Physics Lecture Notes - 9

EM Waves and Optics

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Introduction to Electromagnetic Waves

Electromagnetic waves are waves that are capable of traveling through the vacuum of outer space. Unlike mechanical waves, they do not require a medium to propagate, which makes them fundamentally different from waves like sound waves or water waves. These waves are produced by the motion of electrically charged particles and have electrical and magnetic fields associated with them.

An electromagnetic wave can be characterized by its wavelength λ or frequency f, and its speed v. In vacuum, all electromagnetic waves travel at the same speed, the speed of light $c = 3.0 \times 10^8$ m/s, irrespective of their frequency and wavelength. The relation between speed, frequency, and wavelength in vacuum is given by:

$$c = f\lambda \tag{1}$$

Electromagnetic waves include, in order of increasing frequency and decreasing wavelength: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays. This collection of different types of electromagnetic waves is known as the electromagnetic spectrum. Each type of wave has a different range of frequencies and wavelengths associated with it and can be used in different applications.

Properties of Electromagnetic Waves

Some of the important properties of electromagnetic waves include:

- They do not require a medium to propagate.
- They are transverse waves, meaning the oscillations are perpendicular to the direction of energy propagation.
- The electric field and magnetic field in an electromagnetic wave are always perpendicular to each other and to the direction of propagation.
- The speed of electromagnetic waves in vacuum is constant, equal to the speed of light c.
- They can exhibit behaviors like interference and diffraction, similar to light waves.

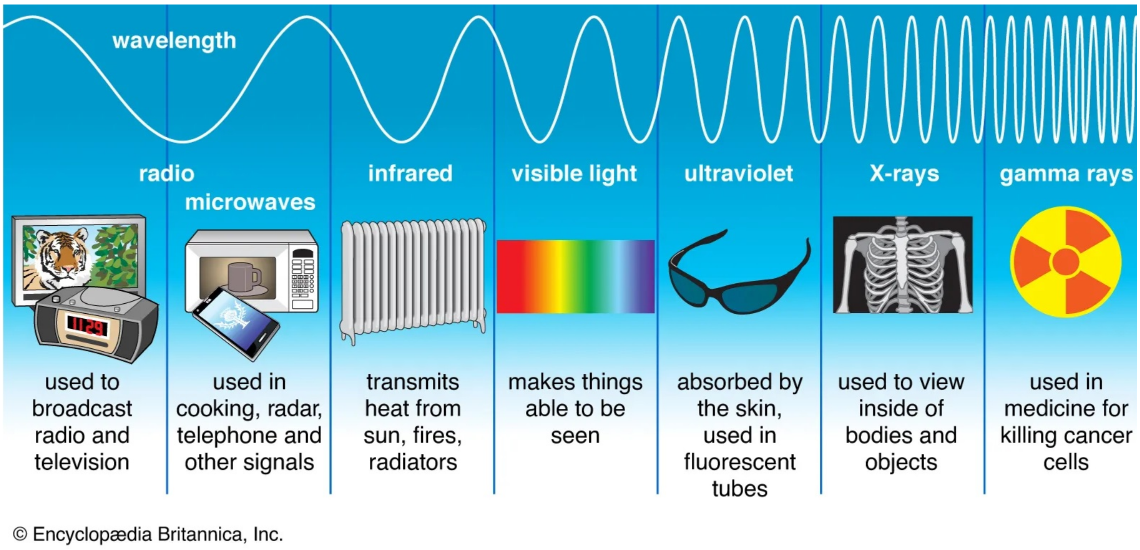
The Electromagnetic Spectrum

The electromagnetic spectrum is the range of all possible frequencies of electromagnetic radiation, arranged from the lowest frequency (longest wavelength) to the highest frequency (shortest wavelength). The main regions of the electromagnetic spectrum, in order of increasing frequency, are:

- Radio waves: These have the longest wavelength and are used in radio and television communication.
- Microwaves: Used in microwave ovens and radar technology.
- Infrared radiation: Emitted by warm objects, and used in thermal imaging.

