Experiment 33 - The Stern-Gerlach Experiment

In[9]:=
```
<< CurveFit`
```
```
CurveFit for Mathematica v7.x thru v11.x, Version 1.96, 4/4/2018
Caltech Sophomore Physics Labs, Pasadena, CA
```

In[35]:=
```
SetDirectory[NotebookDirectory[]];
```

In[∘]:=
```
(* We enter our peak position data and format it for analysis with CurveFit. *)
zmax = {{"S Detector_current(10^-13A)", "Delta_z"},
   {0.459, 15.540 - 14.570}, {0.459, 15.520 - 14.560}, {0.459, 15.510 - 14.570},
   {1.203, 15.480 - 14.860}, {1.203, 15.450 - 14.830}, {1.203, 15.420 - 14.832},
   {0.341, 15.300 - 14.550}, {0.341, 15.280 - 14.570}, {0.341, 15.320 - 14.550},
   {0.851, 15.730 - 14.310}, {0.851, 15.710 - 14.330}, {0.851, 15.720 - 14.280},
   {2.183, 15.900 - 13.900}, {2.183, 15.600 - 13.750}, {2.183, 15.958 - 13.950}};
profile851 = {{"S Detector_current", "peak_position(mm)"}, {0.68, 15.000},
   {0.70, 14.800}, {0.80, 14.600}, {0.90, 14.400}, {0.90, 14.200}, {0.90, 14.000},
   {0.70, 15.200}, {0.82, 15.400}, {0.90, 15.600}, {0.92, 15.800}, {0.90, 16.000}};
profile0 = {{"S Detector_current", "peak_position(mm)"}, {2.17, 15.000},
   {1.85, 15.200}, {1.25, 15.400}, {0.90, 15.600}, {0.80, 15.800}, {0.77, 16.000},
   {1.80, 14.800}, {1.30, 14.600}, {0.90, 14.400}, {0.80, 14.200}, {0.70, 14.000}};
Export["zmax.dat", zmax];
Export["profile851.dat", profile851];
Export["profile0.dat", profile0];
```

0-Field Profile

In[∘]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[∘]:=
```
SwitchXXandYY[]
```

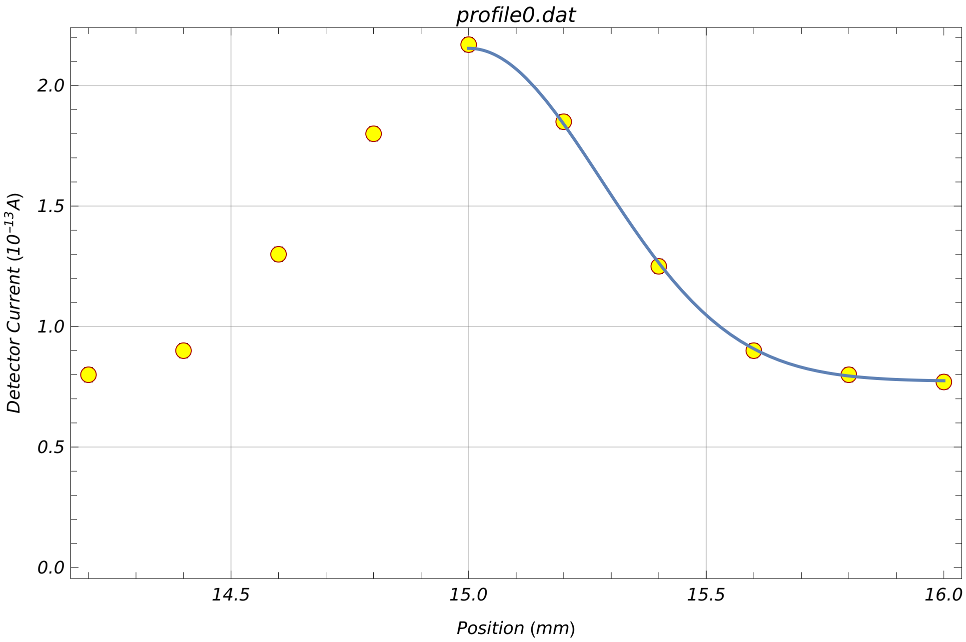
In[∘]:=
```
AssignYsigmas[0.025 ](* We assign an uncertainty of 0.025 mA to our
 detector current profile as a function of position, due to the smallest
 division on the calibrated voltmeter corresponding to 0.05 mA. *)
```

