Experiment 12 - Electron Diffraction

In[93]:=
```
SetDirectory[NotebookDirectory[]];
```

Linear Relation

We observe from the pre-lab that $\lambda = \frac{hc}{\sqrt{2\,mc^2\,eV}}$ and under a small-angle approximation $\frac{r}{x} = \frac{\hat{K}}{2\pi}\frac{\lambda}{a_0}$. This gives $r = \left(\frac{\hat{K}}{2\pi}\frac{hcx}{a_0\sqrt{2\,mc^2\,e}}\right)V^{\frac{-1}{2}}$ the linear relation between r and $V^{\frac{-1}{2}}$.

We first calculate $\hat{K}$ for the features we are concerned with:

For graphite: $\hat{K}_1 = \frac{4\pi}{\sqrt{3}}$; $\hat{K}_2 = 4\pi$; $\hat{K}_3 = \frac{8\pi}{\sqrt{3}}$

For aluminium: $\hat{K}_1 = 2\pi\sqrt{3}$; $\hat{K}_2 = 4\pi$; $\hat{K}_3 = 4\pi\sqrt{2}$; $\hat{K}_4 = 8\pi$

In[481]:=
```
(* We set some constants to help with the analysis later *)
graphitek1 = 4 π/√3;
graphitek2 = 4 π;
graphitek3 = 8 π/√3;
aluminiumk1 = 2 π √3 ;
aluminiumk2 = 4 π;
aluminiumk3 = 4 π √2 ;
aluminiumk4 = 8 π;
graphitea0 = 0.24612 * 10^-9 (*m*);
aluminiuma0 = 4.050 * 10^-10 (*m*);
x = 0.178 (*m*);
c = 2.998 * 10^8 (*ms^-1*);
mc2 = 0.511 * 10^6 (*J*);
```

This allows us to define the following function to calculate h with uncertainty from our fit data.

In[497]:=
```
h[slope_, error_, featureK_, featurea0_] :=
 {ScientificForm[(slope * 2 π * featurea0 * Sqrt[2 * mc2]) / (featureK * x * c), 3],
  ScientificForm[(error * 2 π * featurea0 * Sqrt[2 * mc2]) / (featureK * x * c), 2]}
```

Data Analysis

In[13]:=
```
<< CurveFit`
```

```
CurveFit for Mathematica v7.x thru v10.x, Version 1.95, 1/2016
Caltech Sophomore Physics Labs, Pasadena, CA
```

Graphite

First (closest to direct beam) diffraction feature

In[14]:=
```
(* Import and prepare the data for analysis *)
```

In[356]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

```
/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/graphite1.dat
File comment header:
V (kV) d1 (cm)
```

LoadFile: Data sorted in increasing x order.

```
Read 15 data points.
```

In[357]:=
```
CalculateYsigmas[]
```

```
Sorted data in order of increasing X values.
Calculated and assigned Y uncertainties.
```

In[358]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.      *)
```

$$\text{xnew[x\_, y\_] := } (1000 * x)^{\frac{-1}{2}}$$

$$\text{ynew[x\_, y\_] := } \frac{y}{2} * 0.01$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 15 points transformed.

In[361]:=
```
LinearFit[]
```

n = 15

y(x) = a + b x

Fit of (x,y) (unweighted)

a= 0.00146151	b= 0.961029	
σ_a= 0.00048536	σ_b= 0.0397627	Std. deviation= 0.000196185

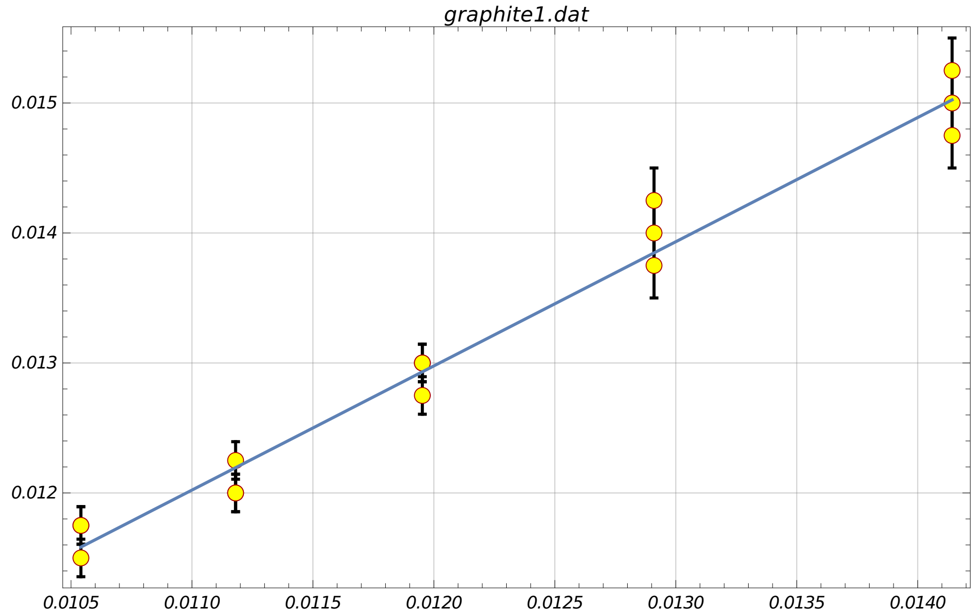
Fit of (x,y±σ_y)

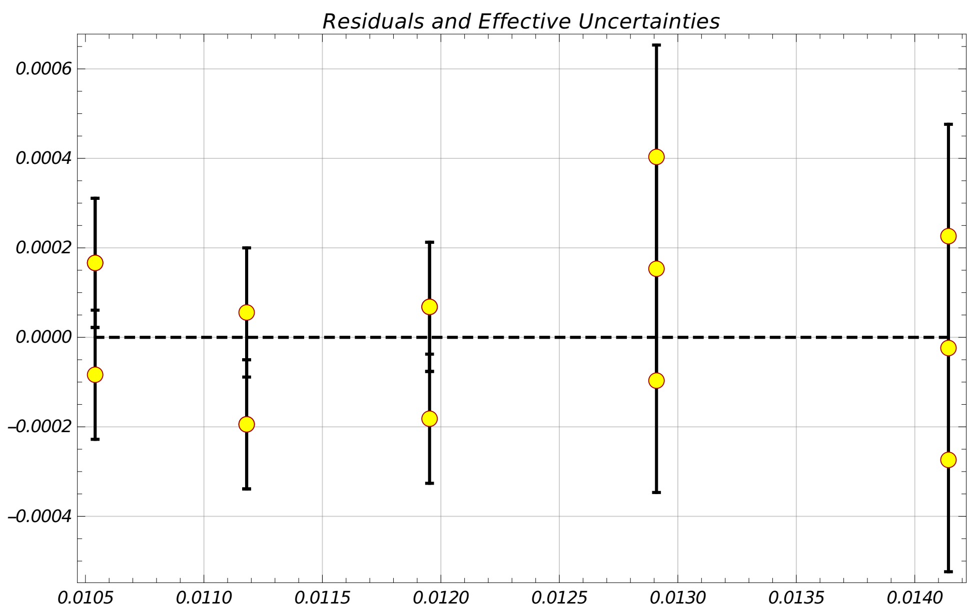
a= 0.00151375	b= 0.955306	
σ_a= 0.000478305	σ_b= 0.0409109	χ^2/(n−2) = 1.07379

In[362]:=