- Visible light: The small portion of the spectrum that human eyes are sensitive to.
- Ultraviolet radiation: Used in sterilization and in tanning lamps, but harmful in large doses.
- X-rays: Used in medical imaging and in exploring the structure of matter at the atomic level.
- Gamma rays: These have the shortest wavelength, highest frequency, and highest energy. They are produced by nuclear reactions and certain types of radioactive decay.

Each region of the electromagnetic spectrum has unique properties and uses, owing to the different energy levels of the waves.



Generation of Electromagnetic Waves

Electromagnetic waves are generated by accelerating electric charges. This can occur in a variety of situations, including an oscillating electric current in a wire or a moving electron in an atom. In both cases, the acceleration of the charge causes the emission of electromagnetic waves.

Accelerating Charges

When an electric charge is at rest, it produces an electric field but no magnetic field. However, when the charge begins to move, it generates both an electric field and a magnetic field. If the charge is moving at a constant speed, the fields will be steady. But if the charge accelerates or decelerates, the fields will change with time, creating a wave of changing electric and magnetic fields that propagates through space. This is an electromagnetic wave.

The accelerating charge causes the electric field to change, which in turn causes the magnetic field to change. This changing magnetic field then causes the electric field to change again, and so on. This self-sustaining process gives rise to an electromagnetic wave that propagates away from the accelerating charge.

Antenna Theory

An antenna is a device that converts electric power into electromagnetic waves, and vice versa. It does this by using the principle of the accelerating charge.

An antenna typically consists of a conductive wire or rod. When an alternating current is applied to the antenna, the electrons in the antenna wire begin to oscillate back and forth. This oscillation means the electrons are constantly accelerating and decelerating, which as explained above, leads to the generation of electromagnetic waves.

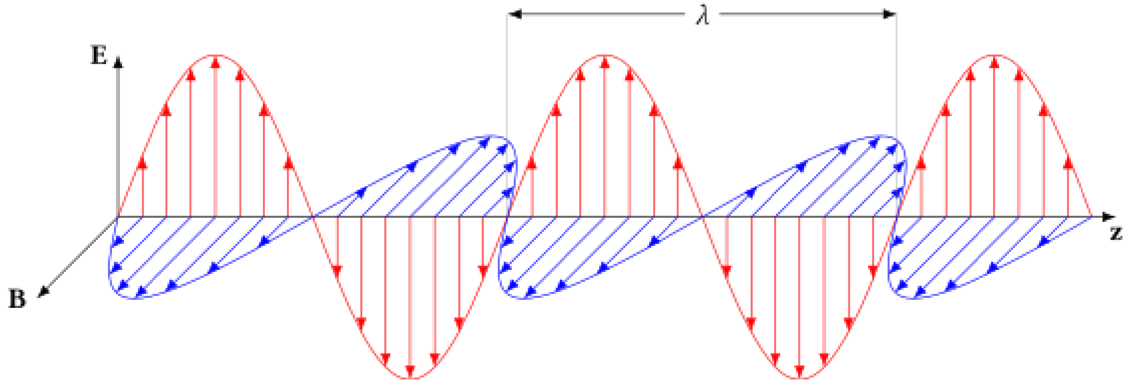
The frequency of the electromagnetic waves produced by an antenna is the same as the frequency of the alternating current that is applied to the antenna. Therefore, by controlling the frequency of the current, we can control the frequency of the electromagnetic waves produced by the antenna.

Properties of Electromagnetic Waves

Electromagnetic waves, including light, have several fundamental properties such as their speed, ability to be polarized, and behaviors including reflection, refraction, and diffraction.