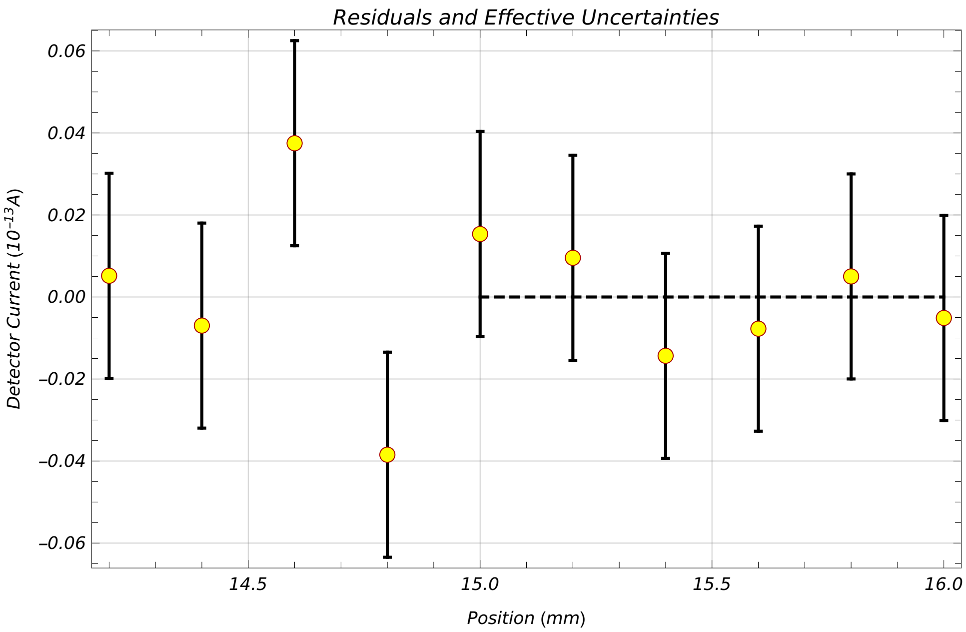
In[•]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to keep." ]},
 Print[x];

 XRangeKeep[Sequence @@ x]
]
```

In[•]:=
```
GaussianCFit[]
```

In[•]:=
```
LinearDifferencePlot[FrameLabel → {"Position (mm)", "Detector Current (10^-13A)"}]
```

Out[•]=





$y(x) =$

$$y_{max}\ \exp\left(\frac{-(x-\mu)^2}{2\ \mathrm{sigma}^2}\right) + c$$

$y_{max}=$ 1.38167	$c=$ 0.77297	
$\sigma_{y_{max}}=$ 0.0223511	$\sigma_c=$ 0.0151737	
$\mu=$ 15.0004	sigma= 0.277922	
$\sigma_\mu=$ 0.00453293	$\sigma_{sigma}=$ 0.00636805	$\chi^2/(n-4)=$ 0.960973

```
(* I am unsure as to why CurveFit refuses to plot half of the fitted function,
but we have obtained σ and observe a reasonably good χ̃2. *)
```

Processing data

Convolved Z_{max} corrected to theoretical value

```
In[•]:= data = Import[
          "/home/yovan/Documents/Coursework/2_Smore_Year/3_Spring_2018/Ph7/Experiment
            33/zmax.dat"]

In[•]:= zmaxdata = Table[{data[[i]][[1]], data[[i]][[2]]/2}, {i, 2, Length[data]}]