```
LinearDifferencePlot[]
```

Out[362]=





$y(x) = a + b\ x$

a= 0.00151375	b= 0.955306	
σ_a= 0.000478305	σ_b= 0.0409109	$\chi^2/(n-2)$= 1.07379

In[506]:=

```
(* Calculating h *)
hg1 = h[0.955306, 0.0409109, graphitek1, graphitea0]
```

Out[506]= $\{3.86\times10^{-15},\ 1.7\times10^{-16}\}$

Second Diffraction Feature

In[14]:=
```
(* Import and prepare the data for analysis *)
```

In[377]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/graphite2.dat

File comment header:

V (kV) d2 (cm)

LoadFile: Data sorted in increasing x order.

Read 15 data points.

In[378]:=
```
CalculateYsigmas[]
```

Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.

In[379]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.      *)
```

$$xnew[x\_, y\_] := (1000 * x)^{\frac{-1}{2}}$$

$$ynew[x\_, y\_] := \frac{y}{2} * 0.01$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 15 points transformed.

In[382]:=
```
LinearFit[]
```

```
n = 15

y(x) = a + b x

Fit of (x,y)  (unweighted)

a=              b=
-0.00526492     2.54134
σa=             σb=          Std. deviation=
0.00281902      0.230949     0.00113951

Fit of (x,y±σy)

a=              b=
-0.00676163     2.63722
σa=             σb=          χ2/(n-2)=
0.000383471     0.0312764    41.566
```

```
(* We observe an abnormally high χ̃2 and an outlier at the V =
 9 kV data point. We remove this data point and attempt another fit. *)
```

Change the plot from LinearDataPlot[ ] to something else like LogDataPlot[ ] if you wish. If the plot has a log X-axis (like LogLogDataPlot[ ]), then the Log option to SetXRange[ ] must be changed to Log → True

In[383]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
    Label -> "Set the X values for the range you wish to remove." ]},
 Print[x];

 XRangeRemove[Sequence @@ x]
]
```

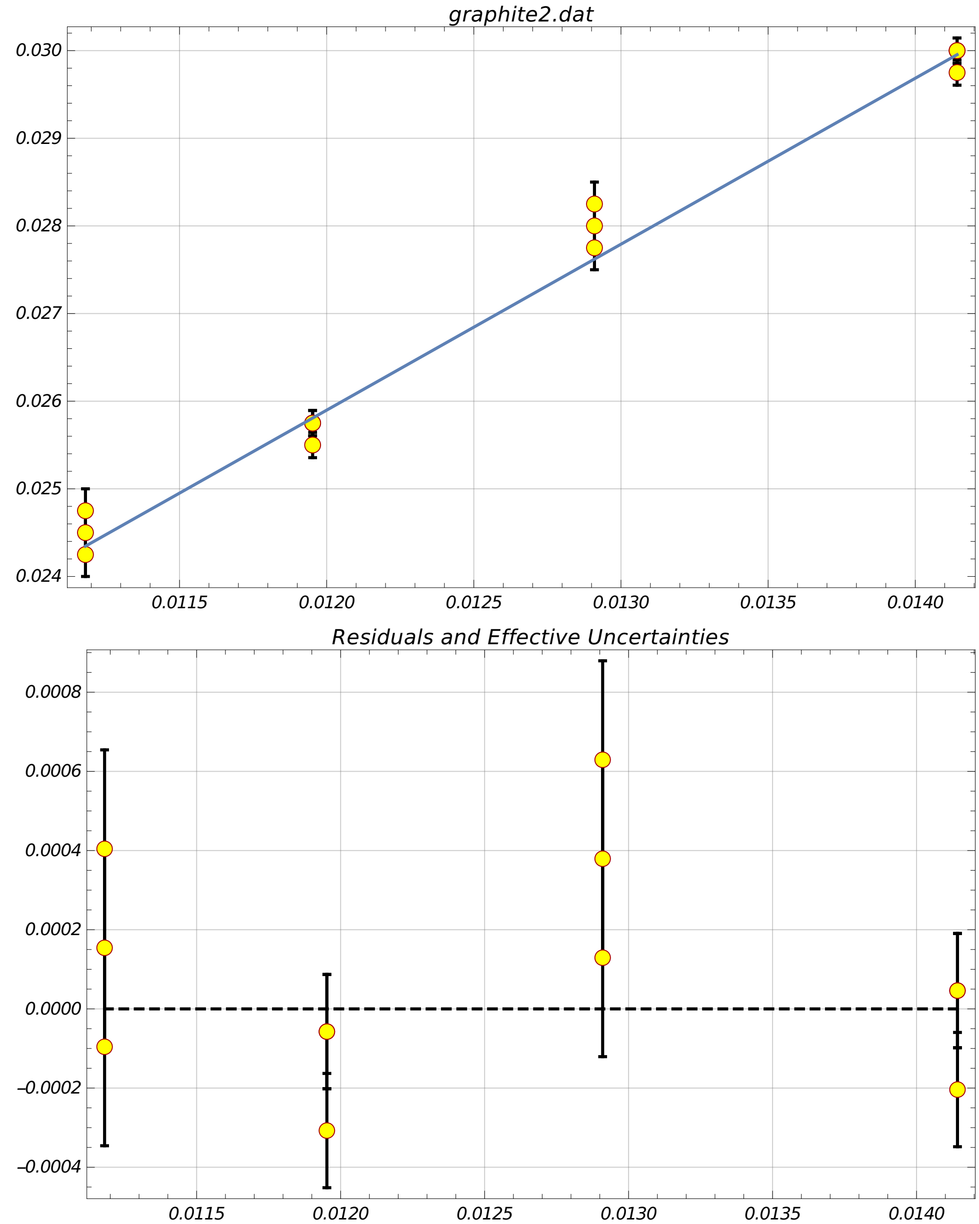
```
{0.0104659, 0.0106175}

n= 12

3 points removed.
```

In[384]:=
```
LinearFit[]
```

```
n = 12

y(x) = a + b x

Fit of (x,y)  (unweighted)

a=              b=
0.00338906      1.88358
σa=             σb=          Std. deviation=
0.000930216     0.0738571    0.000283473

