Physics Lecture Notes - 3

Dynamics

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Newton's Laws of Motion

Newton's three laws of motion describe the relationship between a body's motion and the forces acting upon it:

1. **First Law (Law of Inertia)**: An object at rest stays at rest, and an object in motion stays in motion with the same speed and direction, unless acted upon by a net external force.
2. **Second Law**: The acceleration of an object is directly proportional to the net external force acting on it and inversely proportional to its mass:

$$\vec{F}_{net} = m\vec{a}$$

3. **Third Law (Action and Reaction)**: For every action, there is an equal and opposite reaction.

Friction and Tension

Friction is a force that opposes the relative motion between two surfaces in contact. There are two types of friction: static friction and kinetic friction. Static friction acts on an object when it is not moving, while kinetic friction acts on a moving object.

The maximum static friction is given by:

$$f_s \leq \mu_s N$$

The kinetic friction is given by:

$$f_k = \mu_k N$$

Here, f_s and f_k are the static and kinetic friction, respectively; μ_s and μ_k are the coefficients of static and kinetic friction, respectively; and N is the normal force.

Tension is a force that acts along a rope, cable, or wire when it is stretched. Tension always pulls objects and has the same magnitude throughout the rope, cable, or wire.

Examples:

1: A 5 kg box is on a horizontal table. A force of 15 N is applied horizontally on the box. The coefficient of kinetic friction between the box and the table is 0.2. Find the acceleration of the box.

Solution: First, let's find the normal force, N, which is equal to the weight of the box:

$$N = mg = 5\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s}^2 = 49\,\mathrm{N}$$

Next, we find the kinetic friction force, f_k:

$$f_k = \mu_k N = 0.2 \cdot 49\,\mathrm{N} = 9.8\,\mathrm{N}$$

Now, we apply Newton's second law to find the acceleration, a:

$$F_{net} = m\vec{a}$$

$$15\,\mathrm{N} - 9.8\,\mathrm{N} = 5\,\mathrm{kg} \cdot a$$

$$a = \frac{5.2\,\mathrm{N}}{5\,\mathrm{kg}} = 1.04\,\mathrm{m/s}^2$$

The acceleration of the box is approximately $1.04\,\mathrm{m/s}^2$.

2: A 10 kg object is hanging from a rope attached to the ceiling. Calculate the tension in the rope.

Solution: We can use Newton's second law to analyze the forces acting on the object. In this case, the object is in equilibrium (not accelerating), so the net force is zero:

$$F_{net} = T - mg = 0$$

Here, T is the tension in the rope, and mg is the weight of the object. Solving for T, we get:

$$T = mg = 10\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s}^2 = 98\,\mathrm{N}$$

The tension in the rope is 98 N.

3: A 2 kg block is pushed horizontally by a force of 10 N. Find the acceleration of the block.

Solution: Since there is no friction, we can directly apply Newton's second law to find the acceleration, a:

$$F_{net} = m\vec{a}$$

$$10\,\mathrm{N} = 2\,\mathrm{kg} \cdot a$$

$$a = \frac{10\,\mathrm{N}}{2\,\mathrm{kg}} = 5\,\mathrm{m/s}^2$$

The acceleration of the block is $5\,\mathrm{m/s}^2$.

4: A 1500 kg car is moving with a constant speed of 20 m/s. Calculate the net force acting on the car.

Solution: Since the car is moving with a constant speed, its acceleration is zero. According to Newton's second law:

$$F_{net} = m\vec{a}$$

Since $a = 0$, we have:

$$F_{net} = 0$$

The net force acting on the car is 0 N.

5: A 3 kg object is acted upon by two forces: $\vec{F}_1 = (4\,\mathrm{N})\hat{\imath} + (3\,\mathrm{N})\hat{\jmath}$ and $\vec{F}_2 = (-1\,\mathrm{N})\hat{\imath} + (6\,\mathrm{N})\hat{\jmath}$. Calculate the net force and the acceleration of the object.

