Ph 6 - Experiment 7 (The Kelvin Absolute Voltmeter)

In[1]:= `<< CurveFit``

```
CurveFit for Mathematica v7.x thru v10.x, Version 1.95, 1/2016

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```

In[129]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
   LoadFile[name]]
]
```

```
/home/ybadal/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph 6/Lab 2/Kelvin.dat

File comment header:

Mass (mg) Voltage (V)
```
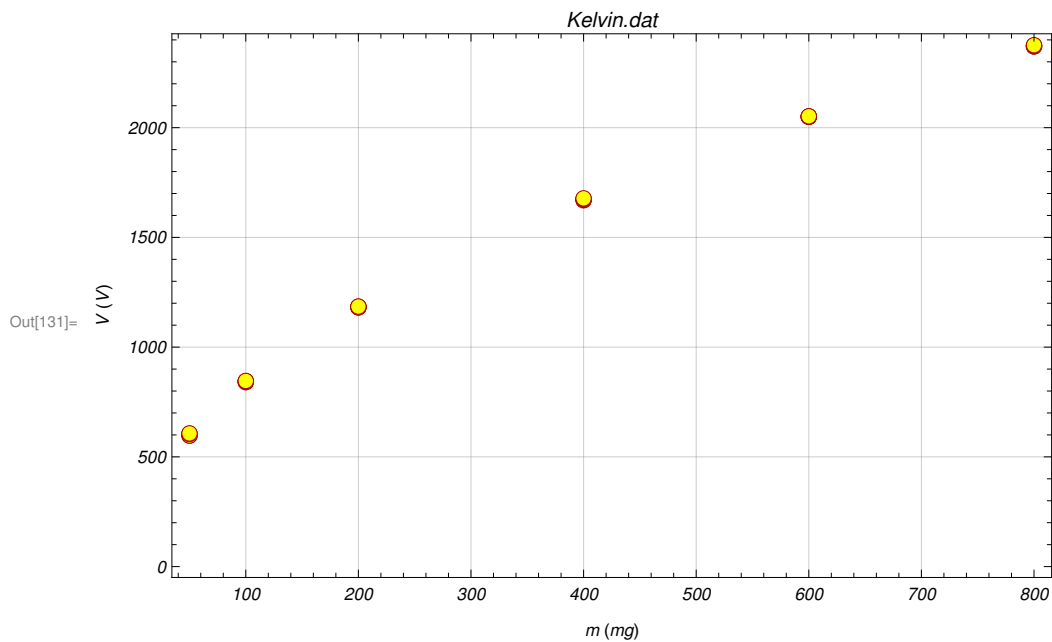
··· LoadFile: Data sorted in increasing x order.

```
Read 30 data points.
```

In[130]:= `CalculateYsigmas[]`

```
Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.
```

In[131]:= **LinearDataPlot[FrameLabel → {"m (mg)", "V (V)"}]**

Out[131]=

Kelvin.dat

(* Converting into V/kg *)

In[132]:= **xnew[x_, y_] := x * 10^{-6}**
ynew[x_, y_] := y
DataTransform[]
(* Use Undo[] if you don't like the results. *)

All 30 points transformed.

In[135]:= **(* Squaring voltage values to obtain expected linear relationship *)**
xnew[x_, y_] := x
ynew[x_, y_] := y^2
DataTransform[]
(* Use Undo[] if you don't like the results. *)

All 30 points transformed.

In[138]:= **LinearDataPlot[FrameLabel → {"V² (V²)", "m (kg)"}]**

Out[138]=

Kelvin.dat
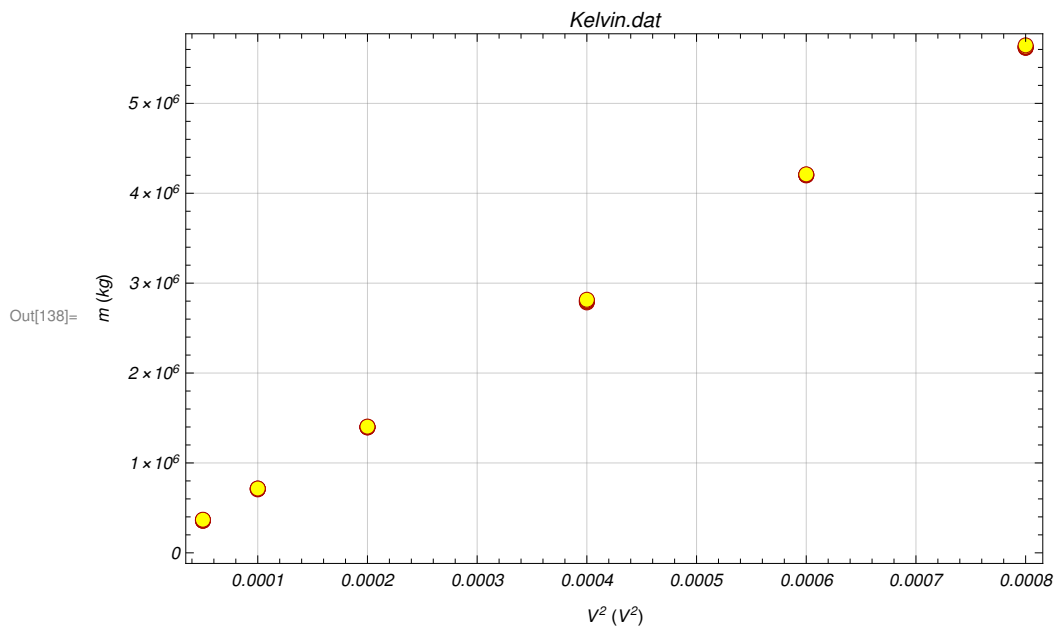
Fitting the V^2 against m data against a linear model

