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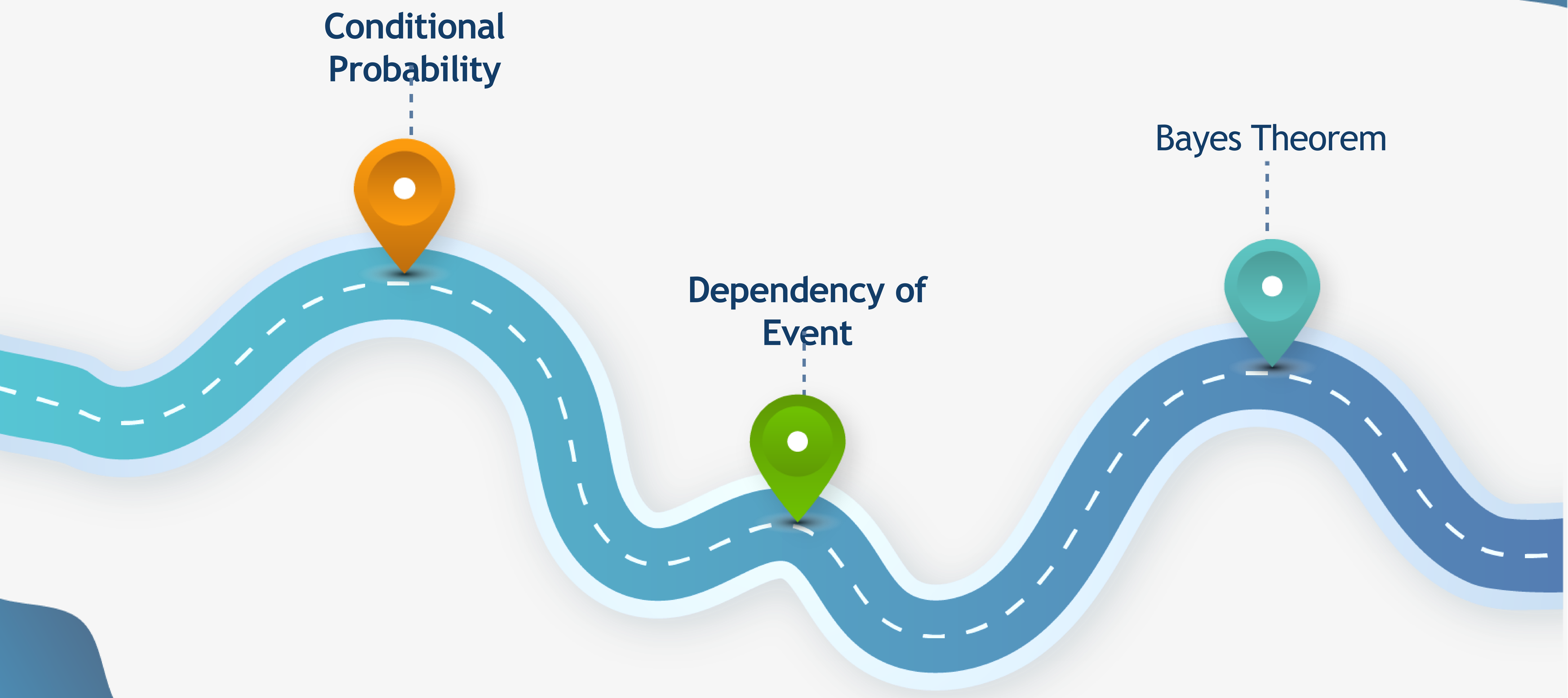
Data Science Fundamentals
Probability Theory 2


Conditional Probability
Dependency of Event
Bayes Theorem

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Conditional Probability

What is Conditional Probability?

- **Conditional Probability**: corresponds to updating one's belief when new information becomes available.
- **Definition**: Let A and B be general events with respect to some sample space and suppose P(A) > 0.

