Experiment 9 - Ph6

In this experiment, we use a mass spectrometer to check the validity of the Lorentz force law for the dynamics of certain ions as described in the notes. We then determine a value for e/m_{AMU}, the ratio of the electron charge to the atomic mass unit.

We will do all calculations using the mass of a neutral atom of each isotope in place of the mass of the corresponding ion. Observe that the mass of an electron is approximately 0.000549 amu, such that our assumption induces an error of 0.0024% in the mass of the Sodium-23 ion that we use for our calculations. This is the lightest element we will be working with, and for heavier elements the error is even smaller. This assumption is therefore not a significant source of error, particularly given the 0.10 - 0.23% error induced in the mass term by the $m = W \times m_{\text{AMU}}$ assumption, where W is the (integer) atomic weight.

In[1]:=
```
(* We import data for the B and V values at the peak for each isotope. *)
SetDirectory[NotebookDirectory[]];
data = Import["raw_data.csv", "CSV"];
data // TableForm
```

Out[3]//TableForm=

Isotope	B	V
Cesium133	0.5298	546.7
Cesium133	0.5209	527.4
Cesium133	0.5405	569.4
Rubidium85	0.4239	549.2
Rubidium85	0.413	521.3
Rubidium85	0.4161	529.5
Rubidium87	0.4239	536.15
Rubidium87	0.4321	557.2
Rubidium87	0.4283	548.8
Rubidium87	0.4384	573.3
Potassium39	0.2874	549.4
Potassium39	0.297	587.2
Potassium39	0.2913	565.5
Potassium41	0.2873	520.6
Potassium41	0.2962	556.2
Potassium41	0.2913	531.
Sodium23	0.2196	542.9
Sodium23	0.2217	553.8
Sodium23	0.216	525.4
Chlorine35	0.2718	562.9
Chlorine35	0.2807	595.8
Chlorine35	0.2625	520.3
Chlorine37	0.2718	531.
Chlorine37	0.2807	565.2
Chlorine37	0.2648	501.5

In[4]:=
```
(* We transform the raw data into W and
 B^2/V pairs to prime it for analysis with CurveFit. *)
```

In[5]:=
```
processed = Join[{{"S", "W", "(B^2)/V"}, {"S"}},
   Table[{IsotopeData[data[[i]][[1]], "MassNumber"], data[[i]][[2]]^2/data[[i]][[3]]},
    {i, 2, Length[data]}]];
Export["data.dat", processed];
```

In[7]:=
```
(* We now use CurveFit for the rest of the analysis *)
```

In[8]:=
```
<< CurveFit`
```

```
CurveFit for Mathematica v7.x thru v10.x, Version 1.95, 1/2016
Caltech Sophomore Physics Labs, Pasadena, CA
```

In[9]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

```
/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 4/data.dat
File comment header:
    W     (B^2)/V

... LoadFile: Data sorted in increasing x order.
Read 25 data points.
```

In[10]:=
```
(* We use CurveFit to use (B,V) pairs corresponding
 to the same isotope to estimate the uncertainty in B²/V. *)
```

In[11]:=
```
CalculateYsigmas[]
```

```
Sorted data in order of increasing X values.
Calculated and assigned Y uncertainties.
```

In[12]:=
```
LinearFit[]
```

```
n = 25
y(x) = a + b x
Fit of (x,y)  (unweighted)
```

