Frequency Count of Nested Loops and Recursive Statements (CCDSALG)

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NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2]
[3]
[4]
```

Line 1 will be executed only 1 time.

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I
[3]
[4]
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I
[3] I
[4] I
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

Value of j	Condition	Enter Loop?
1	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I
[3] II
[4] II
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

Value of j	Condition	Enter Loop?
2	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I
[3] III
[4] III
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

Value of j	Condition	Enter Loop?
3	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I
[3] IIII-
[4] III-
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

Value of j	Condition	Enter Loop?
4	<= 3	No

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]      for j = 1 to n
[4]           print(i, j)
```

```
[1] I
[2] I-I
[3] IIII-
[4] III-
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] IIII-I
[4] III-I
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

Value of j	Condition	Enter Loop?
1	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] IIII-II
[4] III-II
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

Value of j	Condition	Enter Loop?
2	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] IIII-III
[4] III-III
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

Value of j	Condition	Enter Loop?
3	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] IIII-IIII-
[4] III-III-
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

Value of j	Condition	Enter Loop?
4	<= 3	No

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] IIII-IIII-
[4] III-III-
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] IIII-IIII-I
[4] III-III-I
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
1	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] IIII-IIII-II
[4] III-III-II
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
2	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] IIII-IIII-III
[4] III-III-III
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
3	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] IIII-IIII-IIII
[4] III-III-III
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
4	<= 3	No

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I
[2] I-I-I-I
[3] IIII-IIII-IIII
[4] III-III-III
```

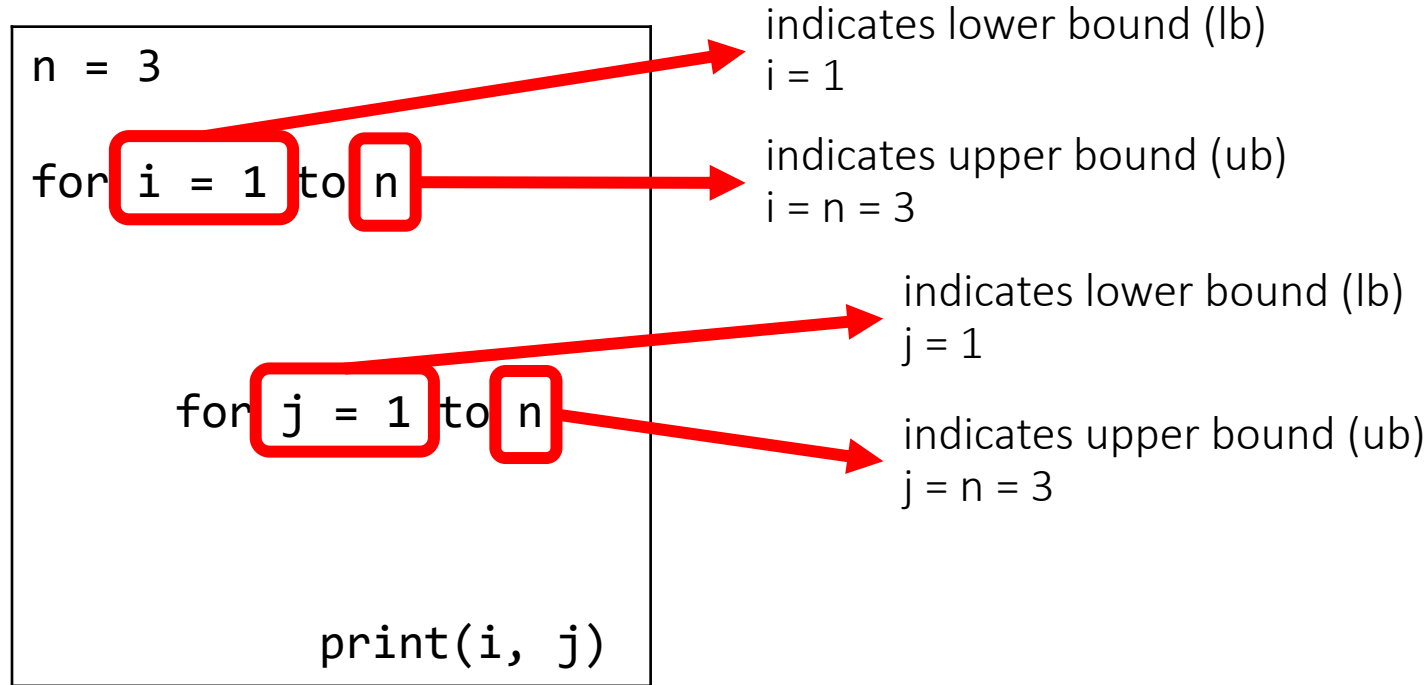| Value of i	Condition	Enter Loop?
4	<= 3	No

