Physics Lecture Notes - 8

AC Current and EM Waves

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Introduction to Alternating Current (AC)

Definition and Comparison with Direct Current (DC)

Alternating Current (AC) and Direct Current (DC) are the two fundamental types of current flow in electric circuits. In a Direct Current circuit, the electric charge flows in a single direction. This is the type of current provided by batteries and is used by most electronic devices.

On the other hand, the electric charge in an Alternating Current circuit periodically reverses direction. The alternating movement is typically sinusoidal, but it can also be triangular or square. The electrical supply grids in most countries use Alternating Current due to its advantages in power transmission over long distances.

The mathematical representation of Alternating Current is given by:

$$I(t) = I_0 \sin(\omega t + \phi)$$

where: - $I(t)$ is the instantaneous current, - I_0 is the peak current, - ω is the angular frequency, and - ϕ is the phase of the current.

Explanation of AC Generation - The Dynamo Effect

Alternating Current is generated using the dynamo effect, which is the generation of an electromotive force (and subsequently, current) in a closed circuit by changing the magnetic field through the circuit. This is achieved in practice by rotating a coil within a magnetic field, or by rotating a magnetic field within a stationary coil.

The mathematical representation of the electromotive force induced due to the dynamo effect is given by Faraday's law of electromagnetic induction, which can be stated as:

$$\varepsilon = -N\frac{d\Phi}{dt}$$

where: - ε is the induced electromotive force, - N is the number of turns in the coil, - Φ is the magnetic flux through a single loop, and - t is the time.

Problems

1. A sinusoidal alternating current has a peak current of 5A and a frequency of 60 Hz. Write down the equation representing the current as a function of time.

2. An AC generator has a coil with 100 turns and an area of 0.1 m^2. The coil is rotated at 60 Hz in a magnetic field of 0.5 T. What is the maximum value of the induced EMF?

3. An alternating current is represented by the equation $I(t) = 5\sin(100\pi t)$, where I is in amps and t is in seconds. Determine the peak current, the frequency, the angular frequency, and the period of the current.

4. A coil of wire with 500 turns is placed in a magnetic field which varies with time as $B = B_0 \cos(\omega t)$, where $B_0 = 0.2$ T and $\omega = 2\pi \times 50$ rad/s. If the area of the coil is 0.1 m^2, find the maximum value of the induced emf.

5. The coil of an AC generator has a radius of 0.15 m and consists of 200 turns. It is rotating at an angular speed of 30 rad/s in a uniform magnetic field of 0.8 T. What is the peak voltage of the generator?

Solutions

1. The equation representing the current as a function of time can be written as: $I(t) = I_0 \sin(\omega t)$, where $\omega = 2\pi f$, so the current is $I(t) = 5\sin(120\pi t)$.

2. The maximum value of the induced EMF is given by Faraday's law as $\varepsilon_{max} = NAB\omega$, where $\omega = 2\pi f$. Substituting the given values gives $\varepsilon_{max} = 100 \times 0.1 \times 0.5 \times 2\pi \times 60 \approx 1885$ V.

3. The peak current is 5A, the angular frequency is 100π rad/s, the frequency is $\omega/(2\pi) = 50$ Hz, and the period is $1/f = 0.02$ s.

4. The maximum value of the induced emf is given by $\varepsilon_{max} = NAB\omega$. Substituting the given values gives $\varepsilon_{max} = 500 \times 0.1 \times 0.2 \times 2\pi \times 50 = 1000\pi$ V.

5. The peak voltage of the generator is given by $\varepsilon_{max} = NAB\omega$. Substituting the given values gives $\varepsilon_{max} = 200 \times \pi \times (0.15)^2 \times 0.8 \times 30 \approx 1700$ V.

Fundamental Concepts in AC

Peak Voltage and RMS Voltage

The maximum value of the alternating current or voltage is called the peak value, often denoted by I_0 or V_0. However, since the AC values are changing sinusoidally, it's more practical to use a representative value. One such representative value is the root mean square (RMS) value, defined as the square root of the mean of the squares of the instantaneous values during one complete cycle. The RMS value of an AC is 0.707 times the peak value, or $I_{rms} = I_0/\sqrt{2}$ and $V_{rms} = V_0/\sqrt{2}$.

Phase Difference

Phase difference is a measure of the difference in phase between two waves. In AC circuits, the phase difference is often between the voltage and the current and is usually denoted by ϕ. A positive phase difference means the current leads the voltage, and a negative phase difference means the current lags the voltage.

Frequency and Time Period

The frequency (f) of an AC is the number of cycles it completes in one second. It is measured in hertz (Hz). The time taken to complete one cycle is called the time period (T), and $f = 1/T$.