  The conditional probability of B given A is defined as:

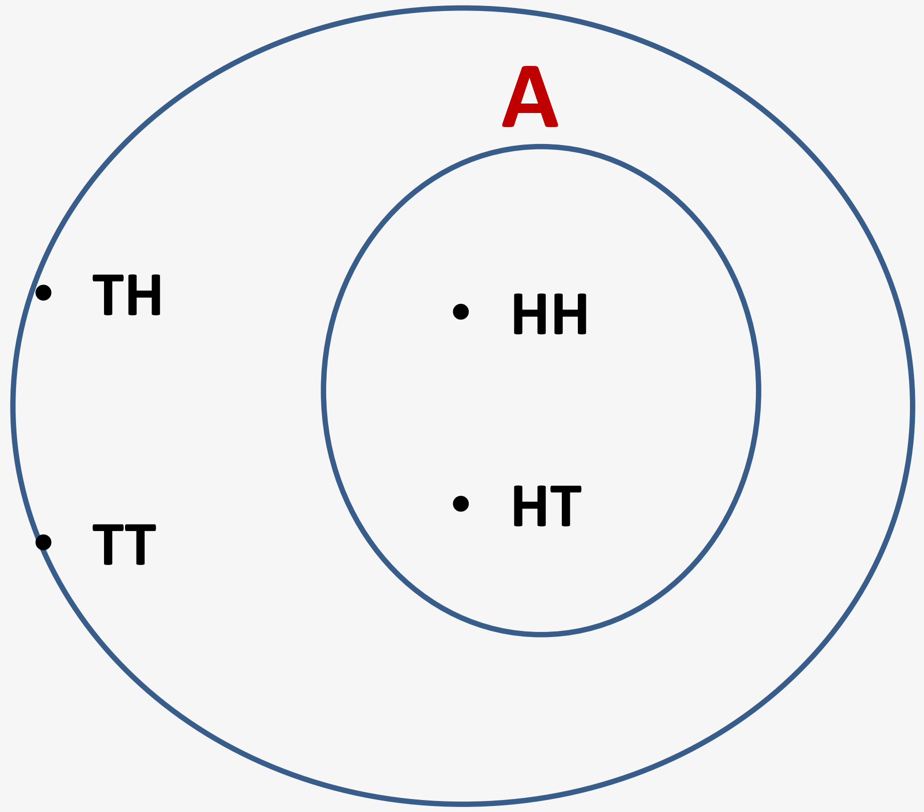
$$p(B|A) = \frac{P(A \cap B)}{P(A)}$$

Examples

Two-coin flips. The first flip is headed. Probability of two heads?

Ω ={HH, HT,TH, TT}; Uniform probability space.

Event A= first flip is heads: A={HH,HT}.



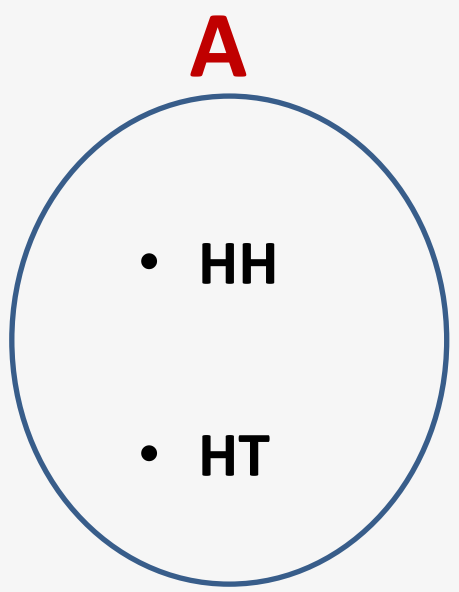
Examples

- **Solution:**

Event B = two heads. = { HH}

The probability of two heads if the first flip is heads= P(B/A)=1/2

The probability of B given A is 1/2.



Examples

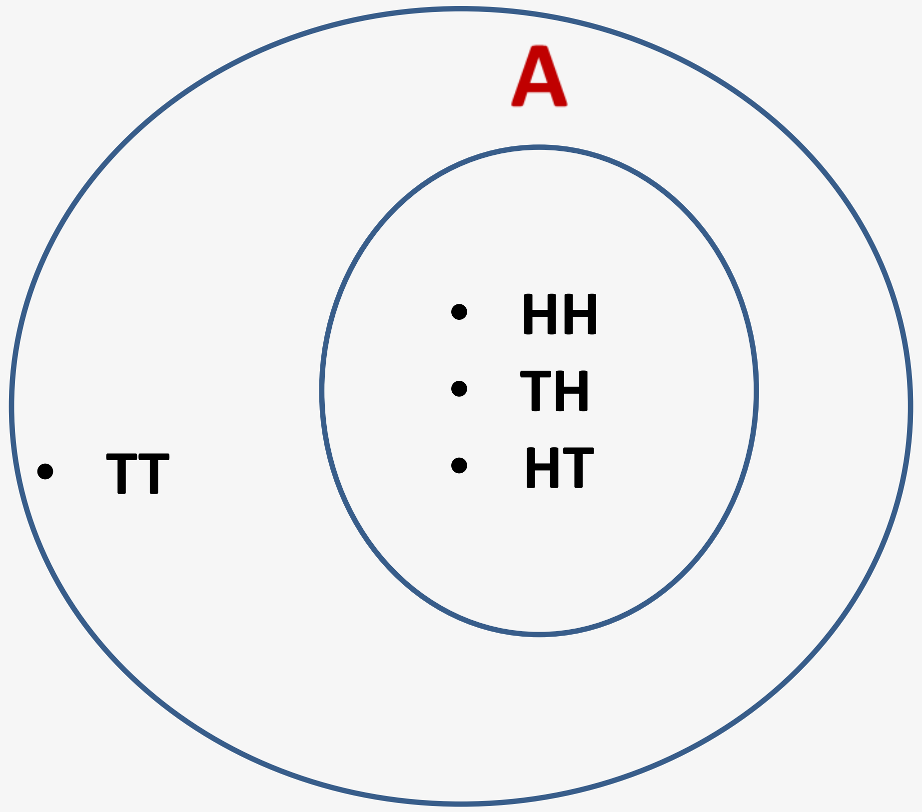
Two-coin flips. At least one of the flips is heads. Probability of two heads?

