Portfolio Optimization based on Tech Companies Stocks

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Introduction

Portfolio optimization has been a cornerstone in financial markets since its inception by Harry Markowitz in the 1950s. Markowitz's Modern Portfolio Theory (MPT) revolutionized investment strategies by introducing the concept of risk-return trade-off, suggesting that optimal portfolios can be constructed to maximize returns for a given level of risk. This theory has profoundly influenced financial practices, leading to the development of diverse portfolio management techniques aimed at achieving optimal asset allocation. [1]

The advent of machine learning (ML) has further transformed financial analysis. ML's ability to process and analyze large datasets has made it an invaluable tool in deciphering complex market patterns and trends. Over time, ML techniques have evolved from simple linear models to sophisticated algorithms capable of handling nonlinear relationships and high-dimensional data. This evolution signifies a paradigm shift in financial analysis, moving towards more data-driven, predictive approaches. [2]

This paper aims to explore the integration of machine learning optimization methods in portfolio optimization, a domain traditionally dominated by MPT and similar strategies. By comparing traditional models with advanced ML techniques, the research seeks to ascertain whether ML algorithms can offer enhanced performance in terms of risk-adjusted returns. This investigation is crucial, as it could potentially redefine portfolio management strategies, aligning them with the contemporary, data-centric landscape of financial markets. The findings are expected to provide valuable insights for both academic researchers and financial practitioners.

Related Work

The concept of portfolio optimization has a rich history, originating with Harry Markowitz's Modern Portfolio Theory (MPT) in 1952. MPT introduced the critical idea of constructing a portfolio to maximize returns for a given level of risk, using mean-variance analysis. This seminal theory laid the groundwork for further developments in portfolio optimization, leading to advancements such as the Capital Asset Pricing Model (CAPM) and the Black-Litterman model. These models enhanced understanding of risk-return trade-offs and asset pricing, playing a pivotal role in investment decision-making processes. [3]

Subsequent developments in this field saw the incorporation of various factors and constraints into optimization models, such as transaction costs, market impact, and liquidity considerations. This evolution reflects the increasing complexity of financial markets and the need for more sophisticated models to capture these nuances. The literature in this area is vast, with researchers continually proposing modifications to existing models to better suit the dynamic nature of financial markets.

Machine learning (ML) has emerged as a powerful tool in financial modeling, primarily due to its ability to handle large volumes of data and uncover complex patterns. Early applications of ML in finance were focused on predictive models for asset prices and market trends, using algorithms like linear regression and time-series analysis. However, the scope of ML in finance has significantly broadened, embracing more advanced techniques such as neural networks, support vector machines, and reinforcement learning. [4]

These sophisticated ML techniques have proven particularly effective in areas like algorithmic trading, risk management, and, more recently, in portfolio optimization. Neural networks, for example, have been used to model non-linear relationships between assets, while reinforcement learning has been applied to develop dynamic portfolio management strategies. The literature on ML applications in finance is growing rapidly, reflecting both the advancements in ML techniques and the increasing availability of financial data.

Comparing traditional portfolio optimization methods with ML-based approaches reveals distinct advantages and limitations of each. Traditional models, grounded in economic theory, provide a solid framework for understanding market behavior and investor preferences. However, they often rely on assumptions like market efficiency and normal distribution of returns, which may not always hold true in real-world scenarios.

In contrast, ML methods offer flexibility and adaptability, capable of capturing complex, non-linear relationships without being bound by strict theoretical assumptions. Studies have shown that ML can enhance portfolio performance, particularly in terms of risk-adjusted returns (refer to specific studies for citation). However, challenges such as overfitting, model interpretability, and the need for extensive data remain concerns in ML applications. [5]

Datasets

The dataset utilized in this research comprises historical stock price data for various companies. The dataset, sourced from a reliable financial database, provides daily stock price information, including closing price, volume, opening price, high, and low values for each trading day. For instance, the data for Apple Inc. (AAPL) includes entries from July 2023, with the closing price on July 17th, 2023, being $193.99 and a trading volume of 50,520,160. This comprehensive dataset offers a valuable resource for analyzing stock price trends and conducting portfolio optimization studies.