The Sine Wave Representation

The sine wave is a geometric waveform that oscillates (moves up, down or side-to-side) periodically, and is defined by the function $y = A\sin(B(x - C)) + D$ where A is the amplitude (peak value), B affects the period (width), C is the phase shift (moves left and right) and D is the vertical shift (moves up and down).

Examples

1. Given a sinusoidal voltage source with peak voltage 200V, what is the RMS voltage?

2. An alternating current is described by the equation $I(t) = I_0 \cos(\omega t + \phi)$ where $I_0 = 5$ A, $\omega = 2\pi \times 50$ rad/s, and $\phi = \pi/6$ rad. What is the phase difference in degrees?

3. A generator produces an alternating voltage of frequency 60 Hz. What is the time period of the voltage?

4. An AC voltage is represented by the function $V(t) = V_0 \sin(\omega t)$. If $V_0 =$ 200 V and $f = 50$ Hz, write down the equation for the voltage as a function of time.

Solutions

1. The RMS voltage is given by $V_{rms} = V_0/\sqrt{2}$. Substituting the given peak voltage, $V_0 = 200$ V, we get $V_{rms} = 200/\sqrt{2} \approx 141.42$ V.

2. The phase difference is given in the equation as $\phi = \pi/6$ rad. To convert this into degrees, we use the conversion factor $180/\pi$. Thus, $\phi = (\pi/6) \times (180/\pi) = 30$ degrees.

3. The time period is the reciprocal of the frequency, $T = 1/f$. Substituting the given frequency, $f = 60$ Hz, we get $T = 1/60 \approx 0.0167$ seconds or 16.7 ms.

4. The angular frequency ω is $2\pi f$. Given that $f = 50$ Hz, we have $\omega = 2\pi \times 50 = 100\pi$ rad/s. Therefore, the equation of the voltage as a function of time is $V(t) = 200\sin(100\pi t)$.

Capacitors and AC Circuits

Understanding the Farad Unit

The Farad (F) is the SI unit for capacitance. Capacitance is a measure of how much electric charge is stored for a given electric potential. It is named after Michael Faraday, an English scientist who made significant contributions in the fields of electromagnetism and electrochemistry.

A capacitor has a capacitance of one Farad when a charge of one Coulomb is stored as a result of applying a potential difference of one Volt across it. The mathematical definition can be expressed as follows:

$$1\,\mathrm{F} = 1\,\mathrm{C/V} \tag{1}$$

where C represents Coulomb, the unit for electric charge, and V represents Volt, the unit for electric potential.

Practically, Farad is a large unit, so microfarads (μF), nanofarads (nF), and picofarads (pF) are commonly used.

AC Circuits

In an Alternating Current (AC) circuit, the current and voltage change in a sinusoidal manner with time. Unlike Direct Current (DC) circuits where the current only flows in one direction, in AC circuits, the current alternates its direction periodically. This is the type of electricity that is supplied to homes and businesses, primarily because it is more efficient to generate and transmit over long distances.

In an AC circuit, the voltage V and current I at any time t can be represented as:

$$V = V_0 \sin(\omega t + \phi_V)$$
$$I = I_0 \sin(\omega t + \phi_I)$$

where V_0 and I_0 are the peak voltage and current, ω is the angular frequency, t is time, and ϕ_V and ϕ_I are the phase angles of voltage and current, respectively.

Understanding Reactance

Reactance is a measure of how a circuit component reacts to the change in current. It is a concept specific to alternating current circuits, as in direct current circuits, current and voltage are constant with time. In alternating current circuits, where current and voltage change with time, the circuit elements respond differently.

There are two types of reactance: capacitive reactance and inductive reactance.

Capacitive Reactance (X_C): In a circuit with capacitance, the current leads the voltage by 90 degrees. The capacitive reactance is inversely proportional to the frequency of the alternating current and the capacitance of the component. It can be calculated using the formula:

$$X_C = \frac{1}{\omega C} \tag{2}$$

where ω is the angular frequency and C is the capacitance. As the frequency or the capacitance increases, the capacitive reactance decreases.

Inductive Reactance (X_L): In a circuit with inductance, the voltage leads the current by 90 degrees. The inductive reactance is directly proportional to the frequency of the alternating current and the inductance of the component. It can be calculated using the formula:

$$X_L = \omega L \tag{3}$$

where L is the inductance. As the frequency or the inductance increases, the inductive reactance also increases.

Impedance, which we introduced earlier, is a combination of resistance R, inductive reactance X_L, and capacitive reactance X_C, and it can be represented as follows:

$$Z = R + j(X_L - X_C) \tag{4}$$

where j is the imaginary unit.

The current and voltage in an AC circuit are generally out of phase, and the phase difference depends on the relative values of R, X_L, and X_C.

