

# Algorithmique et recherche opérationnelle

**INFO-F310**

Projet : CPLEX LP

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## 1 Introduction

Pour ce projet, il est demandé d'écrire un script python qui, à partir d'un fichier d'instance indiquant un ensemble de panneaux de longueur variable à découper dans des planches de longueur fixe, génère une instance de programme linéaire en langage CPLEX LP qui détermine un plan de découpe de panneaux qui minimise le nombre de planches utilisées.

## 2 Modèle

Indices :

$M = \{1, \dots, m\}$  planches

$N = \{1, \dots, n\}$  panneaux

Constantes :

$l_i$  = longueur du panneau  $i$

$L$  = longueur des planches  $j$

Variables de décision :

$x_{i,j} \in \{0, 1\}$  telle que

$$x_{i,j} = \begin{cases} 1 & \text{si une planche } j \text{ est prise pour une découpe du panneau } i \\ 0 & \text{sinon} \end{cases}$$

$m_j \in \{0, 1\}$  telle que

$$m_j = \begin{cases} 1 & \text{si une planche } j \text{ est prise pour une découpe} \\ 0 & \text{sinon} \end{cases}$$

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**Formulation 1** Formulation linéaire

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$$\min \quad \sum_{j \in M} m_j \quad (1)$$

$$\text{s.t.} \quad \sum_{j \in M} x_{i,j} = 1 \quad \forall i \in N \quad (2)$$

$$\sum_{i \in N} l_i x_{i,j} \leq L m_j \quad \forall j \in M \quad (3)$$

$$x_{i,j} \in \{0, 1\} \quad \forall i \in N, \quad \forall j \in M \quad (4)$$

$$m_j \in \{0, 1\} \quad \forall j \in M \quad (5)$$

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Les contraintes (2) assurent que chaque planche a été correctement découpée. D'autre part, (3) assurent que la somme des longueurs d'un ensemble de panneaux découpés à partir d'une planche ne peut pas dépasser la longueur de cette dernière. Finalement, les contraintes (4) impliquent que la somme des longueurs des panneaux découpés doit être inférieure ou égale à la somme totale des longueurs des planches utilisées.

### 3 Fichier CPLEX LP généré

Minimize

$$\text{obj: } m\_1 + m\_2 + m\_3 + m\_4 + m\_5 + m\_6 + m\_7 + m\_8$$

Subject To

$$\begin{aligned} c\_1\_1: & 4.0 \, m\_1 - 1.5 \, x\_1\_1(1.5) - 1.5 \, x\_2\_1(1.5) - 1.5 \, x\_3\_1(1.5) - 1.5 \, x\_4\_1(1.5) - 0.75 \\ & x\_5\_1(0.75) - 0.75 \, x\_6\_1(0.75) - 0.75 \, x\_7\_1(0.75) - 0.75 \, x\_8\_1(0.75) - 0.75 \, x\_9\_1(0.75) - \\ & 0.75 \, x\_10\_1(0.75) - 0.22 \, x\_11\_1(0.22) - 0.22 \, x\_12\_1(0.22) - 0.22 \, x\_13\_1(0.22) - 0.22 \\ & x\_14\_1(0.22) - 0.22 \, x\_15\_1(0.22) - 0.22 \, x\_16\_1(0.22) - 0.22 \, x\_17\_1(0.22) - 0.22 \\ & x\_18\_1(0.22) - 0.22 \, x\_19\_1(0.22) - 0.22 \, x\_20\_1(0.22) - 0.22 \, x\_21\_1(0.22) - 0.22 \\ & x\_22\_1(0.22) \geq 0 \end{aligned}$$