Company	Date	Close/Last	Volume	Open	High	Low
The stock ticker symbol of the company, used to uniquely identify it in the stock market. For example, "AAPL"	The date on which the stock market data was recorded or reported. It represents the trading day for which the stock	The closing price or the last traded price of the company's stock on the given date. It represents the final price at which	The total number of shares of the company's stock traded on the given date. It indicates the level of investor interest and	The opening price of the company's stock on the given date. It is the price at which the first trade occurred during the	The highest price at which the company's stock traded on the given date. It indicates the highest price reached	The lowest price at which the company's stock traded on the given date. It indicates the lowest price reached during the
10 unique values	2013-07-17 2023-07-16	18720 unique values	1.14m 1.07b	18354 unique values	19066 unique values	19087 unique values
AAPL	07/17/2023	$193.99	50520160	$191.90	$194.32	$191.81
AAPL	07/14/2023	$190.69	41616240	$190.23	$191.1799	$189.63
AAPL	07/13/2023	$190.54	41342340	$190.50	$191.19	$189.78
AAPL	07-12-2023	$189.77	60750250	$189.68	$191.70	$188.47
AAPL	07-11-2023	$188.08	46638120	$189.16	$189.30	$186.60

Dataset introduction

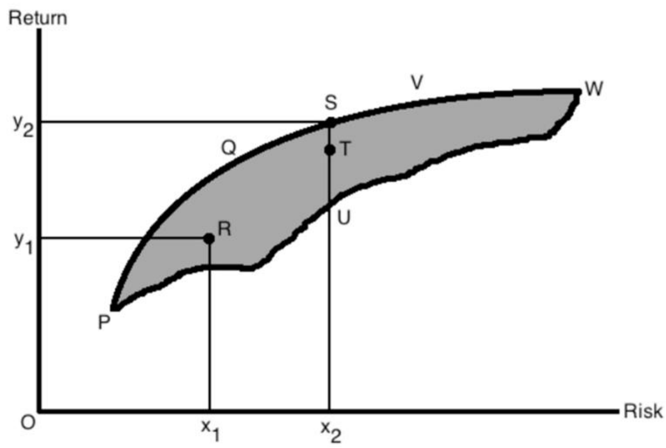
Preprocessing this dataset involved several critical steps. First, the raw data was cleansed to remove any inconsistencies or missing values, ensuring data integrity. For example, any missing trading days were identified and handled appropriately. Next, the stock prices, originally recorded as strings with dollar signs, were converted into numerical values for analysis. This step is crucial for enabling computational operations on the data.

Additionally, the dataset was normalized to account for different scales in stock prices across various companies. Normalization helps in comparing stocks on a similar scale, which is vital for accurate portfolio analysis. Finally, the trading volume, which reflects the liquidity of each stock, was also processed and included in the analysis, as it plays a significant role in investment decisions.

The initial analysis of the dataset involved computing descriptive statistics like the mean, median, standard deviation, and correlation coefficients. These statistics provide an overview of the dataset's characteristics. For instance, the mean and median closing prices offer insights into the central tendency of stock prices, while the standard deviation reflects the price volatility. Correlation coefficients between different stocks' returns are particularly important in portfolio optimization, as they help in understanding the degree to which different stocks move in relation to each other.

Methodology

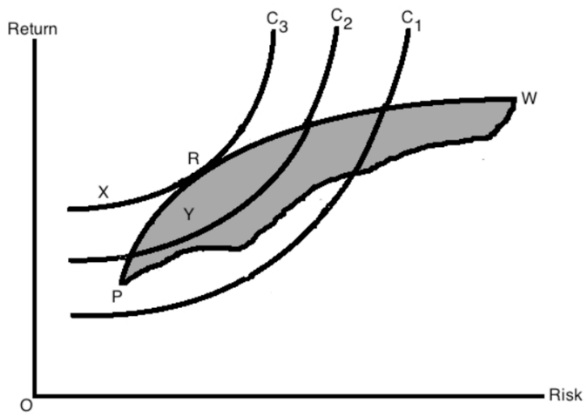
The Gradient-Based Markowitz model represents a modern adaptation of the classic Mean-Variance Optimization framework. This method employs gradient descent, a first-order iterative optimization algorithm, to find the minimum of a function — in this case, the portfolio's variance. The key advantage of the gradient method is its efficiency in handling large-scale optimization problems, common in portfolio management with many assets. [6]



