}$$

- each entry of the matrix is a number.
- A matrix m*n (read “m by n”) has m rows (horizontal lines) and n columns (vertical lines).
- The entry a12 is located in row 1 and column 2 . Index 1 is called the row subscript and index 2 is called the column subscript.

Matrices

- If m = n, then the matrix is called a square matrix.
- The entries where the column subscript is equal to the row subscript are called the main diagonal entries.

- **Examples of matrices**

(a) Size: 1×1

$$[2]$$

(b) Size: 2×2

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(c) Size: 1×4

$$\begin{bmatrix} 1 & -3 & 0 & \frac{1}{2} \end{bmatrix}$$

(d) Size: 3×2

$$\begin{bmatrix} e & \pi \\ 2 & \sqrt{2} \\ -7 & 4 \end{bmatrix}$$

Matrices

- We can use matrices to represent systems of linear equations in two forms: An augmented matrix where the coefficients and constant terms are represented, and a coefficients matrix where only the coefficients are represented

System	Augmented Matrix	Coefficient Matrix
$\begin{aligned} x - 4y + 3z &= 5 \\ -x + 3y - z &= -3 \\ 2x - 4z &= 6 \end{aligned}$	$\begin{bmatrix} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{bmatrix}$	$\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$

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Solving a system using elementary Row operations

Elementary Row Operations

- An elementary row operation on an augmented matrix creates a new augmented matrix corresponding to a new (but equivalent) system of linear equations.

- Two matrices are said to be row-equivalent if one can be obtained from the other by a finite sequence of elementary row operations.

- **Elementary Row Operations**

1. Interchange two rows.
2. Multiply a row by a nonzero constant.
3. Add a multiple of a row to another row.

Elementary Row Operations

(a) Interchange the first and second rows.

Original Matrix	*New Row-Equivalent Matrix*	*Notation*
$\begin{bmatrix} 0 & 1 & 3 & 4 \\ -1 & 2 & 0 & 3 \\ 2 & -3 & 4 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 2 & 0 & 3 \\ 0 & 1 & 3 & 4 \\ 2 & -3 & 4 & 1 \end{bmatrix}$	$R_1 \leftrightarrow R_2$

(b) Multiply the first row by $\frac{1}{2}$ to produce a new first row.

Original Matrix	*New Row-Equivalent Matrix*	*Notation*
$\begin{bmatrix} 2 & -4 & 6 & -2 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix}$	$\left(\frac{1}{2}\right)R_1 \rightarrow R_1$

(c) Add -2 times the first row to the third row to produce a new third row.

Original Matrix	*New Row-Equivalent Matrix*	*Notation*
$\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 2 & 1 & 5 & -2 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 0 & -3 & 13 & -8 \end{bmatrix}$	$R_3 + (-2)R_1 \rightarrow R_3$

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Solving a system using elementary row operations-Example

❖ Examples :

Linear System

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y \quad\quad &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

Associated Augmented Matrix

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$$

Add the first equation to the second equation

Add the first row to the second row to produce a new second row

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 3z &= 5 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{bmatrix} \quad R_2 + R_1 \rightarrow R_2$$

Solving a system using elementary row operations-Example

Add -2 times the first equation to the third equation

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 3z &= 5 \\ -y - z &= -1 \end{aligned}$$

Add -2 times the first row to the third row to produce a new third row.

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{bmatrix} \quad R_3 + (-2)R_1 \rightarrow R_3$$

Add the second equation to the third equation.

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 3z &= 5 \\ 2z &= 4 \end{aligned}$$

Add the second row to the third row to produce a new third row.

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{bmatrix} \quad R_3 + R_2 \rightarrow R_3$$

Solving a system using elementary row operations-Example

Multiply the third equation by ½

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 3z &= 5 \\ z &= 2 \end{aligned}$$

Multiply the third row by ½ to produce a new third row

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \left(\tfrac{1}{2}\right)R_3 \rightarrow R_3$$

- You can now use back substitution to get the same solution as the example we solved before.
- The last matrix is in a row-echelon form that we defined before.
- **The solution is $x = 1 , y = -1, z = 2$**

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Gaussian Elimination

Gaussian Elimination

- It is a procedure for solving systems of linear equations

- **Gaussian Elimination with Back-Substitution**

  1. Write the augmented matrix of the system of linear equations.

  2. Use elementary row operations to rewrite the augmented matrix in row-echelon form.

  3. Write the system of linear equations corresponding to the matrix in row-echelon form, and use back-substitution to find the solution.