Fit of (x,y±σy)

a=              b=
0.00317585      1.89349
σa=             σb=          χ2/(n-2)=
0.000580255     0.0451687    1.90987
```

In[385]:=
```
LinearDifferencePlot[]
```

Out[385]=



$y(x) = a + b\,x$

$a=$	$b=$	
0.00317585	1.89349	
$\sigma_a=$	$\sigma_b=$	$\chi^2/(n-2)=$
0.000580255	0.0451687	1.90987

In[507]:=
```
(* Calculating h *)
hg2 = h[1.839349, 0.0451687, graphitek2, graphitea0]
```

Out[507]= $\{4.29\times10^{-15},\ 1.1\times10^{-16}\}$

Third Diffraction Feature

In[402]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/graphite3.dat

File comment header:

V (kV) d3 (cm)

LoadFile: Data sorted in increasing x order.

Read 15 data points.

In[403]:=
```
CalculateYsigmas[]
```

Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.

In[404]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.      *)
```

$$\text{xnew[x\_, y\_]} := (1000 * x)^{\frac{-1}{2}}$$

$$\text{ynew[x\_, y\_]} := \frac{y}{2} * 0.01$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 15 points transformed.

In[407]:=
```
LinearFit[]
```

n = 15

y(x) = a + b x

Fit of (x,y) (unweighted)

a= 0.00188829	b= 3.10921	
σ_a= 0.00108943	σ_b= 0.0892117	Std. deviation= 0.000440162

Fit of $(x, y \pm \sigma_y)$

a= 0.000109292	b= 3.24072	
σ_a= 0.000461991	σ_b= 0.0367481	$\chi^2/(n-2)$ = 2.75972

```
(* We observe slight anomalous behavior (within error margin) at the V =
 9.0 kV data point. We remove this point to see if we obtain a better fit. *)
```

Change the plot from LinearDataPlot[ ] to something else like LogDataPlot[ ] if you wish. If the plot has a log X-axis (like LogLogDataPlot[ ]), then the Log option to SetXRange[ ] must be changed to Log → True

In[408]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
    Label -> "Set the X values for the range you wish to remove." ]},
 Print[x];

 XRangeRemove[Sequence @@ x]
]
```

{0.0104659, 0.0106645}

n= 12

3 points removed.

In[409]:=
```
LinearFit[]
```

n = 12

y(x) = a + b x

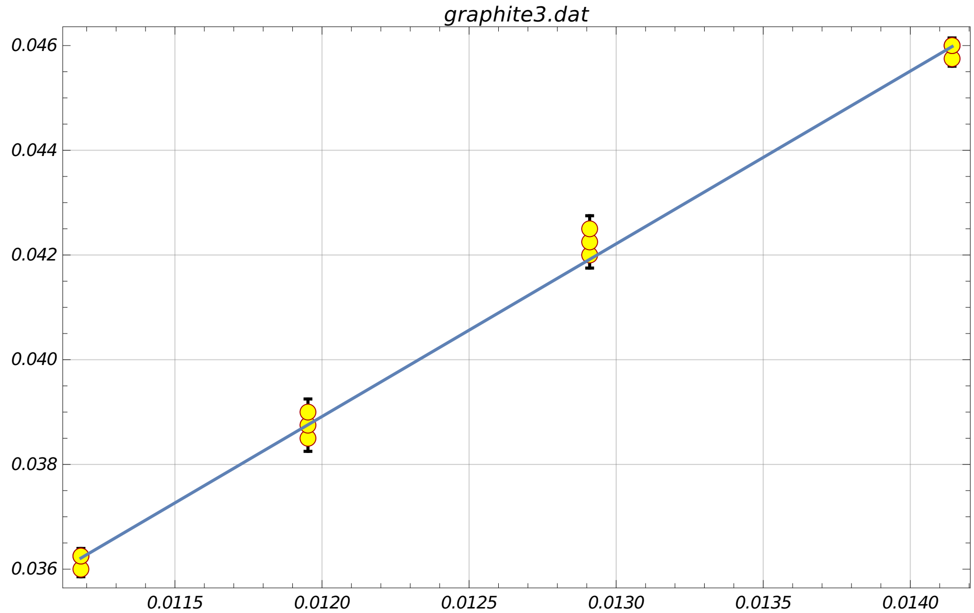
Fit of (x,y) (unweighted)

a=	b=	
-0.00082914	3.31575	
σ_a=	σ_b=	Std. deviation=
0.000833974	0.0662238	0.000254145

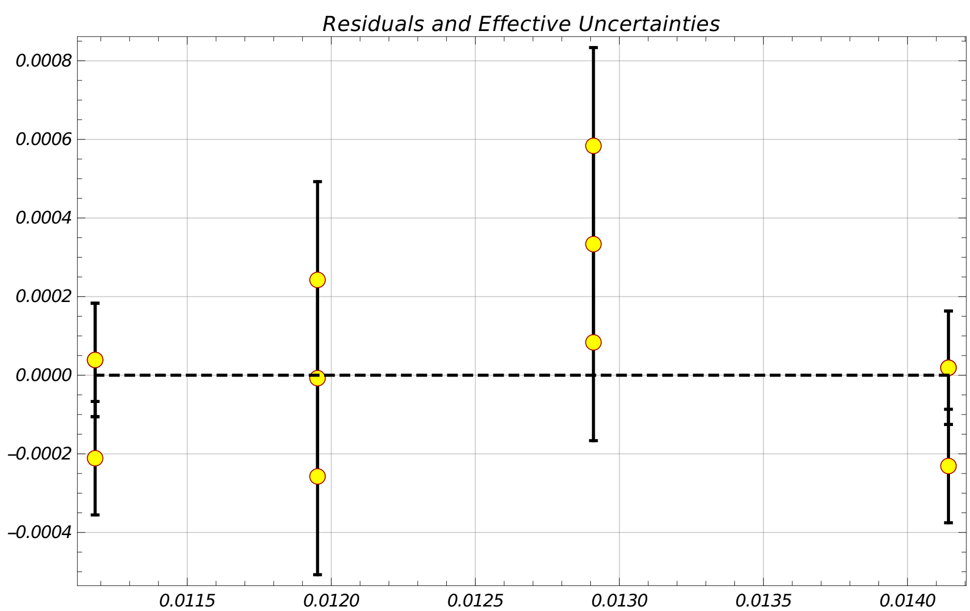
Fit of $(x, y \pm \sigma_y)$

a=	b=	
-0.000668698	3.29863	
σ_a=	σ_b=	$\chi^2/(n-2)$=
0.000494201	0.0390012	1.42234

In[410]:= LinearDifferencePlot[]



Out[410]=



y(x) = a + b x

a=	b=	
-0.000668698	3.29863	
σ_a=	σ_b=	$\chi^2/(n-2)$=
0.000494201	0.0390012	1.42234

In[508]:=
```
(* Calculating h *)
hg3 = h[3.29863, 0.0390012, graphitek3, graphitea0]
```

Out[508]= $\{6.66\times10^{-15}, 7.9\times10^{-17}\}$

Aluminium

First Diffraction Feature

In[14]:=
```
(* Import and prepare the data for analysis *)
```

In[412]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/aluminium1.dat

File comment header:

V (kV) d1 (cm)

LoadFile: Data sorted in increasing x order.

Read 12 data points.

In[413]:=
```
CalculateYsigmas[]
```

Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.

In[414]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.      *)
```