Gradient Method

In this approach, the portfolio's expected return is maximized for a given level of risk, or alternatively, the risk is minimized for a given level of expected return. The gradient method iteratively adjusts the portfolio weights by moving in the direction of the steepest decrease of the risk (variance) function. This process continues until it converges to an optimal set of weights, providing an efficient solution for the allocation of assets in the portfolio.

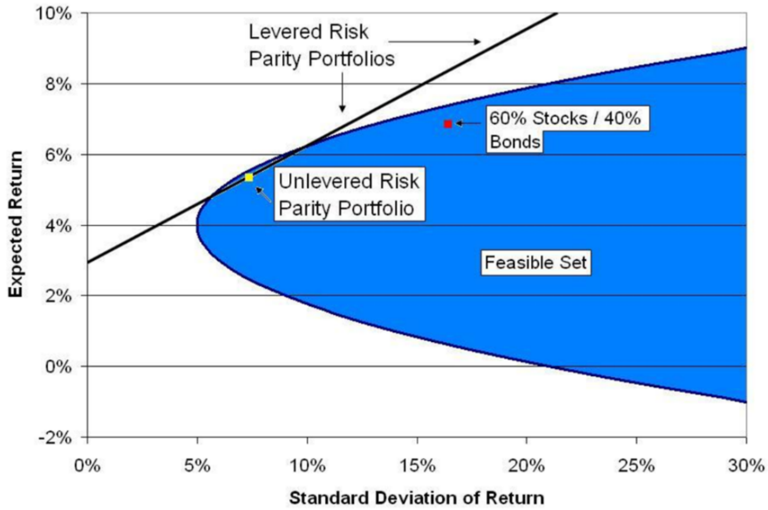
The second technique explored in this study is the proximal gradient method applied to the Markowitz model. The proximal gradient method is an extension of the traditional gradient method, particularly effective in dealing with non-smooth optimization problems. It combines the principles of gradient descent and proximal operators, making it well-suited for problems where the objective function is a sum of two terms: one smooth and one not necessarily smooth. [7]



Proximal Gradient Method

In portfolio optimization, this method can be particularly useful when there are additional constraints or regularization terms involved, such as transaction costs or a limit on the number of assets in the portfolio. The proximal gradient method can navigate these complexities more efficiently than standard optimization techniques, providing a more refined solution in the context of modern portfolio management.

The Frank-Wolfe algorithm, applied to Risk Parity portfolio optimization, offers a different approach compared to traditional Markowitz models. Risk Parity aims to allocate capital such that each asset contributes equally to the overall portfolio risk, promoting diversification and potentially reducing volatility. The Frank-Wolfe algorithm is particularly suited for this approach as it is designed to solve constrained optimization problems efficiently. [8]



