Physics Lecture Notes - 7

Electricity and Magnetism

Electric Field

The electric field is a vector field that associates to each point in space the (electrostatic or Coulomb) force per unit of charge experienced by a test charge placed at that point. The electric field is generated by electrically charged particles and time-varying magnetic fields.

The electric field around a charge q is given by the formula:

$$\vec{E} = \frac{kq}{r^2}\hat{r} \tag{1}$$

where:

- $\vec{E}$ is the electric field vector,
- $k = 8.99 \times 10^9$ $\mathrm{Nm}^2/\mathrm{C}^2$ is Coulomb's constant,
- q is the amount of charge that produces the field,
- r is the distance from the center of the charge to the point in space where you want to determine the electric field,
- $\hat{r}$ is the unit vector that points away from the charge.

Electric Permittivity

Electric permittivity, often denoted by ε, is a physical quantity that describes how an electric field affects and is affected by a medium. It gives us an idea of how much the medium gets polarized due to an applied electric field, and thus how easily a charge can move through the medium.

The permittivity of free space or vacuum permittivity, denoted by ε_0, is a fundamental physical constant. It is the capability of the vacuum to permit electric field lines. Its value is approximately 8.854×10^{-12} F/m (Farads per meter).

The relative permittivity ε_r (also known as the dielectric constant) of a medium is the ratio of the permittivity of the medium to the permittivity of free space. Thus, $\varepsilon = \varepsilon_r \varepsilon_0$.

Unit of Electric Field

The electric field is defined as the force experienced by a unit positive charge due to an electric charge or a group of charges. Mathematically, the electric field $\vec{E}$ is defined as $\vec{E} = \frac{\vec{F}}{q}$, where $\vec{F}$ is the force experienced by a charge q due to the electric field.

The unit of force in the International System of Units (SI) is newton (N) and that of charge is coulomb (C). Therefore, the unit of electric field becomes newton per coulomb (N/C).

In terms of other fundamental constants, the electric field can also be expressed in volts per meter (V/m) since 1 V = 1 Nm/C.

Electric Potential

The electric potential (also known as electrostatic potential) at a point in space due to an electric charge Q is defined as the work done by an external agent in bringing a positive test charge from infinity to that point in the field without any acceleration.

The electric potential V created by a charge Q at a distance r is given by:

$$V = \frac{kQ}{r} \tag{2}$$

The potential energy of a system of charges is then defined as the work required to assemble that system of charges by bringing them in from infinity. If we have a single charge Q, the work W done to bring a test charge q from infinity to a distance r from the charge Q is:

$$W = \int_{\infty}^{r} \vec{F} \cdot d\vec{r} \tag{3}$$

Since $\vec{F} = q\vec{E}$ and $\vec{E} = \frac{kQ}{r^2}\hat{r}$, we can substitute these into the work integral:

$$W = \int_{\infty}^{r} q\frac{kQ}{r^2} dr \tag{4}$$

This integral simplifies to $W = \frac{kqQ}{r}$. This is the work done on the test charge, and by the work-energy theorem, it is also the change in potential energy of the system. Therefore, we have $U = \frac{kqQ}{r}$, which gives us the formula for the electric potential.

Coulomb's Law and Electric Force

Coulomb's law describes the force between two charges. The force experienced by a charge q_1 due to the presence of another charge q_2 at a distance r apart is given by Coulomb's Law:

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} \tag{5}$$

Where:

- $\vec{F}$ is the force experienced,
- $k = 8.99 \times 10^9\ \mathrm{Nm}^2/\mathrm{C}^2$ is Coulomb's constant,
- q_1 and q_2 are the amounts of the two charges,
- r is the distance between the two charges,
- $\hat{r}$ is the unit vector that points from charge q_2 to charge q_1.

The direction of the force is along the line joining the charges. The force is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

Potential Energy of a Two-Charge System

The electric potential energy of a system of two charges is the work done to assemble the system. The work done to bring a charge q_2 from infinity to a distance r from a stationary charge q_1 is given by:

$$U = \frac{kq_1 q_2}{r} \tag{6}$$

Principle of Superposition

The principle of superposition states that the net electric force experienced by a charge due to a collection of other charges is simply the vector sum of the individual forces exerted on that charge by each one of the other charges taken separately.