$$\begin{aligned} c\_1\_2: & 4.0 \, m\_2 - 1.5 \, x\_1\_2(1.5) - 1.5 \, x\_2\_2(1.5) - 1.5 \, x\_3\_2(1.5) - 1.5 \, x\_4\_2(1.5) - 0.75 \\ & x\_5\_2(0.75) - 0.75 \, x\_6\_2(0.75) - 0.75 \, x\_7\_2(0.75) - 0.75 \, x\_8\_2(0.75) - 0.75 \, x\_9\_2(0.75) - \\ & 0.75 \, x\_10\_2(0.75) - 0.22 \, x\_11\_2(0.22) - 0.22 \, x\_12\_2(0.22) - 0.22 \, x\_13\_2(0.22) - 0.22 \\ & x\_14\_2(0.22) - 0.22 \, x\_15\_2(0.22) - 0.22 \, x\_16\_2(0.22) - 0.22 \, x\_17\_2(0.22) - 0.22 \\ & x\_18\_2(0.22) - 0.22 \, x\_19\_2(0.22) - 0.22 \, x\_20\_2(0.22) - 0.22 \, x\_21\_2(0.22) - 0.22 \\ & x\_22\_2(0.22) \geq 0 \end{aligned}$$

$$\begin{aligned} c\_1\_3: & 4.0 \, m\_3 - 1.5 \, x\_1\_3(1.5) - 1.5 \, x\_2\_3(1.5) - 1.5 \, x\_3\_3(1.5) - 1.5 \, x\_4\_3(1.5) - 0.75 \\ & x\_5\_3(0.75) - 0.75 \, x\_6\_3(0.75) - 0.75 \, x\_7\_3(0.75) - 0.75 \, x\_8\_3(0.75) - 0.75 \, x\_9\_3(0.75) - \\ & 0.75 \, x\_10\_3(0.75) - 0.22 \, x\_11\_3(0.22) - 0.22 \, x\_12\_3(0.22) - 0.22 \, x\_13\_3(0.22) - 0.22 \\ & x\_14\_3(0.22) - 0.22 \, x\_15\_3(0.22) - 0.22 \, x\_16\_3(0.22) - 0.22 \, x\_17\_3(0.22) - 0.22 \\ & x\_18\_3(0.22) - 0.22 \, x\_19\_3(0.22) - 0.22 \, x\_20\_3(0.22) - 0.22 \, x\_21\_3(0.22) - 0.22 \\ & x\_22\_3(0.22) \geq 0 \end{aligned}$$

$$\begin{aligned} c\_1\_4: & 4.0 \, m\_4 - 1.5 \, x\_1\_4(1.5) - 1.5 \, x\_2\_4(1.5) - 1.5 \, x\_3\_4(1.5) - 1.5 \, x\_4\_4(1.5) - 0.75 \\ & x\_5\_4(0.75) - 0.75 \, x\_6\_4(0.75) - 0.75 \, x\_7\_4(0.75) - 0.75 \, x\_8\_4(0.75) - 0.75 \, x\_9\_4(0.75) - \\ & 0.75 \, x\_10\_4(0.75) - 0.22 \, x\_11\_4(0.22) - 0.22 \, x\_12\_4(0.22) - 0.22 \, x\_13\_4(0.22) - 0.22 \\ & x\_14\_4(0.22) - 0.22 \, x\_15\_4(0.22) - 0.22 \, x\_16\_4(0.22) - 0.22 \, x\_17\_4(0.22) - 0.22 \\ & x\_18\_4(0.22) - 0.22 \, x\_19\_4(0.22) - 0.22 \, x\_20\_4(0.22) - 0.22 \, x\_21\_4(0.22) - 0.22 \\ & x\_22\_4(0.22) \geq 0 \end{aligned}$$

$$\begin{aligned} c\_1\_5: & 4.0 \, m\_5 - 1.5 \, x\_1\_5(1.5) - 1.5 \, x\_2\_5(1.5) - 1.5 \, x\_3\_5(1.5) - 1.5 \, x\_4\_5(1.5) - 0.75 \\ & x\_5\_5(0.75) - 0.75 \, x\_6\_5(0.75) - 0.75 \, x\_7\_5(0.75) - 0.75 \, x\_8\_5(0.75) - 0.75 \, x\_9\_5(0.75) - \\ & 0.75 \, x\_10\_5(0.75) - 0.22 \, x\_11\_5(0.22) - 0.22 \, x\_12\_5(0.22) - 0.22 \, x\_13\_5(0.22) - 0.22 \\ & x\_14\_5(0.22) - 0.22 \, x\_15\_5(0.22) - 0.22 \, x\_16\_5(0.22) - 0.22 \, x\_17\_5(0.22) - 0.22 \\ & x\_18\_5(0.22) - 0.22 \, x\_19\_5(0.22) - 0.22 \, x\_20\_5(0.22) - 0.22 \, x\_21\_5(0.22) - 0.22 \\ & x\_22\_5(0.22) \geq 0 \end{aligned}$$