Risk Parity Method

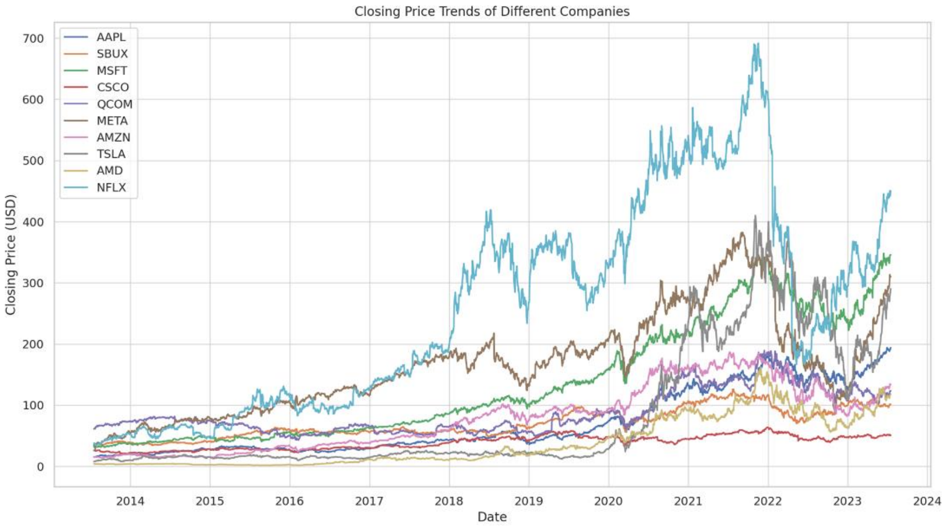
In this methodology, the Frank-Wolfe algorithm iteratively updates the portfolio weights by solving a linear subproblem at each step. This process ensures that the portfolio moves towards equal risk contribution from each asset without violating the constraints. The Frank-Wolfe based Risk Parity model is especially beneficial for long-term investment strategies where risk management is a primary concern, offering a viable alternative to the return-focused strategies of traditional Markowitz optimization.

Experiment

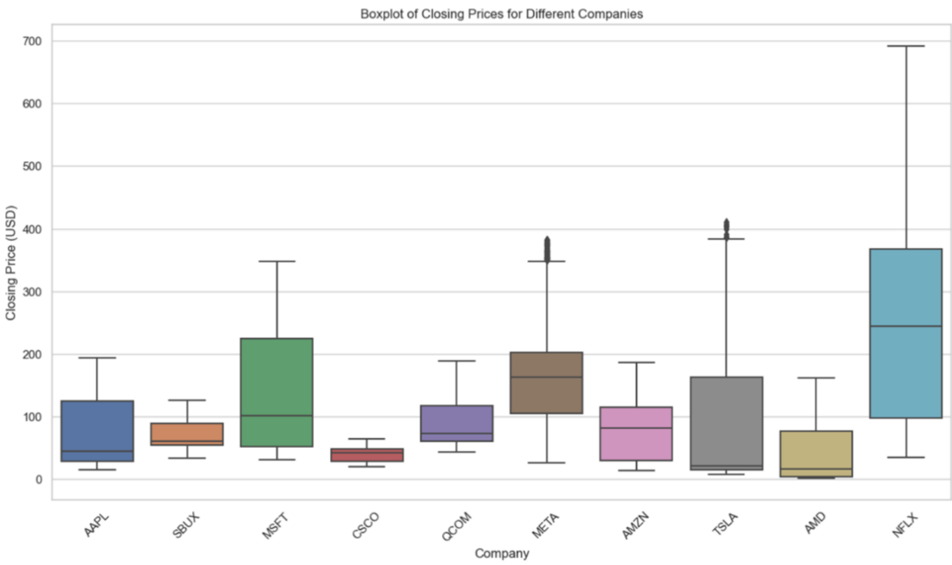
1) Basic Data Analysis:

	Company	Date	Close/Last	Volume	Open	High	Low	Daily Return
2515	AAPL	2013-07-18	15.4199	218632537	15.4779	15.5311	15.3789	NaN
2514	AAPL	2013-07-19	15.1768	268548901	15.4679	15.4993	15.1554	-0.015765
2513	AAPL	2013-07-22	15.2254	207648981	15.3379	15.3482	15.1953	0.003202
2512	AAPL	2013-07-23	14.9639	354477618	15.2143	15.2486	14.9539	-0.017175
2511	AAPL	2013-07-24	15.7325	591624923	15.6761	15.8782	15.5450	0.051364