In[•]:= σ = 0.277922

In[•]:= solveTable = Table[ZconvMax[0.2 + 0.001 z, σ], {z, 1500}];

In[•]:= solver[obs_] :=
         0.2 + 0.001 * Flatten[Position[Abs[solveTable - obs], _? (# < 0.001 &)]][[1]]

In[•]:= modifiedZmax = Join[{{"S Detector_current(10^-13A)", "Corrected_Zmax(mm)"}},
          Table[{zmaxdata[[i]][[1]], solver[zmaxdata[[i]][[2]]]}, {i, 1, Length[zmaxdata]}]]

In[•]:= Export[
         "/home/yovan/Documents/Coursework/2_Smore_Year/3_Spring_2018/Ph7/Experiment
           33/corrected_Zmax.dat", modifiedZmax]

Out[•]= /home/yovan/Documents/Coursework/2_Smore_Year/3_Spring_2018/Ph7/Experiment
          33/corrected_Zmax.dat
```

Coil current data transformed to B_z data

```
In[•]:= data = Import[
          "/home/yovan/Documents/Coursework/2_Smore_Year/3_Spring_2018/Ph7/Experiment
            33/corrected_Zmax.dat"]

In[•]:= modData =
         Join[{data[[1]]}, Table[{B[data[[i]][[1]]], data[[i]][[2]]}, {i, 2, Length[data]}]]

In[•]:= Export[
         "/home/yovan/Documents/Coursework/2_Smore_Year/3_Spring_2018/Ph7/Experiment
           33/finalZmax.dat", modData]
```

Position of peaks

Having obtained $\sigma = 0.27792$mm for the Gaussian zero-field beam, we can use the Mathematica notebook provided for the Stern-Gerlach experiment to correct our data for z_{max} to the theoretical z_{max} for a

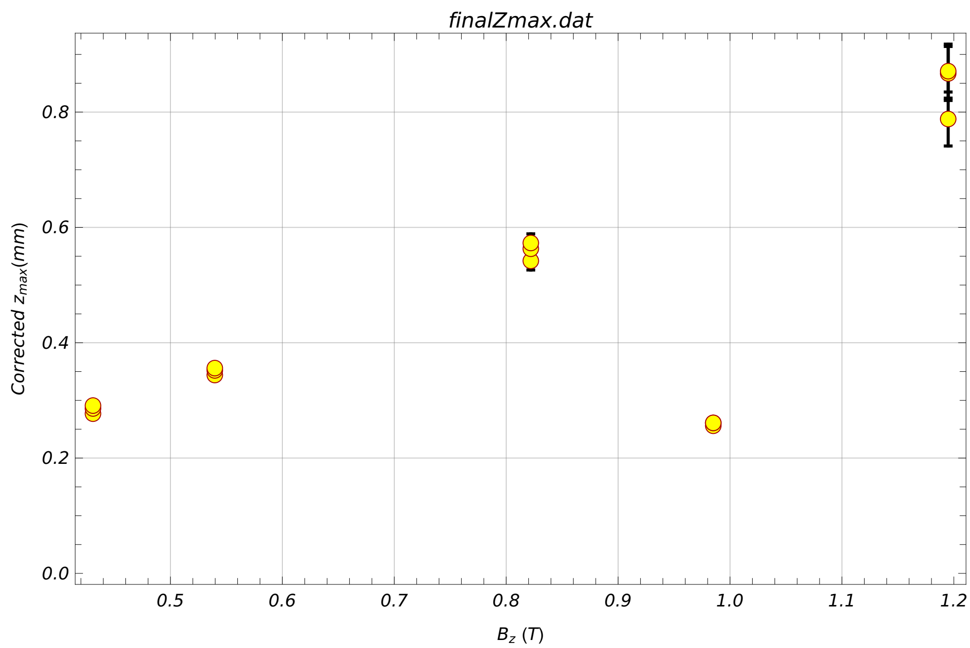
0-width beam. The new data is saved, and we then use the Magnetic field calculator provided to transform our current values into B-field values. This processed data is save, and can be used for the analysis:

In[∘]:=
```
With[ {name = SystemDialogInput["FileOpen",
     {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[∘]:=
```
CalculateYsigmas[]
```