Solution: First, we find the net force acting on the object by summing the two force vectors:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = (4\,\mathrm{N} - 1\,\mathrm{N})\hat{\imath} + (3\,\mathrm{N} + 6\,\mathrm{N})\hat{\jmath} = (3\,\mathrm{N})\hat{\imath} + (9\,\mathrm{N})\hat{\jmath}$$

Now, we apply Newton's second law to find the acceleration, $\vec{a}$:

$$\vec{F}_{net} = m\vec{a}$$

We can write this equation for each component:

$$3\,\mathrm{N} = 3\,\mathrm{kg} \cdot a_x$$
$$9\,\mathrm{N} = 3\,\mathrm{kg} \cdot a_y$$

Solving for a_x and a_y, we get:

$$a_x = \frac{3\,\mathrm{N}}{3\,\mathrm{kg}} = 1\,\mathrm{m/s}^2$$
$$a_y = \frac{9\,\mathrm{N}}{3\,\mathrm{kg}} = 3\,\mathrm{m/s}^2$$

The acceleration of the object is $\vec{a} = (1\,\mathrm{m/s}^2)\hat{\imath} + (3\,\mathrm{m/s}^2)\hat{\jmath}$.

Circular Motion

An object moving in a circle with constant speed experiences centripetal acceleration directed towards the center of the circle. The centripetal force (F_c) required to maintain circular motion can be calculated as:

$$F_c = \frac{mv^2}{r}$$

where m is the mass, v is the speed, and r is the radius of the circle.

Examples:

6: A 0.5 kg object is attached to a string and is moving in a horizontal circle with a radius of 1 m. If the object is moving with a constant speed of 2 m/s, find the tension in the string.

Solution: Since the object is moving in a circle, it experiences a centripetal acceleration:

$$a_c = \frac{v^2}{r} = \frac{(2\,\mathrm{m/s})^2}{1\,\mathrm{m}} = 4\,\mathrm{m/s}^2$$

Now, we apply Newton's second law in the radial direction:

$$F_{net} = T = ma_c$$

Solving for T, we get:

$$T = 0.5\,\mathrm{kg} \cdot 4\,\mathrm{m/s}^2 = 2\,\mathrm{N}$$

The tension in the string is 2 N.

7: A car is driving around a curve with a radius of 50 m at a constant speed of 20 m/s. Calculate the centripetal acceleration of the car.

Solution: The centripetal acceleration can be found using the formula:

$$a_c = \frac{v^2}{r} = \frac{(20\,\mathrm{m/s})^2}{50\,\mathrm{m}} = \frac{400\,\mathrm{m}^2/\mathrm{s}^2}{50\,\mathrm{m}} = 8\,\mathrm{m/s}^2$$

The centripetal acceleration of the car is 8 m/s**²**.

8: A 75 kg person is standing at the edge of a merry-go-round with a radius of 3 m. The merry-go-round makes one full rotation every 10 seconds. Calculate the force the person experiences towards the center of the merry-go-round.

Solution: First, we find the angular velocity, ω:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{10\,\mathrm{s}} = \frac{\pi}{5\,\mathrm{rad/s}}$$

Now, we calculate the linear velocity, v:

$$v = r\omega = 3\,\mathrm{m} \cdot \frac{\pi}{5\,\mathrm{rad/s}} = \frac{3\pi}{5\,\mathrm{m/s}}$$

Next, we find the centripetal acceleration, a_c:

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{3\pi}{5\,\mathrm{m/s}}\right)^2}{3\,\mathrm{m}} = \frac{9\pi^2}{25\,\mathrm{m/s}^2}$$

Finally, we apply Newton's second law in the radial direction:

$$F_{net} = F_c = ma_c$$

Solving for F_c, we get:

$$F_c = 75\,\mathrm{kg} \cdot \frac{9\pi^2}{25\,\mathrm{m/s}^2} = \frac{27\pi^2}{5\,\mathrm{N}}$$

The force the person experiences towards the center of the merry-go-round is approximately $\frac{27\pi^2}{5}$ N (rounded to two decimal places, this is approximately 53.08 N).

Work, Energy, and Power

Work-Energy Theorem

Work (W) is the product of the force (F) applied to an object and the displacement (s) of the object in the direction of the force:

$$W = Fs\cos\theta$$

where θ is the angle between the force and displacement vectors. The work-energy theorem states that the work done on an object is equal to the change in its kinetic energy:

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Conservation of Energy

The principle of conservation of energy states that energy cannot be created or destroyed, only converted from one form to another. In a closed system, the total mechanical energy (the sum of kinetic and potential energy) remains constant.

Potential and Kinetic Energy

Potential energy (PE) is the energy an object possesses due to its position in a force field, such as a gravitational or electric field. For an object near the Earth's surface, the gravitational potential energy can be calculated as:

$$PE = mgh$$

where m is the mass, g is the gravitational acceleration, and h is the height above a reference point.