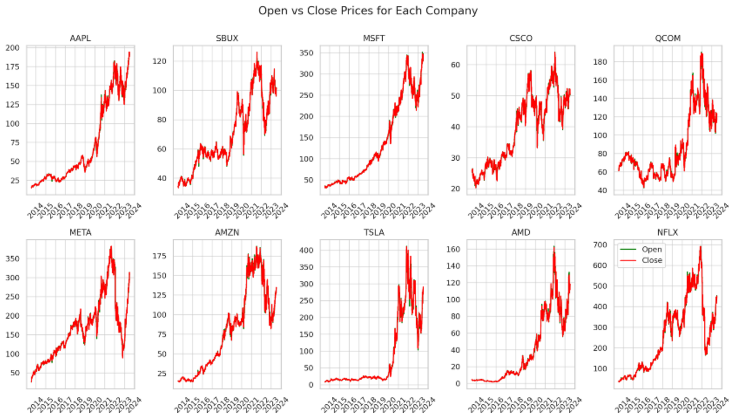
Dataframe of Stocks



Price Trend Analysis



Boxplot Analysis



Open vs Close Price Analysis

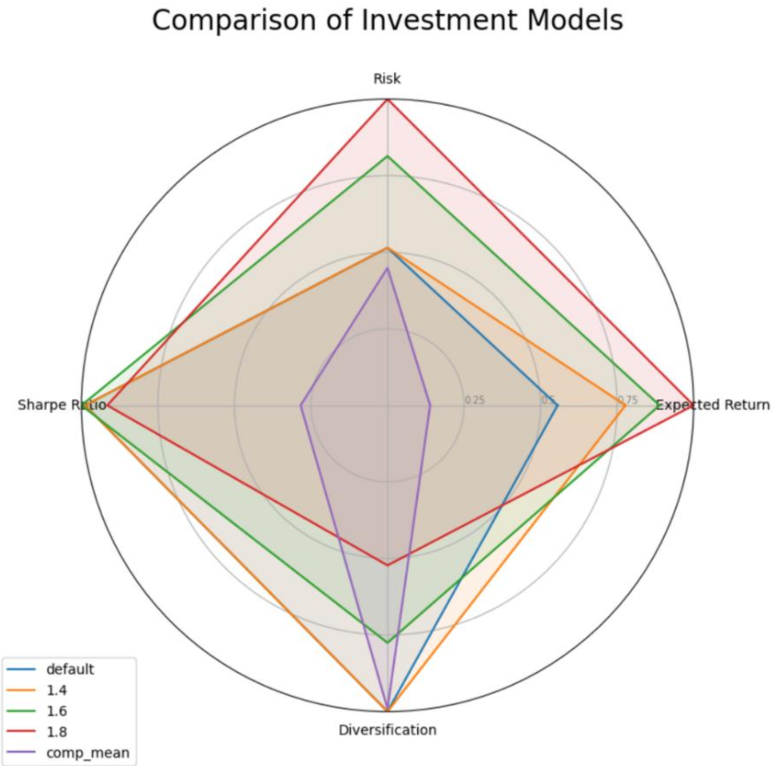
2) Gradient Method using different expectation:

$$\min_{\mathbf{w}} \left(\mathbf{w}^T \Sigma \mathbf{w}\right) \quad \text{subject to} \quad \begin{aligned} &\mathbf{w}^T \mu \geq \text{expected return}, \\ &\sum_i w_i = 1, \\ &w_i \geq 0, \quad \forall i. \end{aligned}$$

Optimization Problem

	Expected Return	Risk	Sharpe Ratio	Diversification
default	0.001831	0.019400	0.094378	0.669910
1.4	0.002563	0.019400	0.094378	0.669910
1.6	0.002930	0.030679	0.095492	0.744083
1.8	0.003296	0.037693	0.087436	0.827500
comp_mean	0.000459	0.016920	0.027108	0.673026