Let's consider a system of n charges. The total force $\vec{F}_{\text{net}}$ experienced by charge q_1 due to $n-1$ other charges is given by:

$$\vec{F}_{\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} \tag{7}$$

Where $\vec{F}_{12}$ is the force on charge q_1 due to charge q_2, $\vec{F}_{13}$ is the force on charge q_1 due to charge q_3 and so on.

Suppose we have three charges: q_1, q_2, and q_3. The force on q_1 due to q_2 is $\vec{F}_{12}$ and the force on q_1 due to q_3 is $\vec{F}_{13}$. According to the principle of superposition, the net force $\vec{F}_{\text{net}}$ on q_1 is given by:

$$\vec{F}_{\text{net}} = \vec{F}_{12} + \vec{F}_{13} \tag{8}$$

Electricity in Parallel Plates

Consider two parallel plates of area A separated by a distance d, with a potential difference V between them. One plate carries a charge $+Q$ and the other carries a charge $-Q$.

Electric Field

The electric field $\vec{E}$ between the plates is uniform and directed from the positive plate to the negative plate. It can be calculated by using the formula:

$$\vec{E} = \frac{V}{d} \tag{9}$$

where V is the potential difference (voltage) between the plates and d is the separation between the plates.

Potential Energy

The potential energy U of a charge q placed in the electric field between the plates is given by:

$$U = qV \tag{10}$$

Example

Suppose we have two parallel plates separated by a distance of 0.02 m and have a potential difference of 50 V between them.

Question 1: What is the electric field between the plates?
By using the formula of the electric field, we have:

$$\vec{E} = \frac{V}{d} = \frac{50\,\text{V}}{0.02\,\text{m}} = 2500\,\text{N/C} \tag{11}$$

Question 2: What is the potential energy of a charge of 2×10^{-6} C placed between the plates?
By using the formula of the potential energy, we have:

$$U = qV = (2 \times 10^{-6}\,\text{C})(50\,\text{V}) = 0.1\,\text{J} \tag{12}$$

Consider two parallel plates each of area A separated by a distance d. One plate carries a charge $+Q$ and the other carries a charge $-Q$.

Electric Field

The electric field $\vec{E}$ produced by the positively charged plate is given by

$$\vec{E}_+ = \frac{\sigma}{\varepsilon_0}\hat{n} \tag{13}$$

where $\sigma = \frac{Q}{A}$ is the surface charge density of the plate and $\hat{n}$ is a unit vector normal to the surface pointing away from the positive plate, and ε_0 is the permittivity of free space.

Similarly, the electric field $\vec{E}_-$ produced by the negatively charged plate is

$$\vec{E}_- = -\frac{\sigma}{\varepsilon_0}\hat{n} \tag{14}$$

The total electric field $\vec{E}$ inside the capacitor (between the plates) is the vector sum of $\vec{E}_+$ and $\vec{E}_-$, and hence

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\sigma}{\varepsilon_0}\hat{n} - \frac{\sigma}{\varepsilon_0}\hat{n} = \frac{2\sigma}{\varepsilon_0}\hat{n} = \frac{2Q}{A\varepsilon_0}\hat{n} \tag{15}$$

Electric Potential

The electric potential difference V between the plates is the work done per unit positive charge in moving a test charge from the negative plate to the positive plate against the electric field, and is given by

$$V = \vec{E} \cdot d = \frac{2Q}{A\varepsilon_0}d \tag{16}$$

where $\vec{E} \cdot d$ is the dot product of the electric field vector and the displacement vector (from the negative plate to the positive plate).

Definition of Current and Movement of Electric Charges

Electric current is defined as the rate of flow of electric charges. If a net charge ΔQ passes through a cross-sectional area of a conductor in a time interval Δt, the average current I during this interval is defined as:

$$I_{avg} = \frac{\Delta Q}{\Delta t} \tag{17}$$

In the limit that Δt approaches zero, the instantaneous current is defined as:

$$I = \frac{dQ}{dt} \tag{18}$$

The unit of electric current is the Ampere (A), which is equivalent to 1 Coulomb/second.