xnew[x_, y_] := $(1000 * x)^{\frac{-1}{2}}$

ynew[x_, y_] := $\frac{y}{2} * 0.01$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 12 points transformed.

In[417]:=
```
LinearFit[]
```

n = 12

y(x) = a + b x

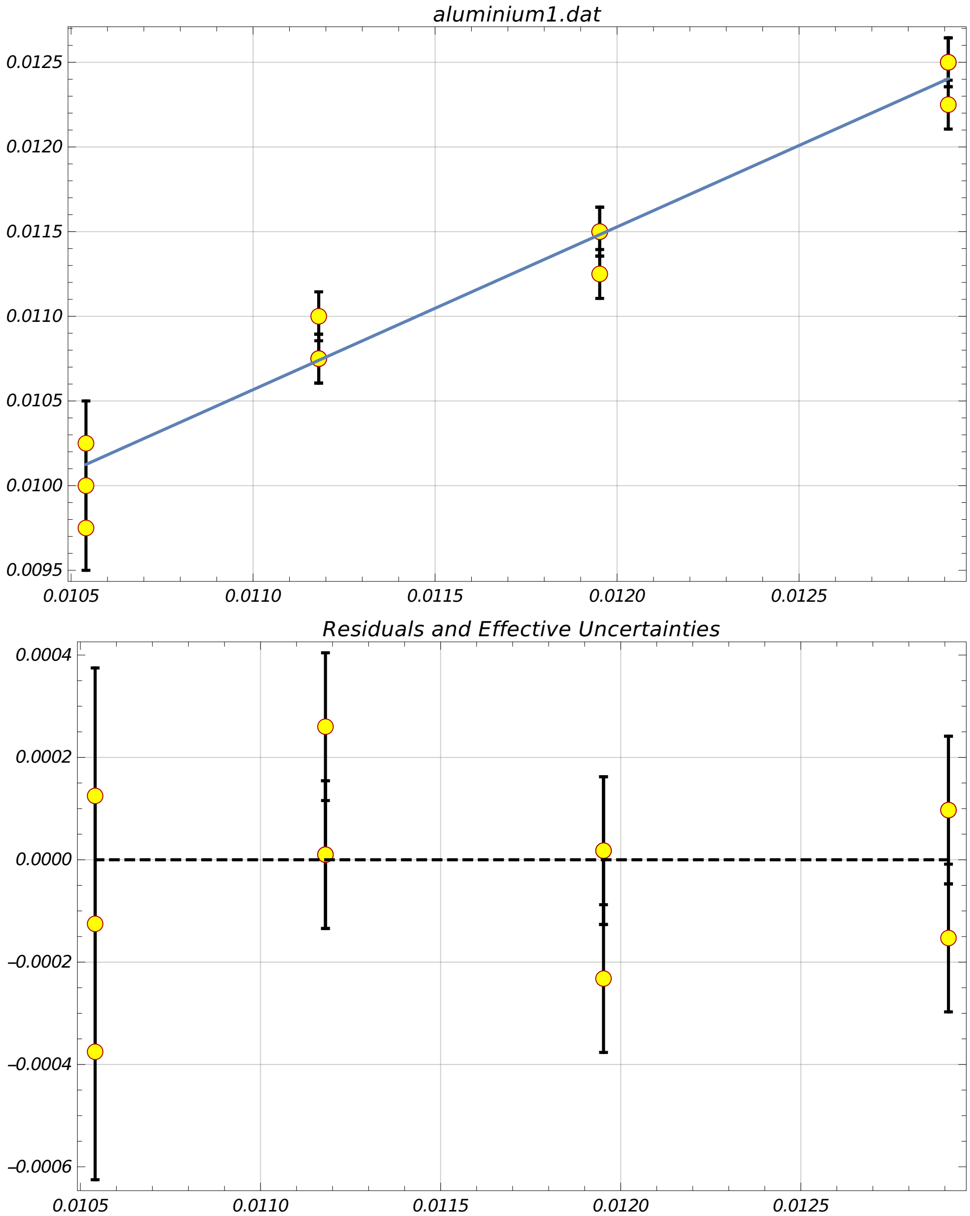
Fit of (x,y) (unweighted)

a= -0.000373444	b= 0.990918	
σ_a= 0.00068478	σ_b= 0.0586499	Std. deviation= 0.000179785

Fit of ($x, y \pm \sigma_y$)

a= -9.4456×10^{-6}	b= 0.961454	
σ_a= 0.000675513	σ_b= 0.0567942	$\chi^2/(n-2)$= 1.06526

In[418]:= `LinearDifferencePlot[]`



Out[418]=

y(x) = a + b x

a= -9.4456×10^{-6}	b= 0.961454	
σ_a= 0.000675513	σ_b= 0.0567942	$\chi^2/(n-2)$= 1.06526

In[509]:=
```
(* Calculating h *)
ha1 = h[0.961454, 0.0567942, aluminiumk1, aluminiuma0]
```

Out[509]= $\{4.26\times10^{-15}, 2.5\times10^{-16}\}$

Second Diffraction Feature

In[14]:=
```
(* Import and prepare the data for analysis *)
```

In[420]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/aluminium2.dat

File comment header:

V (kV) d2 (cm)

LoadFile: Data sorted in increasing x order.

Read 12 data points.

In[421]:=
```
CalculateYsigmas[]
```

Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.

