Experiment 32a - Compton Scattering

In[1]:=
```
<< CurveFit`
```
```
CurveFit for Mathematica v7.x thru v11.x, Version 1.96, 4/4/2018
Caltech Sophomore Physics Labs, Pasadena, CA
```

In[6]:=
```
SetDirectory[NotebookDirectory[]]
```

Energy calibration of MCA

NaI detector

We construct an energy calibration function for the NaI scintillator using the full-energy peaks in the calibration data. We begin by removing noise from our calibration data.

In[88]:=
```
calFiles = StringJoin["32a/", #] & /@
   {"ba133_cal_na.mca", "co60_cal_na.mca", "cs137_cal_na.mca", "na22_cal_na.mca"};
errFiles = StringJoin["err_", #] & /@ {"ba133_cal_na.mca",
    "co60_cal_na.mca", "cs137_cal_na.mca", "na22_cal_na.mca"};
errFiles = StringJoin["32a/", #] & /@ errFiles;
```

In[91]:=
```
(* Add Poisson count errors to datafiles and export *)
For[i = 1, i ≤ Length[calFiles], i++, (LoadFile[calFiles[[i]]];
   MakePoisson[Xerrors -> True];
   SaveFile[errFiles[[i]]])];
```

In[92]:=
```
(* Add poisson count errors to noise and export *)
LoadFile["32a/noise_na.mca"];
MakePoisson[Xerrors → True];
SaveFile["32a/err_noise_na.mca"];
```

In[108]:=
```
(* Removing noise by normalizing over runtime,
and subtracting noise and propagating errors. *)
runtimes = Table[
   StringSplit[Import[calFiles[[i]]][[1]]][[2]][[5]], {i, 1, Length[calFiles]}];
runNoise = StringSplit[Import["32a/noise_na.mca"][[1]]][[2]][[5]];
```

In[110]:=
```
norm = For[i = 1, i ≤ Length[errFiles], i++, (file = Import[errFiles[[i]]];
   normFile = Table[{file[[j]][[1]], file[[j]][[2]] / ToExpression[runtimes[[i]]],
      file[[j]][[3]] / ToExpression[runtimes[[i]]], file[[j]][[4]]}, {j, 1, Length[file]}];
   Export[StringJoin["32a/norm_", StringJoin[FileBaseName[calFiles[[i]]]],
     ".dat"], normFile];)];
```

In[111]:=
```
file = Import["32a/err_noise_na.mca"];
normFile = Table[{file[[j]][[1]], file[[j]][[2]] / ToExpression[runNoise],
    file[[j]][[3]] / ToExpression[runNoise], file[[j]][[4]]}, {j, 1, Length[file]}];
Export[StringJoin["32a/norm_", StringJoin["noise_na"], ".dat"], normFile];
```

In[112]:=
```
(* Subtracting normalized noise from files *)
normList =
  StringJoin["32a/norm_", StringJoin[FileBaseName[#]], ".dat"] & /@ calFiles;
```

In[113]:=
```
normNoise = Import["32a/norm_noise_na.dat"];
```

In[114]:=
```
For[i = 1, i ≤ Length[normList], i++, (file = Import[normList[[i]]];
   corrected = Table[{file[[j]][[1]], file[[j]][[2]] - normNoise[[j]][[2]],
      Sqrt[file[[j]][[3]]^2 + normNoise[[j]][[3]]^2],
      Sqrt[file[[j]][[4]]^2 + normNoise[[j]][[4]]^2]}, {j, 1, Length[file]}];
   Export[StringJoin["32a/corrected_", StringJoin[FileBaseName[calFiles[[i]]]],
     ".dat"], corrected];)];
```

We now fit the full-energy peaks in the calibration data to construct our energy calibration function for the NaI detector.

Ba-133

In[115]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[122]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
    Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[125]:=
```
GaussianCFit[]
```

```
calData = {}; (* {energy in keV, channel, sigchannel, errenergy=0} *)
```

In[127]:=
```
calData = AppendTo[calData, {32, mean, sigmean, 0}];
```

In[134]:=
```
Undo[]
```

In[137]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[138]:=
```
GaussianCFit[]
```

In[139]:=
```
calData = AppendTo[calData, {81, mean, sigmean, 0}];
```

In[145]:=
```
Undo[]
```

In[149]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[150]:=
```
GaussianCFit[]
```

In[157]:=
```
calData = AppendTo[calData, {356, mean, sigmean, 0}];
```

Cs-137

In[158]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[165]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[166]:=
```
GaussianCFit[]
```

```
In[168]:= calData = AppendTo[calData, {32, mean, sigmean, 0}];
```

```
In[172]:= Undo[]
```

```
In[173]:= With[{x = SetXRange[ LinearDataPlot[], Log -> False,
              Label -> "Set the X values for the range you wish to keep." ]},
           Print[x];

           XRangeKeep[Sequence @@ x]
          ]
```

```
In[175]:= GaussianCFit[]
```

```
In[176]:= calData = AppendTo[calData, {661, mean, sigmean, 0}];
```

Na-22

```
In[179]:= With[ {name = SystemDialogInput["FileOpen",
              {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
           If[ name =!= $Canceled,
            LoadFile[name]]
          ]
```

```
In[180]:= With[{x = SetXRange[ LinearDataPlot[], Log -> False,
              Label -> "Set the X values for the range you wish to keep." ]},
           Print[x];

           XRangeKeep[Sequence @@ x]
          ]
```

```
In[181]:= GaussianCFit[]
```

```
In[182]:= calData = AppendTo[calData, {511, mean, sigmean, 0}];
```

The Na-22 data seems very prone to noise, therefore we only use the 0.522 MeV annihilation peak for calibration (the others features are buried in noise. Although the non noise-corrected spectrum is cleaner, this is an indication that the calibration data collected for Na-22 is of poor quality so we neglect the other features in the calibration.).

Co-60

```
In[186]:= With[ {name = SystemDialogInput["FileOpen",
              {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
           If[ name =!= $Canceled,
            LoadFile[name]]
          ]
```

In[187]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
    Label -> "Set the X values for the range you wish to keep."]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[188]:=
```
GaussianCFit[]
```

In[189]:=
```
calData = AppendTo[calData, {1173, mean, sigmean, 0}];
```

In[190]:=
```
Undo[]
```

In[191]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
    Label -> "Set the X values for the range you wish to keep."]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[192]:=
```
GaussianCFit[]
```

In[193]:=
```
calData = AppendTo[calData, {1332, mean, sigmean, 0}];
```

Constructing the calibration function

In[218]:=
```
(* Combine 32 keV datapoints. *)
calData[[1]] = {32, Mean[{calData[[1]][[2]], calData[[4]][[2]]}],
   Sqrt[calData[[1]][[3]]^2 + calData[[4]][[3]]^2] / 2, 0};
```

In[223]:=
```
calData = Delete[calData, 4];
```

In[224]:=
```
Export["32a/cal_data.dat", calData];
```

In[327]:=
```
calTable = Join[{{"Energy (keV)", "Channel Number"}},
   Table[{StringReplace["%1", {"%1" → ToString[NumberForm[calData[[i]][[1]], 3]]}],
     StringReplace["%1 ± %2",
      {"%1" → ToString[NumberForm[calData[[i]][[2]], 3]], "%2" →
        ToString[NumberForm[calData[[i]][[3]], 2]]}]}, {i, 1, Length[calData]}]];
Grid[calTable, Alignment → Left, Spacings → {2, 1}, Frame → All,
 ItemStyle → "Text", Background → {{Gray, None}, {LightGray, None}}]
```

Out[328]=

Energy (keV)	Channel Number
32	38.4 ± 0.026
81	66.2 ± 0.047
356	250. ± 0.083
661	460. ± 0.092
511	358. ± 0.4
1173	834. ± 0.48
1332	944. ± 0.47

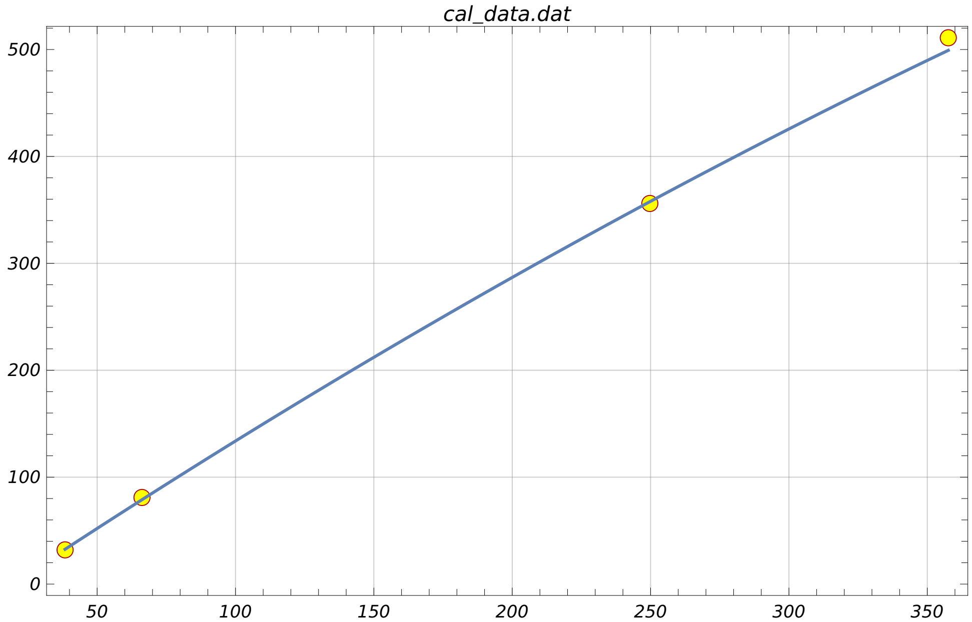
In[241]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[243]:=
```
SwitchXXandYY[]
```

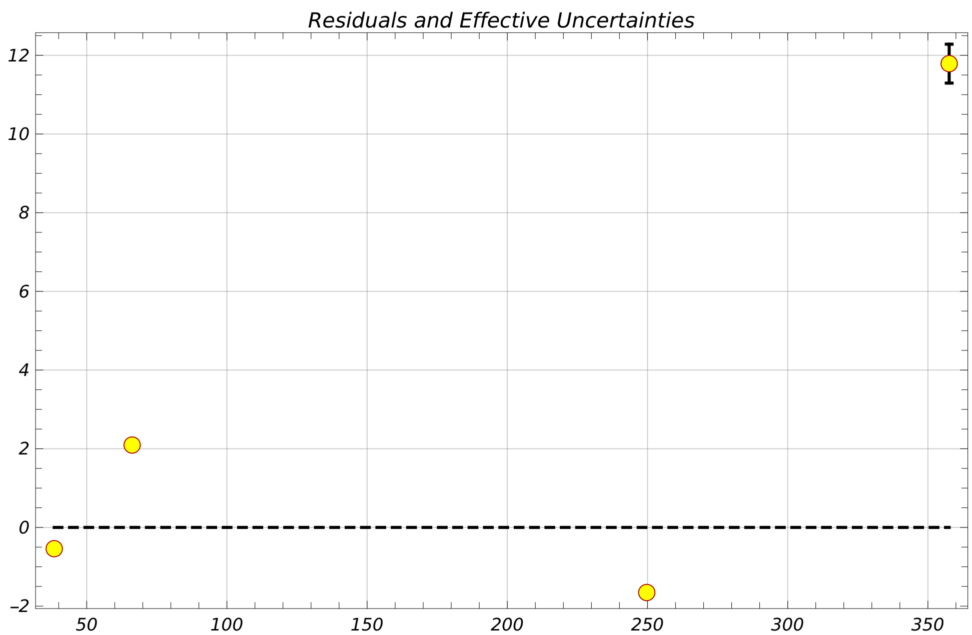
In[244]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
    Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[246]:=
```
QuadraticFit[]
```

In[247]:= `LinearDifferencePlot[]`



Out[247]=



$y(x) = a + b\ x + c\ x^2$

a= -33.3313	b= 1.74233	c= -0.000707733	
σ_a= 0.119738	σ_b= 0.00289677	σ_c= 9.43175×10^{-6}	$\chi^2/(n-3)$= 1666.42

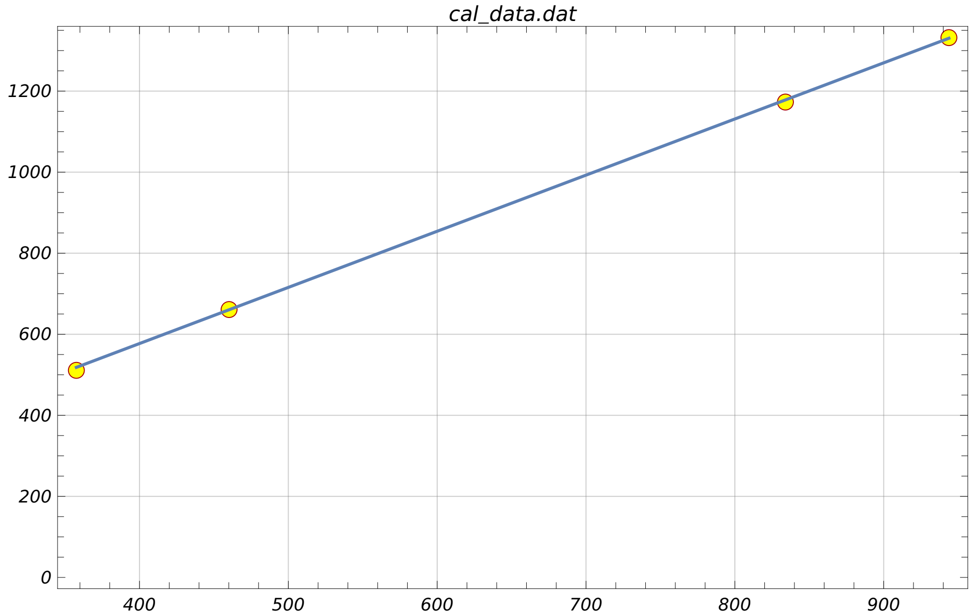
In[248]:= `Undo[]`

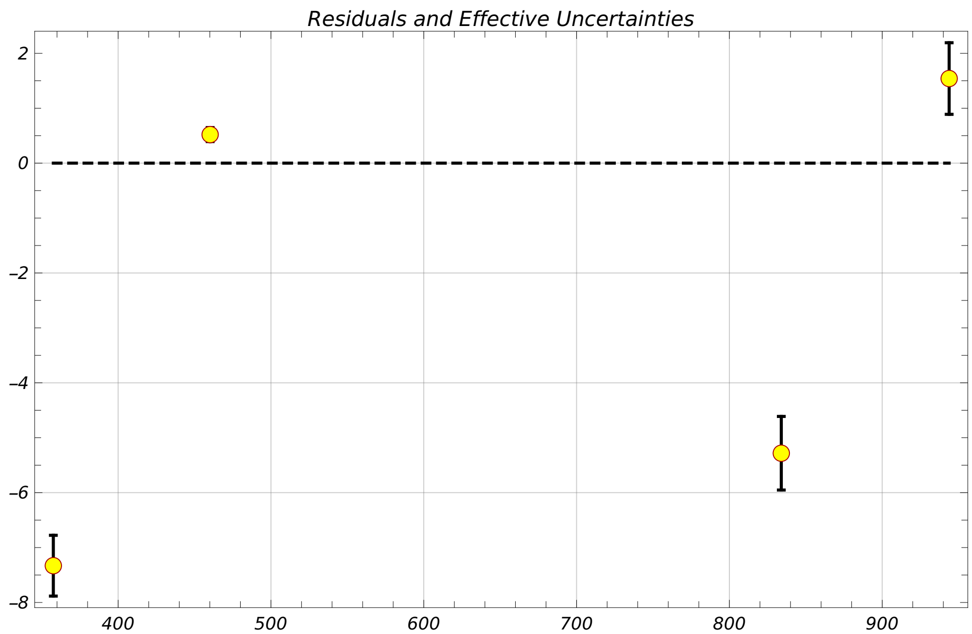
In[249]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[250]:=
```
LinearFit[]
```

In[251]:= `LinearDifferencePlot[]`

Out[251]=





$y(x) = a + b\ x$

a= 22.943	b= 1.38535	
σ_a= 0.535681	σ_b= 0.001081	$\chi^2/(n-2)$= 129.9

In[255]:=
```
calibration[channel_] :=
  {Piecewise[{{-33.331281248295426` + 1.7423347894608627` channel[[1]] +
       -0.0007077325790811842` channel[[1]]^2, channel[[1]] < 300},
     {22.943046857142885` + 1.3853510522657388` channel[[1]], channel[[1]] ≥ 300}}],
   Piecewise[{{Sqrt[0.11973843181390834`^2 + channel[[1]]^2 0.002896772825801799`^2 +
        1.7423347894608627` channel[[2]]^2 + (-0.0007077325790811842` channel[[1]]^2)^2
         ((9.431753627773405`*^-6 / -0.0007077325790811842`)^2 + (2 channel[[2]] / channel[[1]])^2)], channel[[1]] < 300},
     {Sqrt[0.5356805976904518`^2 + channel[[1]]^2 0.0010809983159092101`^2 +
        channel[[2]]^2 1.3853510522657388`^2], channel[[1]] ≥ 300}}]};
```

Energy as a function of scattering angle

The following work is done in the “Exp32.nb” notebook provided for analyzing scattering events.