Evaluation Matrix



Comparison of different expectation

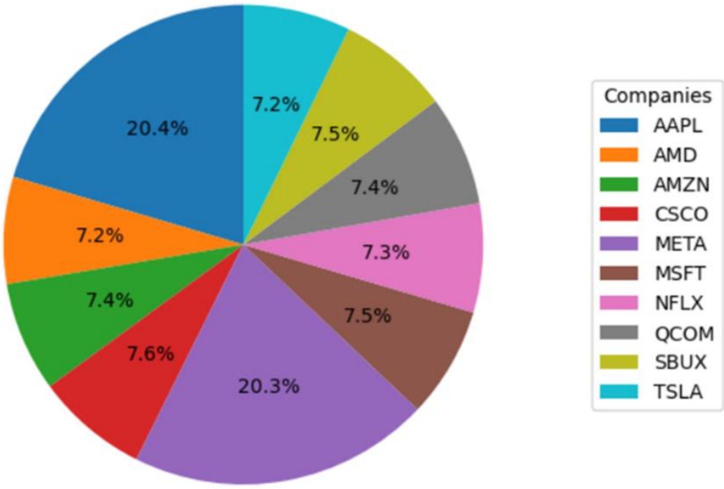
3) Proximal Gradient Method:

$$
\begin{aligned}
\min_{w} \quad & w^T \Sigma w \\
\text{s.t.} \quad & \mu^T w \geq 0.8 \times \text{median}(\mu) \\
& \sum w_i = 1 \\
& w_{\text{AAPL}} \geq 0.20 \\
& w_{\text{META}} \geq 0.20 \\
& w_i \geq 0 \quad \text{for all } i
\end{aligned}
$$

Optimization Problem

	Metric	Value
0	Expected Return	0.001892
1	Risk Control	0.019548
2	Sharpe Ratio	-0.158986

Evaluation Matrix



Pie Plot Analysis

4) Frank-Wolfe:

a. Find a direction $\mathbf{s}_k$ by solving the linear problem:

$$\mathbf{s}_k = \arg\min_{\mathbf{s} \in \mathcal{C}} \langle \nabla f(\mathbf{x}_k), \mathbf{s} \rangle$$

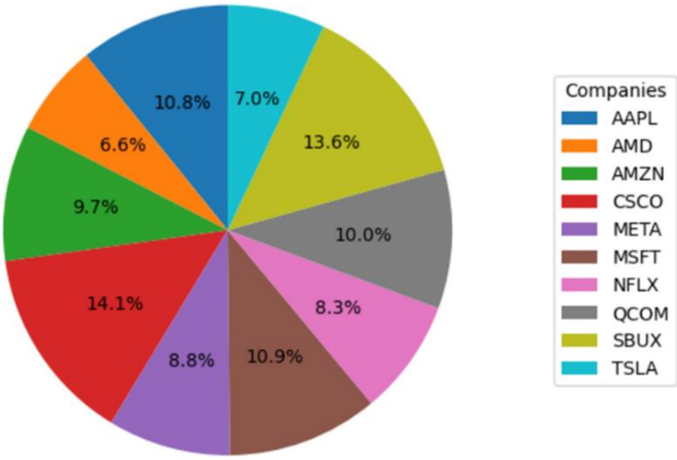
b. Update the solution with a suitable step size γ_k (often determined by line search):

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \gamma_k(\mathbf{s}_k - \mathbf{x}_k)$$

Optimization Problem

(	Asset	Optimized Weight (%)	Actual Risk Contribution (%)	Ideal Risk Contribution (%)
0	AAPL	10.943112	9.991041	10.0
1	AMD	6.205831	9.979666	10.0
2	AMZN	9.683858	10.016868	10.0
3	CSCO	15.046974	10.008809	10.0
4	META	8.701927	9.987588	10.0
5	MSFT	11.384166	9.984720	10.0
6	NFLX	8.006371	10.003142	10.0
7	QCOM	9.803683	10.026176	10.0
8	SBUX	13.570853	10.017363	10.0
9	TSLA	6.653225	9.984627	10.0

Actual & Ideal Risk Contribution



Pie Plot Analysis

Conclusion

The Gradient-Based Markowitz model, leveraging the gradient descent method, has shown notable efficacy in optimizing portfolios. Its primary strength lies in efficiently handling large datasets, a common scenario in modern financial markets with a vast array of assets. The iterative nature of the gradient method ensures a steady approach towards the optimal solution, making it a reliable choice for portfolio optimization. However, its performance is contingent on the choice of the initial point and the learning rate, which can impact the convergence speed and the risk of getting trapped in local minima.