In[422]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.     *)
```

$$xnew[x\_, y\_] := (1000 * x)^{\frac{-1}{2}}$$

$$ynew[x\_, y\_] := \frac{y}{2} * 0.01$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 12 points transformed.

In[425]:=
```
LinearFit[]
```

$n = 12$

$y(x) = a + b\ x$

Fit of (x,y) (unweighted)

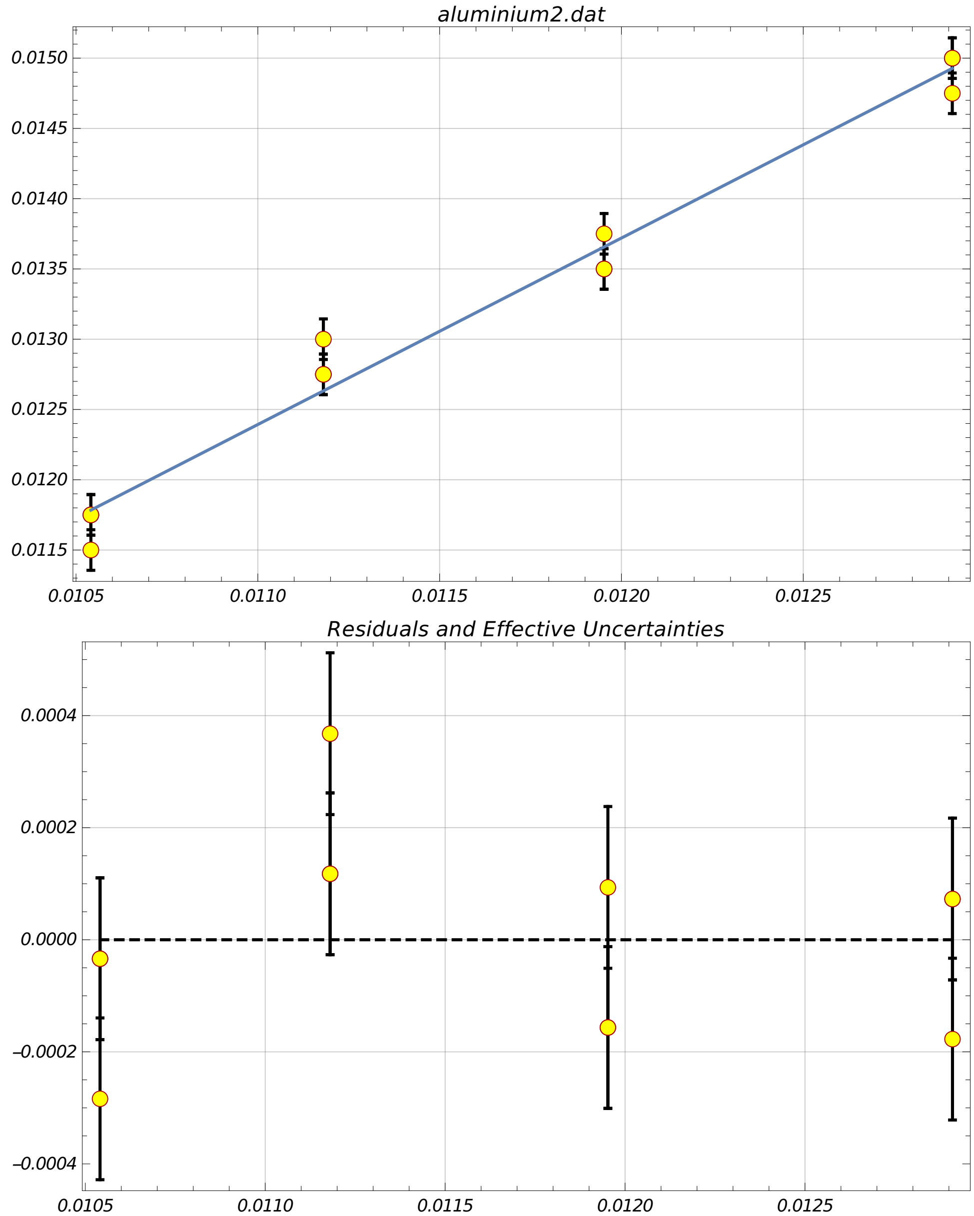
$a=$ -0.00220291	$b=$ 1.3269	
$\sigma_a=$ 0.000708172	$\sigma_b=$ 0.0606243	Std. deviation= 0.000185891

Fit of $(x, y\pm\sigma_y)$

$a=$ -0.00220291	$b=$ 1.3269	
$\sigma_a=$ 0.000549919	$\sigma_b=$ 0.0470845	$\chi^2/(n-2)=$ 1.65867

In[426]:=
```
LinearDifferencePlot[]
```

Out[426]=



y(x) = a + b x

a=	b=	
-0.00220291	1.3269	
σ_a=	σ_b=	$\chi^2/(n-2)$ =
0.000549919	0.0470845	1.65867

In[510]:=
```
(* Calculating h *)
ha2 = h[1.3269, 0.0470845, aluminiumk2, aluminiuma0]
```

Out[510]= $\{5.09\times10^{-15}, 1.8\times10^{-16}\}$

Third Diffraction Feature

In[14]:=
```
(* Import and prepare the data for analysis *)
```

In[428]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

```
/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/aluminium3.dat

File comment header:

V (kV) d3 (cm)
```

LoadFile: Data sorted in increasing x order.

```
Read 12 data points.
```

In[429]:=
```
CalculateYsigmas[]
```

```
Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.
```

In[430]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.      *)
```

$$\text{xnew[x\_, y\_] := } (1000 * x)^{\frac{-1}{2}}$$

$$\text{ynew[x\_, y\_] := } \frac{y}{2} * 0.01$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

```
All 12 points transformed.
```

In[433]:=
```
LinearFit[]
```

n = 12

y(x) = a + b x

Fit of (x,y) (unweighted)