$$\begin{aligned}
c\_1\_6: & 4.0 \, m\_6 - 1.5 \, x\_1\_6(1.5) - 1.5 \, x\_2\_6(1.5) - 1.5 \, x\_3\_6(1.5) - 1.5 \, x\_4\_6(1.5) - 0.75 \\
& x\_5\_6(0.75) - 0.75 \, x\_6\_6(0.75) - 0.75 \, x\_7\_6(0.75) - 0.75 \, x\_8\_6(0.75) - 0.75 \, x\_9\_6(0.75) - \\
& 0.75 \, x\_10\_6(0.75) - 0.22 \, x\_11\_6(0.22) - 0.22 \, x\_12\_6(0.22) - 0.22 \, x\_13\_6(0.22) - 0.22 \\
& x\_14\_6(0.22) - 0.22 \, x\_15\_6(0.22) - 0.22 \, x\_16\_6(0.22) - 0.22 \, x\_17\_6(0.22) - 0.22 \\
& x\_18\_6(0.22) - 0.22 \, x\_19\_6(0.22) - 0.22 \, x\_20\_6(0.22) - 0.22 \, x\_21\_6(0.22) - 0.22 \\
& x\_22\_6(0.22) \geq 0 \\
c\_1\_7: & 4.0 \, m\_7 - 1.5 \, x\_1\_7(1.5) - 1.5 \, x\_2\_7(1.5) - 1.5 \, x\_3\_7(1.5) - 1.5 \, x\_4\_7(1.5) - 0.75 \\
& x\_5\_7(0.75) - 0.75 \, x\_6\_7(0.75) - 0.75 \, x\_7\_7(0.75) - 0.75 \, x\_8\_7(0.75) - 0.75 \, x\_9\_7(0.75) - \\
& 0.75 \, x\_10\_7(0.75) - 0.22 \, x\_11\_7(0.22) - 0.22 \, x\_12\_7(0.22) - 0.22 \, x\_13\_7(0.22) - 0.22 \\
& x\_14\_7(0.22) - 0.22 \, x\_15\_7(0.22) - 0.22 \, x\_16\_7(0.22) - 0.22 \, x\_17\_7(0.22) - 0.22 \\
& x\_18\_7(0.22) - 0.22 \, x\_19\_7(0.22) - 0.22 \, x\_20\_7(0.22) - 0.22 \, x\_21\_7(0.22) - 0.22 \\
& x\_22\_7(0.22) \geq 0 \\
c\_1\_8: & 4.0 \, m\_8 - 1.5 \, x\_1\_8(1.5) - 1.5 \, x\_2\_8(1.5) - 1.5 \, x\_3\_8(1.5) - 1.5 \, x\_4\_8(1.5) - 0.75 \\
& x\_5\_8(0.75) - 0.75 \, x\_6\_8(0.75) - 0.75 \, x\_7\_8(0.75) - 0.75 \, x\_8\_8(0.75) - 0.75 \, x\_9\_8(0.75) - \\
& 0.75 \, x\_10\_8(0.75) - 0.22 \, x\_11\_8(0.22) - 0.22 \, x\_12\_8(0.22) - 0.22 \, x\_13\_8(0.22) - 0.22 \\
& x\_14\_8(0.22) - 0.22 \, x\_15\_8(0.22) - 0.22 \, x\_16\_8(0.22) - 0.22 \, x\_17\_8(0.22) - 0.22 \\
& x\_18\_8(0.22) - 0.22 \, x\_19\_8(0.22) - 0.22 \, x\_20\_8(0.22) - 0.22 \, x\_21\_8(0.22) - 0.22 \\
& x\_22\_8(0.22) \geq 0 \\