Example:

Suppose 20 Coulombs of charge pass through a wire in 4 seconds.

$$I_{avg} = \frac{\Delta Q}{\Delta t} = \frac{20\,\mathrm{C}}{4\,\mathrm{s}} = 5\,\mathrm{A} \tag{19}$$

So, the current in the wire is 5 Amperes.

Example:

If the charge flow in a wire varies with time according to the equation $Q = 5t^2$ (where Q is in Coulombs and t is in seconds), the instantaneous current at time $t = 3\,\mathrm{s}$ is:

$$I = \frac{dQ}{dt} = \frac{d(5t^2)}{dt} = 10t,$$
$$I(t = 3\,\mathrm{s}) = 10 \times 3 = 30\,\mathrm{A}.$$

So, the instantaneous current at $t = 3\,\mathrm{s}$ is 30 Amperes.

Magnetic Field

A magnetic field is a vector field that describes the magnetic influence on moving electric charges, electric currents, and magnetic materials. It is represented by the symbol $\vec{B}$.

A charge that is moving in a magnetic field experiences a force that is perpendicular to both its velocity and the magnetic field. The magnetic field $\vec{B}$ is defined from the Lorentz Force Law, and specifically from the magnetic force on a moving charge:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{20}$$

where $\vec{F}$ is the force acting on the particle, q is the charge of the particle, $\vec{E}$ is the electric field, $\vec{v}$ is the velocity of the particle, and $\vec{B}$ is the magnetic field.

Unit of Magnetic Field

The unit of magnetic field in the International System of Units (SI) is the tesla (T). One tesla is equal to one newton per ampere-meter (N/A·m) which can be understood as the magnetic flux density which is force per unit charge per unit velocity.

Another commonly used unit, especially in magnetic field of the Earth measurements, is the gauss (G). However, it is not an SI unit. To convert from gauss to tesla, we use the relation $1\,\mathrm{T} = 10^4\,\mathrm{G}$.

Derivation of the Unit of Magnetic Field (Tesla)

The Lorentz force law describes the effect of a magnetic field on a moving charge. Mathematically, it is expressed as:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Where:

- $\vec{F}$ is the force experienced by the charge (in Newtons, N),
- q is the charge (in Coulombs, C),
- $\vec{v}$ is the velocity of the charge (in meters per second, m/s), and
- $\vec{B}$ is the magnetic field (in Tesla, T).

Rearranging the equation for $\vec{B}$ gives:

$$\vec{B} = \frac{\vec{F}}{q\vec{v}}$$

The right-hand side units are $\frac{\mathrm{N}}{\mathrm{C \cdot m/s}}$. Since the unit of current (I) is charge per unit time (C/s or Amperes, A), we can express the unit of magnetic field as:

$$\mathrm{T} = \frac{\mathrm{N}}{\mathrm{A \cdot m}}$$

So, one Tesla is defined as one Newton per Ampere per meter.

Creation of Magnetic Fields

Magnetic fields are created by magnetic materials and electric currents:

Magnetic Fields from Magnetic Materials

Magnetic fields originate from magnetic materials due to the alignment of magnetic domains within the material. When these domains are aligned in the same direction, the material exhibits a net magnetic field. This is seen in permanent magnets.

For example, an iron bar magnet has a magnetic field because the spins of its unpaired electrons are aligned in the same direction, creating a net magnetic field. The lines of magnetic force run from the north pole of the magnet to the south pole.

Magnetic Fields from Electric Currents

According to Ampère's law, an electric current produces a magnetic field around it. The direction of the magnetic field is given by the right-hand rule. If you wrap your fingers around the current-carrying wire in the direction of the current (from positive to negative), your thumb will point in the direction of the magnetic field.

For example, consider a straight wire carrying a current. The magnetic field lines will be concentric circles around the wire, with the direction given by the right-hand rule.

Magnetic Fields from Changing Electric Fields

According to Faraday's law of electromagnetic induction, a changing electric field produces a magnetic field. This is the principle behind the operation of many electrical devices, such as transformers and electric generators.

For example, in a transformer, an alternating current in the primary coil creates a changing magnetic field. This changing magnetic field, in turn, induces an electric current in the secondary coil.