AC in Circuits

Behavior in Resistive, Capacitive and Inductive Loads

In an AC circuit, the response of different elements to the alternating current depends on their nature.

- **Resistive Load:** In a purely resistive AC circuit, the voltage and current are in phase, meaning they reach their peak values at the same time. The power in a resistive load is always positive, signifying that power is always being absorbed.
- **Capacitive Load:** In a purely capacitive AC circuit, the current leads the voltage by a phase angle of 90 degrees, meaning the current reaches its peak value one quarter of a cycle before the voltage. Reactive power is consumed and then returned to the source, no real power is dissipated.
- **Inductive Load:** In a purely inductive AC circuit, the current lags the voltage by a phase angle of 90 degrees, meaning the current reaches its peak value one quarter of a cycle after the voltage. As in the capacitive load, reactive power is consumed and then returned to the source.

Reactance and Impedance

Reactance is a measure of how much a circuit resists the flow of current due to the effect of inductors and capacitors. Capacitive reactance $X_C = 1/(\omega C)$ and inductive reactance $X_L = \omega L$, where ω is the angular frequency, C is the capacitance, and L is the inductance. Impedance is the total opposition to current flow in an AC circuit, and it's a complex quantity given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Power in AC Circuits: Real, Reactive, and Apparent Power

In an AC circuit, power can be classified into real power (P), reactive power (Q), and apparent power (S). Real power is the capacity of the circuit for performing work in a particular time. Reactive power is the power oscillated between source and load, which does not perform any work. Apparent power is the product of the current and voltage in the circuit.

- **Real Power (P):** $P = IV\cos(\theta)$ where θ is the phase difference between the voltage and the current. The unit of real power is the watt (W).
- **Reactive Power (Q):** $Q = IV\sin(\theta)$ The unit of reactive power is the volt-ampere reactive (VAR).
- **Apparent Power (S):** $S = IV = \sqrt{P^2 + Q^2}$ The unit of apparent power is the volt-ampere (VA).

Power Factor and Its Significance

The power factor of an AC electrical power system is defined as the ratio of the real power flowing to the load to the apparent power, and it is a dimensionless number between -1 and 1. Low power factor circuits require higher current to deliver a certain amount of real power, which can cause power losses. Therefore, improving the power factor can significantly improve the efficiency of the system.

Examples

1. A $50\,\Omega$ resistor, a $30\,\mu F$ capacitor and a $500\,mH$ inductor are connected in series to an AC source of $230\,V$ and $50\,Hz$. Find the impedance of the circuit and the phase angle.
2. A household uses a number of electrical appliances which operate at an average power factor of 0.8. The average power consumption is $3\,kW$. Calculate the apparent power and the reactive power.
3. An AC circuit has an impedance of $50 + j80\,\Omega$. The circuit is connected to a $100\,V$, $50\,Hz$ supply. Find the current and the power factor of the circuit.

Solutions

1.
   - The capacitive reactance $X_C = 1/(2\pi fC) = 1/(2\pi \times 50 \times 30 \times 10^{-6}) = 106.1\,\Omega$
   - The inductive reactance $X_L = 2\pi fL = 2\pi \times 50 \times 500 \times 10^{-3} = 157.1\,\Omega$
   - The impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (157.1 - 106.1)^2} = 64.6\,\Omega$
   - The phase angle $\theta = \arctan((X_L - X_C)/R) = \arctan((157.1 - 106.1)/50) = 45.6°$
2.
   - The real power $P = 3\,kW = 3000\,W$
   - The power factor $\cos(\theta) = 0.8$
   - The apparent power $S = P/\cos(\theta) = 3000/0.8 = 3750\,VA$
   - The reactive power $Q = \sqrt{S^2 - P^2} = \sqrt{3750^2 - 3000^2} = 2250\,VAR$
3.
   - The impedance $Z = 50 + j80\,\Omega = \sqrt{50^2 + 80^2} = 94\,\Omega$
   - The phase angle $\theta = \arctan(80/50) = 58.0°$
   - The current $I = V/Z = 100/94 = 1.06\,A$
   - The power factor $\cos(\theta) = \cos(58.0°) = 0.53$

AC in Transformers

A transformer is a device used in AC circuits for the purpose of changing the voltage and current levels. They are very crucial for power transmission over long distances.

Working Principle of a Transformer

The basic principle on which the transformer works is Faraday's law of electromagnetic induction, which states that a change in the magnetic field within a closed loop of wire induces an electromotive force (EMF) in the wire.

A transformer consists of two coils, the primary coil and the secondary coil, which are wound around a common iron core. When an AC voltage is applied to the primary coil, it creates a changing magnetic field in the iron core. This changing magnetic field induces an EMF in the secondary coil according to Faraday's law.