Kinetic energy (KE) is the energy an object possesses due to its motion. The kinetic energy of an object with mass m and velocity v can be calculated as:

$$KE = \frac{1}{2}mv^2$$

EXAMPLES:

9: A 10 kg box is being pushed across a horizontal surface by a horizontal force of 30 N. The box moves 5 meters. Calculate the work done by the applied force.

Solution: Work done is calculated using the formula:

$$W = Fd\cos\theta$$

In this case, the force is in the same direction as the displacement, so $\theta = 0$, and $\cos\theta = 1$. Therefore:

$$W = (30\,\mathrm{N})(5\,\mathrm{m})(1) = 150\,\mathrm{J}$$

The work done by the applied force is 150 J.

10: A 3 kg block is lifted vertically upwards at a constant speed of 2 m/s for 4 seconds. Calculate the work done by the gravitational force acting on the block.

Solution: First, calculate the height the block is lifted:

$$h = vt = (2\,\mathrm{m/s})(4\,\mathrm{s}) = 8\,\mathrm{m}$$

The gravitational force acting on the block is:

$$F_g = mg = (3\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 29.43\,\mathrm{N}$$

Since the gravitational force acts opposite to the displacement, $\theta = 180°$, and $\cos\theta = -1$. Now, we calculate the work done by the gravitational force:

$$W = Fd\cos\theta = (29.43\,\mathrm{N})(8\,\mathrm{m})(-1) = -235.44\,\mathrm{J}$$

The work done by the gravitational force is -235.44 J.

11: A 60 kg person runs up a flight of stairs with a vertical height of 5 m in 3 seconds. Calculate the person's power output.

Solution: First, calculate the potential energy gained by the person:

$$U = mgh = (60\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(5\,\mathrm{m}) = 2943\,\mathrm{J}$$

Now, we find the power output using the formula:

$$P = \frac{W}{t} = \frac{2943\,\mathrm{J}}{3\,\mathrm{s}} = 981\,\mathrm{W}$$

The person's power output is 981 W.

12: A 0.5 kg ball is attached to a 1.5 m long string and is swung in a vertical circle. The ball is released when it is at the highest point of the circle. Calculate the minimum speed of the ball at the bottom of the circle to ensure it completes the circle.

Solution: At the top of the circle, the gravitational potential energy is at its maximum and the kinetic energy is at its minimum. For the ball to complete the circle, the centripetal force at the top of the circle must be equal to or greater than the gravitational force. This means that:

$$m\frac{v^2}{r} \geq mg$$

Solving for the minimum speed at the top, v, we get:

$$v \geq \sqrt{rg} = \sqrt{(1.5\,\mathrm{m})(9.81\,\mathrm{m/s^2})} \approx 3.83\,\mathrm{m/s}$$

Now, we calculate the minimum kinetic energy at the bottom of the circle to have this speed at the top using conservation of energy:

$$\frac{1}{2}mv_{bottom}^2 - mgh = \frac{1}{2}mv_{top}^2$$

Substituting the values and solving for v_{bottom}, we get:

$$\frac{1}{2}(0.5\,\mathrm{kg})v_{bottom}^2 - (0.5\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(3\,\mathrm{m}) = \frac{1}{2}(0.5\,\mathrm{kg})(3.83\,\mathrm{m/s})^2$$

$$v_{bottom} \approx 5.42\,\mathrm{m/s}$$

The minimum speed of the ball at the bottom of the circle to ensure it completes the circle is approximately 5.42 m/s.

13: A roller coaster car of mass 800 kg is moving in a vertical loop with a radius of 20 m. At the top of the loop, the car has a speed of 15 m/s. Calculate the normal force acting on the car at the top of the loop.

Solution: First, we calculate the gravitational force acting on the car at the top of the loop:

$$F_g = mg = (800\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 7848\,\mathrm{N}$$

Next, we find the centripetal force required to keep the car in the loop:

$$F_c = m\frac{v^2}{r} = (800\,\mathrm{kg})\frac{(15\,\mathrm{m/s})^2}{20\,\mathrm{m}} = 9000\,\mathrm{N}$$

At the top of the loop, the normal force and the gravitational force act in the same direction, so we can write:

$$F_{net} = F_c = F_g + F_N$$

Solving for the normal force, F_N, we get:

$$F_N = F_c - F_g = 9000\,\mathrm{N} - 7848\,\mathrm{N} = 1152\,\mathrm{N}$$

The normal force acting on the car at the top of the loop is 1152 N.