In[∘]:=
```
LinearDataPlot[FrameLabel → {"Bz  (T)", "Corrected zmax (mm)"}]
```

Out[∘]=



We observe an outlier, which we have to remove seeing as including it prevents us from doing any sort of linear fit. It is probably due to measurement error, seeing as the delayed response makes it difficult to reliably identify a peak, and makes it easy to mistake small fluctuations for features in the detector current profile.

Even with the outlier removed, we can observe an increase in the uncertainty as $B_z$increases: as the peaks become broader, it becomes difficult to reliably find the peaks using the analog voltmeter.

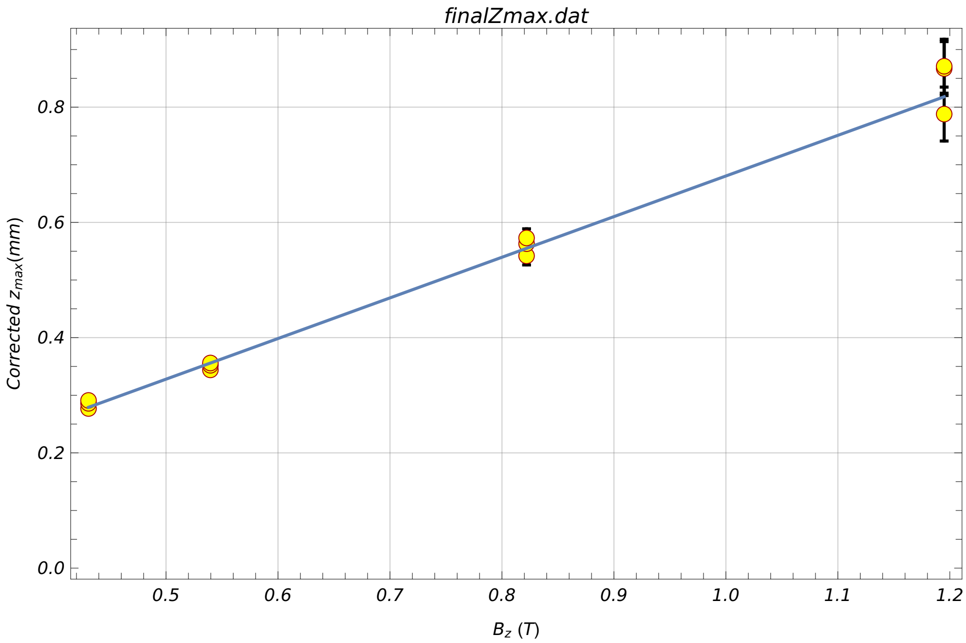
In[∘]:=
```
With[{x = SetXRange[ LinearDataPlot[], Log -> False,
     Label -> "Set the X values for the range you wish to remove." ]},
 Print[x];

 XRangeRemove[Sequence @@ x]
]
```