In[139]:= **LinearFit[]**

```
n = 30

y(x)  = a + b x

Fit of (x,y)   (unweighted)

a=            b=
3775.75       7.01542 × 10⁹

σa=           σb=                    Std. deviation=
3516.48       7.82246 × 10⁶         11628.6

Fit of (x,y±σy)

a=            b=
6941.72       6.99929 × 10⁹

σa=           σb=                    χ²/(n−2) =
1446.83       4.22734 × 10⁶         2.828
```
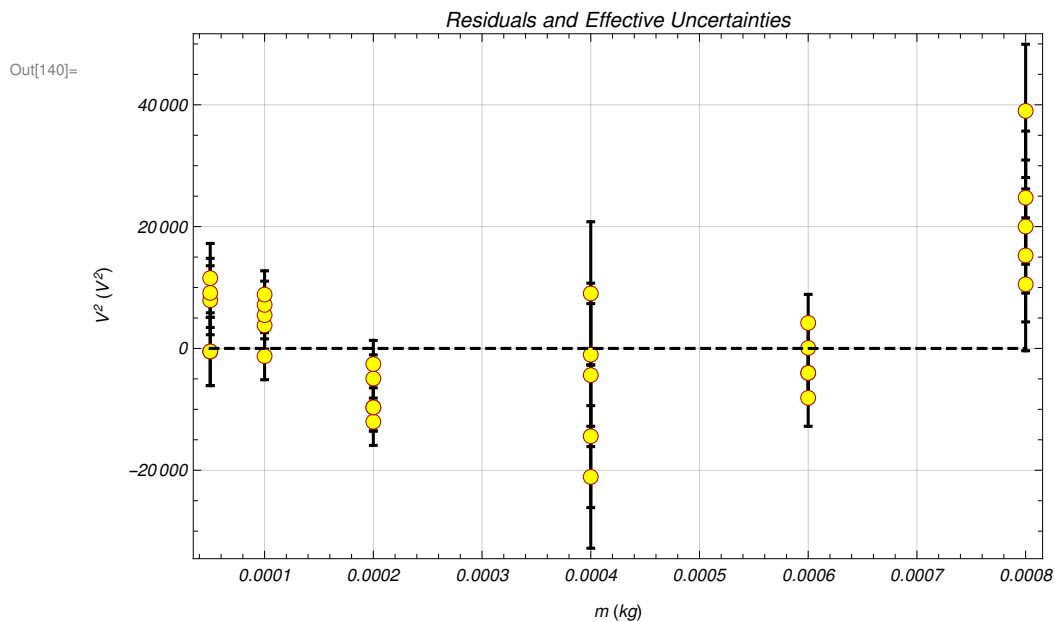
In[140]:= `LinearDifferencePlot[FrameLabel → {"m (kg)", "V² (V²)"}]`

Kelvin.dat

Out[140]=

Residuals and Effective Uncertainties
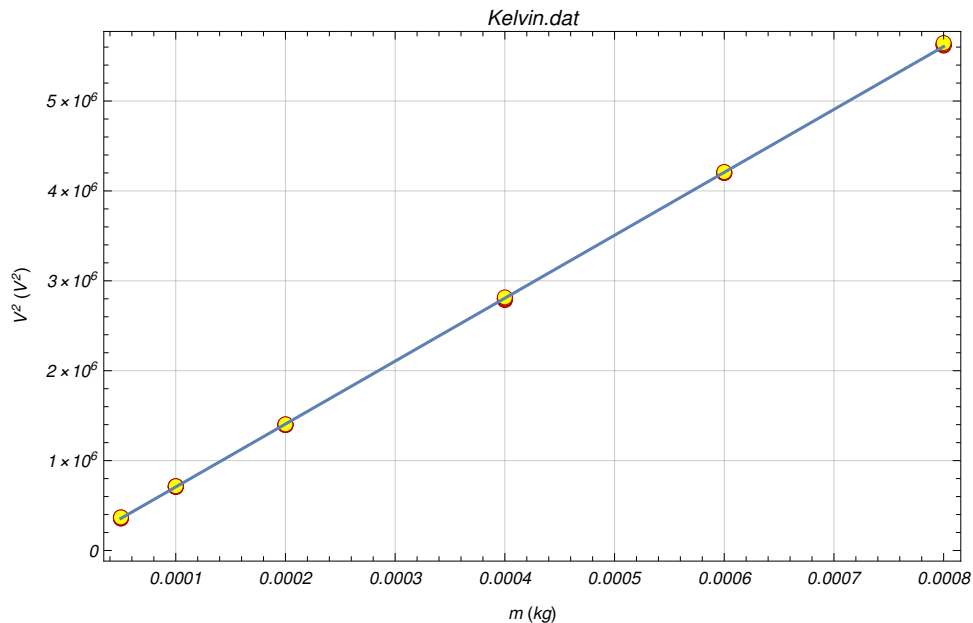

```
y(x)  =  a  +  b x
a=          b=
6941.72     6.99929 × 10⁹
```