40 degrees

In[•]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

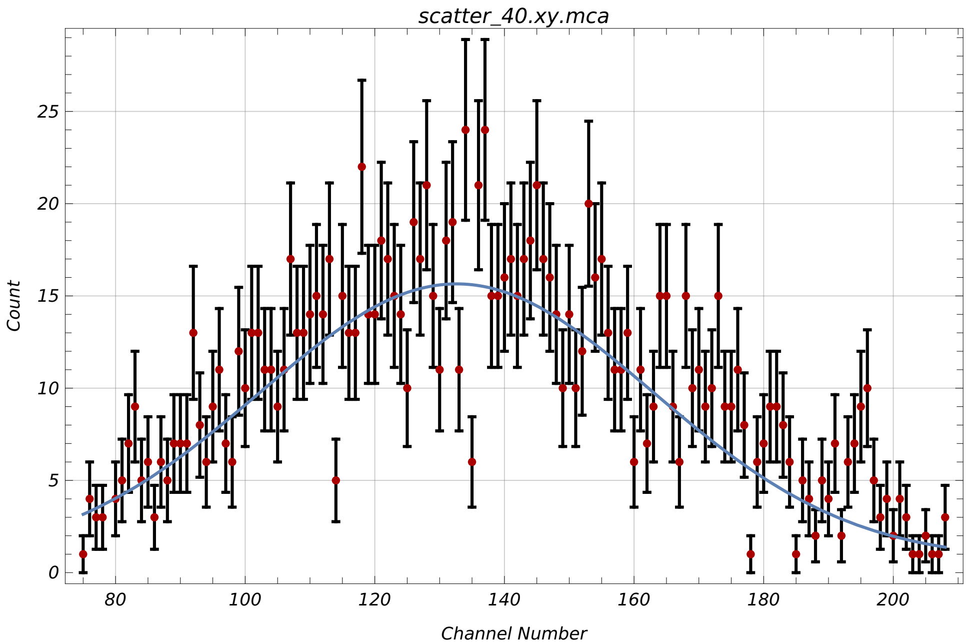
In[•]:=
```
KeepXY[]
```

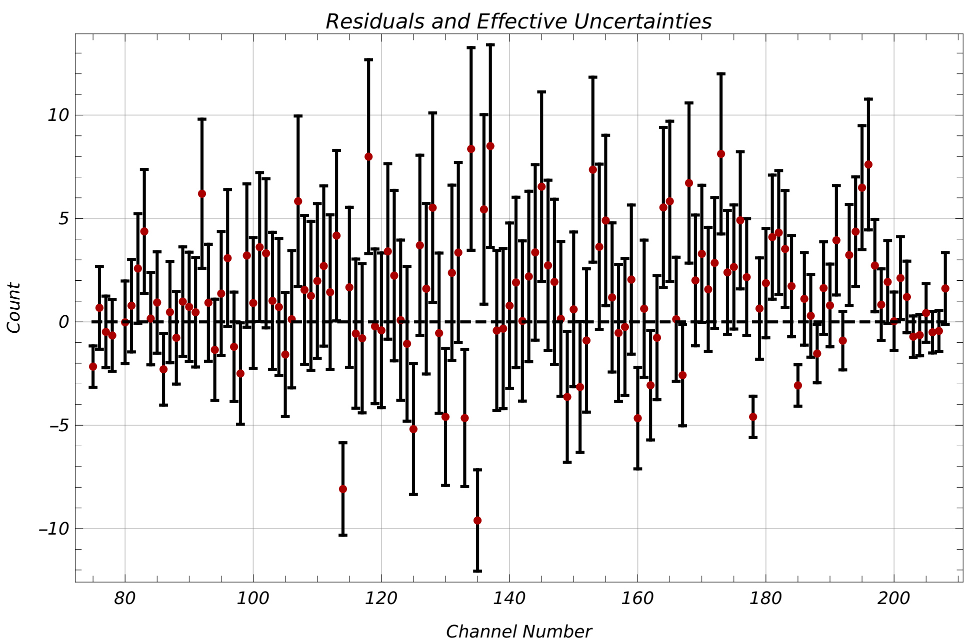
In[•]:=
```
xy2x[]
```

In[•]:=
```
GaussianCFit[]
```

In[•]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`

Out[•]=





y(x) =

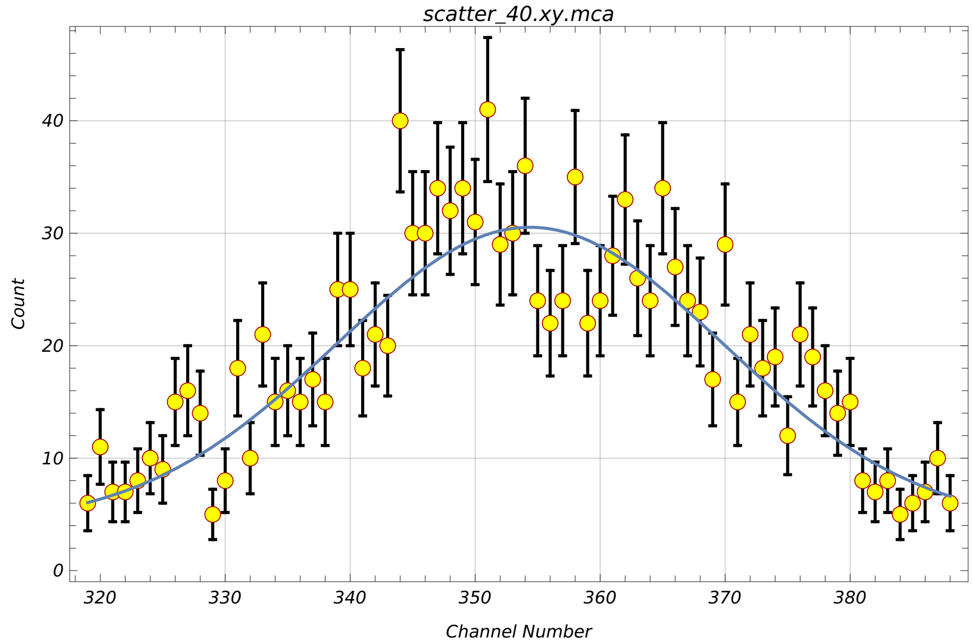
$$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$$

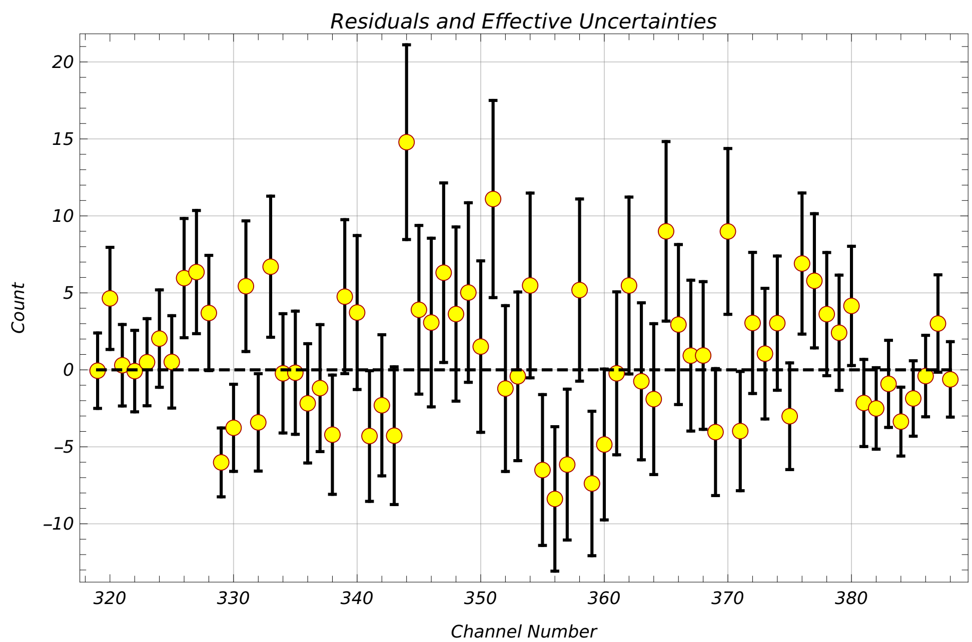
$y_{max}=$ 14.9556	c= 0.695793	
$\sigma_{y_{max}}=$ 0.774352	$\sigma_c=$ 0.632289	
$\mu=$ 132.636	sigma= 30.3599	
$\sigma_\mu=$ 1.11627	$\sigma_{\text{sigma}}=$ 2.31373	$\chi^2/(n-4)=$ 1.34211

```
In[ ]:= xData = {};

In[ ]:= AppendTo[xData, {40, mean, sigmean}];

In[ ]:= Undo[]

In[ ]:= xy2y[]

In[ ]:= GaussianCFit[]
```

In[∘]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`

Out[∘]=





$$y(x) = y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\,\mathrm{sigma}^2}\right) + c$$

y_{max} = 26.4203	c = 4.10615	
$\sigma_{y_{max}}$ = 2.08407	σ_c = 1.73134	
μ = 354.386	sigma = 15.4963	
σ_μ = 0.647671	σ_{sigma} = 1.81144	$\chi^2/(n-4)$ = 1.10695

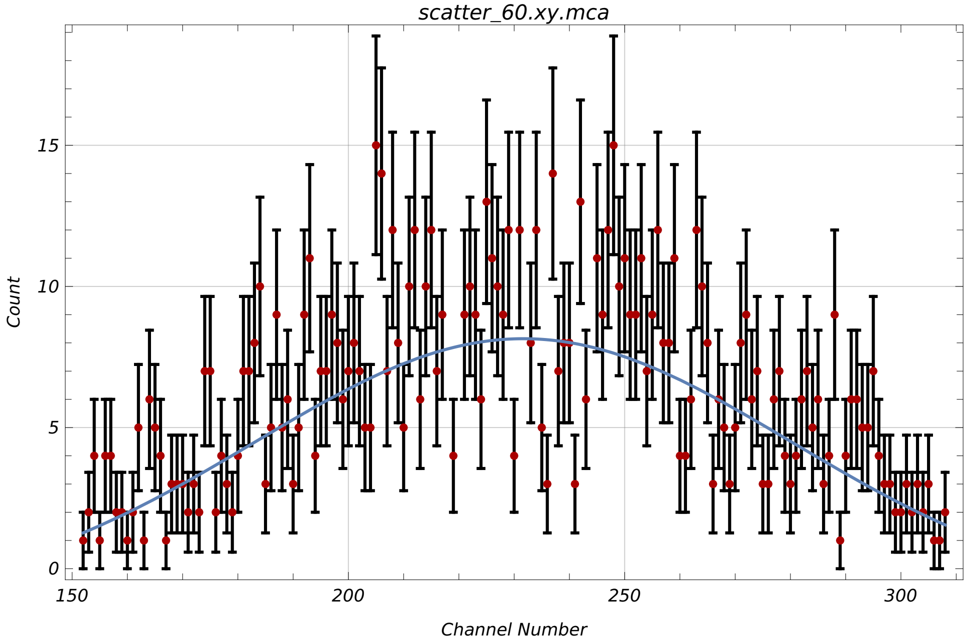
```
In[•]:= yData = {};

In[•]:= AppendTo[yData, {40, mean, sigmean}];
```

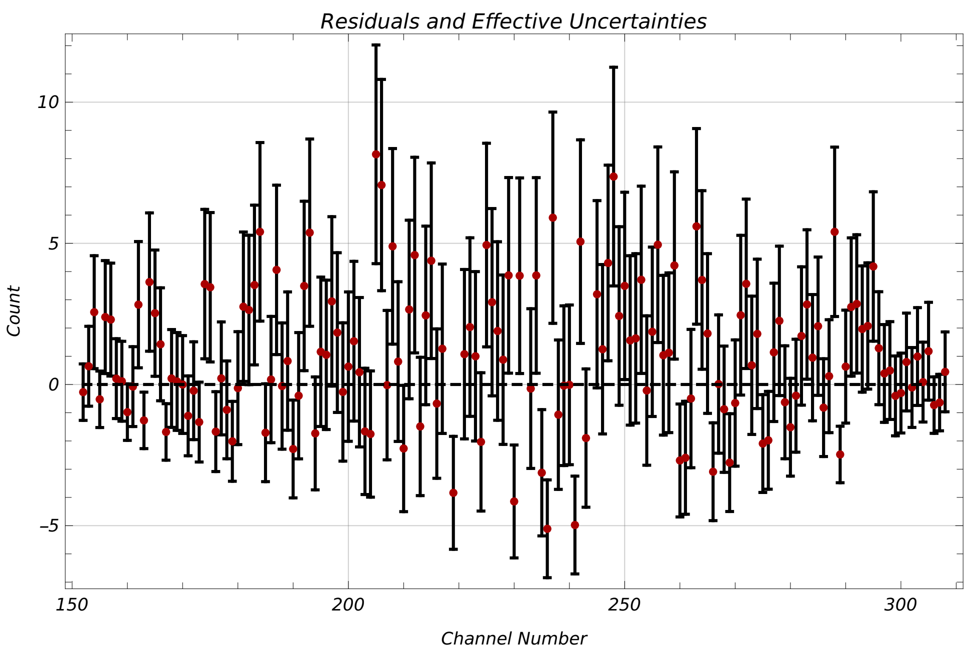
60 degrees

```
In[•]:= With[ {name = SystemDialogInput["FileOpen",
            {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
         If[ name =!= $Canceled,
          LoadFile[name]]
        ]

In[•]:= KeepXY[]

In[•]:= xy2x[]

In[•]:= NRangeRemove[#] & /@ { 67, 68, 79, 90}

In[•]:= GaussianCFit[]
```