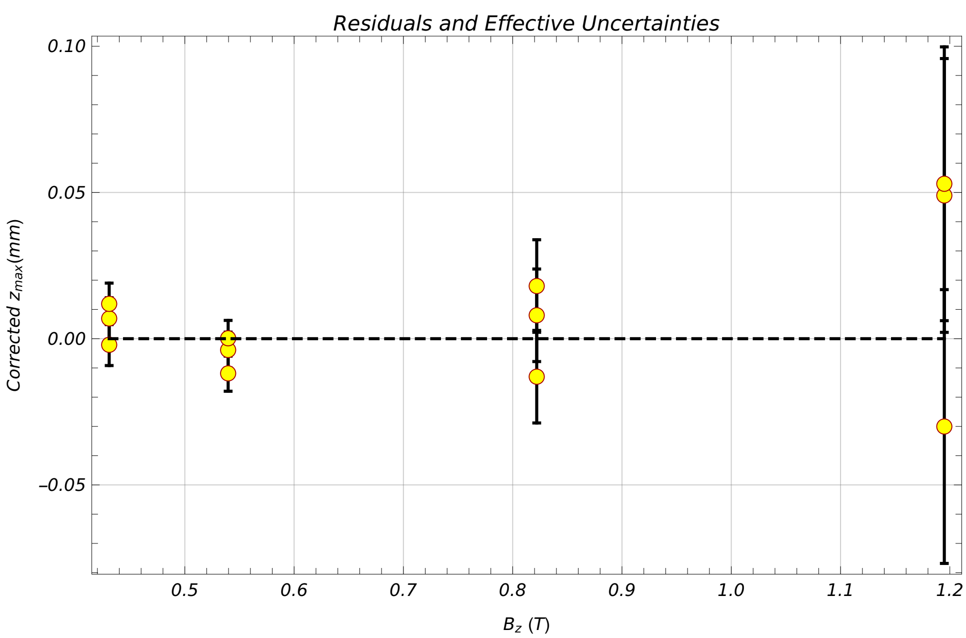
```
(* We now carry out a linear fit on our data. *)
```

In[•]:=
```
LinearFit[]
```

In[•]:=
```
LinearDifferencePlot[FrameLabel → {"Bz (T)", "Corrected zmax (mm)"}]
```

Out[•]=






```
y(x) = a + b x
a=           b=
-0.0247287   0.705271
σa=          σb=          χ2/(n-2)=
0.0114416    0.021234     1.3031
```

We observe a reasonably good $\tilde{\chi}^2$ of 1.30, and an offset of -0.0025±0.0014 mm, consistent with zero within the 10% error in our calibration. Therefore, our results agree up to accuracy with the prediction that z_{max} is linear in B.

Estimating μ_B

We expect $z_{\max} = \frac{\mu_B}{3}\left[l_2\left(\frac{l_2}{2} + l_3\right)\frac{1}{2\,k\,T}\right] 1.76\,B_z$. We can then estimate $\mu_B$by $\mu_B = \frac{6\,k\,T\,b}{1.76\left[l_2\left(\frac{l_2}{2}+l_3\right)\right]}$ where b is the slope of our linear fit.

In[∘]:=
```
l2 = 12.7 (* cm *);
l3 = 7.9 (* cm *);
k = 1.3806 * 10^-16 (* erg K^-1 *);
T = (273 + 116) (* K*);
b = {0.00000705271, 0.00000021234} (* cm G^-1 *);
μ[b_] := (6 k T b)/(1.76 (l2 (l2/2 + l3))) (* erg G^-1 *)
NumberForm[μb = μ[b], 3]
```

Out[∘]//NumberForm=

$\{7.13 \times 10^{-21},\ 2.15 \times 10^{-22}\}$

In SI units, this corresponds to $\mu_B = (7.13 \pm 0.21) \times 10^{-24}$ J T^{-1}. However, we now need to account for the uncertainty in our calibration for converting coil current to B-field data. This corresponds to a 10% uncertainty in our proportionality factor, which we add in quadrature to the uncertainty in μ_B:

In[∘]:=
```
μb[[2]] = μb[[1]] * Sqrt[(μb[[2]]/μb[[1]])^2 + (0.10)^2];
NumberForm[μb, 3]
```

Out[∘]//NumberForm=

$\{7.13 \times 10^{-21},\ 7.45 \times 10^{-22}\}$

We therefore obtain $\mu_B = (7.13 \pm 0.75) \times 10^{-24}$ J T^{-1} as our estimate for μ_B. We observe that this is within 3 standard errors of the literature value of 9.274×10^{-24} J T^{-1}. To accuracy of the experiment, our results therefore do not allow us to reject the hypothesis that $z_{\max} = \frac{\mu_B}{3}\left[l_2\left(\frac{l_2}{2} + l_3\right)\frac{1}{2\,k\,T}\right] 1.76\,B_z$.