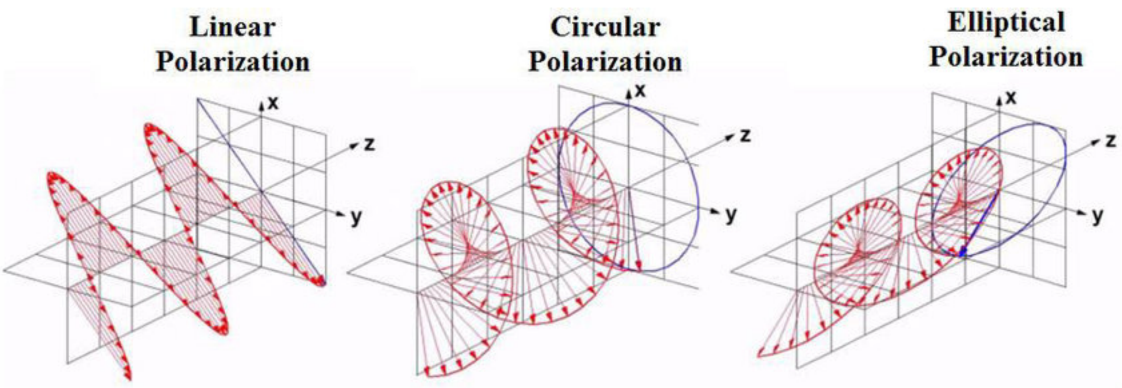
Speed of Light

The speed at which electromagnetic waves propagate in a vacuum is a fundamental constant of nature known as the speed of light, denoted by c. Its value is approximately 3.00×10^8 meters per second. It's worth noting that in materials other than vacuum, such as glass or water, electromagnetic waves travel at a speed less than c due to interactions with the material.



Polarization

Polarization is a property of electromagnetic waves that describes the direction in which the electric field oscillates. In a polarized wave, the electric field oscillates in a specific direction, whereas in an unpolarized wave, the electric field oscillates in all directions perpendicular to the direction of propagation. Polarization can be linear, circular, or elliptical, depending on the nature of the oscillation.



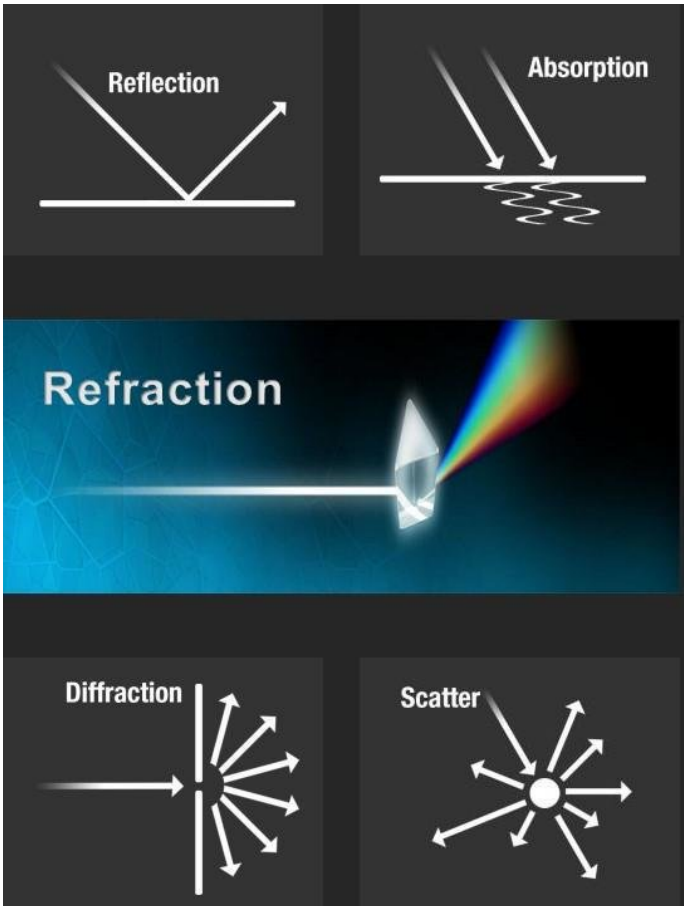
Reflection, Refraction, and Diffraction

When electromagnetic waves encounter a boundary between two different media, several things can happen:

- **Reflection:** The wave can be reflected, or bounce off the boundary. The law of reflection states that the angle of incidence equals the angle of reflection.