c\_2\_1: & x\_1\_1(1.5) + x\_1\_2(1.5) + x\_1\_3(1.5) + x\_1\_4(1.5) + x\_1\_5(1.5) + x\_1\_6(1.5) + \\
& x\_1\_7(1.5) + x\_1\_8(1.5) = 1 \\
c\_2\_2: & x\_2\_1(1.5) + x\_2\_2(1.5) + x\_2\_3(1.5) + x\_2\_4(1.5) + x\_2\_5(1.5) + x\_2\_6(1.5) + \\
& x\_2\_7(1.5) + x\_2\_8(1.5) = 1 \\
c\_2\_3: & x\_3\_1(1.5) + x\_3\_2(1.5) + x\_3\_3(1.5) + x\_3\_4(1.5) + x\_3\_5(1.5) + x\_3\_6(1.5) + \\
& x\_3\_7(1.5) + x\_3\_8(1.5) = 1 \\
c\_2\_4: & x\_4\_1(1.5) + x\_4\_2(1.5) + x\_4\_3(1.5) + x\_4\_4(1.5) + x\_4\_5(1.5) + x\_4\_6(1.5) + \\
& x\_4\_7(1.5) + x\_4\_8(1.5) = 1 \\
c\_2\_5: & x\_5\_1(0.75) + x\_5\_2(0.75) + x\_5\_3(0.75) + x\_5\_4(0.75) + x\_5\_5(0.75) + x\_5\_6(0.75) + \\
& x\_5\_7(0.75) + x\_5\_8(0.75) = 1 \\
c\_2\_6: & x\_6\_1(0.75) + x\_6\_2(0.75) + x\_6\_3(0.75) + x\_6\_4(0.75) + x\_6\_5(0.75) + x\_6\_6(0.75) + \\
& x\_6\_7(0.75) + x\_6\_8(0.75) = 1 \\
c\_2\_7: & x\_7\_1(0.75) + x\_7\_2(0.75) + x\_7\_3(0.75) + x\_7\_4(0.75) + x\_7\_5(0.75) + x\_7\_6(0.75) + \\
& x\_7\_7(0.75) + x\_7\_8(0.75) = 1 \\
c\_2\_8: & x\_8\_1(0.75) + x\_8\_2(0.75) + x\_8\_3(0.75) + x\_8\_4(0.75) + x\_8\_5(0.75) + x\_8\_6(0.75) + \\
& x\_8\_7(0.75) + x\_8\_8(0.75) = 1 \\
c\_2\_9: & x\_9\_1(0.75) + x\_9\_2(0.75) + x\_9\_3(0.75) + x\_9\_4(0.75) + x\_9\_5(0.75) + x\_9\_6(0.75) + \\
& x\_9\_7(0.75) + x\_9\_8(0.75) = 1 \\
c\_2\_10: & x\_10\_1(0.75) + x\_10\_2(0.75) + x\_10\_3(0.75) + x\_10\_4(0.75) + x\_10\_5(0.75) + \\
& x\_10\_6(0.75) + x\_10\_7(0.75) + x\_10\_8(0.75) = 1 \\
c\_2\_11: & x\_11\_1(0.22) + x\_11\_2(0.22) + x\_11\_3(0.22) + x\_11\_4(0.22) + x\_11\_5(0.22) + \\
& x\_11\_6(0.22) + x\_11\_7(0.22) + x\_11\_8(0.22) = 1 \\
c\_2\_12: & x\_12\_1(0.22) + x\_12\_2(0.22) + x\_12\_3(0.22) + x\_12\_4(0.22) + x\_12\_5(0.22) + \\
& x\_12\_6(0.22) + x\_12\_7(0.22) + x\_12\_8(0.22) = 1
\end{aligned}$$