However, we have a large standard error (~ 11%) in our estimate, which is indicative of uncertainties inherent to the experiment (uncertainty in calibration, notably, along with the uncertainties associated with the apparatus such as the divisions on the detector voltmeter) and due to the quality of the data (experimental errors in finding peaks due to the delayed response). Furthermore, we have a sparse dataset, which does not allow our analysis to confirm the hypothesis with reasonable confidence.

Non-zero magnetic field beam profile

We load a non-zero magnetic field beam profile for coil current = 0.851 A, which by our calibration corresponds to 0.822 T. We use our model to predict the Z-distribution:

In[•]:=
```
μbval = 9.274 * 10^-21 (* erg G^-1 *);
bz = 8220 (* G *);
zmaxprofile = (μbval 1.76)/(6 k T) (l2 (l2/2 + l3)) bz (* cm *)
```

$$zmaxprofile = \frac{\mu bval\ 1.76}{6\,k\,T}\left(l2\left(\frac{l2}{2}+l3\right)\right) bz$$

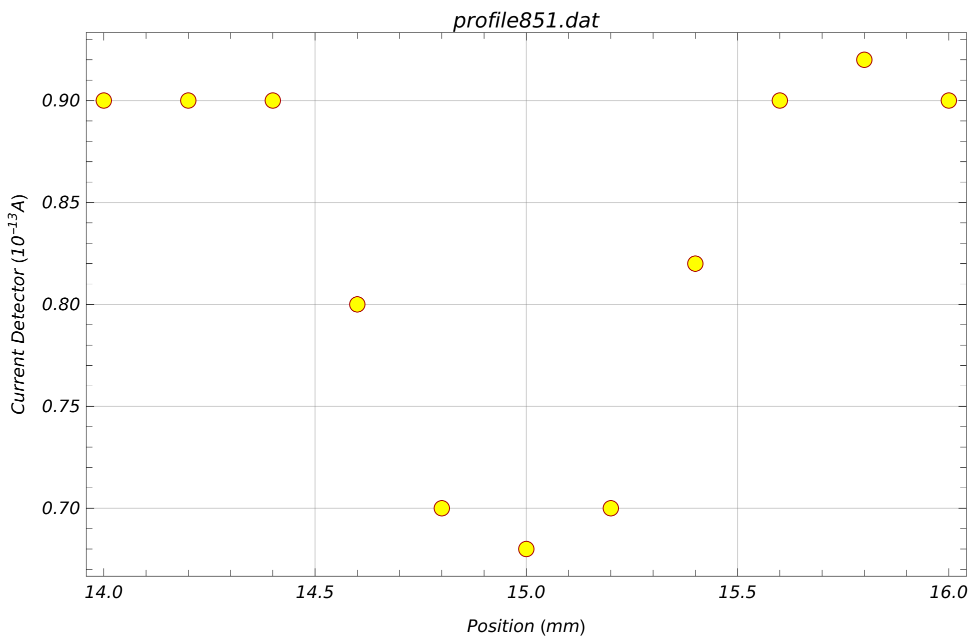
Out[•]= 0.0753532

In[•]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

In[•]:=
```
SwitchXXandYY[]
```

In[•]:=
```
LinearDataPlot[FrameLabel → {"Position (mm)", "Current Detector (10^-13A)"}]
```

Out[•]=



(* We use the provided Notebook to create a table of values for the theoretical distribution with finite width σ = 0.2779 mm, z_{max} =0.753 mm. For a Gaussian, σ = 0.2779 mm corresponds to a FWHM of 0.6544 mm. *)

In[24]:=
```
With[ {name = SystemDialogInput["FileOpen",
    {DataFileName, {"data files" -> {"*.dat", "*.mca"}, "all files" -> {"*"}}}]},
 If[ name =!= $Canceled,
  LoadFile[name]]
]
```

(* We scale and shift the data to be able to compare the theoretical distribution with our dataset. *)

In[•]:=
```
? CurveFit`DataTransform
```

In[29]:=
```
(* change the right-hand sides of the following function
   definitions of xnew[] and ynew[] to perform the data
   transformation you need, then evaluate this cell.     *)

xnew[ x_, y_ ] := x + 15
ynew[ x_, y_ ] := y/(0.252 - 0.0486) (0.92 - 0.68) + (0.68 - 0.0486) - 0.01

DataTransform[]

(* Use Undo[] if you don't like the results. *)
```

All 201 points transformed.