Magnetic Field of a Linear Current

According to Ampère's law, a current-carrying wire generates a magnetic field around it. The direction of the magnetic field at any point around the wire is tangent to a circle centered at the wire, and its magnitude is given by:

$$B = \frac{\mu_0 I}{2\pi r} \tag{21}$$

where B is the magnetic field, μ_0 is the permeability of free space, I is the current, and r is the distance from the wire.

Magnetic Field in a Loop and Solenoids

Magnetic Field in a Loop

A current-carrying loop generates a magnetic field around it. The direction of the magnetic field is given by the right-hand rule, where if you wrap your fingers in the direction of the current, your thumb will point in the direction of the magnetic field. At the center of a current-carrying loop, the magnetic field (B) is given by the formula:

$$B = \frac{\mu_0 I}{2R} \tag{22}$$

where μ_0 is the permeability of free space, I is the current, and R is the radius of the loop.

Solenoids

A solenoid is a coil of many wire loops closely packed together. The magnetic field inside a long solenoid is nearly uniform and is directed along the axis of the solenoid. It is given by the formula:

$$B = \mu_0 n I \tag{23}$$

where n is the number of turns per unit length. Outside the solenoid, the magnetic field is weak and can be assumed to be zero for an ideal solenoid.

Importance of Solenoids in Engineering

Solenoids play a crucial role in a variety of engineering applications. They are used to convert electrical energy into mechanical energy, which can be used to control a switch or valve, move or position a machine component, and more. Solenoids are also used in many electronic devices, such as doorbells, cars (to start the engine), and pinball machines. In addition, solenoids are used in the medical field for magnetic resonance imaging (MRI) machines. They are used in physics and engineering labs for producing a uniform magnetic field.

Magnetic Permeability

Magnetic permeability, often denoted by μ, is a physical quantity that describes how a magnetic field affects and is affected by a medium. It is a measure of how much the medium gets magnetized due to an applied magnetic field, and thus how easily a magnetic field can pass through the medium.

The permeability of free space or vacuum permeability, denoted by μ_0, is a fundamental physical constant. It represents the extent to which a magnetic field can pass through the vacuum. Its value is approximately $4\pi \times 10^{-7}$ Tm/A (Tesla meter per Ampere).

The relative permeability μ_r of a medium is the ratio of the permeability of the medium to the permeability of free space. Thus, $\mu = \mu_r \mu_0$.

Magnetic Field of a Circular Current

For a circular loop of wire carrying a current, the magnetic field at the center of the loop is given by:

$$B = \frac{\mu_0 I}{2R} \tag{24}$$

where R is the radius of the loop.

Problems and Solutions

Problem 1

A long straight wire carries a current of $2A$. Calculate the magnetic field $1m$ from the wire.

Solution 1

The magnetic field of a current-carrying wire is given by $B = \frac{\mu_0 I}{2\pi r}$. Substituting $I = 2A$, $r = 1m$, and $\mu_0 = 4\pi \times 10^{-7} Tm/A$, we find that $B = 4 \times 10^{-7} T$.

Problem 2

A circular loop of wire with a radius of $0.5m$ carries a current of $3A$. Calculate the magnetic field at the center of the loop.

Solution 2

The magnetic field at the center of a loop is given by $B = \frac{\mu_0 I}{2R}$. Substituting $I = 3A$, $R = 0.5m$, and $\mu_0 = 4\pi \times 10^{-7} Tm/A$, we find that $B = 3.6 \times 10^{-6} T$.

Problem 3

A solenoid has 1000 turns per meter and carries a current of $0.5A$. Calculate the magnetic field inside the solenoid.

Solution 3

The magnetic field inside a solenoid is given by $B = \mu_0 n I$. Substituting $n = 1000 turns/m$, $I = 0.5A$, and $\mu_0 = 4\pi \times 10^{-7} Tm/A$, we find that $B = 6.28 \times 10^{-4} T$.

Problem 4

The magnetic field $2cm$ from a wire is $5 \times 10^{-5}T$. What current is the wire carrying?