In[∘]:=
```
LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]
```

Out[∘]=





$y(x) =$

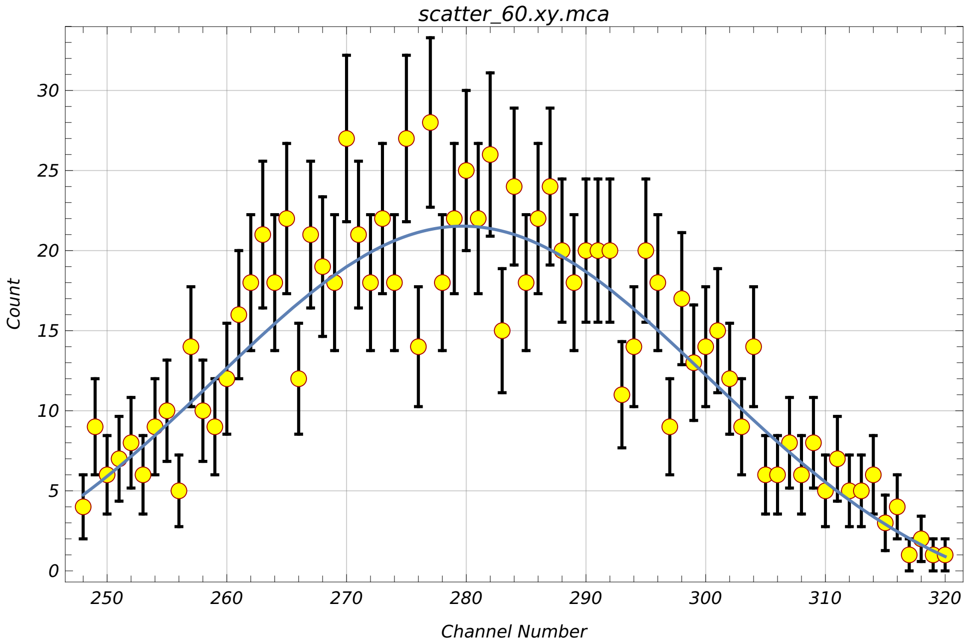
$y_{max} \exp\left(\frac{-(x-\mu)^2}{2\, sigma^2}\right) + c$

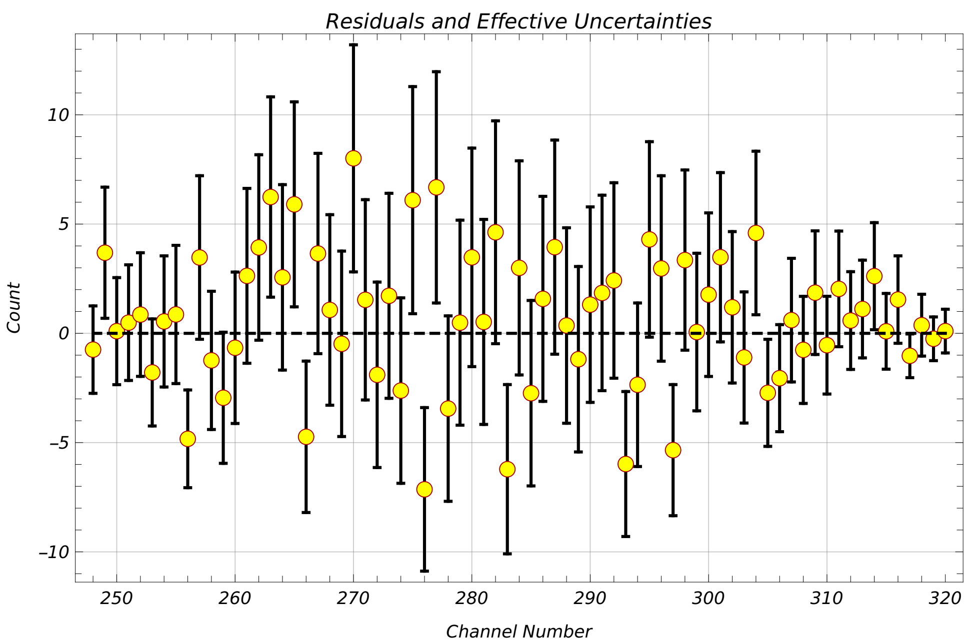
$y_{max}=$ 9.29206	$c=$ -1.14376	
$\sigma_{y_{max}}=$ 3.66407	$\sigma_c=$ 3.90427	
$\mu=$ 231.659	sigma= 48.527	
$\sigma_\mu=$ 1.69465	$\sigma_{sigma}=$ 15.8216	$\chi^2/(n-4)=$ 1.05839

```
In[•]:= AppendTo[xData, {60, mean, sigmean}];

In[•]:= Undo[]

In[•]:= xy2y[]

In[•]:= GaussianCFit[]
```

In[ ]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`

Out[ ]=





$$y(x) = y_{max} \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$$

y_{max}= 24.2247	c= -2.69883	
$\sigma_{y_{max}}$= 2.41656	σ_c= 2.82003	
μ= 279.698	sigma= 20.6341	
σ_μ= 0.692554	σ_{sigma}= 2.62177	$\chi^2/(n-4)$= 0.745444

In[•]:=
```
AppendTo[yData, {60, mean, sigmean}];
```

80 degrees

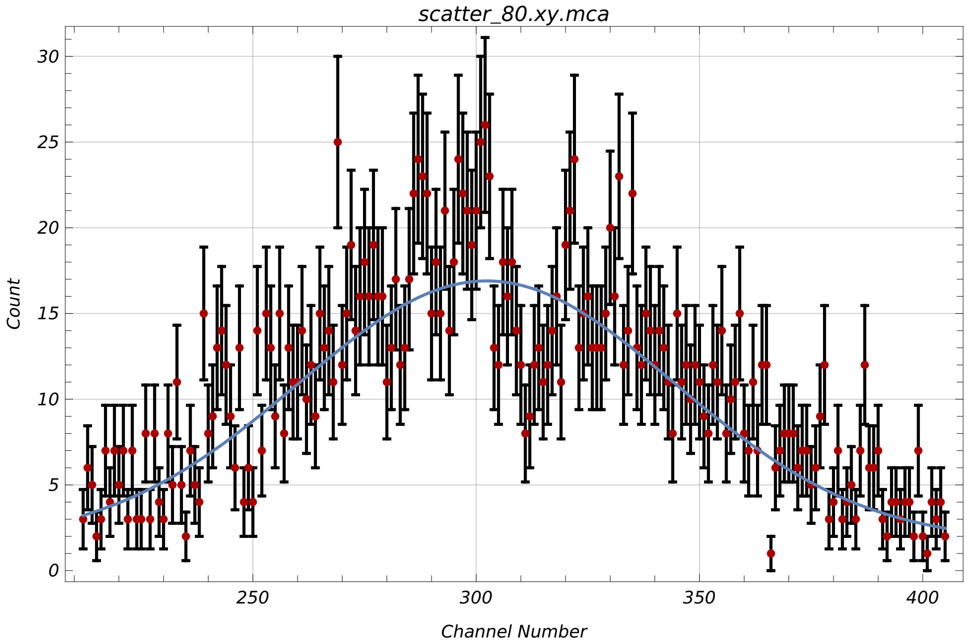
In[•]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[•]:=
```
KeepXY[]
```

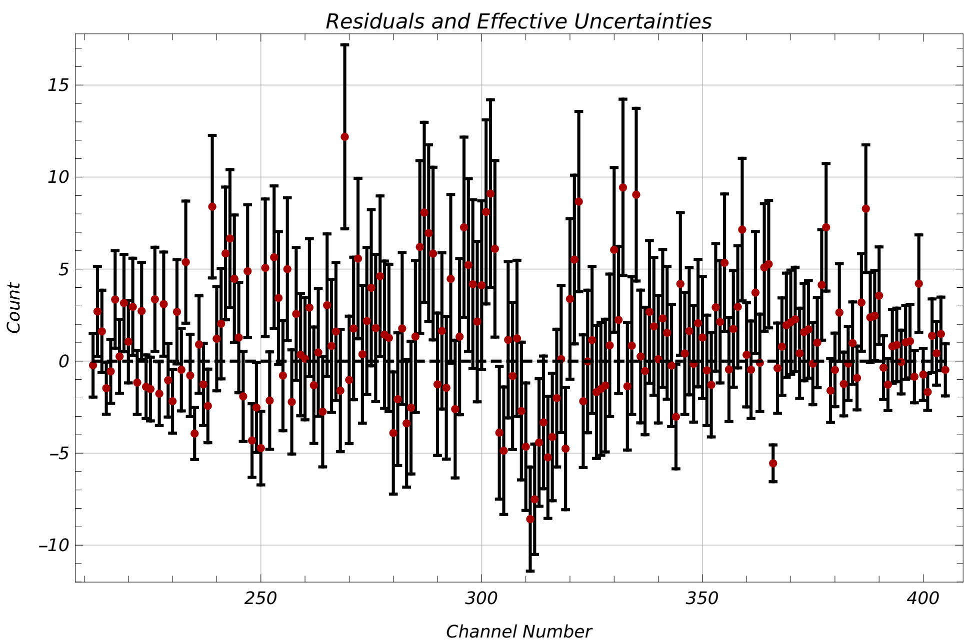
In[•]:=
```
xy2x[]
```

In[•]:=
```
GaussianCFit[]
```

In[•]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`



Out[•]=



$y(x) =$

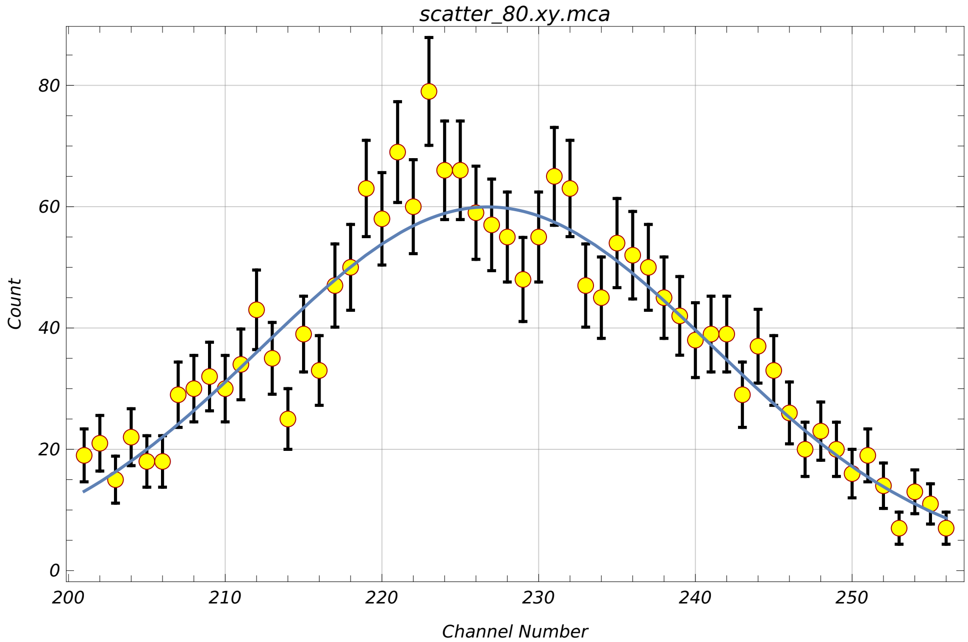
$y_{max} \exp\left(\frac{-(x-\mu)^2}{2\,\mathrm{sigma}^2}\right) + c$

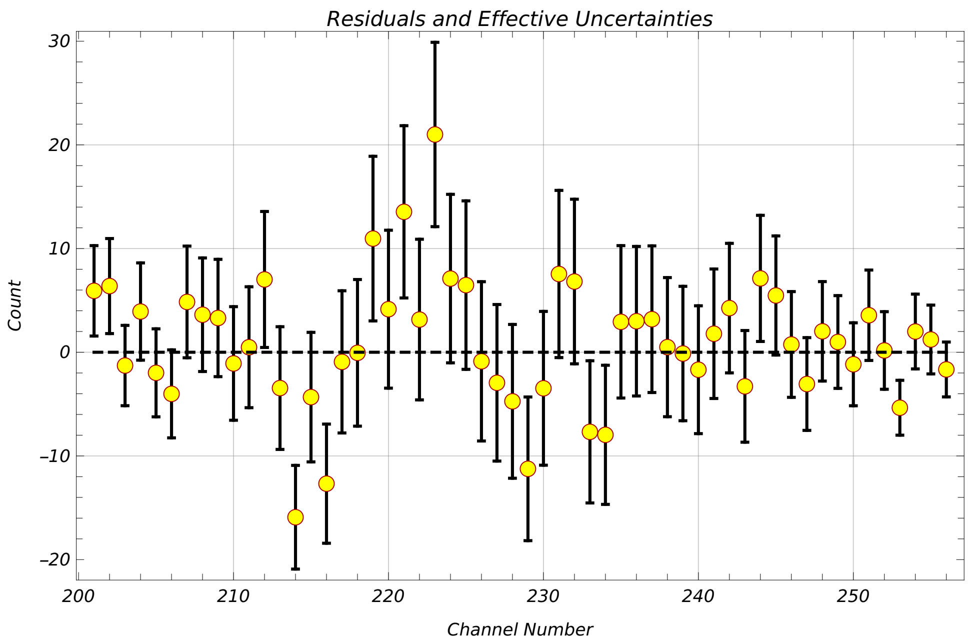
y_{max}= 15.207	c= 1.69228	
$\sigma_{y_{max}}$= 0.761096	σ_c= 0.663175	
μ= 302.345	sigma= 42.1452	
σ_μ= 1.36054	σ_{sigma}= 3.19759	$\chi^2/(n-4)$= 1.21439

```
In[•]:= AppendTo[xData, {80, mean, sigmean}];

In[•]:= Undo[]

In[•]:= xy2y[]

In[•]:= GaussianCFit[]
```

In[∘]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`

Out[∘]=





y(x) =

$y_{max} \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$

y_{max}= 58.7817	c= 1.17445	
$\sigma_{y_{max}}$= 4.93753	σ_c= 4.13823	
μ= 226.749	sigma= 14.4076	
σ_μ= 0.387345	σ_{sigma}= 1.56972	$\chi^2/(n-4)$= 1.04349

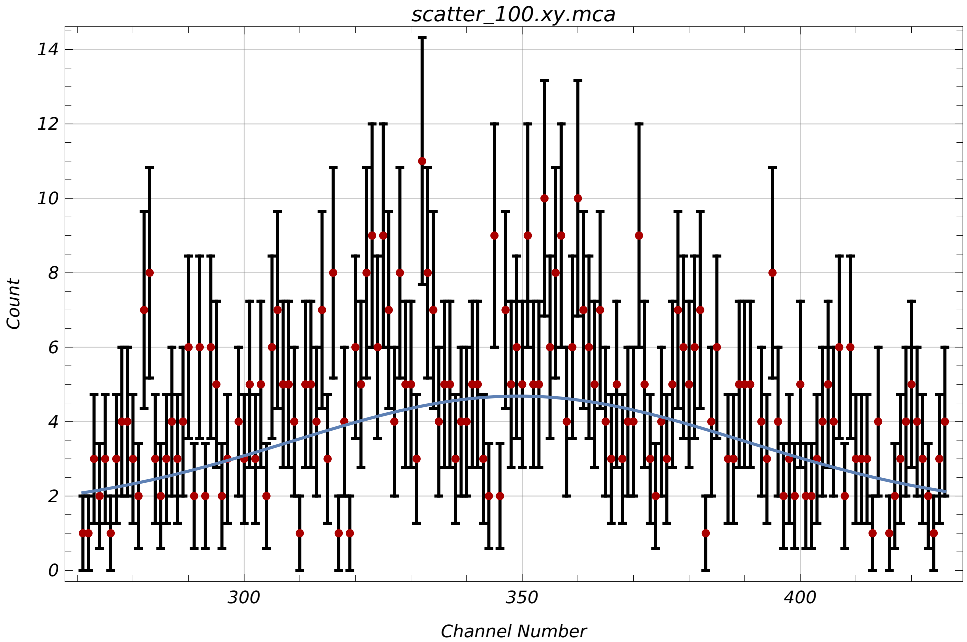
```
In[]:= AppendTo[yData, {80, mean, sigmean}];
```