- Refraction: The wave can be refracted, or bent as it passes from one medium into another. The law of refraction, or Snell's law, relates the angles of incidence and refraction to the speeds of light in the two media.

- Diffraction: The wave can be diffracted, or spread out as it passes through a narrow opening or around an obstacle. The degree of diffraction depends on the wavelength of the wave and the size of the opening or obstacle.



Applications of Electromagnetic Waves

Electromagnetic waves have a wide range of applications, some of which are highlighted below:

In Communications: Radio, TV, Mobile

The ability to generate and detect electromagnetic waves has revolutionized communication. Radio and TV broadcasts use different frequency bands of the electromagnetic spectrum to transmit audio and video signals. In mobile communications, electromagnetic waves are used to transmit voice and data signals between devices and cell towers.

In Medicine: X-Rays, MRI

Medical imaging technologies often rely on the properties of electromagnetic waves. For example, X-rays use high-frequency electromagnetic waves to produce images of the internal structures of the body. Magnetic Resonance Imaging (MRI) machines use radio waves in combination with strong magnetic fields to generate detailed images of the body.

In Astronomy: Various Types of Telescopes

Different types of telescopes are used to observe electromagnetic waves of different frequencies from space. Optical telescopes collect visible light, radio telescopes collect radio waves, X-ray telescopes collect X-rays, and so on. These observations give astronomers information about distant celestial bodies and phenomena.

Optics

Optics is the branch of physics that deals with the behavior and properties of light, including its interactions with matter and the construction of instruments that use or detect it.

Geometric Optics

Geometric optics, or ray optics, describes light propagation in terms of rays. The ray in geometric optics is an abstraction useful for approximating the paths along which light propagates in certain classes of circumstances.

The two laws of geometric optics are:

- **Law of Reflection:** The angle of incidence equals the angle of reflection. Mathematically, this is expressed as $\theta_i = \theta_r$, where θ_i is the angle of incidence and θ_r is the angle of reflection.
- **Law of Refraction (Snell's law):** The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant that depends on the two materials. Mathematically, this is expressed as $\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{n_t}{n_i}$, where θ_i and θ_t are the angles of incidence and refraction respectively, and n_i and n_t are the refractive indices of the two materials.

Lenses and Mirrors

Lenses and mirrors are used to focus or disperse light. The basic formulas for lenses and mirrors are the lens/mirror equation and the magnification equation.

Lens/Mirror equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$, where f is the focal length, d_o is the object distance, and d_i is the image distance.

Magnification equation: $m = -\frac{d_i}{d_o}$, where m is the magnification, d_i is the image distance, and d_o is the object distance.

Wave Optics

Wave optics, or physical optics, deals with phenomena that can be explained using the wave nature of light.

Interference: The superposition of two or more waves results in an interference pattern. For constructive interference, the path difference between the two waves must be a whole number of wavelengths. For destructive interference, the path difference must be a half-integer number of wavelengths.

Diffraction: Diffraction is the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle.

Polarization: Polarization is the property of waves that can oscillate with more than one orientation. For electromagnetic waves like light, polarization can be linear or circular.

Acoustics

Acoustics is the branch of physics that deals with the study of mechanical waves in gases, liquids, and solids including vibration, sound, ultrasound, and infrasound.

Sound waves: Sound waves are pressure waves that can be represented as $y = A\sin(kx - wt)$, where A is the amplitude, k is the wave number, w is the angular frequency, x is the distance, and t is the time.