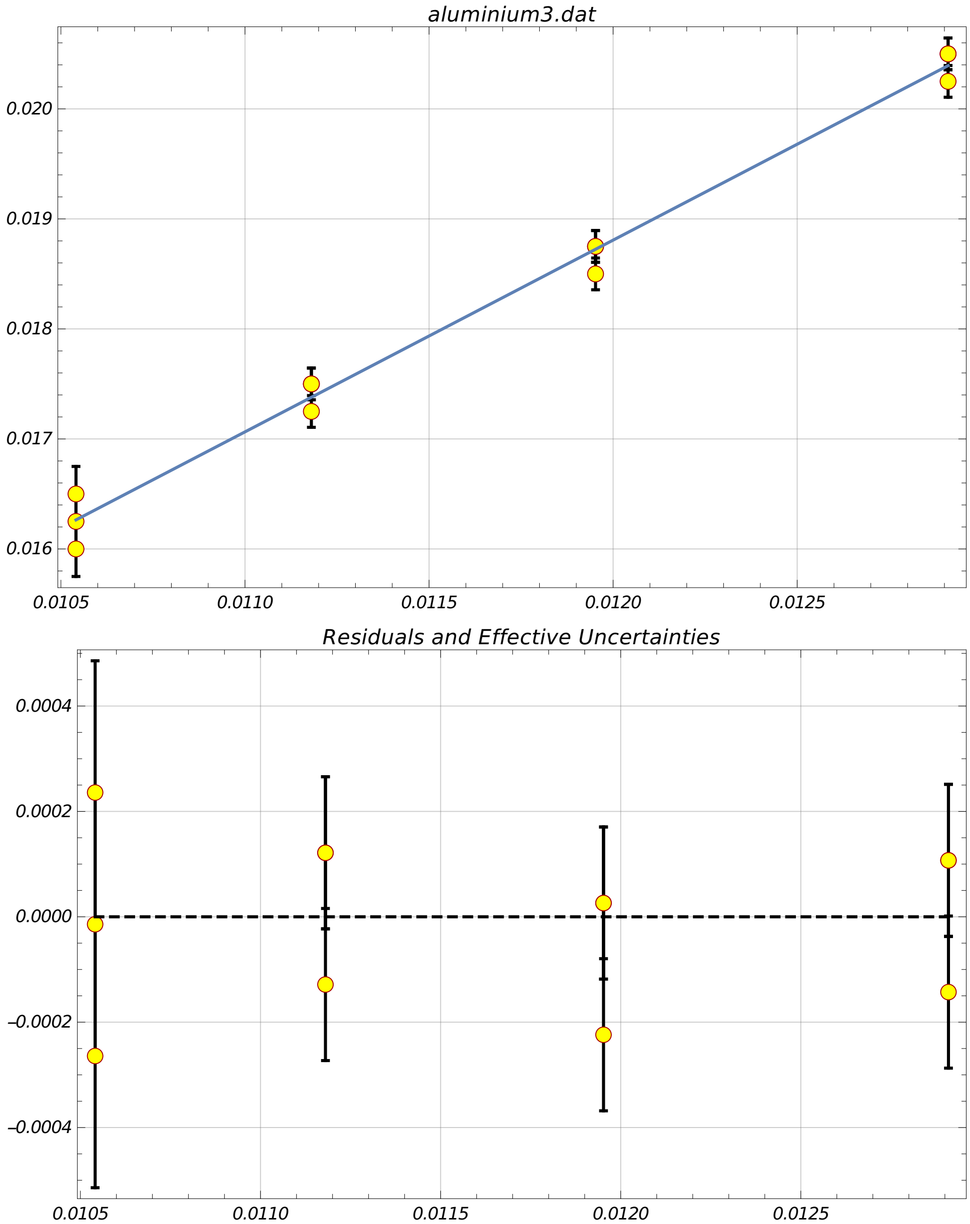
a= −0.00214772	b= 1.74613	
σ_a= 0.000621888	σ_b= 0.0532458	Std. deviation= 0.00016319

Fit of $(x,y\pm\sigma_y)$

a= −0.00210654	b= 1.7428	
σ_a= 0.000675506	σ_b= 0.0567936	$\chi^2/(n-2)$= 0.877183

In[434]:= `LinearDifferencePlot[]`

Out[434]=



$y(x) = a + b\,x$

a=	b=	
-0.00210654	1.7428	
σ_a=	σ_b=	$\chi^2/(n-2)$=
0.000675506	0.0567936	0.877183

In[511]:=
```
(* Calculating h *)
ha3 = h[1.7428, 0.0567936, aluminiumk3, aluminiuma0]
```

Out[511]= $\{4.73\times10^{-15},\ 1.5\times10^{-16}\}$

Fourth Diffraction Feature

In[14]:=
```
(* Import and prepare the data for analysis *)
```

In[436]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/aluminium4.dat

File comment header:

V (kV) d4 (cm)

... LoadFile: Data sorted in increasing x order.

Read 12 data points.

In[437]:=
```
CalculateYsigmas[]
```

Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.