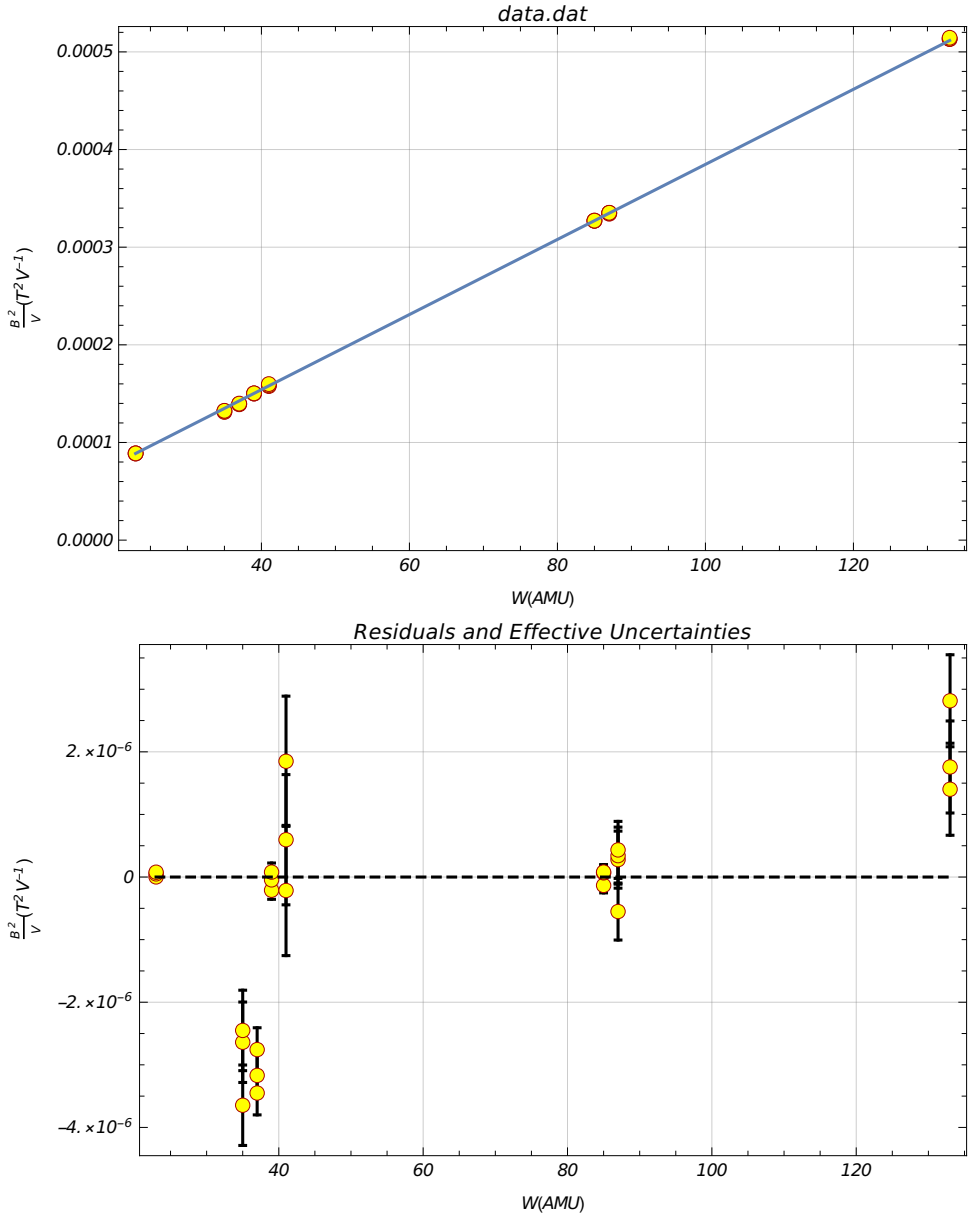
a=	b=	
-1.74672×10^{-6}	3.87237×10^{-6}	
σ_a=	σ_b=	Std. deviation=
5.69852×10^{-7}	8.09107×10^{-9}	1.41832×10^{-6}

```
Fit of (x,y±σy)
```

a=	b=	
3.22344×10^{-7}	3.84468×10^{-6}	
σ_a=	σ_b=	$\chi^2/(n-2)$ =
3.79739×10^{-8}	1.0721×10^{-9}	15.1099