$$\begin{aligned}
c\_2\_13: & x\_13\_1(0.22) + x\_13\_2(0.22) + x\_13\_3(0.22) + x\_13\_4(0.22) + x\_13\_5(0.22) + \\
& x\_13\_6(0.22) + x\_13\_7(0.22) + x\_13\_8(0.22) = 1 \\
c\_2\_14: & x\_14\_1(0.22) + x\_14\_2(0.22) + x\_14\_3(0.22) + x\_14\_4(0.22) + x\_14\_5(0.22) + \\
& x\_14\_6(0.22) + x\_14\_7(0.22) + x\_14\_8(0.22) = 1 \\
c\_2\_15: & x\_15\_1(0.22) + x\_15\_2(0.22) + x\_15\_3(0.22) + x\_15\_4(0.22) + x\_15\_5(0.22) + \\
& x\_15\_6(0.22) + x\_15\_7(0.22) + x\_15\_8(0.22) = 1 \\
c\_2\_16: & x\_16\_1(0.22) + x\_16\_2(0.22) + x\_16\_3(0.22) + x\_16\_4(0.22) + x\_16\_5(0.22) + \\
& x\_16\_6(0.22) + x\_16\_7(0.22) + x\_16\_8(0.22) = 1 \\
c\_2\_17: & x\_17\_1(0.22) + x\_17\_2(0.22) + x\_17\_3(0.22) + x\_17\_4(0.22) + x\_17\_5(0.22) + \\
& x\_17\_6(0.22) + x\_17\_7(0.22) + x\_17\_8(0.22) = 1 \\
c\_2\_18: & x\_18\_1(0.22) + x\_18\_2(0.22) + x\_18\_3(0.22) + x\_18\_4(0.22) + x\_18\_5(0.22) + \\
& x\_18\_6(0.22) + x\_18\_7(0.22) + x\_18\_8(0.22) = 1 \\
c\_2\_19: & x\_19\_1(0.22) + x\_19\_2(0.22) + x\_19\_3(0.22) + x\_19\_4(0.22) + x\_19\_5(0.22) + \\
& x\_19\_6(0.22) + x\_19\_7(0.22) + x\_19\_8(0.22) = 1 \\
c\_2\_20: & x\_20\_1(0.22) + x\_20\_2(0.22) + x\_20\_3(0.22) + x\_20\_4(0.22) + x\_20\_5(0.22) + \\
& x\_20\_6(0.22) + x\_20\_7(0.22) + x\_20\_8(0.22) = 1 \\
c\_2\_21: & x\_21\_1(0.22) + x\_21\_2(0.22) + x\_21\_3(0.22) + x\_21\_4(0.22) + x\_21\_5(0.22) + \\
& x\_21\_6(0.22) + x\_21\_7(0.22) + x\_21\_8(0.22) = 1 \\
c\_2\_22: & x\_22\_1(0.22) + x\_22\_2(0.22) + x\_22\_3(0.22) + x\_22\_4(0.22) + x\_22\_5(0.22) + \\
& x\_22\_6(0.22) + x\_22\_7(0.22) + x\_22\_8(0.22) = 1
\end{aligned}$$