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to n
[4]         print(i, j)
```

```
[1] I                   =
1
[2] I-I-I-I         = 4
[3] IIII-IIII-IIII  = 3(4)
[4] III-III-III     = 3(3)
```

In this example, the frequency count of the statements inside the inner loop is not dependent on the value of the counter value of the outer loop.

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n

        for j = 1 to n

            print(i, j)
```

indicates lower bound (lb)
i = 1

indicates upper bound (ub)
i = n = 3

indicates lower bound (lb)
j = 1

indicates upper bound (ub)
j = n = 3

How can we get the number of times it looped without manually counting it?

n = 3
 	
for i = 1 to n
 	
        for j = 1 to n
 	
                print(i, j)
	1

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



        for j = 1 to n



        print(i, j)
```

$$1$$

$$\left(\sum_{i=1}^{3} <count\ of\ iterator> \right) + 1$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



        for j = 1 to n



        print(i, j)
```

$$1$$

$$\left(\sum_{i=1}^{3} 1 \right) + 1 = (3 - 1 + 1) + 1 = 4$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n




        for j = 1 to n



        print(i, j)
```

Body 1

1

$$\left(\sum_{i=1}^{3} 1 \right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3} < count\ of\ Body\ 1 >$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n




        for j = 1 to n



                print(i, j)
```

Body 1

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{3} 1\right) + 1\right) = \sum_{i=1}^{3}((3 - 1 + 1) + 1) = \sum_{i=1}^{3} 4$$

$$= 4(3 - 1 + 1) = 4(3) = 12$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



    for j = 1 to n

Body 1

Body 2    print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{3} 1\right) + 1\right) = 4(3) = 12$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n

    for j = 1 to n

        print(i, j)
```

Body 1

Body 2

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{3} 1\right) + 1\right) = 3(4) = 12$$

$$\sum_{i=1}^{3} < count\ of\ Body\ 2 >$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



    for j = 1 to n



        print(i, j)
```

Body 1

Body 2

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{3} 1\right) + 1\right) = 3(4) = 12$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} 1 = \sum_{i=1}^{3}(3 - 1 + 1) = 3(3 - 1 + 1) = 3(3) = 9$$

How can we get the number of times it looped without manually counting it?

n = 3

for i = 1 to n

    for j = 1 to n

Body 1

Body 2   print(i, j)

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{3} 1\right) + 1\right) = 3(4) = 12$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} 1 = \sum_{i=1}^{3}(3 - 1 + 1) = 3(3 - 1 + 1) = 3(3) = 9$$

How can we get the number of times it looped without manually counting it?

n = 3	1
`for i = `**`1`**` to `**`n`**	$\left(\sum_{i=\mathbf{1}}^{\mathbf{3}} 1\right) + 1 = (\mathbf{3} - \mathbf{1} + 1) + 1 = 4$
`for j = `**`1`**` to `**`n`**	$\sum_{i=\mathbf{1}}^{\mathbf{3}}\left(\left(\sum_{j=\mathbf{1}}^{\mathbf{3}} 1\right) + 1\right) = 3(4) = 12$
Body 1	
Body 2 `print(i, j)`	$\sum_{i=\mathbf{1}}^{\mathbf{3}}\sum_{j=\mathbf{1}}^{\mathbf{3}} 1 = 3(3) = 9$

Frequency Count	1 + 4 + 12 + 9 = 26

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]      for j = 1 to i
[4]           print(i, j)
```

What happens if the frequency count of the inner loop is controlled by the counter variable of the outer loop? In this example, the upper bound of the inner loop is changed to the counter variable of the outer loop.

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2]
[3]
[4]
```

Line 1 will be executed only 1 time.