The Proximal Gradient-Based Markowitz model extends the capabilities of traditional gradient methods, especially in handling non-smooth objective functions. This is particularly beneficial in scenarios where the portfolio optimization problem involves complex constraints or irregular return distributions. By incorporating a proximal term, this method demonstrates improved convergence properties over standard gradient methods. Nevertheless, the choice of the proximal parameter is critical and can be challenging to optimize, potentially affecting the algorithm's efficiency.

The application of the Frank-Wolfe algorithm in Risk Parity portfolio optimization has provided a compelling alternative to return-centric models. Its main advantage is the ability to manage risk more evenly across a portfolio, contributing to potentially lower overall volatility. This approach aligns well with long-term investment strategies where risk management is paramount. However, the Frank-Wolfe algorithm can be slower in convergence compared to other methods, and its performance heavily depends on the nature of the constraints in the optimization problem.

Comparatively, each algorithm caters to specific requirements and scenarios in portfolio optimization. The Gradient-Based Markowitz is well-suited for scenarios demanding rapid solutions in high-dimensional spaces. In contrast, the Proximal Gradient-Based Markowitz offers a more robust solution in the presence of complex constraints. The Frank-Wolfe Based Risk Parity stands out in its ability to achieve a balanced risk distribution, which is crucial for certain investment strategies.

One potential area of improvement across these algorithms is the integration of adaptive learning rates, which could enhance the convergence speed and accuracy. Additionally, incorporating elements of machine learning, such as reinforcement learning, could enable the models to adapt to changing market conditions dynamically.

Exploring hybrid models that combine the strengths of different optimization methods could lead to more robust and versatile portfolio management solutions. For instance, a blend of Gradient-Based and Frank-Wolfe methods might balance the speed of convergence with risk parity considerations. [9]

Future research could focus on adapting these algorithms to better handle real-world constraints, such as transaction costs, regulatory requirements, and liquidity constraints. This would enhance the practical applicability of the models in actual investment scenarios.

Lastly, tailoring these optimization models to individual investor profiles, considering factors like risk tolerance and investment horizons, could make them more relevant and beneficial for personal portfolio management.

In conclusion, the Gradient-Based, Proximal Gradient-Based, and Frank-Wolfe Based Risk Parity models each present unique advantages and limitations in portfolio optimization. Their future development, potentially through hybridization and integration with machine learning techniques, holds great promise for advancing the field of financial portfolio management.

Reference

[1] Fisher L. Using modern portfolio theory to maintain an efficiently diversified portfolio[J]. Financial Analysts Journal, 1975, 31(3): 73-85.

[2] Ban G Y, El Karoui N, Lim A E B. Machine learning and portfolio optimization[J]. Management Science, 2018, 64(3): 1136-1154.

[3] Markowitz H M. Foundations of portfolio theory[J]. The journal of finance, 1991, 46(2): 469-477.

[4] Bartram S M, Branke J, De Rossi G, et al. Machine learning for active portfolio management[J]. The Journal of Financial Data Science, 2021, 3(3): 9-30.

[5] Ma Y, Han R, Wang W. Portfolio optimization with return prediction using deep learning and machine learning[J]. Expert Systems with Applications, 2021, 165: 113973.

[6] Alrabadi D W H. Portfolio optimization using the generalized reduced gradient nonlinear algorithm: An application to Amman Stock Exchange[J]. International Journal of Islamic and Middle Eastern Finance and Management, 2016, 9(4): 570-582.

[7] Bertsimas D, Cory-Wright R. A scalable algorithm for sparse and robust portfolios[J]. arXiv, 2018, 2018.

[8] Di Lorenzo D, Liuzzi G, Rinaldi F, et al. A concave optimization-based approach for sparse portfolio selection[J]. Optimization Methods and Software, 2012, 27(6): 983-1000.

[9] Pian T H H. Hybrid models[M]//Numerical and computer methods in structural mechanics. Academic Press, 1973: 59-78.