In[33]:=
```
With[ {name = SystemDialogInput["FileSave", DataFileName]},
 If[ name =!= $Canceled, SaveFile[name]]
]
```

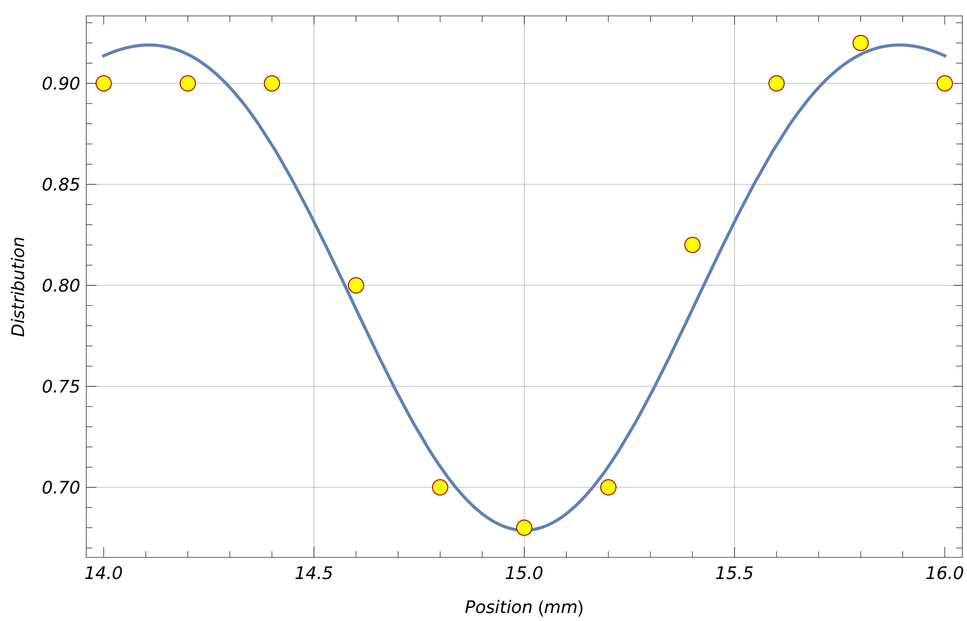
In[36]:=
```
scaledTheoretical = DeleteCases[Import["scaled_theoretical851.dat"], 0., Infinity];
```

In[37]:=
```
distribution = Interpolation[scaledTheoretical];
```

In[38]:=
```
profile = Drop[Import["profile851.dat"], 1];
flipped = Table[{profile[[i]][[2]], profile[[i]][[1]]}, {i, 1, Length[profile]}];
```

In[43]:=
```
plt1 = Plot[distribution[x], {x, 14, 16}, PlotRange → All];
plt2 = ListPlot[flipped];
Show[plt1, plt2, FrameLabel → {"Position (mm)", "Distribution"}]
```

Out[45]=



We have found parameters for our model to match our non-zero profile at coil current of 0.851 A, then offset and scaled our theoretical predictions to match our measurements for the profile (distribution as a function of position). We then used an interpolating function to recover the theoretical distribution, and plotted it against our measured profile.

We observe that the qualitative shape matches quite well. However, the peaks appear wider and z_{max}appears larger in the theoretical distribution than in our measurement. This is probably due to error in calibration of the magnet, but may also be due to an error in our determination of σ (for instance, if we took the profile while the B-field was not quite zeroed), which we have used to calculate the convolved Z-distribution above.