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I
[3]
[4]
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I
[3] I
[4] I
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

Value of j	Condition	Enter Loop?
1	<= 1	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I
[3] II-
[4] I-
```

Value of i	Condition	Enter Loop?
1	<= 3	Yes

Value of j	Condition	Enter Loop?
2	<= 1	No

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] II-
[4] I-
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] II-I
[4] I-I
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

Value of j	Condition	Enter Loop?
1	<= 2	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] II-II
[4] I-II
```

Value of i	Condition	Enter Loop?
2	<= 3	Yes

Value of j	Condition	Enter Loop?
2	<= 2	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I
[3] II-III-
[4] I-II-
```

Value of i	Condition	Enter Loop?
2	**<= 3**	**Yes**

Value of j	Condition	Enter Loop?
3	**<= 2**	**No**

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] II-III-
[4] I-II-
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] II-III-I
[4] I-II-I
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
1	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] II-III-II
[4] I-II-II
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
2	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] II-III-III
[4] I-II-III
```

Value of i	Condition	Enter Loop?
3	<= 3	Yes

Value of j	Condition	Enter Loop?
3	<= 3	Yes

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I-I
[3] II-III-IIII
[4] I-II-III
```

Value of `i`	Condition	Enter Loop?
3	**<= 3**	**Yes**

Value of `j`	Condition	Enter Loop?
4	**<= 3**	**No**

NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I
[2] I-I-I-I
[3] II-III-IIII
[4] I-II-III
```

Value of i	Condition	Enter Loop?
4	<= 3	No

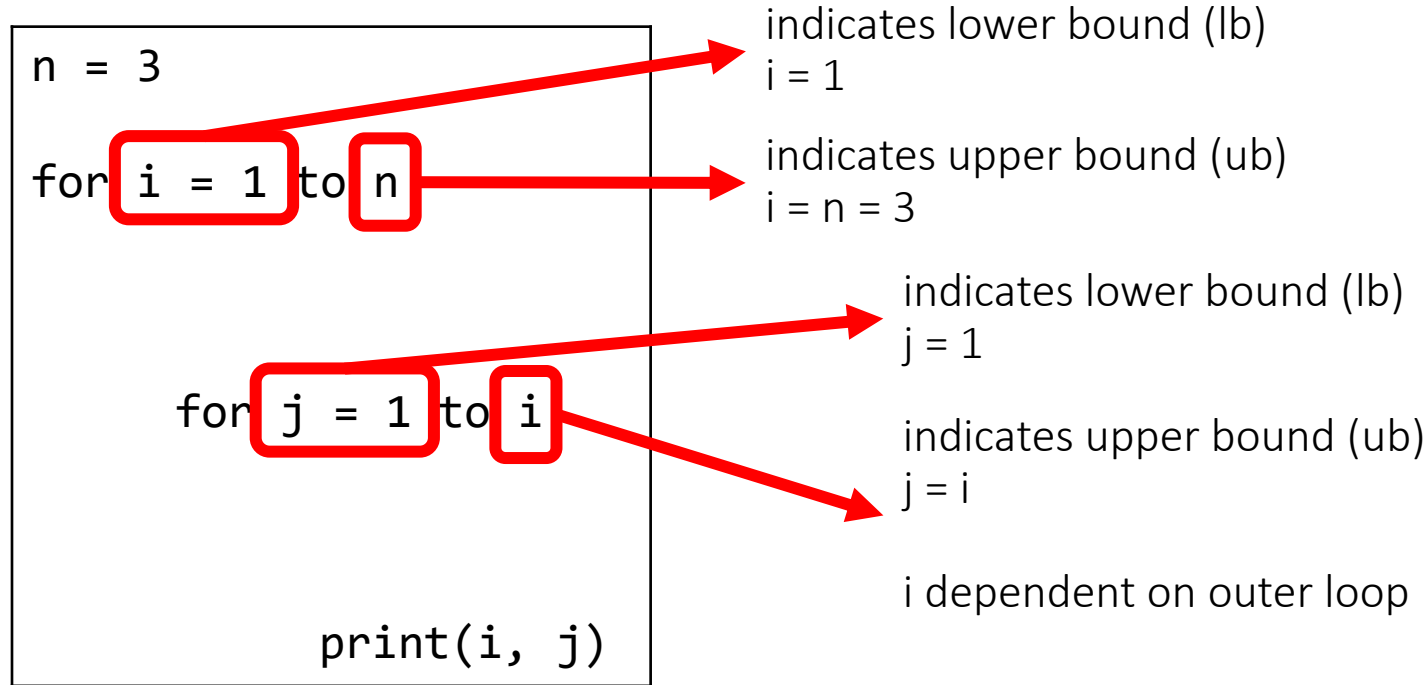# NESTED LOOP

```
[1] n = 3
[2] for i = 1 to n
[3]     for j = 1 to i
[4]         print(i, j)
```

```
[1] I                  = 1
[2] I-I-I-I            = 4
[3] II-III-IIII        = 2+3+4
[4] I-II-III           = 1+2+3
```

Notice that the frequency count of the inner loop is affected by the current value of the counter variable of the outer loop. How to represent this?