In[13]:=
```
LinearDifferencePlot[FrameLabel → {"W(AMU)", "B²/V (T²V⁻¹)"}]
```

Out[13]=



$y(x) = a + b\,x$

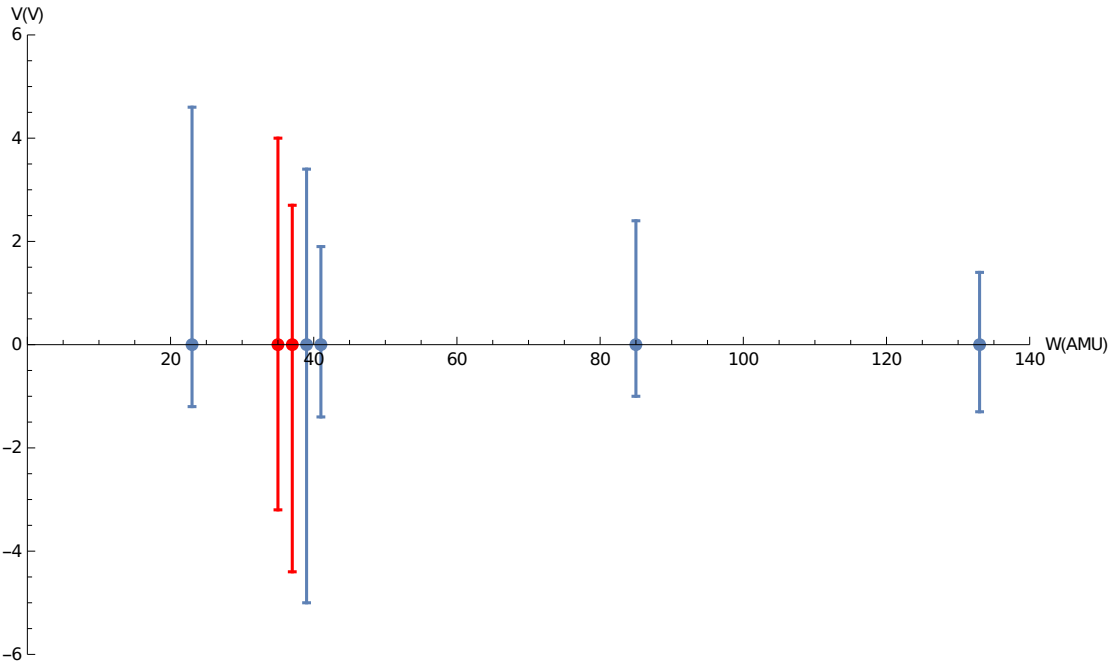
a =	b =		
3.22344×10^{-7}	3.84468×10^{-6}		
σ_a =	σ_b =	$\chi^2/(n-2)$ =	
3.79739×10^{-8}	1.0721×10^{-9}	15.1099	

In[39]:=
```
(* We now visualize the Full Width at Half Maximum by plotting the difference
  between the half-maximum range and the measurement value, against W *)
QuitCurveFit
Quit[]
```

In[1]:=
```
Needs["ErrorBarPlots`"]
```

In[28]:=
```
alkali = ErrorListPlot[{{{133, 0}, ErrorBar[{545.4 - 546.7, 548.1 - 546.7}]},
    {{85, 0}, ErrorBar[{548.2 - 549.2, 551.6 - 549.2}]},
    {{39, 0}, ErrorBar[{544.4 - 549.4, 552.8 - 549.4}]},
    {{41, 0}, ErrorBar[{519.2 - 520.6, 522.5 - 520.6}]},
    {{23, 0}, ErrorBar[{541.7 - 542.9, 547.5 - 542.9}]}}, PlotRange →
    {{0, 140}, {-6, 6}}, AxesLabel → {"W(AMU)", "V(V)"}, ImageSize → Large];
halogens = ErrorListPlot[{{{35, 0}, ErrorBar[{559.7 - 562.9, 566.9 - 562.9}]},
    {{37, 0}, ErrorBar[{526.6 - 531.0, 533.7 - 531.0}]}},
   PlotRange → {{0, 140}, {-6, 6}}, AxesLabel → {"W(AMU)", "V(V)"},
   ImageSize → Large, PlotStyle → Red];
Show[alkali, halogens]
```