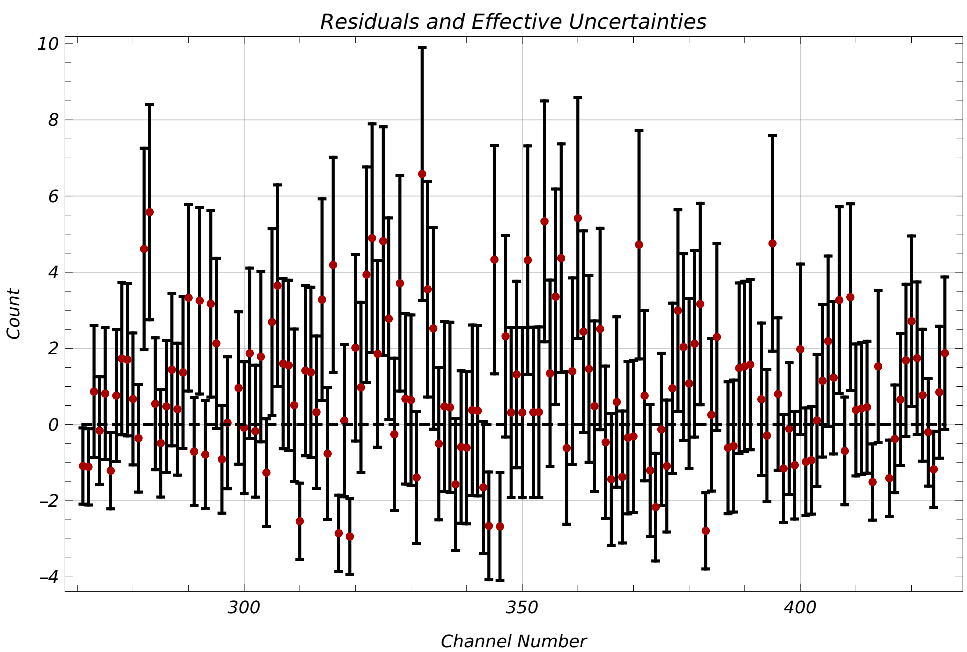
100 degrees

```
In[]:= With[ {name = SystemDialogInput["FileOpen",
          {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
        If[ name =!= $Canceled,
         LoadFile[name]]
       ]

In[]:= KeepXY[]

In[]:= xy2x[]

In[]:= NRangeRemove[#] & /@ {28}

In[]:= GaussianCFit[]
```

In[•]:=
```
LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]
```

Out[•]=





$$y(x) = y_{max} \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$$

y_{max} = 3.08151	c = 1.60418	
$\sigma_{y_{max}}$ = 2.43344	σ_c = 0.876048	
μ = 349.291	sigma = 40.6876	
σ_μ = 4.44023	σ_{sigma} = 3649.19	$\chi^2/(n-4)$ = 0.988598

In[∘]:=
```
AppendTo[xData, {100, mean, sigmean}];
```

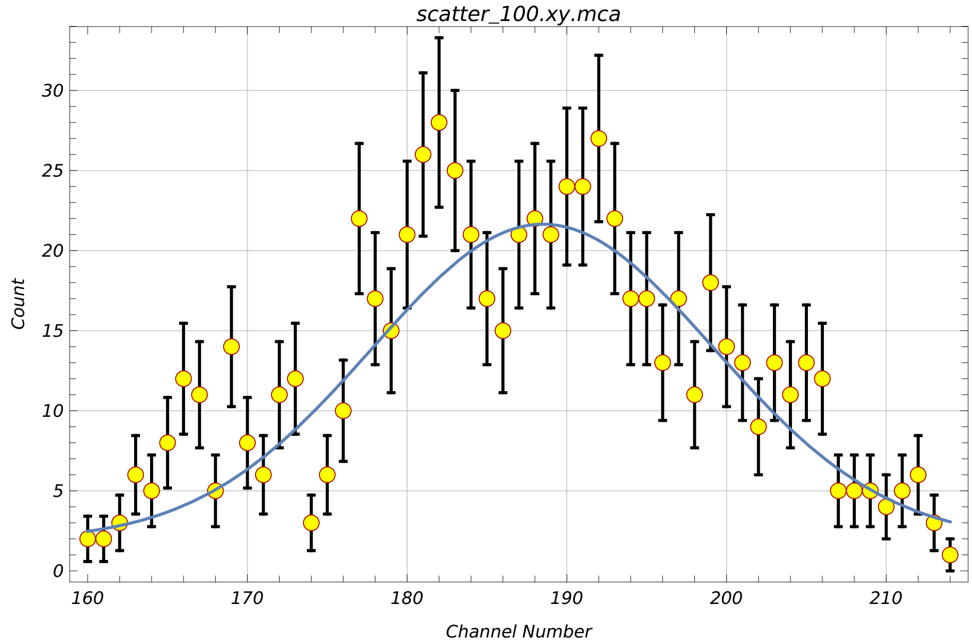
In[∘]:=
```
Undo[]
```

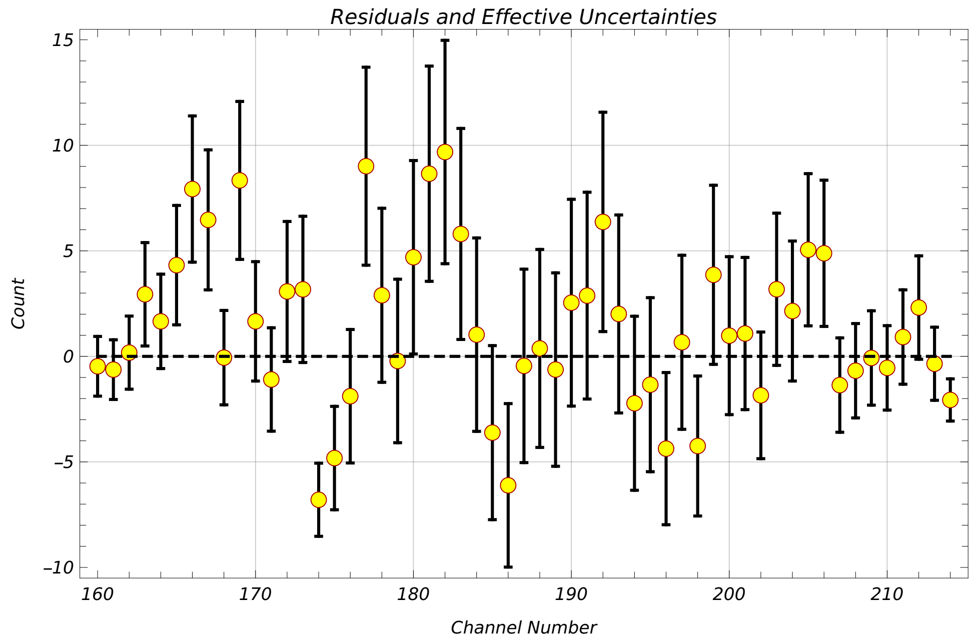
In[∘]:=
```
xy2y[]
```

In[∘]:=
```
GaussianCFit[]
```

In[•]:=
```
LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]
```

Out[•]=





y(x) =

$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\,\mathrm{sigma}^2}\right) + c$

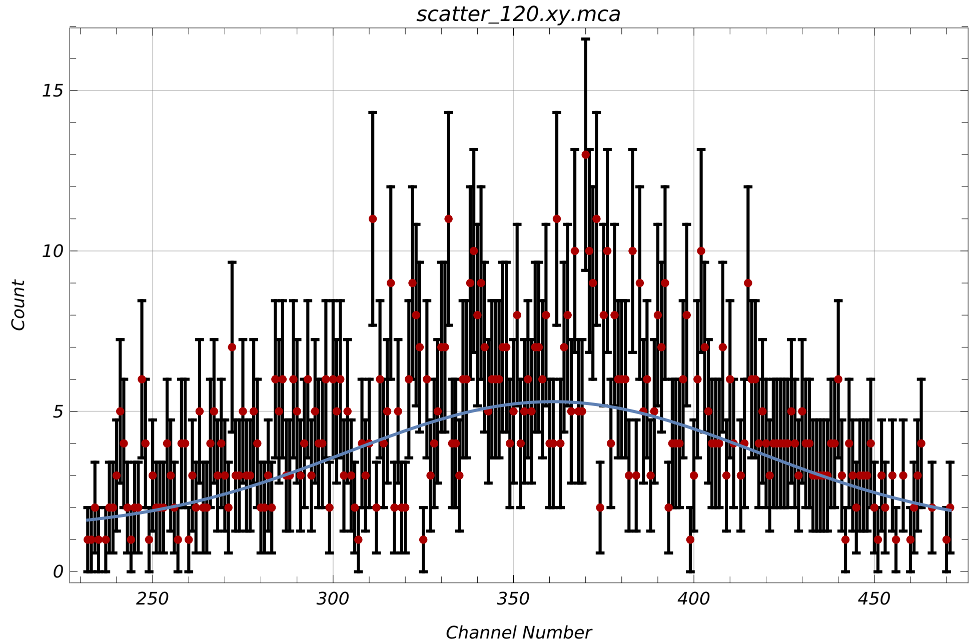
y_{max}= 19.7508	c= 1.8952	
$\sigma_{y_{max}}$= 1.28756	σ_c= 0.92032	
μ= 188.513	sigma= 10.7196	
σ_μ= 0.574773	σ_{sigma}= 1.24045	$\chi^2/(n-4)$= 1.46893

```
In[•]:= AppendTo[yData, {100, mean, sigmean}];
```

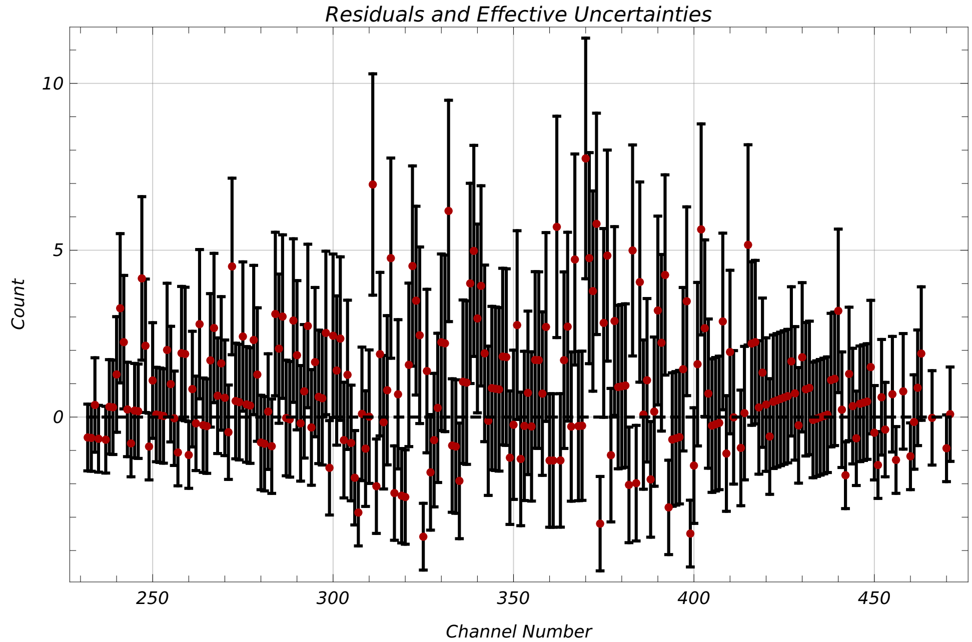
120 degrees

```
In[•]:= With[ {name = SystemDialogInput["FileOpen",
            {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
         If[ name =!= $Canceled,
          LoadFile[name]]
        ]

In[•]:= KeepXY[]

In[•]:= xy2x[]

In[•]:= GaussianCFit[]
```

In[]:=
```
LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]
```



Out[]=



$y(x)$ =

$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\ \text{sigma}^2}\right) + c$

y_{max}= 3.99783	c= 1.30416	
$\sigma_{y_{max}}$= 1.27369	σ_c= 0.584287	
μ= 360.741	sigma= 56.8461	
σ_μ= 4.24652	σ_{sigma}= 22.1567	$\chi^2/(n-4)$= 0.880585

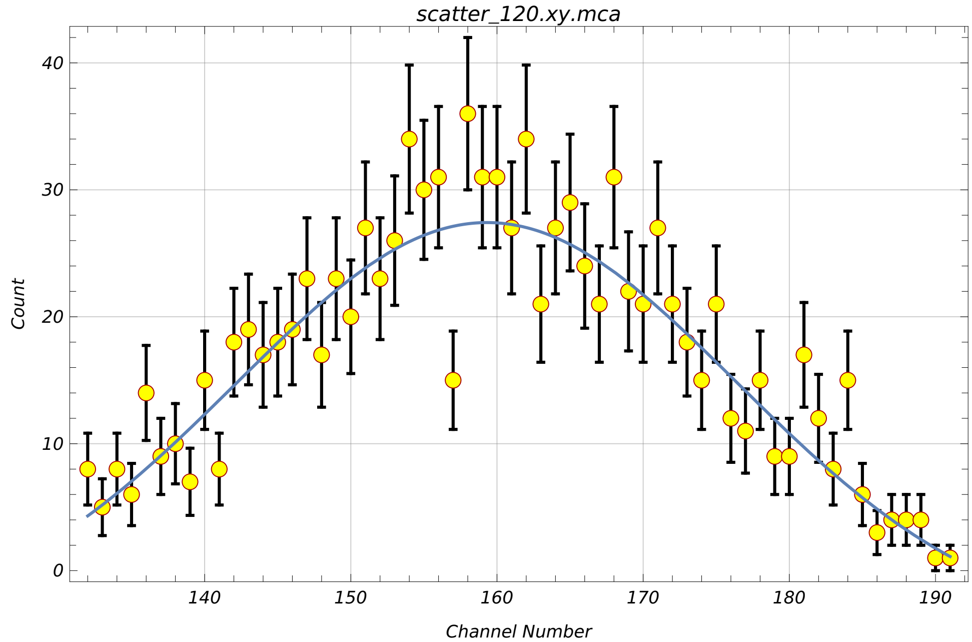
In[•]:=
```
AppendTo[xData, {120, mean, sigmean}];
```

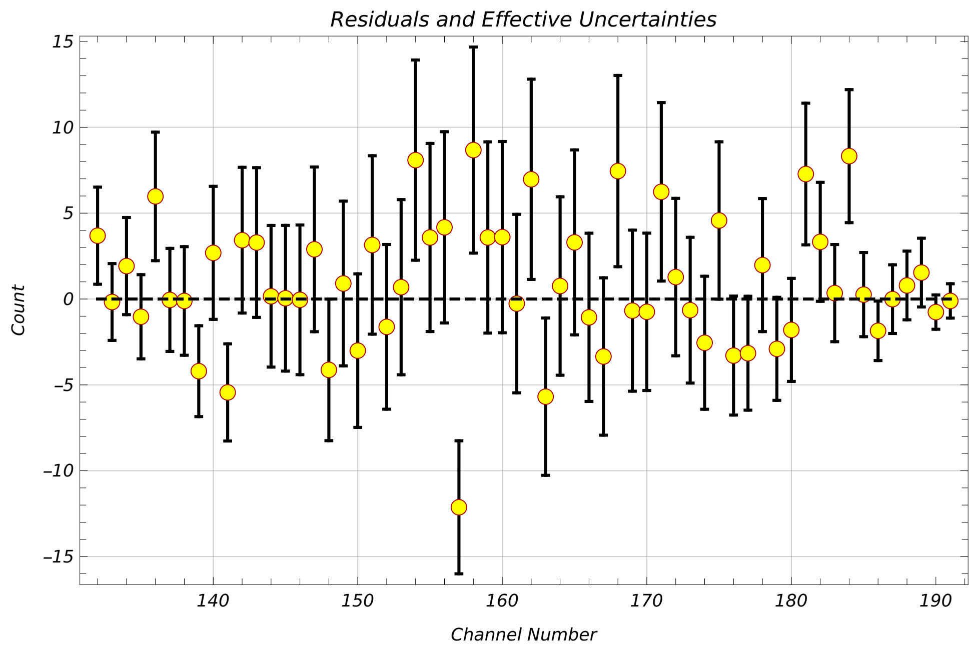
In[•]:=
```
Undo[]
```

In[•]:=
```
xy2y[]
```

In[•]:=
```
GaussianCFit[]
```

In[∘]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`





Out[∘]=

$y(x) =$

$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\,\mathrm{sigma}^2}\right) + c$

y_{max}= 32.1203	c= -4.69935	
$\sigma_{y_{max}}$= 3.6568	σ_c= 4.22351	
μ= 159.316	sigma= 17.1331	
σ_μ= 0.496917	σ_{sigma}= 2.34467	$\chi^2/(n-4)$= 0.96631