Solution 4

Rearranging the equation for the magnetic field of a wire to solve for I, we have $I = \frac{2\pi r B}{\mu_0}$. Substituting $r = 0.02m$, $B = 5 \times 10^{-5}T$, and $\mu_0 = 4\pi \times 10^{-7}Tm/A$, we find that $I = 1.59A$.

Problem 5

A wire is bent into a circle with a radius of $10cm$ and carries a current of $1.5A$. What is the magnetic field at a point on the axis of the circle, $10cm$ from the center of the circle?

Solution 5

The magnetic field of a circular loop at a distance z along the axis is given by $B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$. Substituting $I = 1.5A$, $R = 0.1m$, $z = 0.1m$, and $\mu_0 = 4\pi \times 10^{-7}Tm/A$, we find that $B = 3 \times 10^{-6}T$.

Electric Fields and Potential

Problem 1

Two charges $q_1 = 5\mu C$ and $q_2 = -3\mu C$ are located at the origin and the point $(0, 2m)$ on the y-axis respectively. What is the electric field and potential at a point P located at $(3m, 0)$ on the x-axis?

Solution 1

The electric field and potential due to a point charge Q at a distance r are given by $E = \frac{kQ}{r^2}$ and $V = \frac{kQ}{r}$, where k is Coulomb's constant. Therefore, the electric field and potential at point P due to both charges are obtained by superposing the contributions from each charge.

Magnetism

Problem 2

A long solenoid with $n = 100$ turns per meter carries a current of 5 A. Calculate the magnetic field inside the solenoid.

Solution 2

The magnetic field inside a long solenoid carrying current I is given by $B = \mu_0 n I$, where μ_0 is the permeability of free space and n is the number of turns per meter.

Magnetic Fields and Forces

Problem 3

An electron with speed $v = 10^7$ m/s enters a region with a uniform magnetic field of $B = 0.5$ T perpendicular to the velocity of the electron. Calculate the radius of the circular path that the electron will follow.

Solution 3

The radius r of the path of a charged particle with charge q moving with speed v perpendicular to a magnetic field B is given by $r = \frac{mv}{|q|B}$, where m is the mass of the particle.

Electromagnetic Induction

Problem 4

A coil with $N = 100$ turns and area $A = 1$ m^2 is oriented with its plane perpendicular to a magnetic field of $B = 0.2$ T. The coil is flipped so that its plane is now parallel to the field. This flip takes 0.1 seconds. What is the average induced emf in the coil during the flip?

Solution 4

The average induced emf in the coil is given by $\epsilon_{avg} = \frac{\Delta\Phi_B}{\Delta t}$, where $\Delta\Phi_B$ is the change in magnetic flux and Δt is the change in time.

Electric Circuits

An electric circuit is a path in which electrons from a voltage or current source flow. Electric current flow in a closed circuit involves continuous movement of free-electrons, driven by the electric field, around a loop.

Current, Voltage, and Resistance

Current

Electric current (I) is the rate of charge flow past a given point in an electric circuit, measured in Amperes (A). Mathematically, it is represented as $I = \frac{dQ}{dt}$, where Q is the electric charge and t is time.

Voltage

Voltage, also called electric potential difference, is the electric potential energy per unit charge, measured in volts (V). It is given by $V = \frac{W}{Q}$, where W is the work done by the force and Q is the charge.

Resistance

Resistance (R) is a measure of the opposition to current flow in an electric circuit. Resistance is measured in Ohms (Ω). It is given by $R = \frac{V}{I}$, where V is the voltage and I is the current.

Ohm's Law

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points, and inversely proportional to the resistance between them. Mathematically, it is represented as $V = I \cdot R$.

Example: If a current of 2 A flows through a resistor of 5 Ω, then the voltage across the resistor is $V = 2A \cdot 5\Omega = 10V$.

Series and Parallel Circuits

Series Circuits

In a series circuit, the same current flows through each component. The total resistance, R_{total}, is the sum of the individual resistances: $R_{total} = R_1 + R_2 + R_3 + ...$

Example: If three resistors with resistances 1 Ω, 2 Ω, and 3 Ω are connected in series, then the total resistance is $1\Omega + 2\Omega + 3\Omega = 6\Omega$.