Doppler effect: The frequency of a wave depends on the relative speed of the source and the observer. When the source and the observer are moving closer together, the observed frequency is higher than the source frequency. When the source and the observer are moving away from each other, the observed frequency is lower than the source frequency. This is known as the Doppler effect. The formula for the Doppler effect for sound waves is

$$f' = f\frac{v + v_o}{v + v_s}$$

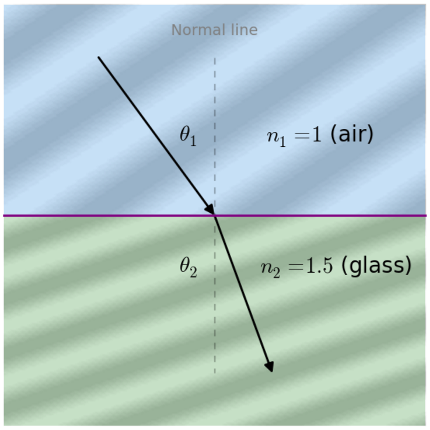
where f' is the observed frequency, f is the source frequency, v is the speed of sound, v_o is the speed of the observer, and v_s is the speed of the source.

Standing waves and resonance: Standing waves are produced by the superposition of two waves of the same frequency traveling in opposite directions. The points of maximum amplitude are known as antinodes, and the points where the amplitude is zero are known as nodes.

Resonance occurs when an object is made to vibrate at its natural frequency, leading to a significant increase in amplitude.

Example Problems

1. **Reflection and Refraction:** A light ray traveling in air strikes a glass surface at an angle of 45° to the normal. If the refractive index of glass is 1.5, calculate the angle of refraction.



2. **Lenses and Mirrors:** An object is placed 10 cm in front of a convex mirror of focal length -5 cm. Determine the position and nature of the image formed.

3. **Interference:** In a double-slit experiment, the slits are 0.02 mm apart and are illuminated by light of wavelength 500 nm. To find the angle for the first-order maximum, use the formula for double-slit interference: $d\sin(\theta) = m\lambda$, where d is the distance between the slits, θ is the angle of the maximum from the central maximum, m is the order number, and λ is the wavelength of the light. Here, you need to find θ when $m = 1$.

4. **Polarization:** Unpolarized light is incident on a polarizer-analyzer pair. The transmission axis of the analyzer is at 30° to that of the polarizer. The transmitted intensity I of light through the analyzer can be calculated by $I = I_0 \cos^2(\theta)$, where I_0 is the initial intensity and θ is the angle between the transmission axes. Here, you need to calculate the fraction of the incident intensity that is transmitted, given $\theta = 30°$.

5. **Acoustics:** A sound wave travels at a speed of 343 m/s in air. Its frequency is 440 Hz (the frequency of the A4 note on a piano). The speed v of a wave is given by $v = f\lambda$, where f is the frequency and λ is the wavelength. Here, you need to find λ, given $v = 343$ m/s and $f = 440$ Hz.

6. **Doppler Effect:** A car is moving towards a stationary observer at a speed of 30 m/s. The car is honking its horn at a frequency of 500 Hz, and the speed of sound in air is 343 m/s. The observed frequency f' is given by $f' = f\frac{v+v_0}{v+v_s}$, where v is the speed of sound, v_0 is the speed of the observer, v_s is the speed of the source, and f is the source frequency. Here, the observer is stationary ($v_0 = 0$) and the source is moving towards the observer ($v_s = -30$ m/s).

7. **Standing Waves and Resonance:** A 1.0 m long string fixed at both ends vibrates in its second harmonic mode. The speed of waves on the string is 200 m/s. The frequency of the nth harmonic on a string of length L with wave speed v is given by $f_n = \frac{nv}{2L}$. Here, you need to find the frequency f_2 of the second harmonic, given $L = 1.0$ m and $v = 200$ m/s.

Solutions

1. Using Snell's law, we get $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$. Here, $n_1 = 1$ (for air), $n_2 = 1.5$ (for glass), and $\theta_1 = 45°$. Solving for θ_2 gives us $\theta_2 \approx 27.6°$.