BINARY

m\_1  
m\_2  
m\_3  
m\_4  
m\_5  
m\_6  
m\_7  
m\_8  
x\_1\_1(1.5)  
x\_1\_2(1.5)  
x\_1\_3(1.5)  
x\_1\_4(1.5)  
x\_1\_5(1.5)  
x\_1\_6(1.5)  
x\_1\_7(1.5)  
x\_1\_8(1.5)  
x\_2\_1(1.5)  
x\_2\_2(1.5)  
x\_2\_3(1.5)  
x\_2\_4(1.5)  
x\_2\_5(1.5)  
x\_2\_6(1.5)

$x_{2\_7}(1.5)$   
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 $x_{3\_1}(1.5)$   
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x\_16\_7(0.22)  
x\_16\_8(0.22)  
x\_17\_1(0.22)  
x\_17\_2(0.22)  
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x\_17\_7(0.22)  
x\_17\_8(0.22)  
x\_18\_1(0.22)  
x\_18\_2(0.22)  
x\_18\_3(0.22)  
x\_18\_4(0.22)  
x\_18\_5(0.22)  
x\_18\_6(0.22)  
x\_18\_7(0.22)

```

x_18_8(0.22)
x_19_1(0.22)
x_19_2(0.22)
x_19_3(0.22)
x_19_4(0.22)
x_19_5(0.22)
x_19_6(0.22)
x_19_7(0.22)
x_19_8(0.22)
x_20_1(0.22)
x_20_2(0.22)
x_20_3(0.22)
x_20_4(0.22)
x_20_5(0.22)
x_20_6(0.22)
x_20_7(0.22)
x_20_8(0.22)
x_21_1(0.22)
x_21_2(0.22)
x_21_3(0.22)
x_21_4(0.22)
x_21_5(0.22)
x_21_6(0.22)
x_21_7(0.22)
x_21_8(0.22)
x_22_1(0.22)
x_22_2(0.22)
x_22_3(0.22)
x_22_4(0.22)
x_22_5(0.22)
x_22_6(0.22)
x_22_7(0.22)
x_22_8(0.22)
END

```

## 4 Log de GLPK obtenu

Problem:

Rows: 30

Columns: 184 (184 integer, 184 binary)

Non-zeros: 360

Status: INTEGER OPTIMAL  
Objective: obj = 4 (MINimum)

No.	Row name	Activity	Lower bound	Upper bound
1	c_1_1	0	0	
2	c_1_2	0.21	0	
3	c_1_3	1.75	0	
4	c_1_4	0	0	
5	c_1_5	0	0	
6	c_1_6	0	0	
7	c_1_7	0.78	0	
8	c_1_8	0.12	0	
9	c_2_1	1	1	=
10	c_2_2	1	1	=
11	c_2_3	1	1	=
12	c_2_4	1	1	=
13	c_2_5	1	1	=
14	c_2_6	1	1	=
15	c_2_7	1	1	=
16	c_2_8	1	1	=
17	c_2_9	1	1	=
18	c_2_10	1	1	=
19	c_2_11	1	1	=
20	c_2_12	1	1	=
21	c_2_13	1	1	=
22	c_2_14	1	1	=
23	c_2_15	1	1	=
24	c_2_16	1	1	=
25	c_2_17	1	1	=
26	c_2_18	1	1	=
27	c_2_19	1	1	=
28	c_2_20	1	1	=
29	c_2_21	1	1	=

30 c\_2\_22 1 1 =

No.	Column name	Activity	Lower bound	Upper bound
1	m_1 *	0	0	1
2	m_2 *	1	0	1
3	m_3 *	1	0	1
4	m_4 *	0	0	1
5	m_5 *	0	0	1
6	m_6 *	0	0	1
7	m_7 *	1	0	1
8	m_8 *	1	0	1
9	x_1_1(1.5) *	0	0	1
10	x_2_1(1.5) *	0	0	1
11	x_3_1(1.5) *	0	0	1
12	x_4_1(1.5) *	0	0	1
13	x_5_1(0.75) *	0	0	1
14	x_6_1(0.75) *	0	0	1
15	x_7_1(0.75) *	0	0	1
16	x_8_1(0.75) *	0	0	1
17	x_9_1(0.75) *	0	0	1
18	x_10_1(0.75) *	0	0	1
19	x_11_1(0.22) *	0	0	1
20	x_12_1(0.22) *	0	0	1
21	x_13_1(0.22) *	0	0	1
22	x_14_1(0.22) *	0	0	1
23	x_15_1(0.22) *	0	0	1
24	x_16_1(0.22) *	0	0	1
25	x_17_1(0.22) *	0	0	1
26	x_18_1(0.22) *	0	0	1
27	x_19_1(0.22) *	0	0	1
28	x_20_1(0.22) *	0	0	1
29	x_21_1(0.22) *	0	0	1
30	x_22_1(0.22) *	0	0	1