The ratio of the voltages across the primary and secondary coils is directly proportional to the ratio of the number of turns in the coils, given by

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \tag{5}$$

where V_P and V_S are the voltages across the primary and secondary coils respectively, and N_P and N_S are the number of turns in the primary and secondary coils respectively.

Ideal vs Real Transformers

In an ideal transformer, it is assumed that all the magnetic field lines produced by the primary coil pass through the secondary coil, i.e., the transformer is 100% efficient. However, in a real transformer, some of the magnetic field lines do not pass through the secondary coil. This is known as magnetic leakage and results in the transformer's efficiency being less than 100%.

Furthermore, the resistance of the coils and the magnetization of the core also lead to energy losses in a real transformer.

Significance in Power Transmission

Transformers are crucial in power transmission. They step up the voltage level from the power station to a high level for transmission over long distances, which reduces power loss due to resistance in the wires.

When the power reaches the consumer, transformers step down the voltage to safer levels for use in homes and businesses. Without transformers, the transmission of electricity over long distances would not be practical or efficient.

Examples on Transformers

1. A transformer has 100 turns on the primary coil and 500 turns on the secondary coil. If the input voltage is 120V, what is the output voltage?

2. A step-up transformer increases the voltage from 110V to 220V. If there are 400 turns in the primary coil, how many turns are there in the secondary coil?

3. The primary coil of a transformer has 2000 turns and the secondary coil has 500 turns. If the input voltage is 240V and the input current is 5A, calculate the output voltage and output current.

4. A transformer with 400 turns on the primary coil is used to step down a voltage from 200V to 50V. What is the number of turns on the secondary coil?

5. A transformer is 90% efficient and steps up voltage from 100V to 200V. If the primary coil current is 4A, what is the secondary coil current?

Solutions

1. The output voltage can be calculated using the transformer equation, $V_S = V_P \frac{N_S}{N_P}$. Here, V_P is the input voltage, N_P is the number of turns in the primary coil, N_S is the number of turns in the secondary coil, and V_S is the output voltage. Substituting the given values, we have $V_S = 120V \frac{500}{100} = 600V$.

2. Again, using the transformer equation, the number of turns in the secondary coil can be calculated as $N_S = N_P \frac{V_S}{V_P}$. Substituting the given values, we get $N_S = 400 \frac{220V}{110V} = 800$ turns.

3. Here, we use the transformer equations for both voltage and current. For voltage, $V_S = V_P \frac{N_S}{N_P} = 240V \frac{500}{2000} = 60V$. For current, we use the relation $I_P V_P = I_S V_S$, implying $I_S = I_P \frac{V_P}{V_S} = 5A \frac{240V}{60V} = 20A$.

4. Using the transformer equation for voltage, the number of turns on the secondary coil can be calculated as $N_S = N_P \frac{V_S}{V_P} = 400 \frac{50V}{200V} = 100$ turns.

5. First, calculate the output power using the efficiency, $\eta = \frac{P_{out}}{P_{in}}$, which gives $P_{out} = \eta P_{in} = 0.9(100V \times 4A) = 360W$. Then, the current in the secondary coil can be calculated using $P_{out} = V_S I_S$, implying $I_S = \frac{P_{out}}{V_S} = \frac{360W}{200V} = 1.8A$.

Problem Set

1. An AC generator produces a peak voltage of 170 V. Calculate the RMS voltage.

2. A certain AC voltage has a frequency of 50 Hz and a peak voltage of 200 V. Write the equation of the sine wave representing this voltage.

3. In an AC circuit with only inductive reactance, the current lags the voltage by 90°. If the voltage is represented by $V = V_0 \sin(wt)$, write the equation representing the current.

4. An AC circuit consists of a resistor with resistance 4 Ω, an inductor with inductive reactance 3 Ω, and a capacitor with capacitive reactance 5 Ω. If the peak voltage supplied to the circuit is 230 V, calculate the peak current in the circuit.

5. In a certain AC circuit, the real power is 150 W, the reactive power is 90 VAR, and the apparent power is 180 VA. Calculate the power factor of the circuit.

6. A transformer has 800 turns in the primary coil and 200 turns in the secondary coil. If the input voltage is 220 V, calculate the output voltage and the turns ratio.

7. A step-down transformer has a turns ratio of 10:1, and the primary coil is connected to a 230 V supply. Calculate the secondary voltage. If the primary current is 2 A, what is the secondary current?

8. In a transformer, the primary coil has 350 turns and the secondary coil has 1750 turns. If the input voltage is 110 V, find the output voltage. Also, if the input current is 5 A, find the output current assuming the transformer to be ideal.