Power

Power is the rate at which work is done or energy is transferred. It is defined as the amount of work done per unit time or the rate of change of energy with respect to time. The SI unit of power is the watt (W), where 1 watt is equal to 1 joule per second (J/s).

The formula for calculating power is given by:

$$P = \frac{W}{\Delta t}$$

where P is the power, W is the work done, and Δt is the time interval.

14: A 60 kg person climbs a 10 m high ladder in 8 seconds. Calculate the power output of the person during the climb.

Solution: First, we calculate the gravitational potential energy gained by the person:

$$U = mgh = (60\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(10\,\mathrm{m}) = 5886\,\mathrm{J}$$

Next, we use the formula for power to find the power output:

$$P = \frac{W}{\Delta t} = \frac{5886\,\mathrm{J}}{8\,\mathrm{s}} = 735.75\,\mathrm{W}$$

The power output of the person during the climb is approximately 735.75 W.

15: An electric motor has a power output of 1500 W and is used to lift a 200 kg crate at a constant speed. Calculate the vertical speed of the crate.

Solution: First, we find the gravitational force acting on the crate:

$$F_g = mg = (200\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1962\,\mathrm{N}$$

Next, we use the formula for power and work:

$$P = \frac{W}{\Delta t} = \frac{F_g \cdot d}{\Delta t}$$

Since the crate is lifted at a constant speed, the force applied by the motor is equal to the gravitational force acting on the crate:

$$F_g = F_{motor}$$

Then, we can calculate the vertical speed of the crate:

$$\frac{1500\,\mathrm{W}}{1962\,\mathrm{N}} = \frac{d}{\Delta t}$$

$$v = \frac{d}{\Delta t} \approx 0.765\,\mathrm{m/s}$$

The vertical speed of the crate is approximately 0.765 m/s.

Problem Set - 3

1. A 4 kg box is initially at rest on a horizontal surface. A horizontal force of 12 N is applied to the box. Calculate the acceleration of the box.
2. Two boxes are connected by a rope on a horizontal surface. The mass of the first box is 6 kg, and the mass of the second box is 10 kg. If a horizontal force of 64 N is applied to the second box, find the acceleration of the system and the tension in the rope.
3. A 5 kg block is placed on a 30° incline. Calculate the acceleration of the block down the incline if the coefficient of kinetic friction between the block and the incline is 0.2.
4. A 50 kg crate is pulled horizontally across the floor with a force of 250 N. If the crate is displaced 12 m, how much work is done by the applied force?
5. A 3 kg object is dropped from a height of 20 m. Calculate the kinetic energy and velocity of the object just before it hits the ground.
6. A 60 kg person climbs a 10 m high ladder in 8 seconds. Calculate the power output of the person during the climb.
7. A 0.5 kg ball on the end of a 1 m string is swung in a horizontal circle. If the ball has a speed of 4 m/s, what is the tension in the string?
8. A car of mass 1000 kg is moving around a circular bend with a radius of 50 m at a constant speed of 20 m/s. Calculate the centripetal force acting on the car and the friction force between the car's tires and the road.
9. An object of mass 2 kg is moving in a vertical circle with a radius of 1.5 m. At the top of the circle, the object has a speed of 5 m/s. Calculate the tension in the string at the top of the circle.
10. A 2 kg block is pushed horizontally with a force of 10 N against a wall. The coefficient of static friction between the block and the wall is 0.6. Determine if the block will move.
11. A 70 kg person jumps off a 5 m high platform into a pool. Calculate the potential energy lost and the kinetic energy gained by the person during the fall.
12. A 300 kg roller coaster car is moving along a horizontal track at a speed of 15 m/s. It then climbs a 25 m high hill. Assuming no friction, calculate the final speed of the roller coaster car at the top of the hill.
13. An object of mass 3 kg is attached to a string and is swung in a vertical circle with a radius of 0.8 m. If the tension in the string at the bottom of the circle is 45 N, calculate the speed of the object at the bottom of the circle.
14. A cyclist does 1200 J of work in 20 seconds to pedal up a hill. Calculate the cyclist's power output during this time.
15. An electric winch lifts a 150 kg load vertically at a constant speed of 1.5 m/s. Calculate the power required to operate the winch.