31	x_1_2(1.5)	*	0	0	1
32	x_2_2(1.5)	*	0	0	1
33	x_3_2(1.5)	*	0	0	1
34	x_4_2(1.5)	*	0	0	1
35	x_5_2(0.75)	*	0	0	1
36	x_6_2(0.75)	*	1	0	1
37	x_7_2(0.75)	*	1	0	1
38	x_8_2(0.75)	*	0	0	1
39	x_9_2(0.75)	*	0	0	1
40	x_10_2(0.75)	*	1	0	1
41	x_11_2(0.22)	*	1	0	1
42	x_12_2(0.22)	*	1	0	1
43	x_13_2(0.22)	*	1	0	1
44	x_14_2(0.22)	*	1	0	1
45	x_15_2(0.22)	*	1	0	1
46	x_16_2(0.22)	*	1	0	1
47	x_17_2(0.22)	*	1	0	1
48	x_18_2(0.22)	*	0	0	1
49	x_19_2(0.22)	*	0	0	1
50	x_20_2(0.22)	*	0	0	1
51	x_21_2(0.22)	*	0	0	1
52	x_22_2(0.22)	*	0	0	1
53	x_1_3(1.5)	*	0	0	1
54	x_2_3(1.5)	*	0	0	1
55	x_3_3(1.5)	*	0	0	1
56	x_4_3(1.5)	*	0	0	1
57	x_5_3(0.75)	*	1	0	1
58	x_6_3(0.75)	*	0	0	1
59	x_7_3(0.75)	*	0	0	1
60	x_8_3(0.75)	*	1	0	1
61	x_9_3(0.75)	*	1	0	1
62	x_10_3(0.75)	*	0	0	1
63	x_11_3(0.22)	*	0	0	1
64	x_12_3(0.22)	*	0	0	1

65	x_13_3(0.22) *	0	0	1
66	x_14_3(0.22) *	0	0	1
67	x_15_3(0.22) *	0	0	1
68	x_16_3(0.22) *	0	0	1
69	x_17_3(0.22) *	0	0	1
70	x_18_3(0.22) *	0	0	1
71	x_19_3(0.22) *	0	0	1
72	x_20_3(0.22) *	0	0	1
73	x_21_3(0.22) *	0	0	1
74	x_22_3(0.22) *	0	0	1
75	x_1_4(1.5) *	0	0	1
76	x_2_4(1.5) *	0	0	1
77	x_3_4(1.5) *	0	0	1
78	x_4_4(1.5) *	0	0	1
79	x_5_4(0.75) *	0	0	1
80	x_6_4(0.75) *	0	0	1
81	x_7_4(0.75) *	0	0	1
82	x_8_4(0.75) *	0	0	1
83	x_9_4(0.75) *	0	0	1
84	x_10_4(0.75) *	0	0	1
85	x_11_4(0.22) *	0	0	1
86	x_12_4(0.22) *	0	0	1
87	x_13_4(0.22) *	0	0	1
88	x_14_4(0.22) *	0	0	1
89	x_15_4(0.22) *	0	0	1
90	x_16_4(0.22) *	0	0	1
91	x_17_4(0.22) *	0	0	1
92	x_18_4(0.22) *	0	0	1
93	x_19_4(0.22) *	0	0	1
94	x_20_4(0.22) *	0	0	1
95	x_21_4(0.22) *	0	0	1
96	x_22_4(0.22) *	0	0	1
97	x_1_5(1.5) *	0	0	1
98	x_2_5(1.5) *	0	0	1