Solution:

Ω ={HH,HT,TH,TT}; uniform.

Event A= at least one flip is head. A={HH,HT,TH}.

New sample space: A; uniform still. Event B = two heads.



The probability of two heads if at least one flip is head. The probability of B given A is 1/3.

Examples

A card from a deck of 52 cards is missing, but we don't know which one. One card is drawn arbitrarily from the remaining 51 cards. Find the probability that it is a spade. Suppose A= the missing card could be a spade; B = the card chosen from the imputed deck may be a spade.

Solution:

$$p(B) = p(B|A)p(A) + p(B|A^C)p(A^C) = \frac{12}{51} * \frac{1}{4} + \frac{13}{51} * \frac{3}{4} = \frac{1}{4}$$

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Dependency of Events

Dependency of Events

Independence of events corresponds to a lack of probabilistic information in one event A about some other event B.

Definition: Two events A & B are called independent if

$$P(B|A) = P(B) \Leftrightarrow P(A|B)P(A) \Leftrightarrow P(A \cap B) = P(A)P(B)$$

Example

In the experiment of rolling a fair dice twice. Suppose event A is the first roll and it is an even number, and event B which represents the sum of the two rolls is an even number.

Solution:

$P(B)=P(A)=0.5$

$P(A\cap B)=0.25$

$P(B|A)= (P(A\cap B))/(P(A))=(\mathbf{0.25})/(\mathbf{0.5})=\mathbf{0.5}=\boldsymbol{P}(\boldsymbol{B})$

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Bayes Theorem

What is Bayes Theorem?

It is a formula used to calculate a conditional probability when its opposite is known.

Calculating P(A|B) when P(B|A) is known for example.

Definition: Let $\{A_1, A_2, \ldots, A_m\}$ be a partition of a sample space Ω. Let B be some fixed event.

Then $$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{m} P(B|A_i)P(A_i)}$$

Example

Suppose that the questions in a multiple-choice exam have five alternatives each, of which a student picks one because of the correct alternative. A student either knows the truly correct alternative with a probability of 0.7 or he randomly picks one among the five alternatives as his choice. Suppose a particular problem was answered correctly. what's the probability that the student really knew the right answer?

14

Example

- **Solution**:

Define A = the student knew the correct answer and B = the student answered the question correctly. We want to compute P(A|B) .

$$(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} = \frac{1 * 0.7}{1 * 0.7 + 0.2 * 0.3}$$
$$= 0.921$$

Example

- A girl is known to speak the truth 2 out of 5 times. she throws a die and reports that the number obtained is one. Find the probability that the number obtained is one ?

16

Example

Solution:

- The probability that the girl tells the truth is =P(A)=2/5
- The probability that the girl lies is = P(B)=1−P(A)=3/5
- probability of getting one=1/6
- probability of not getting one =5/6

P(A|B)=P(B)P(B|A)/P(A)

the required probability =(1/6)(2/5)/((1/6)(2/5)+(5/6)(3/5))=2/17

References

1. www.bing.com. (2022b). microsoft - Search. [online] Available at: https://www.bing.com/ck/a?
2. Javed, O.A. (n.d.). Probability and Statistics for Engineers and Scientist - 9th Edition (by Walpole, Mayers, Ye).

THANK YOU