In[438]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.      *)
```

$$\text{xnew[x\_, y\_] := } (1000 * x)^{\frac{-1}{2}}$$

$$\text{ynew[x\_, y\_] := } \frac{y}{2} * 0.01$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 12 points transformed.

In[441]:=
```
LinearFit[]
```

$n = 12$

$y(x) = a + b\,x$

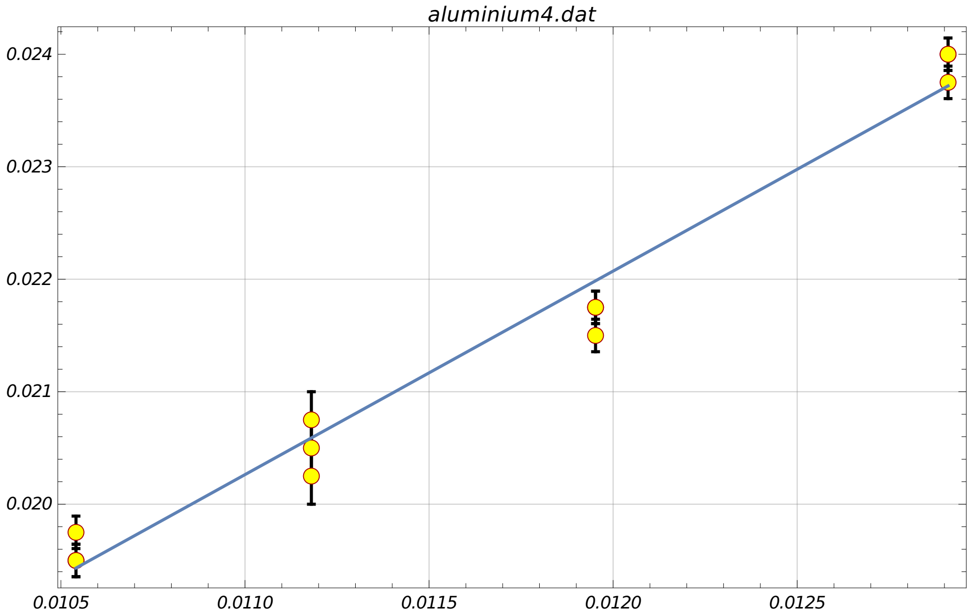
Fit of (x,y) (unweighted)

$a=$ 0.000244407	$b=$ 1.81801	
$\sigma_a=$ 0.00105171	$\sigma_b=$ 0.0900916	Std. deviation= 0.00027611

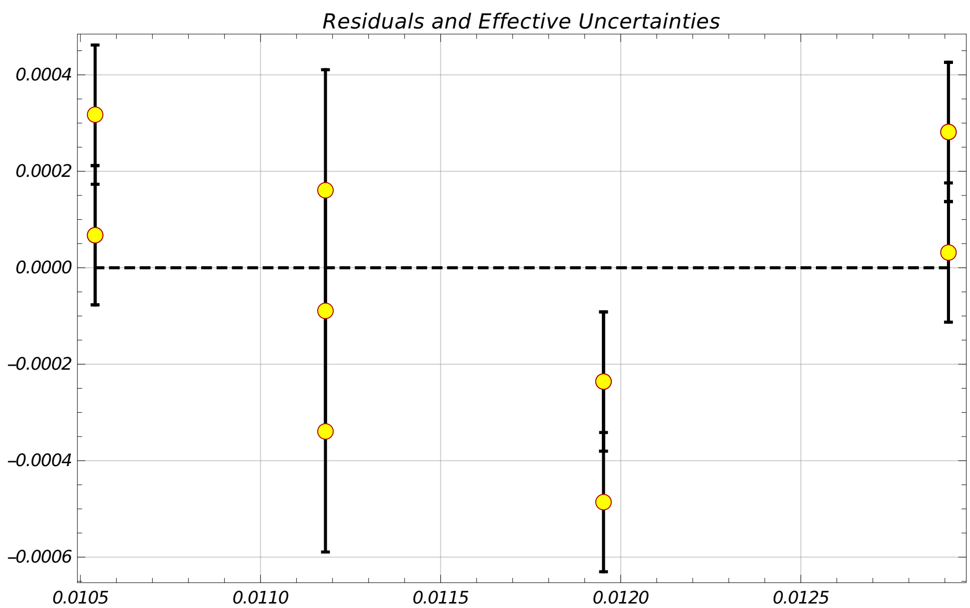
Fit of $(x, y \pm \sigma_y)$

$a=$ 0.000362479	$b=$ 1.80915	
$\sigma_a=$ 0.000570581	$\sigma_b=$ 0.0484501	$\chi^2/(n-2)=$ 3.19905

In[442]:= `LinearDifferencePlot[]`



Out[442]=



y(x) = a + b x

a= 0.000362479	b= 1.80915	
σ_a= 0.000570581	σ_b= 0.0484501	$\chi^2/(n-2)$ = 3.19905

No obviously anomalous point is found, but the high error involved in measuring the very dim fourth diffraction feature and high $\tilde{\chi}^2$ indicate that we should discount this feature when calculating h.

All data

We attempt to fit the entire data set by first scaling data from each diffraction feature by its

$\frac{\hat{k}}{2\pi a_0}$.

In[459]:=
```
graphite1data = Import["graphite1.dat"];
graphite1 =
  Table[{graphite1data[[i]][[1]], 0.01 * graphite1data[[i]][[2]] * 2 π graphitea0 / graphitek1},
   {i, 3, Length[graphite1data]}];
graphite2data = Import["graphite2.dat"];
graphite2 =
  Table[{graphite2data[[i]][[1]], 0.01 * graphite2data[[i]][[2]] * 2 π graphitea0 / graphitek2},
   {i, 4, Length[graphite2data]}];
graphite3data = Import["graphite3.dat"];
graphite3 =
  Table[{graphite3data[[i]][[1]], 0.01 * graphite3data[[i]][[2]] * 2 π graphitea0 / graphitek3},
   {i, 4, Length[graphite3data]}];
aluminium1data = Import["aluminium1.dat"];
aluminium1 =
  Table[{aluminium1data[[i]][[1]], 0.01 * aluminium1data[[i]][[2]] * 2 π aluminiuma0 / aluminiumk1},
   {i, 3, Length[aluminium1data]}];
aluminium2data = Import["aluminium2.dat"];
aluminium2 =
  Table[{aluminium2data[[i]][[1]], 0.01 * aluminium2data[[i]][[2]] * 2 π aluminiuma0 / aluminiumk2},
   {i, 3, Length[aluminium2data]}];
aluminium3data = Import["aluminium3.dat"];
aluminium3 =
  Table[{aluminium3data[[i]][[1]], 0.01 * aluminium3data[[i]][[2]] * 2 π aluminiuma0 / aluminiumk3},
   {i, 3, Length[aluminium3data]}];
```

We then join this into one dataset and export as a CurveFit-readable file.

In[471]:=
```
dataset = Join[{{"S V (kV) Scaled r (m^2)"}}, {{"S"}}, graphite1,
   (*graphite2,graphite3,*)aluminium1, aluminium2, aluminium3];
```

In[472]:=
```
Export["dataset.dat", dataset]
```

Out[472]= dataset.dat

In[473]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

/home/yovan/Documents/Coursework/2_Smore_Year/2_Winter_2018/Ph6/Lab 5/dataset.dat

File comment header:

V (kV) Scaled r (m^2)

LoadFile: Data sorted in increasing x order.

Read 51 data points.

In[474]:=
```
CalculateYsigmas[]
```

Sorted data in order of increasing X values.

Calculated and assigned Y uncertainties.

In[475]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.     *)
```