Parallel Circuits

In a parallel circuit, the voltage across each component is the same, and the total current is the sum of the currents through each component. The total resistance R_{total} is given by $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.$

Example: If two resistors with resistances 3 Ω and 6 Ω are connected in parallel, then the total resistance is given by $\frac{1}{R_{total}} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{2\Omega}$, so $R_{total} = 2\Omega$.

Examples

1. A circuit contains a resistor with a resistance of $5\,\Omega$. If a current of $3\,A$ flows through the circuit, what is the voltage across the resistor?
   - **Solution:** Using Ohm's Law $V = I \cdot R$, we can find the voltage: $V = 3\,A \cdot 5\,\Omega = 15\,V$.
2. Three resistors with resistances of $2\,\Omega$, $3\,\Omega$, and $5\,\Omega$ are connected in series. What is the total resistance?
   - **Solution:** In a series circuit, the total resistance is the sum of the individual resistances: $R_{\text{total}} = 2\,\Omega + 3\,\Omega + 5\,\Omega = 10\,\Omega$.
3. Two resistors with resistances of $4\,\Omega$ and $8\,\Omega$ are connected in parallel. What is the total resistance?
   - **Solution:** In a parallel circuit, the total resistance is given by the reciprocal of the sum of the reciprocals of individual resistances: $\frac{1}{R_{\text{total}}} = \frac{1}{4\,\Omega} + \frac{1}{8\,\Omega} = \frac{1}{2\,\Omega}$, so $R_{\text{total}} = 2\,\Omega$.
4. A resistor with a resistance of $10\,\Omega$ is connected to a power supply of $120\,V$. What is the current through the resistor?
   - **Solution:** Using Ohm's Law $V = I \cdot R$, we can find the current: $I = \frac{V}{R} = \frac{120\,V}{10\,\Omega} = 12\,A$.
5. A circuit contains two resistors in series, $R_1 = 3\,\Omega$ and $R_2 = 6\,\Omega$. If the total voltage across the circuit is $18\,V$, what is the voltage across R_2?
   - **Solution:** The total resistance in series is $R_{\text{total}} = R_1 + R_2 = 3\,\Omega + 6\,\Omega = 9\,\Omega$. Using Ohm's Law, the total current is $I = \frac{V}{R} = \frac{18\,V}{9\,\Omega} = 2\,A$. The voltage across R_2 is $V_{R_2} = I \cdot R_2 = 2\,A \cdot 6\,\Omega = 12\,V$.

Problem Set - 7

1. A point charge of $4\mu C$ is at the origin. Calculate the electric field strength at a point $2m$ away from the charge.

2. Two point charges, $q_1 = -1.5nC$ and $q_2 = 3.0nC$, are placed 3 cm apart. Find the electric field at a point midway between the charges.

3. A parallel plate capacitor has a plate separation of $0.02m$ and a plate area of $0.5m^2$. If the plates are charged to a potential difference of 2000 V, what is the electric field strength and what is the surface charge density on each plate?

4. Two identical metallic plates are parallel and close to each other, separated by a distance of 10 mm in air. An electric field of $10^6 V/m$ is required between the plates. What should be the charge density on the plates?

5. Two charges $q1 = 5\mu C$ and $q2 = -3\mu C$ are separated by a distance of 2 m in vacuum. Calculate the electrostatic potential energy of the system.

6. A proton is moved from point A to point B in an electric field. Point A is at a potential of 120 V, and point B is at a potential of 200 V. What is the work done by the electric field on the proton? ($Q_{proton} = 1.6 \times 10^{-19} Coulomb$)

7. A straight wire carries a current of 5 Amperes. Calculate the magnetic field at a point 10 cm from the wire.

8. A circular coil of radius 0.2 m has 500 turns and carries a current of 1.5 Ampere. Calculate the magnetic field at the center of the coil.

9. A series DC circuit contains a resistor with resistance 8Ω, and a capacitor with capacitance $2\mu F$. If the circuit is connected to a DC supply of voltage 12V, calculate the current in the circuit and the charge on the capacitor once the steady state is reached.

10. Three resistors with resistances of 3 Ω, 6 Ω, and 9 Ω are connected in parallel across a 12V battery. Calculate the total current supplied by the battery.