Gaussian Elimination

❖ **Example :Solve the system**

$$\begin{aligned} x_2 + x_3 - 2x_4 &= -3 \\ x_1 + 2x_2 - x_3 \qquad &= 2 \\ 2x_1 + 4x_2 + x_3 - 3x_4 &= -2 \\ x_1 - 4x_2 - 7x_3 - x_4 &= -19 \end{aligned}$$

1. The augmented matrix for this system is

$$\begin{bmatrix} 0 & 1 & 1 & -2 & -3 \\ 1 & 2 & -1 & 0 & 2 \\ 2 & 4 & 1 & -3 & -2 \\ 1 & -4 & -7 & -1 & -19 \end{bmatrix}.$$

Gaussian Elimination

2. Obtain a leading 1 in the upper left corner and zeros elsewhere in the first column.

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 2 & 4 & 1 & -3 & -2 \\ 1 & -4 & -7 & -1 & -19 \end{bmatrix}$$

The first two rows R1↔R2 are interchanged.

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -6 \\ 1 & -4 & -7 & -1 & -19 \end{bmatrix}$$

Adding -2 times to the first row to the third row to produce a new third-row R3+(-2)R1→R3

Gaussian Elimination

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -6 \\ 0 & -6 & -6 & -1 & -21 \end{bmatrix}$$

Adding -1 times the first row to the fourth row to produce a new fourth row.

$$R4 + (-1)\, R1 \rightarrow R4$$

Gaussian Elimination

3. Now that the first column is in the desired form, you should change the second column as shown below

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -6 \\ 0 & 0 & 0 & -13 & -39 \end{bmatrix}$$

Adding 6 times the second row to the fourth row to produce a new fourth row.

$$\boldsymbol{R4} + (\boldsymbol{6})\ \boldsymbol{R2} \rightarrow \boldsymbol{R4}$$

Gaussian Elimination

4. To write the third column in proper form, multiply the third row by 1/3

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & -13 & -39 \end{bmatrix}$$

Multiplying the third row by 1/3 produces a new third row

$$\mathbf{1/3R3 \rightarrow R3}$$

Gaussian Elimination

5. Similarly, to write the fourth column in proper form, you should multiply the fourth row by -1/13

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Multiplying the fourth row by -1/13 produces a new fourth row.

$$-1/13R4 \to R4$$

Gaussian Elimination

The matrix is now in row-echelon form, and the corresponding system of linear equations is as shown below

$$\begin{aligned} x_1 + 2x_2 - x_3 \phantom{{}- 2x_4} &= 2 \\ x_2 + x_3 - 2x_4 &= -3 \\ x_3 - x_4 &= -2 \\ x_4 &= 3 \end{aligned}$$

- Using back-substitution, you can determine that the solution is

$$X1 = -1\,, \qquad x2 = 2\,, \qquad x3 = 1, \qquad x4 = 3$$

Gaussian Elimination

- If during elimination you obtain a row with all zeros, then the system is inconsistent and has no solutions.

- Gauss Jordan Elimination :
  - It is the same as Gaussian Elimination but the reduction is continued until you reach a reduced row-echelon form.

Gaussian Elimination

- Homogenous Systems of linear equations.
  - A system where all the constant terms are zero.
  - All homogenous systems must have at least one solution: The trivial solution where all variables are set to zero.
- Every homogeneous system of linear equations is consistent. Moreover, if the system has fewer equations than variables, then it must have an infinite number of solutions.

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Operations with Matrices

Matrix Operations

- Matrix operations hold the basic arithmetic operations of addition, subtraction, and multiplication of matrices, which help combine two or more matrices to form a single matrix.
- we can also include the Transpose and Inverse of a matrix as operations on matrices that help to transform a specific matrix onto itself.

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Matrix Operations

❖ There are three ways to represent matrices

- ✓ Using uppercase letters such as A, B, C,.
- ✓ Using a representative element enclosed in brackets like:[ai][aj]
- ✓ By a rectangular array of numbers:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Matrix Operations

❖ The Two matrices are equal if their corresponding elements are equal in two matrices.

Two matrices $A = [aij]$ and $B = [bij]$ are equal if they have the same size and $aij = bij$

❖ A matrix that has one column is called a column matrix or a column vector.

❖ A matrix that has one row is called a row matrix or a row vector.

Matrices Addition

- The addition of two matrices is done by adding each corresponding entry in each matrix.
- Both matrices must be of the same size
- **Definition of Matrix Addition**

If A=[aij] and B=[bij] are matrices of size m x n then their sum is the m x n matrix given by

$$A + B = [aij + bij]$$

The sum of two matrices of different sizes is undefined.

Matrices Addition

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & b_{m3} & \cdots & b_{mn} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

$$[a_{ij}]_{m \times n} \qquad [b_{ij}]_{m \times n} \qquad [a_{ij} + b_{ij}]_{m \times n}$$

➢ Example

- $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1+1 & 2+3 \\ 0+(-1) & 1+2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$

Matrices Subtraction

➢ Subtraction of matrices is done by subtracting each corresponding entry in each matrix, and the two matrices must have the same number of rows and columns.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}}_{[a_{ij}]_{m\times n}} - \underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & b_{m3} & \cdots & b_{mn} \end{bmatrix}}_{[b_{ij}]_{m\times n}} = \underbrace{\begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} & \cdots & a_{1n}-b_{1n} \\ a_{21}-b_{21} & a_{22}-b_{22} & \cdots & a_{2n}-b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}-b_{m1} & a_{m2}-b_{m2} & \cdots & a_{mn}-b_{mn} \end{bmatrix}}_{[a_{ij}-b_{ij}]_{m\times n}}$$

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Scalar Multiplication

- You can multiply a matrix A with a scalar c by multiplying each entry in A with scalar c.