99	x_3_5(1.5)	*	0	0	1
100	x_4_5(1.5)	*	0	0	1
101	x_5_5(0.75)	*	0	0	1
102	x_6_5(0.75)	*	0	0	1
103	x_7_5(0.75)	*	0	0	1
104	x_8_5(0.75)	*	0	0	1
105	x_9_5(0.75)	*	0	0	1
106	x_10_5(0.75)	*	0	0	1
107	x_11_5(0.22)	*	0	0	1
108	x_12_5(0.22)	*	0	0	1
109	x_13_5(0.22)	*	0	0	1
110	x_14_5(0.22)	*	0	0	1
111	x_15_5(0.22)	*	0	0	1
112	x_16_5(0.22)	*	0	0	1
113	x_17_5(0.22)	*	0	0	1
114	x_18_5(0.22)	*	0	0	1
115	x_19_5(0.22)	*	0	0	1
116	x_20_5(0.22)	*	0	0	1
117	x_21_5(0.22)	*	0	0	1
118	x_22_5(0.22)	*	0	0	1
119	x_1_6(1.5)	*	0	0	1
120	x_2_6(1.5)	*	0	0	1
121	x_3_6(1.5)	*	0	0	1
122	x_4_6(1.5)	*	0	0	1
123	x_5_6(0.75)	*	0	0	1
124	x_6_6(0.75)	*	0	0	1
125	x_7_6(0.75)	*	0	0	1
126	x_8_6(0.75)	*	0	0	1
127	x_9_6(0.75)	*	0	0	1
128	x_10_6(0.75)	*	0	0	1
129	x_11_6(0.22)	*	0	0	1
130	x_12_6(0.22)	*	0	0	1
131	x_13_6(0.22)	*	0	0	1
132	x_14_6(0.22)	*	0	0	1

133	x_15_6(0.22)	*	0	0	1
134	x_16_6(0.22)	*	0	0	1
135	x_17_6(0.22)	*	0	0	1
136	x_18_6(0.22)	*	0	0	1
137	x_19_6(0.22)	*	0	0	1
138	x_20_6(0.22)	*	0	0	1
139	x_21_6(0.22)	*	0	0	1
140	x_22_6(0.22)	*	0	0	1
141	x_1_7(1.5)	*	0	0	1
142	x_2_7(1.5)	*	0	0	1
143	x_3_7(1.5)	*	1	0	1
144	x_4_7(1.5)	*	1	0	1
145	x_5_7(0.75)	*	0	0	1
146	x_6_7(0.75)	*	0	0	1
147	x_7_7(0.75)	*	0	0	1
148	x_8_7(0.75)	*	0	0	1
149	x_9_7(0.75)	*	0	0	1
150	x_10_7(0.75)	*	0	0	1
151	x_11_7(0.22)	*	0	0	1
152	x_12_7(0.22)	*	0	0	1
153	x_13_7(0.22)	*	0	0	1
154	x_14_7(0.22)	*	0	0	1
155	x_15_7(0.22)	*	0	0	1
156	x_16_7(0.22)	*	0	0	1
157	x_17_7(0.22)	*	0	0	1
158	x_18_7(0.22)	*	0	0	1
159	x_19_7(0.22)	*	0	0	1
160	x_20_7(0.22)	*	0	0	1
161	x_21_7(0.22)	*	0	0	1
162	x_22_7(0.22)	*	1	0	1
163	x_1_8(1.5)	*	1	0	1
164	x_2_8(1.5)	*	1	0	1
165	x_3_8(1.5)	*	0	0	1
166	x_4_8(1.5)	*	0	0	1



167	x_5_8(0.75)	*	0	0	1
168	x_6_8(0.75)	*	0	0	1
169	x_7_8(0.75)	*	0	0	1
170	x_8_8(0.75)	*	0	0	1
171	x_9_8(0.75)	*	0	0	1
172	x_10_8(0.75)	*	0	0	1
173	x_11_8(0.22)	*	0	0	1
174	x_12_8(0.22)	*	0	0	1
175	x_13_8(0.22)	*	0	0	1
176	x_14_8(0.22)	*	0	0	1
177	x_15_8(0.22)	*	0	0	1
178	x_16_8(0.22)	*	0	0	1
179	x_17_8(0.22)	*	0	0	1
180	x_18_8(0.22)	*	1	0	1
181	x_19_8(0.22)	*	1	0	1
182	x_20_8(0.22)	*	1	0	1
183	x_21_8(0.22)	*	1	0	1
184	x_22_8(0.22)	*	0	0	1

Integer feasibility conditions:

KKT.PE: max.abs.err = 8.88e-16 on row 2

max.rel.err = 9.87e-17 on row 2

High quality

KKT.PB: max.abs.err = 0.00e+00 on row 0

max.rel.err = 0.00e+00 on row 0

High quality

End of output

## 5 Conclusion