In[∘]:=
```
AppendTo[yData, {120, mean, sigmean}];
```

130 degrees

In[∘]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
  If[ name =!= $Canceled,
   LoadFile[name]]
 ]
```

In[∘]:=
```
KeepXY[]
```

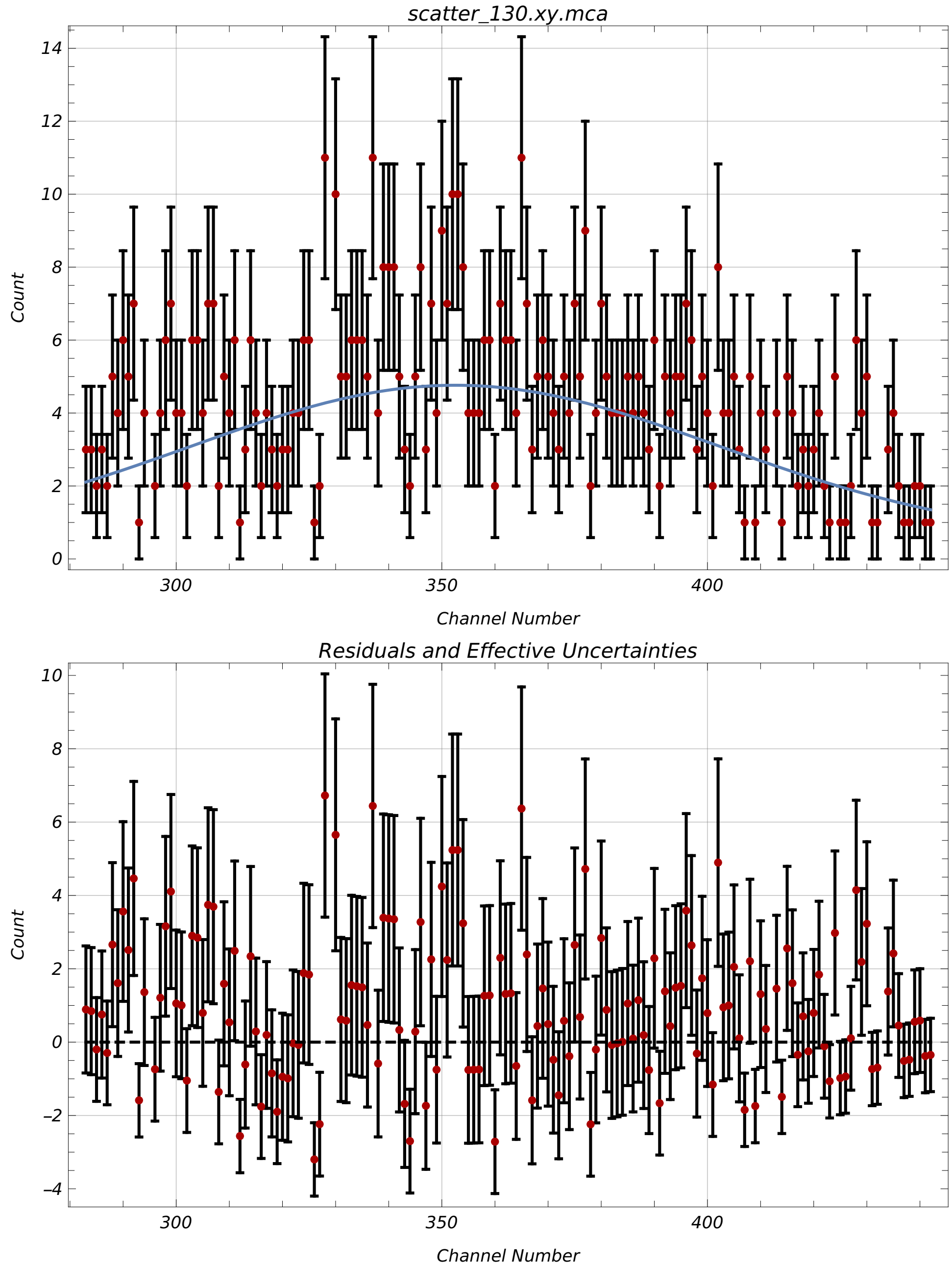
In[∘]:=
```
xy2x[]
```

In[∘]:=
```
NRangeRemove[#] & /@ {13, 46}
```

In[∘]:=
```
GaussianCFit[]
```

In[•]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`

Out[•]=



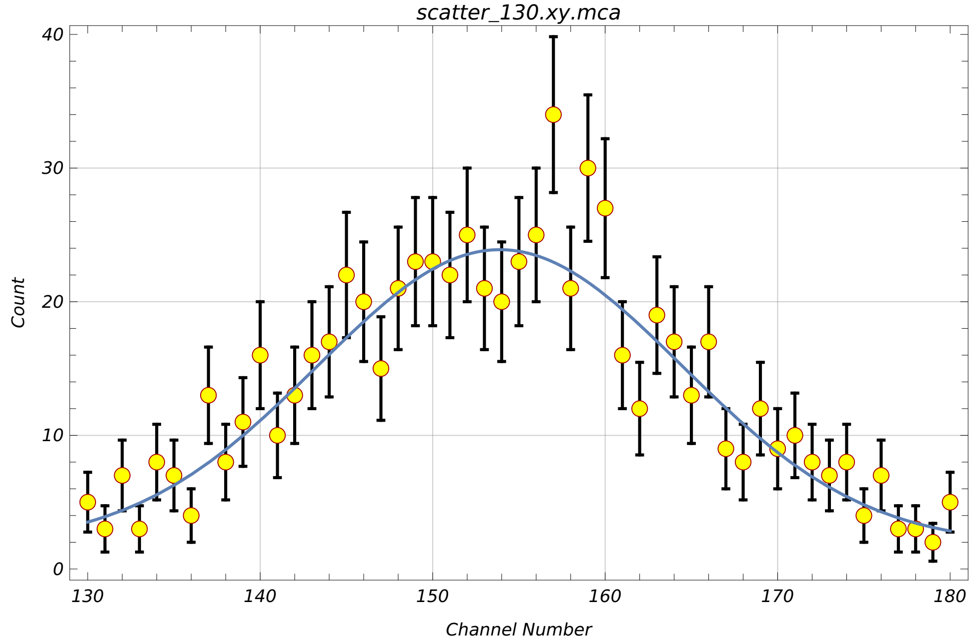
$y(x) =$

$y_{max} \exp\left(\frac{-(x-\mu)^2}{2\,sigma^2}\right) + c$

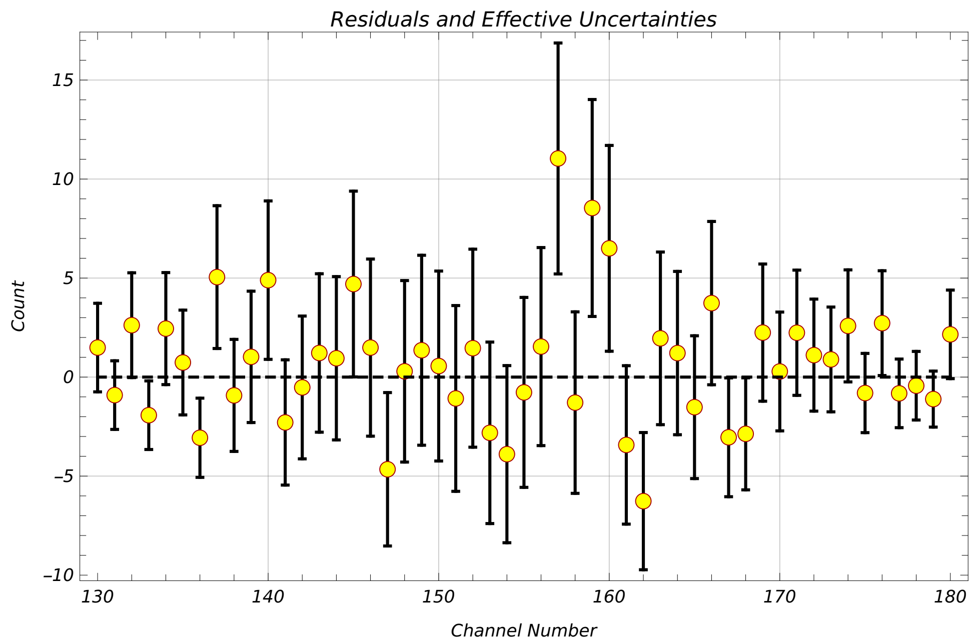
$y_{max}=$ 4.26556	$c=$ 0.496153	
$\sigma_{y_{max}}=$ 4.28188	$\sigma_c=$ 1.06997	
$\mu=$ 352.543	sigma= 49.8594	
$\sigma_\mu=$ 3.65033	$\sigma_{sigma}=$ 40.9078	$\chi^2/(n-4)=$ 0.982867

```
In[•]:= AppendTo[xData, {130, mean, sigmean}];

In[•]:= Undo[]

In[•]:= xy2y[]