How can we get the number of times it looped without manually counting it?

```
n = 3

for  i = 1  to  n

        for  j = 1  to  i

              print(i, j)
```

indicates lower bound (lb)
i = 1

indicates upper bound (ub)
i = n = 3

indicates lower bound (lb)
j = 1

indicates upper bound (ub)
j = i

i dependent on outer loop

How can we get the number of times it looped without manually counting it?

`n = 3`
 	
`for i = 1 to n`
 	
`    for j = 1 to i`
 	
`        print(i, j)`
	1

How can we get the number of times it looped without manually counting it?

n = 3

for i = **1** to **n**

for j = **1** to **i**

Body 1

Body 2   print(i, j)

1

$$\left(\sum_{i=1}^{3} <count\ of\ iterator>\right) + 1$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



    for j = 1 to i
```

Body 1

Body 2   print(i, j)

1

$$\left(\sum_{i=1}^{3} 1 \right) + 1 = (3 - 1 + 1) + 1 = 4$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n

        for j = 1 to i

Body 1

Body 2          print(i, j)
```

$$1$$

$$\left(\sum_{i=1}^{3} 1 \right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3} < count\ of\ Body\ 1 >$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n

    for j = 1 to i

Body 1

Body 2    print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1 \right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3} \left(\left(\sum_{j=1}^{i} 1 \right) + 1 \right)$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n




    for j = 1 to i

Body 1


Body 2       print(i, j)
```

$$1$$

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{i} 1\right) + 1\right) = \sum_{i=1}^{3}((i - 1 + 1) + 1)$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n


        for j = 1 to i

Body 1


Body 2        print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{i} 1\right) + 1\right) = \sum_{i=1}^{3}(i + 1)$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n


    for j = 1 to i

Body 1

Body 2    print(i, j)
```

$$1$$

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{i} 1\right) + 1\right) = \sum_{i=1}^{3}(i + 1) = \sum_{i=1}^{3} i + \sum_{i=1}^{3} 1$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



    for j = 1 to i

Body 1

Body 2        print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1 \right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3} \left(\left(\sum_{j=1}^{i} 1 \right) + 1 \right) = \sum_{i=1}^{3} (i + 1) = \sum_{i=1}^{3} i + \sum_{i=1}^{3} 1$$

$$= \frac{3(3+1)}{2} + (3 - 1 + 1) = \frac{12}{2} + 3 = 9$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



        for j = 1 to i
Body 1

Body 2        print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{i} 1\right) + 1\right) = \frac{12}{2} + 3 = 9$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



    for j = 1 to i


Body 1



Body 2      print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{i} 1\right) + 1\right) = \frac{12}{2} + 3 = 9$$

$$\sum_{i=1}^{3} < count\ of\ Body\ 2 >$$

How can we get the number of times it looped without manually counting it?

```
n = 3

for i = 1 to n



    for j = 1 to i

Body 1

Body 2    print(i, j)
```

1

$$\left(\sum_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$$

$$\sum_{i=1}^{3}\left(\left(\sum_{j=1}^{i} 1\right) + 1\right) = \frac{12}{2} + 3 = 9$$

$$\sum_{i=1}^{3}\sum_{j=1}^{i} 1 = \sum_{i=1}^{3} i - 1 + 1 = \sum_{i=1}^{3} i = \frac{3(3+1)}{2} = \frac{12}{2} = 6$$

How can we get the number of times it looped without manually counting it?

`n = 3`	1
`for i = 1 to n`	$\left(\sum\limits_{i=1}^{3} 1\right) + 1 = (3 - 1 + 1) + 1 = 4$
`for j = 1 to i`	$\sum\limits_{i=1}^{3}\left(\left(\sum\limits_{j=1}^{i} 1\right) + 1\right) = \dfrac{12}{2} + 3 = 9$
Body 1 ←	
Body 2 ← `print(i, j)`	$\sum\limits_{i=1}^{3}\sum\limits_{j=1}^{3} 1 = \dfrac{12}{2} = 6$

Frequency Count	1 + 4 + 9 + 6 = 20

Questions? ☺