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32032 Feedback Quiz, 2022/23, Week 04: Weight enumerator and P_{undetect}

Open-book. 10–15 minutes. Not for credit. To be marked in class.

Some questions below refer to the *weight enumerator*. Recall that the weight enumerator of a linear code C is the polynomial $W_C(x,y)$ in two variables, defined as $W_C(x,y) = \sum_{c \in C} x^{n-w(c)} y^{w(c)}$, or equivalently as $A_0 x^n + A_1 x^{n-1} y + \cdots + A_n y^n$ where n is the length of C and A_i is the number of codevectors of C of weight i.

Consider E_4, the even weight code of length 4 which is a subspace of $\mathbb{F}_2^4$.

Question 1 The weight of E_4 is:

◯ 0 ◯ 1 ◯ 2 ◯ 3 ◯ 4

Question 2 E_4 is a perfect code:

◯ No ◯ Yes

Question 3 Alice needs to trasmit information (a stream of bits) to Bob over a noisy channel. She wants to use the code E_4 for error detection. In order to do that, Alice needs to split her stream of bits into messages and encode each message into a codevector of E_4. How many bits should be in each message?

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5 ◯ 8

Question 4 Write down the weight enumerator of E_4:

$W_{E_4}(x,y) =$ ☐

A codevector of E_4 is sent via BSC(p). What is the probability $P_{\text{undetect}}(E_4)$ of an undetected error?

◯ $(1-p)^4$ ◯ 0 ◯ p^4 ◯ $6(1-p)^2p^2+p^4$ ◯ $(1-p)^3$

Write down the **leading term** of $P_{\text{undetect}}(E_4)$. (The leading term is Ap^d where $A \neq 0$ and $P_{\text{undetect}}(E_4)$ is a polynomial of the form Ap^d+(higher powers of p).

$P_{\text{undetect}}(E_4) \approx$ ☐$p^{\square}$

Question 5 The space $\mathbb{F}_2^4$ is partitioned into cosets of E_4. How many cosets?

◯ 1 ◯ 2 ◯ 4 ◯ 8 ◯ 16