2. We use the formula for double-slit interference: $d \sin(\theta) = m\lambda$. Here, we need to find θ when $m = 1$, $d = 0.02$ mm $= 0.02 \times 10^{-3}$ m, and $\lambda = 500$ nm $= 500 \times 10^{-9}$ m. Thus, we get $\sin(\theta) = \frac{m\lambda}{d} = \frac{1\times500\times10^{-9}}{0.02\times10^{-3}}$. Solving for θ, we find $\theta = \arcsin(\frac{m\lambda}{d})$.

3. We use the formula for the intensity of light transmitted through a polarizer: $I = I_0 \cos^2(\theta)$. Here, we need to find the fraction of the incident intensity that is transmitted, i.e., $\frac{I}{I_0} = \cos^2(30°)$.

4. We use the formula for the speed of a wave: $v = f\lambda$. Here, we need to find λ given $v = 343$ m/s and $f = 440$ Hz. So, we get $\lambda = \frac{v}{f} = \frac{343}{440}$ m.

5. We use the formula for the observed frequency in the Doppler effect: $f' = f\frac{v+v_0}{v+v_s}$. Here, $v = 343$ m/s, $v_0 = 0$ (as the observer is stationary), $v_s = -30$ m/s (as the source is moving towards the observer), and $f = 500$ Hz. Thus, we get $f' = 500\frac{343+0}{343-30}$ Hz.

6. We use the formula for the frequency of the nth harmonic on a string of length L with wave speed v: $f_n = \frac{nv}{2L}$. Here, we need to find f_2 given $L = 1.0$ m and $v = 200$ m/s. Thus, we get $f_2 = \frac{2\times200}{2\times1.0}$ Hz.

Problem Set - 9

1. An EM wave in a vacuum has a frequency of 5×10^{14} Hz. Calculate its wavelength.
2. A radio station broadcasts at a frequency of 88 MHz. What is the wavelength of these radio waves?
3. Light of wavelength 500 nm falls on a double slit with a separation of 0.1 mm. At what angle will the first order maximum be observed?
4. A plane wave hits a plane mirror at an angle of incidence of 30 degrees. What is the angle of reflection?
5. An observer sees a rainbow with an angle of 42 degrees between the line from the sun to the observer and the line from the observer to the rainbow. What is the index of refraction of water for the color seen by the observer?
6. A lens has a focal length of 5 cm. An object is placed 10 cm away from the lens. Where is the image formed and what is its magnification?
7. Light of wavelength 600 nm passes through a single slit of width 0.05 mm. What is the width of the central maximum on a screen 2 m away?
8. A sound wave of frequency 440 Hz travels from air to water. If the speed of sound in air is 343 m/s and in water is 1500 m/s, what is the wavelength of the sound wave in water?
9. A car moving at 20 m/s towards a stationary observer emits a sound of frequency 500 Hz. If the speed of sound in air is 343 m/s, what frequency does the observer hear?
10. **Question:** Why are electromagnetic waves considered transverse waves?

    Answer: Electromagnetic waves are considered transverse waves because their electric and magnetic fields oscillate perpendicular to the direction of propagation.
11. **Question:** What is the significance of the speed of light being constant?

    Answer: The constancy of the speed of light (in a vacuum) is a cornerstone of Einstein's theory of relativity. It implies that the laws of physics are the same for all observers, regardless of their state of motion or their frame of reference.
12. **Question:** How does polarization help in reducing glare?

    Answer: Polarization helps in reducing glare by blocking horizontally polarized light, which is the majority of light reflected off horizontal surfaces like water or roads.
13. **Question:** What is the difference between constructive and destructive interference?

    Answer: Constructive interference occurs when two or more waves combine to produce a wave of larger amplitude, while destructive interference occurs when two or more waves combine to produce a wave of smaller amplitude or even cancel each other out.
14. **Question:** How does the Doppler effect explain the change in frequency of a siren as it moves towards and then away from an observer?

    Answer: As the siren moves towards the observer, the waves are compressed resulting in a higher frequency (higher pitch). As it moves away, the waves are stretched resulting in a lower frequency (lower pitch).