$$xnew[x\_, y\_] := (1000 * x)^{\frac{-1}{2}}$$

$$ynew[x\_, y\_] := \frac{y}{2}$$

```
DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 51 points transformed.

In[478]:=
```
LinearFit[]
```

n = 51

y(x) = a + b x

Fit of (x,y) (unweighted)

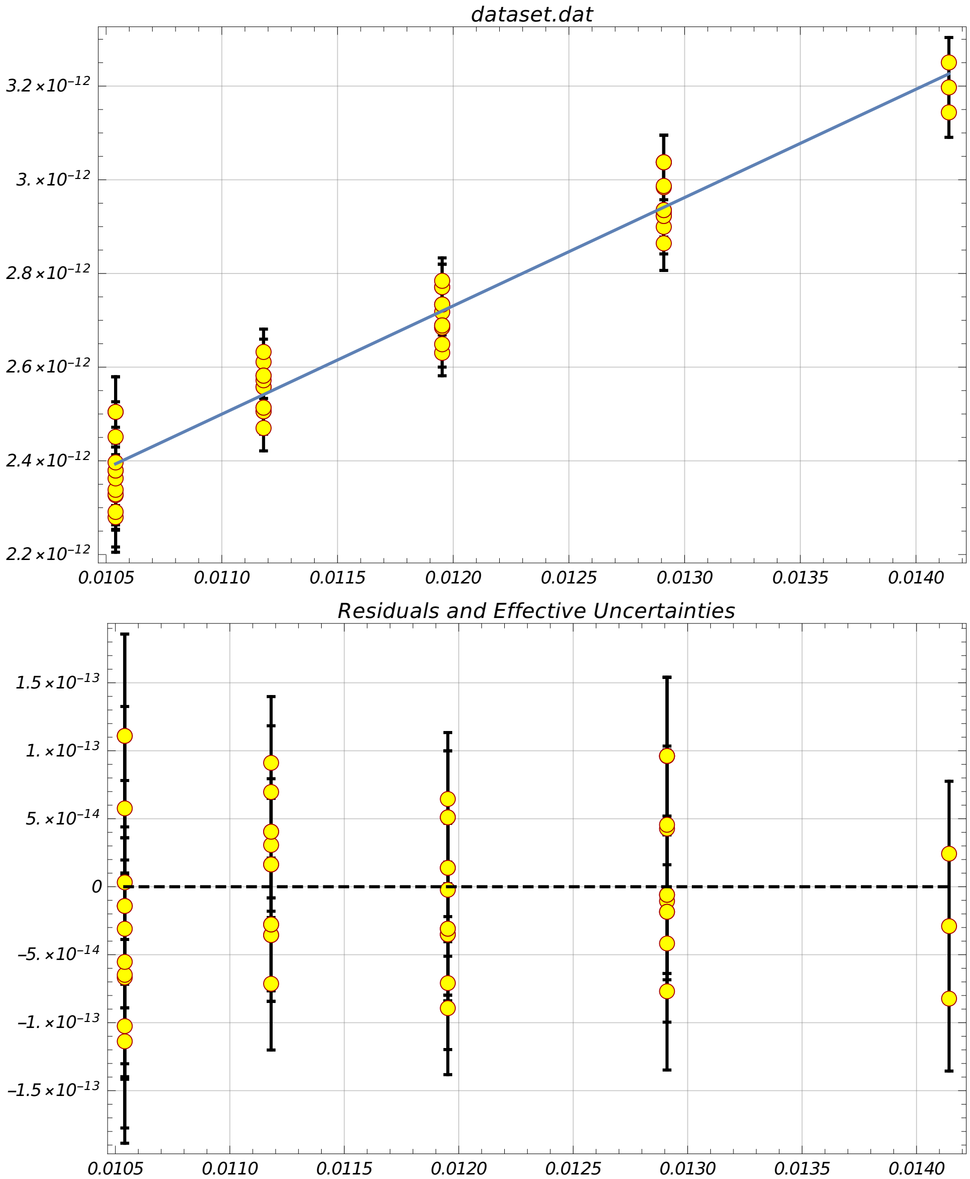
a= -4.93547×10^{-13}	b= 2.69252×10^{-10}	
σ_a= 1.58684×10^{-14}	σ_b= 1.21456×10^{-12}	Std. deviation= 6.91759×10^{-14}

Fit of $(x, y \pm \sigma_y)$

a= -4.3699×10^{-14}	b= 2.31217×10^{-10}	
σ_a= 9.56118×10^{-14}	σ_b= 8.02025×10^{-12}	$\chi^2/(n-2)$ = 1.0021

In[479]:=
```
LinearDifferencePlot[]
```

Out[479]=



y(x) = a + b x

a= -4.3699×10^{-14}	b= 2.31217×10^{-10}	
σ_a= 9.56118×10^{-14}	σ_b= 8.02025×10^{-12}	$\chi^2/(n-2)$ = 1.0021

We define a new function for calculating h in this case:

In[504]:=

$$h2[slope\_, error\_] := \left\{ScientificForm\left[\frac{slope * Sqrt[2 * mc2]}{x * c}, 3\right], ScientificForm\left[\frac{error * Sqrt[2 * mc2]}{x * c}, 2\right]\right\}$$

In[512]:=

$$h2all = h2\left[2.31217 * 10^{-10}, 8.02025 * 10^{-12}\right]$$

Out[512]= $\{4.38\times10^{-15}, 1.5\times10^{-16}\}$

Note that we had to remove the second and third graphite diffraction features from the dataset to get a reasonable collapse of the data points by our scaling $\frac{\hat{k}}{2\pi a_0}$. This indicates a large systematic error in those measurements. All calculated values for h are in eVs.

Conclusion

In[520]:=
```
summary =
  Join[{{"Feature", "h (eVs)", "σh (eVs)"}}, {Join[{"First (Graphite)"}, hg1]},
   {Join[{"Second (Graphite)"}, hg2]}, {Join[{"Third (Graphite)"}, hg3]},
   {Join[{"First (Aluminium)"}, ha1]}, {Join[{"Second (Aluminium)"}, ha2]},
   {Join[{"Third (Aluminium)"}, ha3]}, {Join[{"Overall"}, h2all]}];
Grid[summary, Alignment → Left, Spacings → {2, 1}, Frame → All,
 ItemStyle → "Text", Background → {{Gray, None}, {LightGray, None}}]
```

Out[521]=

Feature	h (eVs)	σ_h (eVs)
First (Graphite)	3.86×10^{-15}	1.7×10^{-16}
Second (Graphite)	4.29×10^{-15}	1.1×10^{-16}
Third (Graphite)	6.66×10^{-15}	7.9×10^{-17}
First (Aluminium)	4.26×10^{-15}	2.5×10^{-16}
Second (Aluminium)	5.09×10^{-15}	1.8×10^{-16}
Third (Aluminium)	4.73×10^{-15}	1.5×10^{-16}
Overall	4.38×10^{-15}	1.5×10^{-16}

For comparison, the NIST accepted value for h is 4.136×10^{-15} eV.

We observe a large scattering of our estimates for h for each feature despite a good (around 1) $\tilde{\chi}^2$ for each individual data set. This is due to two main issues with our data: few reliable data points and a large systematic error for each feature. One data set in particular seem to show significant systematic errors: the third graphite diffraction pattern. This is consistent with what we have found above - the measurements for the third feature of diffraction in graphite is not collapsed onto a collinear set with the rest of the data points by a scaling $\frac{\hat{k}}{2\pi a_0}$as we would expect. This either indicates a large systematic error in our measurements for this feature, or that the small-angle approximation starts to fail for the third feature of graphite and onwards.

While the second diffraction pattern provides a reasonable estimate of h, we observe that its dataset is not collapsed by the scaling either. This dataset was discarded on the basis of internal inconsistencies (we had to remove a data point to obtain a linear fit, and even then the fit was significantly worse than most of the other data sets).

We collapsed the remaining data sets using the scaling and used a linear fit to obtain an estimate of h. We obtain $h = (4.38 \pm 0.15)\times10^{-15}$ eVs, which is within two standard errors of the NIST value. This is a reasonable estimate with a very good $\tilde{\chi}^2$ for the fit, indicating that our collapsed dataset is reasonably free of significant systematic errors (i.e. the systematic errors for each data point were probably independent and random to within the accuracy of the experiment).