  - **Definition of Scalar Multiplication**

If $A = [aij]$ is an m x n matrix and c is a scalar, then the scalar multiple of A by c is the m x n matrix given by

$$\boldsymbol{cA} = [\boldsymbol{caij}]$$

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Scalar Multiplication

- **Example**

- $A = \begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix}$

- $3A = 3\begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(5) \\ 3(2) & 3(-3) \end{bmatrix} = \begin{bmatrix} 3 & 15 \\ 6 & -9 \end{bmatrix}$

○ Subtraction can be modeled as a combination of scalar multiplication and addition: A + (-1)B

Scalar Multiplication

➢ The entry in the ith row and the jth column of the product AB is obtained by multiplying the entries in the ith row of A by the corresponding entries in the jth column of B and then adding the results.

- **Definition of Matrix Multiplication**

If $A = [aij]$ *is an m x n matrix and* $B = [bij]$ *is an n x p matrix, then the product AB is an m x p matrix* ***AB=[cij]***

Where

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdot\cdot\cdot + a_{in}b_{nj}.$$

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Matrix Multiplication

- Two **matrices** A and B are said to be compatible if the number of columns in A is equal to the number of rows in B.

- **Matrix multiplication Formula :**

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} (aj+bm+cp) & (ak+bn+cq) & (al+bo+cr) \\ (dj+em+fp) & (dk+en+fq) & (dl+eo+fr) \\ (gj+hm+ip) & (gk+hn+iq) & (gl+ho+ir) \end{bmatrix}$$

Matrix Multiplication

- **Example**: Find the product AB of the two matrices

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}.$$

$$AB = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}.$$

Matrix Multiplication

- Matrix multiplication is not commutative.

- They can be used to represent systems of linear equations.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned}
$$

Can be written as the equation AX=B

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Properties of Matrices Operations

Properties of Matrices Operations

➢ Properties of Matrices Addition and scalar multiplication

If A, B, and C are $m \times n$ matrices and c and d are scalars, then the following properties are true.

1. $A + B = B + A$ — Commutative property of addition
2. $A + (B + C) = (A + B) + C$ — Associative property of addition
3. $(cd)A = c(dA)$ — Associative property of multiplication
4. $1A = A$ — Multiplicative identity
5. $c(A + B) = cA + cB$ — Distributive property
6. $(c + d)A = cA + dA$ — Distributive property

[3]

➢ When adding real numbers, the number 0 serves as the additive identity. Meaning that A + O = A where A is an equal-sized matrix with zero for all entries.

Properties of Matrices Operations

➢ The matrix O with all zero entries is called a zero matrix.

➢ Properties of Zero Matrices

If A is an m x n matrix and c is a scalar, then the following properties are true.

1. $A + O_{mn} = A$
2. $A + (-A) = O_{mn}$
3. If $cA = O_{mn}$, then $c = 0$ or $A = O_{mn}$.

-A in property two is called the additive inverse of A

Properties of Matrices Operations

Properties of Matrix multiplication

If A, B, and C are matrices (with sizes such that the given matrix products are defined) and is a scalar, then the following properties are true.

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(A + B)C = AB + BC$
4. $c(AB) = (cA)B = A(cB)$

The multiplication AB is not equal to BA. Matrix multiplication is non-commutative

- If $AC = BC$ it does not mean that $A = B$

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The Identity & Transpose of Matrix

The Identity matrix

- It is a matrix with ones across the diagonal and zeros everywhere else

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

- If A is a matrix of size m x n, then the following properties are true.

1. $AI_n = A$
2. $I_mA = A$

The Transpose of a matrix

- The transpose of a matrix is formed by writing its columns as rows

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix},$$

Size: $m \times n$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & \cdots & a_{m1} \\ a_{12} & a_{22} & a_{32} & \cdots & a_{m2} \\ a_{13} & a_{23} & a_{33} & \cdots & a_{m3} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & a_{3n} & \cdots & a_{mn} \end{bmatrix}.$$

Size: $n \times m$

The Transpose of a matrix

❖ **Example:**

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

$$D = \begin{bmatrix} 0 & 1 \\ 2 & 4 \\ 1 & -1 \end{bmatrix} \qquad D^T = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 4 & -1 \end{bmatrix}$$

The Transpose of a matrix

Properties of transposes

If A and B are matrices and c is a scalar, then the following properties are true.

1. $(A^T)^T = A$	Transpose of a transpose
2. $(A + B)^T = A^T + B^T$	Transpose of a sum
3. $(cA)^T = c(A^T)$	Transpose of a scalar multiple
4. $(AB)^T = B^T A^T$	Transpose of a product

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THANK YOU