In[•]:= GaussianCFit[]
```

In[◦]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`



Out[◦]=



$y(x) =$

$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$

$y_{max}=$ 22.0973	$c=$ 1.79622	
$\sigma_{y_{max}}=$ 1.56726	$\sigma_c=$ 1.26034	
$\mu=$ 153.896	$\text{sigma}=$ 10.568	
$\sigma_\mu=$ 0.532607	$\sigma_{\text{sigma}}=$ 1.29822	$\chi^2/(n-4)=$ 0.736339

In[•]:=
```
AppendTo[yData, {130, mean, sigmean}];
```

150 degrees

In[•]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[•]:=
```
KeepXY[]
```

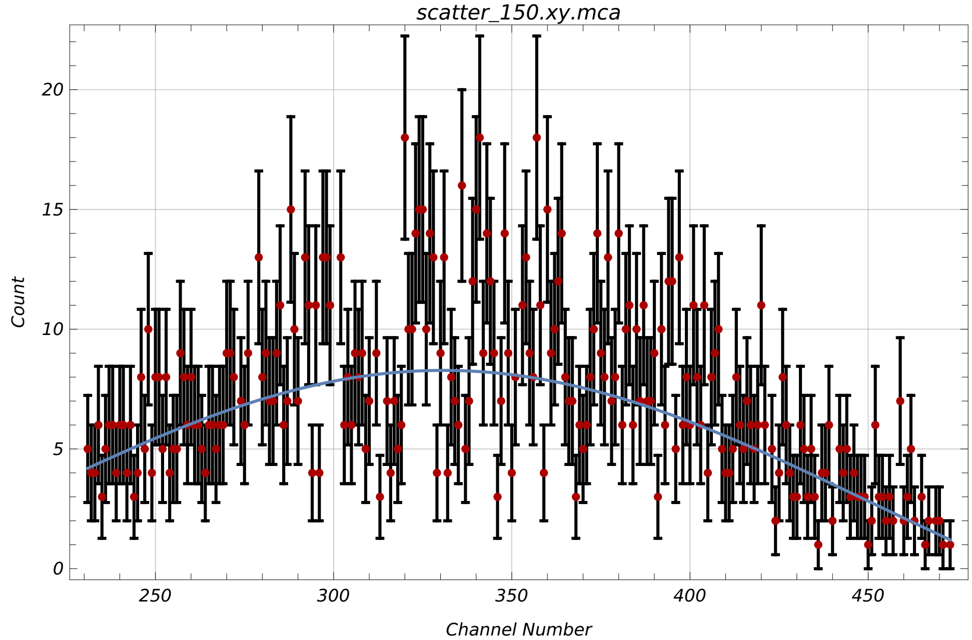
In[•]:=
```
xy2x[]
```

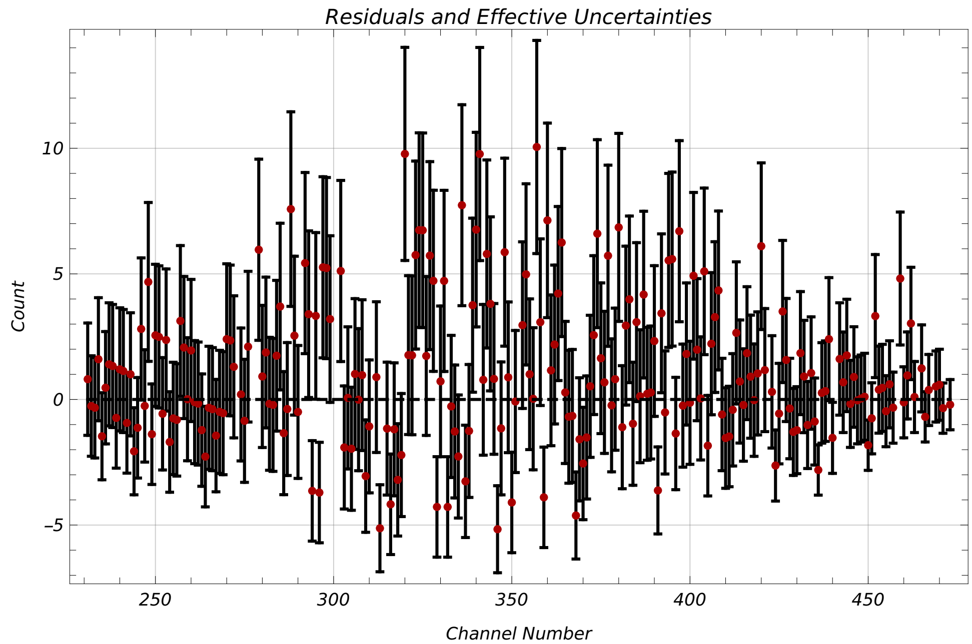
In[•]:=
```
NRangeRemove[#] & /@ {75, 77}
```

In[•]:=
```
GaussianCFit[]
```

In[•]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`

Out[•]=





$y(x)$ =

$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$

y_{max}= 14.07	c= -5.7903	
$\sigma_{y_{max}}$= 85.9889	σ_c= 72.9871	
μ= 330.726	sigma= 120.379	
σ_μ= 3.60567	σ_{sigma}= 274.349	$\chi^2/(n-4)$ = 1.08132

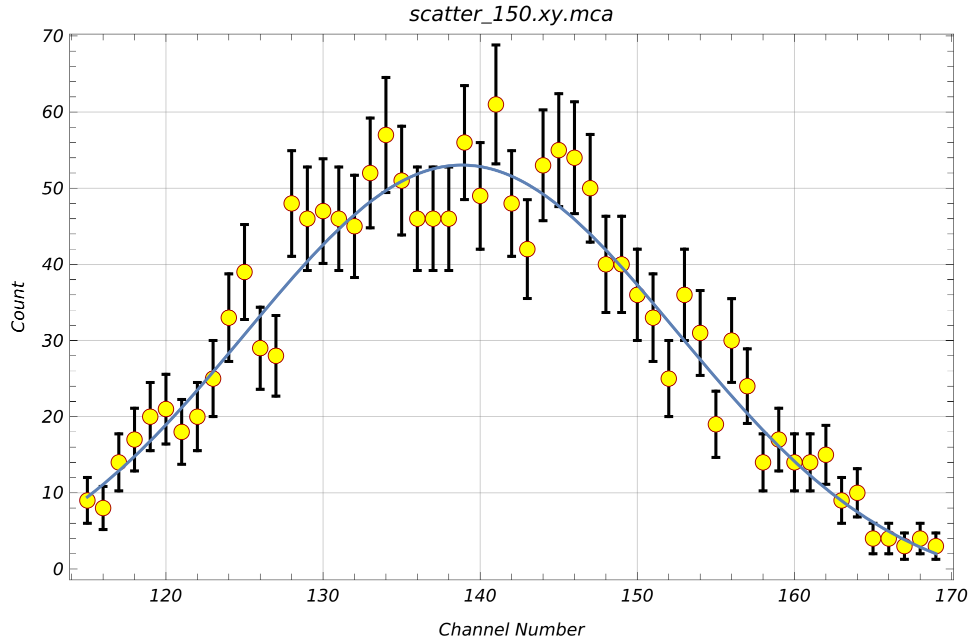
In[•]:=
```
AppendTo[xData, {150, mean, sigmean}];
```

In[•]:=
```
Undo[]
```

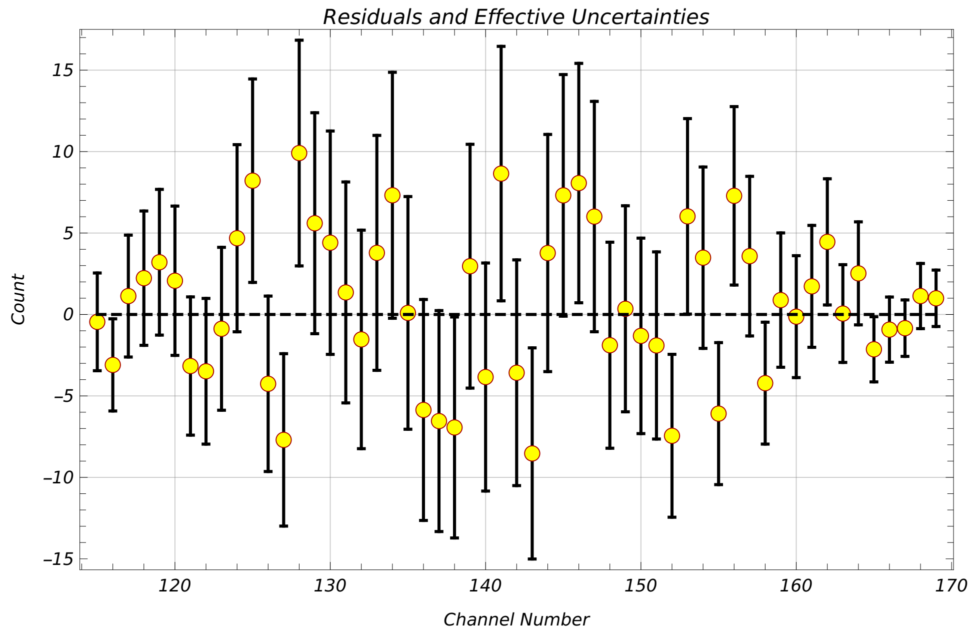
In[•]:=
```
xy2y[]
```

In[•]:=
```
GaussianCFit[]
```

In[∘]:= `LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]`



Out[∘]=



$y(x) =$

$y_{max} \exp\left(\frac{-(x-\mu)^2}{2\,\text{sigma}^2}\right) + c$

y_{max}= 56.1225	c= −3.08235	
$\sigma_{y_{max}}$= 2.57393	σ_c= 2.9097	
μ= 138.836	sigma= 13.7666	
σ_μ= 0.357275	σ_{sigma}= 0.95142	$\chi^2/(n-4)$= 0.716333

In[∘]:= `AppendTo[yData, {150, mean, sigmean}];`

Exporting NaI and Target Full-Energy Data

In[•]:=
```
SetDirectory[NotebookDirectory[]];
```

In[•]:=
```
Export["32a/na_full_energy.dat", yData];
```

In[•]:=
```
Export["32a/target_full_energy.dat", xData];
```

We now recall the NaI and Target full-energy events into this notebook:

In[258]:=
```
yData = Import["32a/na_full_energy.dat"];
```

In[259]:=
```
xData = Import["32a/target_full_energy.dat"];
```

Testing the Compton Energy equation

We tabulate the scattered photon energy for the full energy scattering events detected above using the scatter plots, propagating all errors including the $\Delta\theta = 1°$ and systematic uncertainties due to the calibration. For convenience, we obtain the total uncertainty by adding systematic and random errors in quadrature as defined in the calibration function above.

In[262]:=
```
calEnergy =
  Table[calibration[{yData[[i]][[2]], yData[[i]][[3]]}], {i, 1, Length[yData]}];
```

In[266]:=
```
scatteredEnergy = Table[{π/180 yData[[i]][[1]], calEnergy[[i]][[1]],
     calEnergy[[i]][[2]], π/180 Sqrt[2]}, {i, 1, Length[yData]}] // N;
```

In[268]:=
```
Export["32a/scattered_energy.dat", scatteredEnergy];
```

We tabulate the θ, E' data:

In[280]:=
```
scatteredTable = Join[{{"θ (rad)", "E' (keV)"}}, Table[{StringReplace[
      "%1 ± %2", {"%1" → ToString[NumberForm[scatteredEnergy[[i]][[1]], 3]],
       "%2" → ToString[NumberForm[scatteredEnergy[[i]][[4]], 2]]}], StringReplace[
      "%1 ± %2", {"%1" → ToString[NumberForm[scatteredEnergy[[i]][[2]], 3]],
       "%2" → ToString[NumberForm[scatteredEnergy[[i]][[3]], 2]]}]},
    {i, 1, Length[scatteredEnergy]}]];
Grid[scatteredTable, Alignment → Left, Spacings → {2, 1}, Frame → All,
 ItemStyle → "Text", Background → {{Gray, None}, {LightGray, None}}]
```

Out[281]=

θ (rad)	E' (keV)
0.698 ± 0.025	514. ± 1.1
1.05 ± 0.025	399. ± 1.5
1.4 ± 0.025	325. ± 0.98
1.75 ± 0.025	270. ± 1.
2.09 ± 0.025	226. ± 0.85
2.27 ± 0.025	218. ± 0.88
2.62 ± 0.025	195. ± 0.66

We can now test the Compton energy relationship with scattering angle:

In[269]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

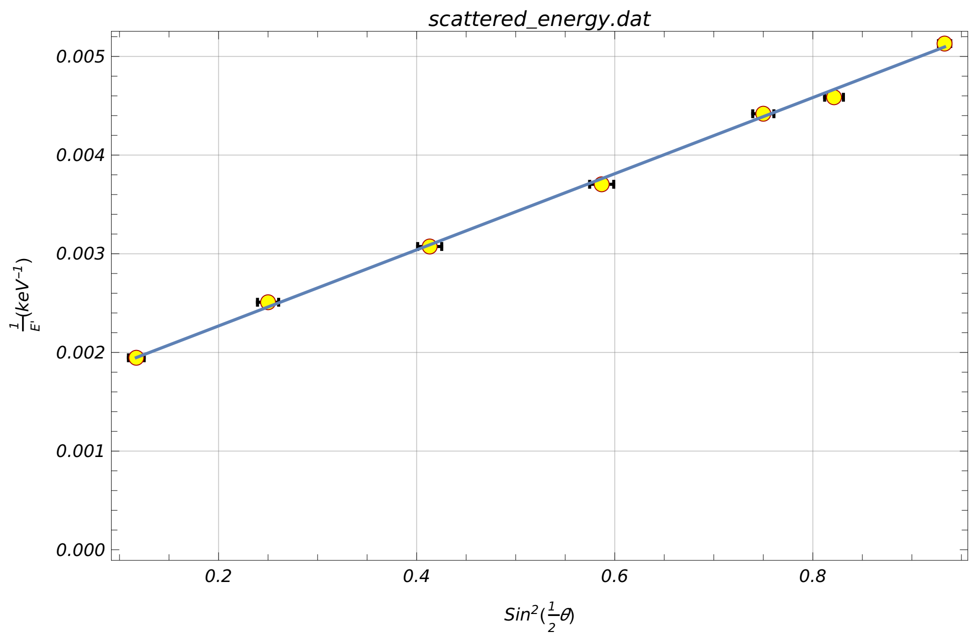
```
(* Transforming data to linear form *)
```

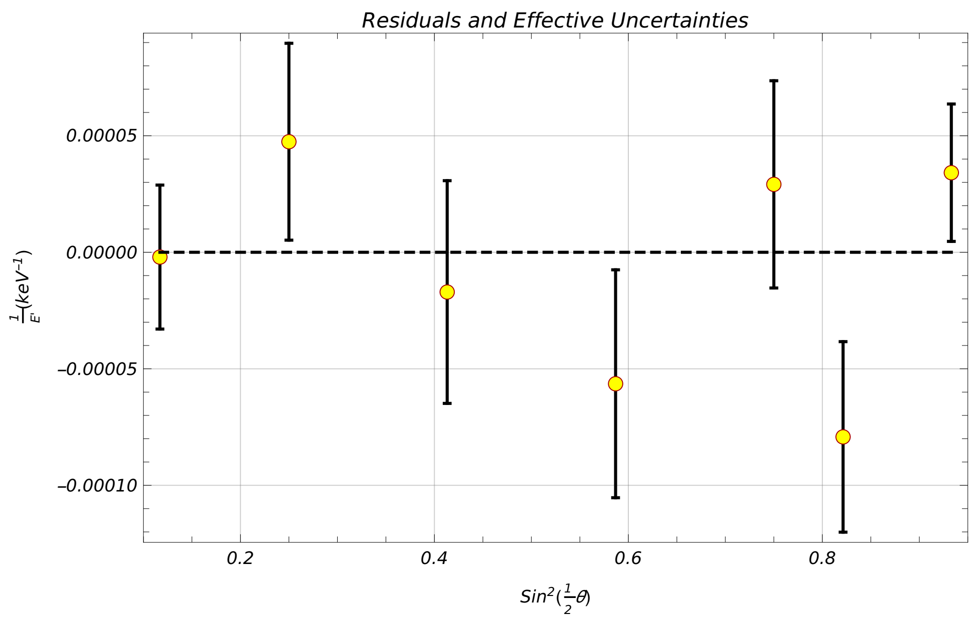
In[270]:=
```
xnew[ x_, y_ ] := Sin[1/2 x]^2
ynew[ x_, y_ ] := 1/y