Out[30]=




```
(* The halogens are shown with the red half-maximum bar *)
```

Comments on fit and resolution

While there are no significant outliers in the data set and the fit visually looks good on the plot of the linear fit and residuals, the high reduced chi-squared of 15.1 suggests that we cannot satisfactorily use our results above alone to confirm the theory. This could be due to the large uncertainty in each measurement of $\frac{B^2}{V}$ due to inaccuracies in the experimental procedure (for example, we identified the peaks by varying the accelerator and using a reading of the detector plate current to find the maximum point. This can be error-prone, particularly for broad peaks.) or to the small number of repetitions carried out for each data point.

We also notice that our fit parameter for the intercept is $(3.22 \pm 0.38) \times 10^{-7}\, T^2\, V^{-1}$, a rela-

tively strong measurement with small uncertainty. This indicates that there is an offset in our data. One possible cause could an incorrect zeroing of the Hall probe.

We nevertheless observe that the linear fit obtained suggests that the theory expressed in (9.10) is valid to within the accuracy of our experiment. We therefore use our fit parameters to estimate e/m_{AMU}.

We obtain a measure of the mass resolution of the spectrometer by finding the full width half maximum with respect to voltage (as stated in the pre-lab, there is a symmetry between m and V in the equations describing this system). We observe from the unexpected asymmetry in the half-maximum range as plotted above that this measurement seems quite prone to error and possibly because of this shows no particular pattern.

Estimating e/m_{AMU}

We obtain from our fit parameters that the slope, say $b = (3.84 \pm 0.0011)\times 10^{-6}\, T^2\, V^{-1}(\text{AMU})^{-1}$. Then we have $\frac{e}{m_{\text{AMU}}} = \frac{1}{b}\left(\frac{2}{R_{\text{eff}}{}^2}\right) =$

In[46]:=
```
ratio = 1/(3.8447 * 10^-6) (2/0.0735^2);
NumberForm[ratio, 4]
```

Out[47]//NumberForm=

9.629×10^7

We can then estimate the total error in $\frac{e}{m_{\text{AMU}}}$ as in the pre-lab:

$\frac{\sigma_{e/m_{\text{AMU}}}}{e/m_{\text{AMU}}} = \sqrt{\left(\frac{\sigma_{\text{fit}}}{\text{fit}}\right)^2 + \left(2\,\frac{\sigma_R}{R}\right)^2 + 2\left(\frac{\sigma_B}{B}\right)} =$

In[44]:=
```
uncertainty = ratio * Sqrt[(0.0010721/3.4468)^2 + (2 * 0.002)^2 + 2 (0.002)^2];
ScientificForm[uncertainty, 2]
```

Out[45]//ScientificForm=

4.7×10^5

Our estimate is therefore $\frac{e}{m_{\text{AMU}}} = (9.629 \pm 0.047)\times 10^7\, C\,\text{kg}^{-1}$. Using the NIST published values for the mass of the proton and charge of the electron, we obtain $\frac{e}{m_{\text{AMU}}} =$

In[48]:=
```
publishedRatio = 1.6021766208 * 10^-19/1.672621898 * 10^-27;
ScientificForm[publishedRatio, 4]
```

Out[49]//ScientificForm=

9.579×10^7

This comes very close to being within the error bounds of our estimate, indicating that the experiment agrees reasonably well with the theory barring issues with the fit as described above, but by itself does not give a conclusive estimate of $\frac{e}{m_{\text{AMU}}}$ or a conclusive assessment

of the validity of the Lorentz Force law for ions.