$\sigma_a =$ $\sigma_b =$ $\chi^2 / (n-2) =$
1446.83 4.22734×10^6 2.828

Conclusions from fit

From the low $\tilde{\chi}^2$ value (2.828) of the linear fit and the relatively small intercept (and therefore small offset), we can deduce that the $V^2 \propto m$ holds over our range of measurements. In fact, our fit determines the proportionality constant up to an error of ± 0.0604%, whereby our residuals are of the order of 10^4 but our transformed data points for V^2 have values on the order of 10^6. We therefore conclude that the

data supports the proposed relationship $V^2 \propto m$ given the fit we have obtained and considering the uncertainties inherent to the system (see last section).

Experimental determination of ϵ_0 and c

Recall the relationship $V^2 = km$, $k = \frac{2\,gd^2}{\pi^2\,\epsilon_0}$. From this, we can find ϵ_0 from the slope k using $\epsilon_0 = \frac{2\,gd^2}{\pi^2\,k}$.

We can then find the uncertainty in our value for ϵ_0 by propagating the errors:

$\frac{\Delta\epsilon_0}{\epsilon_0} = \left[\left(\frac{\Delta g}{g}\right)^2 + \left(\frac{2\Delta d}{d}\right)^2 + \left(\frac{2\Delta r}{r}\right)^2 + \left(\frac{\Delta k}{k}\right)^2\right]^{\frac{1}{2}}$. Using data from the calibration card:

```
In[141]:= g = 9.7957;
       errg = 0.001;
       d = 0.00309;
       errd = 0.00005;
       reff = 0.03073;
       errreff = 0.00001;
       k = 6.99929 * 10^9;
       errk = 4.22733 * 10^6;
       ε = (2 g d^2) / (π reff^2 k)
       errε = ε * Sqrt[ (errg/g)^2 + (2 errd/d)^2 + (2 errreff/reff)^2 + (errk/k)^2 ]
```

Out[149]= 9.00852×10^{-12}

Out[150]= 2.91649×10^{-13}

We therefore have an estimate of $\epsilon_0 = (9.0085 \pm 0.2916) \times 10^{-12}$ Fm^{-1}. From this, we can use the permeability of free space, $\mu_0 = 4\,\pi \times 10^{-7}$ Hm^{-1} to estimate the speed of light in a vacuum, c.

```
In[151]:= μ = 4 π * 10^-7;
       c = 1 / Sqrt[μ ε]
       errc = c * errε/ε
```

Out[152]= 2.97213×10^8

Out[153]= 9.62222×10^6

We then estimate c = $(2.9721 \pm 0.0962) \times 10^8$ ms^{-1}.

Major limiter to accuracy

Given the use of S-1 precision masses and an accurate constant-voltage power supply and careful leveling of the apparatus, it can be concluded that the major limiter to the accuracy of this experiment is the sensitivity to environmental noise inherent to the system.

In fact, by considering a parallel plate capacitor maintained under constant voltage, we observe that a small change in the distance between the plates (for instance, due to vibrations of the table due to sound or transmitted via the floor) will cause the force between the capacitor places to increase, with $\frac{F}{A} = \frac{1}{2} \epsilon_0 \frac{V^2}{d^2}$ as derived in the pre-lab. Therefore, the equilibrium point at which the force equals the weight of the mass used is an unstable equilibrium - when in the neighborhood of the equilibrium point, a small disturbance causing the top plate to move slightly past the equilibrium point will in turn cause the force between the plates to increase, accelerating the top plate downwards, which in turn increases the force between the plates and this feeds into itself until the pin stops the top plate from moving any further.

This unstable equilibrium is useful - our method of taking data points is based around it, in that moving just past the equilibrium results in the top plate moving all the way down and hitting the pin, which is the observation that we use to take data points. However, it also introduces errors in our experiments, as small disturbances when in the neighborhood of the equilibrium point can cause the top plate to move all the way down at an incorrect voltage randomly.