DataTransform[]
```

In[275]:=
```
LinearFit[]
```

In[277]:= LinearDifferencePlot[FrameLabel → {"Sin^2($\frac{1}{2}\theta$)", "$\frac{1}{E'}$(keV^{-1})"}]

Out[277]=





y(x) = a + b x

a= 0.00149672   b= 0.00385771

σ_a= 0.0000289197 σ_b= 0.0000458586 χ^2/(n−2) = 1.6505

We obtain a good linear fit, indicating that the Compton relationship indeed holds. Furthermore,

In[285]:= k0 = $\frac{1}{a}$ {1, $\frac{siga}{a}$} (* keV *)

Out[285]= {668.126, 12.9096}

In[286]:=
```
me = 2/b {1, sigb/b}(* keV *)
```

Out[286]= {518.442, 6.16299}

We therefore have estimates for $k_0 = 668 \pm 13$ keV and $m_e = 518.4 \pm 6.2$ keV, which are within error of the expected values for the full-energy photons for Cs-137 and the rest mass of the electron, respectively. This again indicates consistency with the Compton energy relationship with scattering angle.

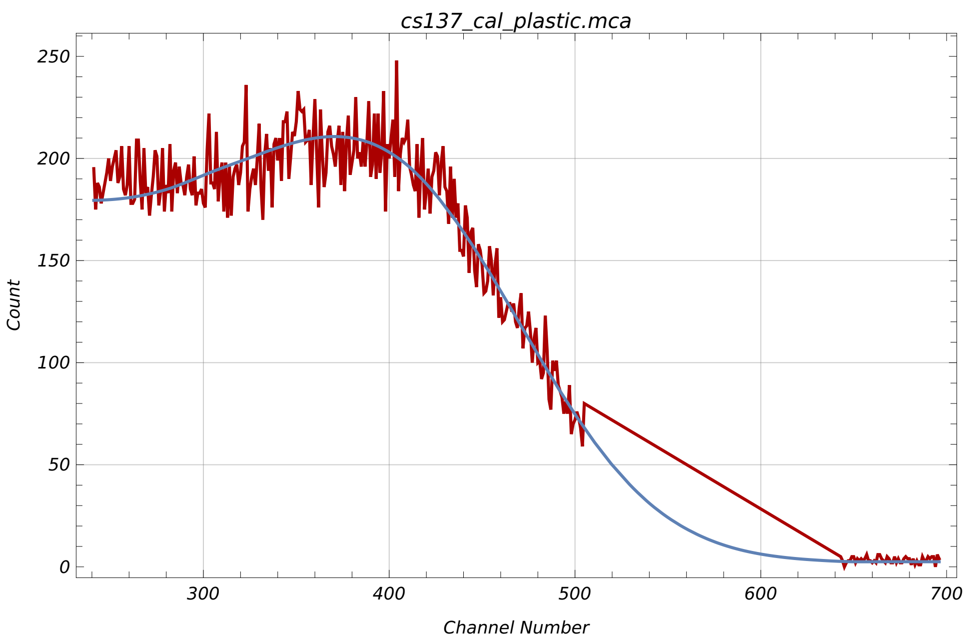
Target scintillator calibration

Unfortunately, only the Cs-137 spectrum has a discernible, single Compton edge. The Na-22 and Ba-133 calibration data have noise issues making it impossible to accurately identify the Compton edge, and the Co-60 spectrum has two full-energy events, making it difficult to distinguish the superposed Compton edges.

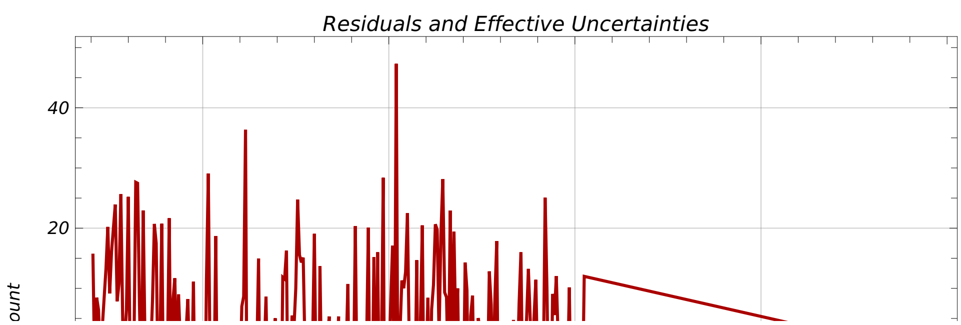
We use the ComptonSpectra1.nb notebook from Experiment 30b and fit the Cs-137 Compton edge:

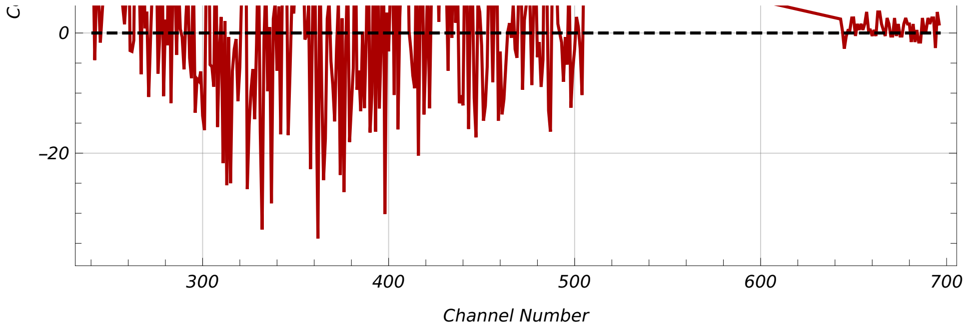
In[•]:=
```
ComptonFit[0.661, 5000, 450]
```

In[•]:=
```
LinearDifferencePlot[FrameLabel → {"Channel Number", "Count"}]
```



Out[•]=





y(x) = Compton spectrum from a photon with energy k_0, scaled in X and Y and convolved with the scintillator resolution e_{pe}(energy/photoelectron).

K0 (MeV) = 0.661	Tedge (MeV) = 0.476728	
EPE (eV/photoelectron) = 9706.37	sigEPE = 360.519	
Xscale (channels/MeV) = 953.657	sigXscale = 1.8013	
Yscale = (counts/channel)/(probability/MeV) 102.133	sigYscale = 0.618745	
Yoffset (counts/channel) = 2.50152	sigYoffset = 0.221614	
Xedge (channel) = 454.635	sigXedge = 0.858731	χ^2 = 0.970863

We obtain a reasonably good fit, and a rough estimate of the calibration given by Xscale above.

In[293]:=
```
calibrationTarget[channel_] :=
  channel[[1]] / 953.6573547506756` * 1000 {1, Sqrt[(1.8013021195385253` / 953.6573547506756`)^2 + (channel[[2]] / channel[[1]])^2]};
```

In[294]:=
```
calEnergyTarget = Table[
   calibrationTarget[{xData[[i]][[2]], xData[[i]][[3]]}], {i, 1, Length[xData]}];
```

In[296]:=
```
targetEnergy = Table[{π/180 xData[[i]][[1]], calEnergyTarget[[i]][[1]],
     calEnergyTarget[[i]][[2]], π/180 Sqrt[2]}, {i, 1, Length[xData]}] // N;
```

In[297]:=
```
Export["32a/target_energy.dat", scatteredEnergy];
```

We tabulate the θ, E data:

In[302]:=
```
targetTable = Join[{{"θ (rad)", "E (keV)"}}, Table[{StringReplace[
      "%1 ± %2", {"%1" → ToString[NumberForm[targetEnergy[[i]][[1]], 3]],
       "%2" → ToString[NumberForm[targetEnergy[[i]][[4]], 2]]}], StringReplace[
      "%1 ± %2", {"%1" → ToString[NumberForm[targetEnergy[[i]][[2]], 3]],
       "%2" → ToString[NumberForm[targetEnergy[[i]][[3]], 2]]}]},
    {i, 1, Length[targetEnergy]}]];
Grid[targetTable, Alignment → Left, Spacings → {2, 1}, Frame → All,
 ItemStyle → "Text", Background → {{Gray, None}, {LightGray, None}}]
```

Out[303]=

θ (rad)	E (keV)
0.698 ± 0.025	139. ± 1.2
1.05 ± 0.025	243. ± 1.8
1.4 ± 0.025	317. ± 1.5
1.75 ± 0.025	366. ± 4.7
2.09 ± 0.025	378. ± 4.5
2.27 ± 0.025	370. ± 3.9
2.62 ± 0.025	347. ± 3.8

Consistency with conservation of energy

We add the scattered photon energy and the outgoing electron energy for each angle, and expect to obtain data points consistent with a constant at 661 keV if the interactions are consistent with energy conservation.

In[304]:=
```
totalEnergy = Table[
   {scatteredEnergy[[i]][[1]], scatteredEnergy[[i]][[2]] + targetEnergy[[i]][[2]],
    Sqrt[scatteredEnergy[[i]][[3]]^2 + targetEnergy[[i]][[3]]^2],
    scatteredEnergy[[i]][[4]]}, {i, 1, Length[scatteredEnergy]}];
```

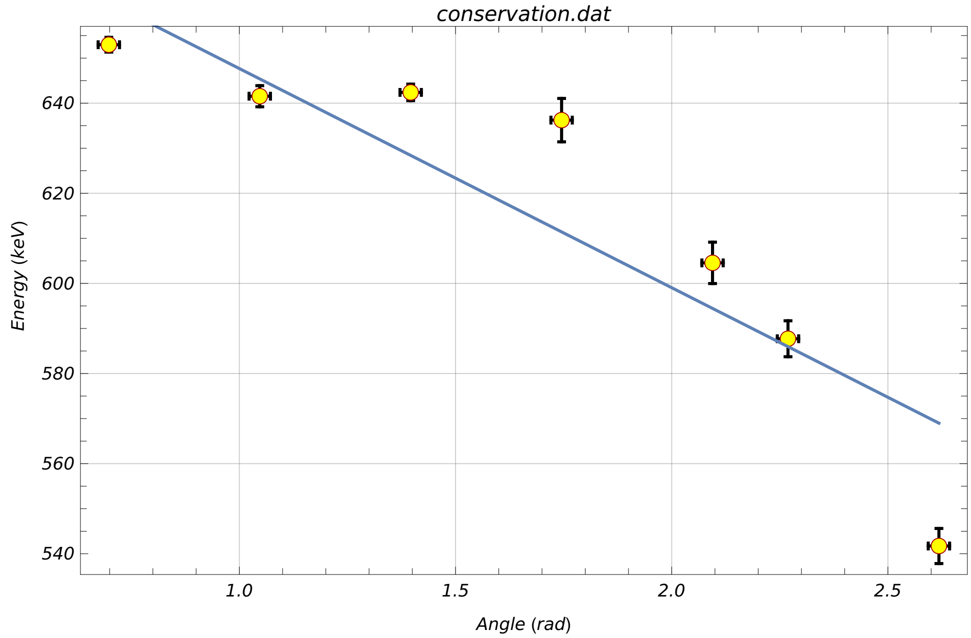
In[306]:=
```
Export["32a/conservation.dat", totalEnergy];
```

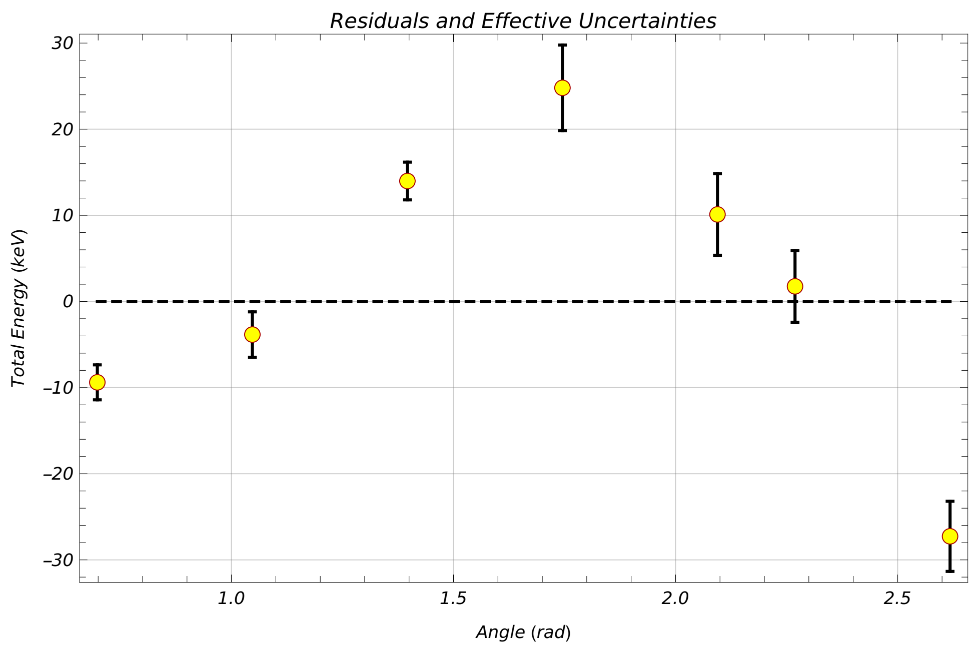
In[307]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[309]:=
```
LinearFit[]
```

In[317]:=
```
LinearDifferencePlot[FrameLabel → {"Angle (rad)", "Total Energy (keV)"}]
```

Out[317]=





y(x) = a + b x

a=	b=	
696.317	−48.6362	
σ_a=	σ_b=	$\chi^2/(n-2)$=
2.90004	1.99282	27.7605

This is clearly not consistent with a constant, indicating that the larger angle data points either contain significant systematic errors, or some of the larger angle data points are skewed by the presence of Compton scatters in the NaI scintillator. This would make sense, seeing as we found it much more difficult to extract the full-energy events from the scatter plots for the large-angle measurements. We remove the large-angle measurements and attempt the same fit.

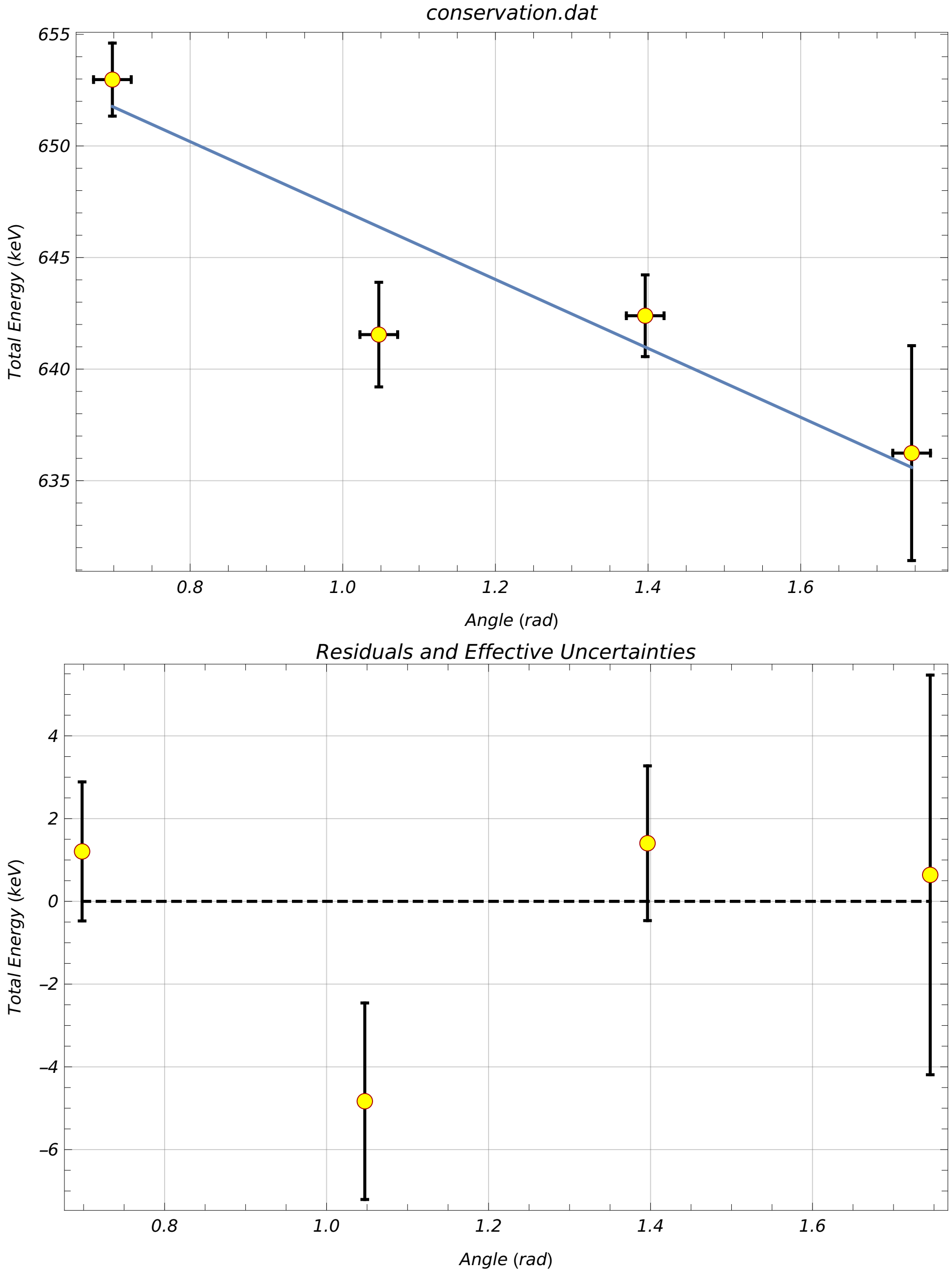
In[318]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[322]:=
```
LinearFit[]
```

In[323]:=

```
LinearDifferencePlot[FrameLabel → {"Angle (rad)", "Total Energy (keV)"}]
```

Out[323]=



y(x) = a + b x

a=	b=	
662.547	-15.4419	
σ_a=	σ_b=	$\chi^2/(n-2)$=
3.54747	3.20908	2.61749

This is still not consistent with a slope of 0 (constant) but we have a much better fit with a much smaller slope, with the intercept within error of the Cs-137 full-energy events. This indeed indicates as suspected earlier, the presence of systematic errors in the larger-angle measurements due to Compton scattering in the NaI scintillator difficult to extract from the full-energy events when converting the scatter plot to energy peak data.

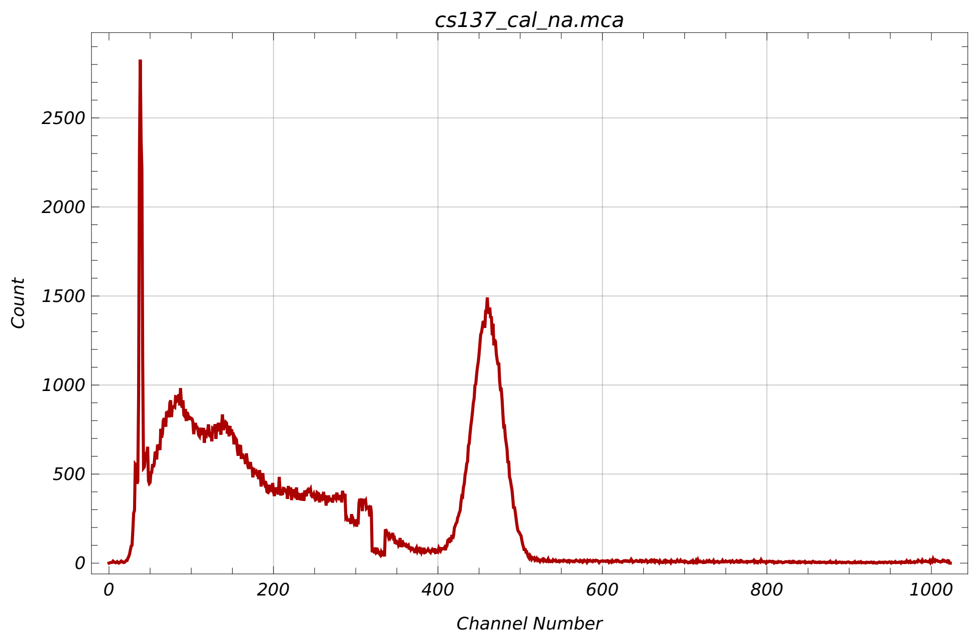
Appendix - Calibration spectra and scatter plot

Typical NaI scintillator calibration spectrum - Cs-137

In[329]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[331]:=
```
LinearDataPlot[FrameLabel → {"Channel Number", "Count"}]
```

Out[331]=

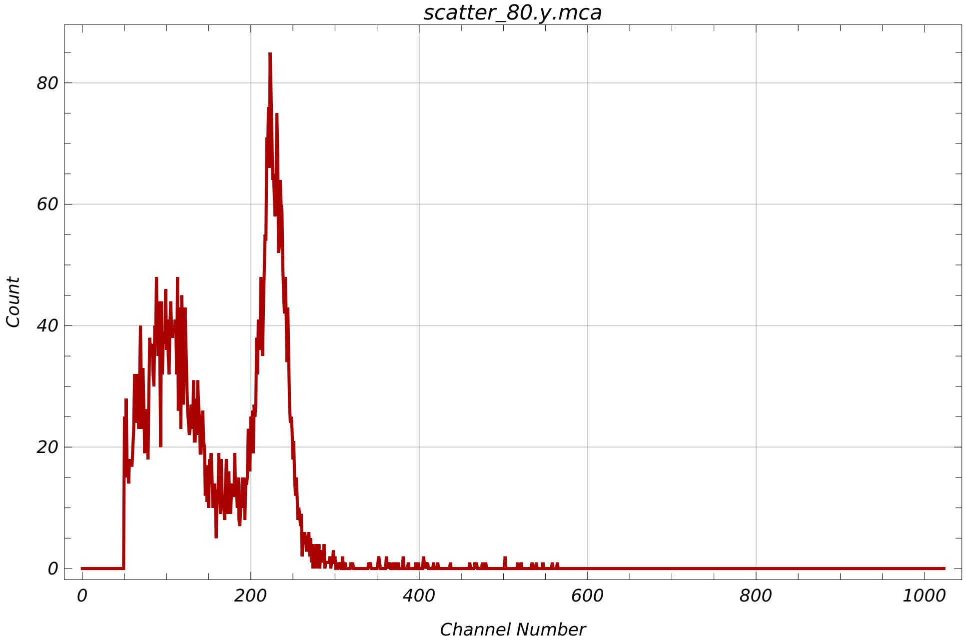


Typical NaI scintillator Compton scattering spectrum (80°) - Cs-137

In[334]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[335]:=
```
LinearDataPlot[FrameLabel → {"Channel Number", "Count"}]
```

Out[335]=

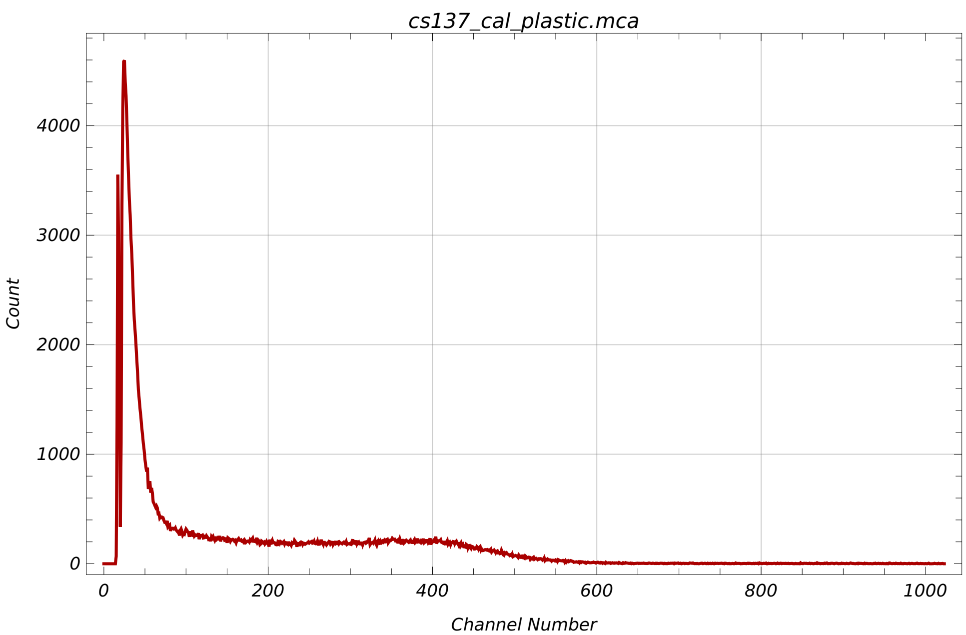


Typical Plastic scintillator calibration spectrum - Cs-137

In[332]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[333]:=
```
LinearDataPlot[FrameLabel → {"Channel Number", "Count"}]
```

Out[333]=

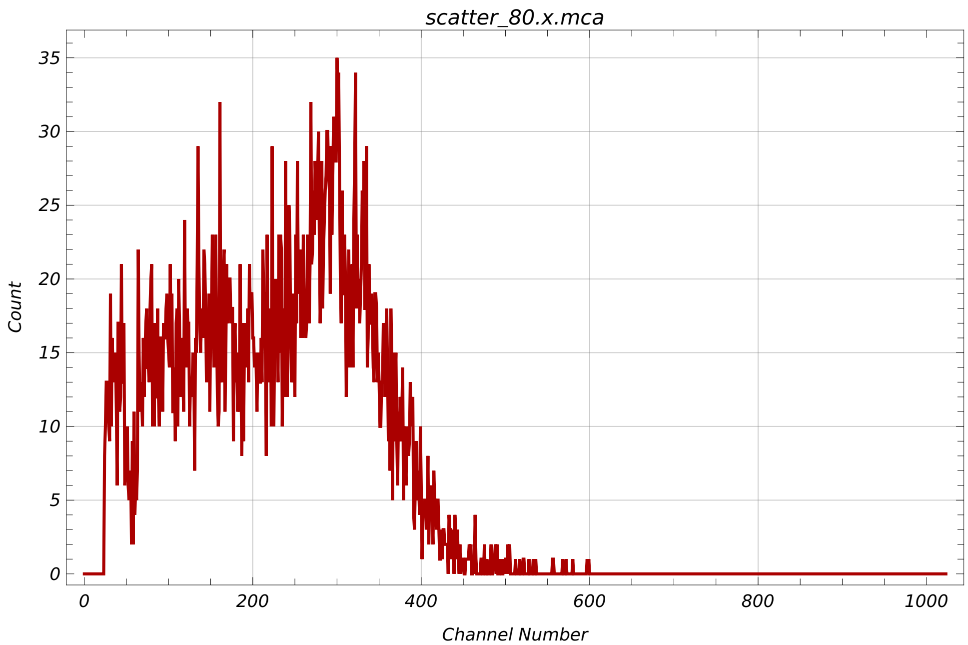


Typical Plastic scintillator Compton scattering spectrum (80°) - Cs-137

In[336]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[337]:=
```
LinearDataPlot[FrameLabel → {"Channel Number", "Count"}]
```

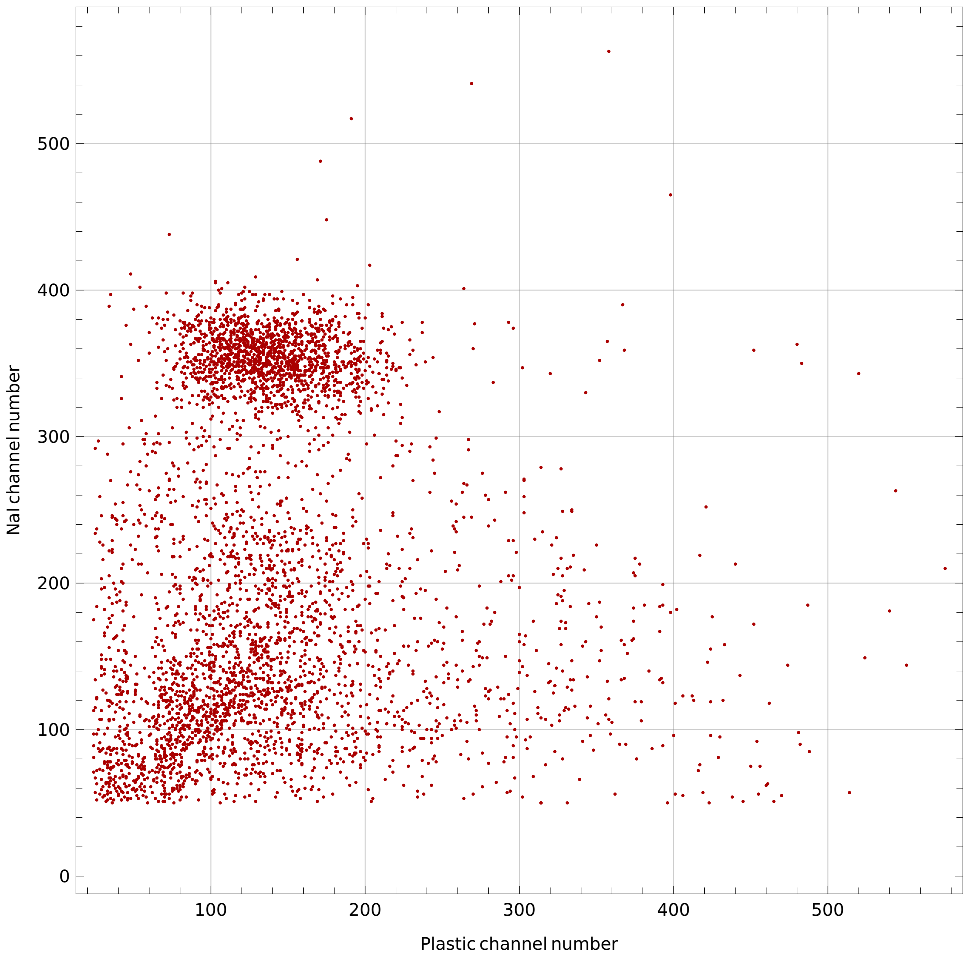
Out[337]=



Typical Compton scatter plots - Cs-137

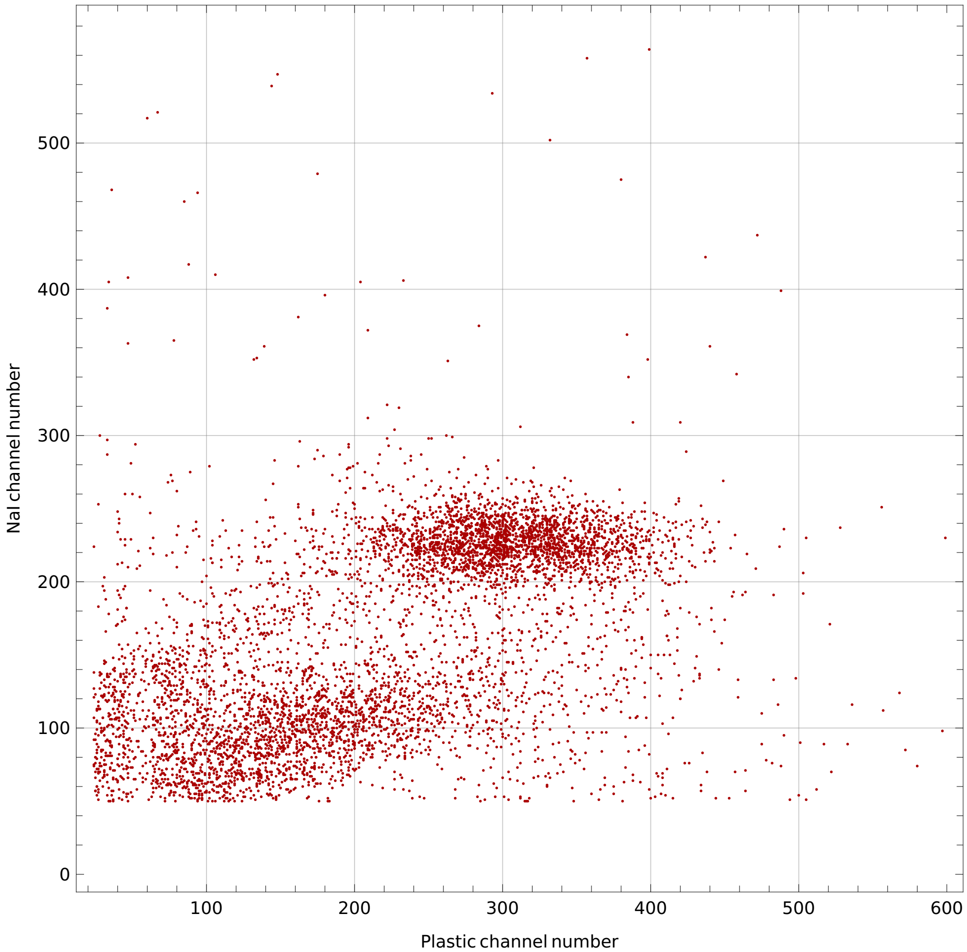
40°

Out[ ]=



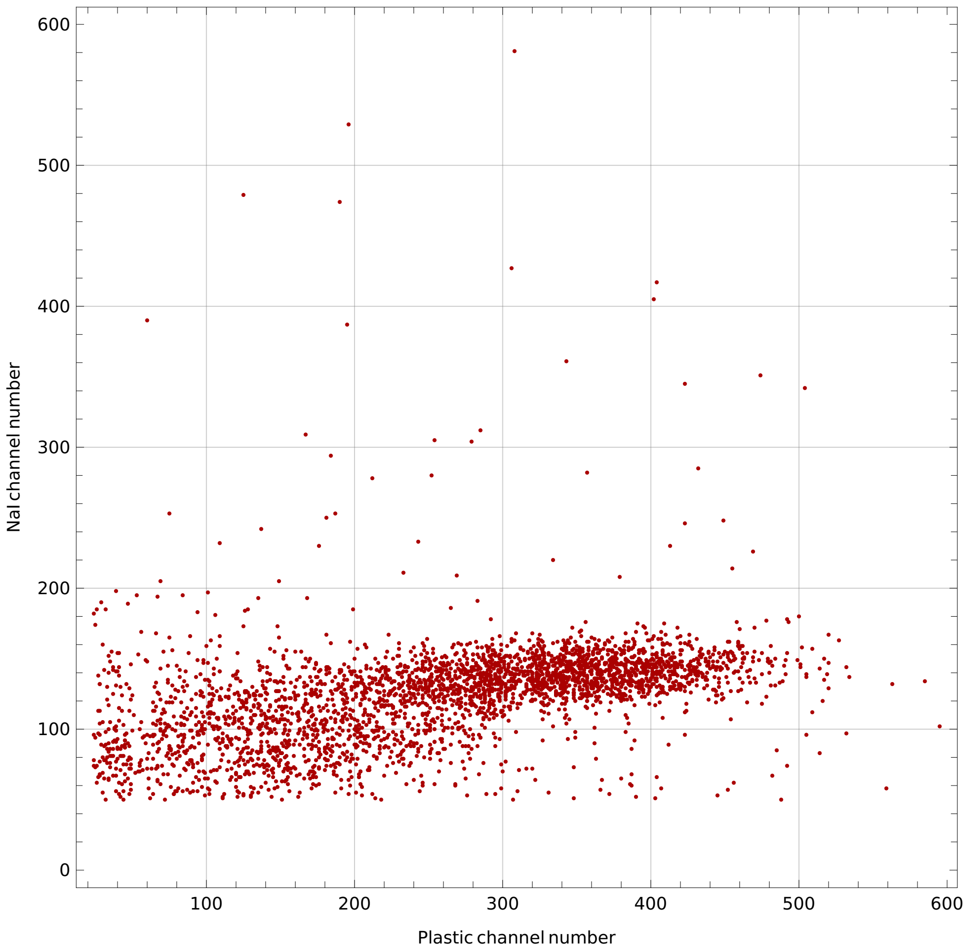
80°

Out[ ]=


NaI channel number
Plastic channel number
0
100
200
300
400
500
600

150°

Out[•]=



NaI channel number
Plastic channel number
0
100
200
300
400
500
600
100
200
300
400
500
600