SECTION A

Answer **ALL** questions in this section (40 marks in total)

A1. Let C be the linear code over the field $\mathbb{F}_5 = \{0, 1, 2, 3, 4\}$ generated by the matrix

$$G = \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 1 \\ 0 & 1 & 3 & 1 & 0 & 2 \\ 2 & 0 & 4 & 0 & 0 & 4 \end{bmatrix}.$$

For each of the statements about the code C, given below, determine if the statement is true and briefly justify your answer. Marks will not be given for true/false answers without any justification.

(a) $\dim C = 6$.

(b) C is a code of weight 4.

(c) $d(C^\perp) = 2$.

(d) C is a cyclic code.

(e) $\sum_{\underline{c} \in C} w(\underline{c}) = 600$.

[10 marks]

A2. Let C be the binary linear code with parity check matrix $H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$.

(a) Construct a table of syndromes for C.

(b) Use your table of syndromes to decode the received vectors 11110 and 10011.

From now on assume that a codevector $\underline{c}$ of C is sent via BSC(p), the binary symmetric channel with bit error rate p.

(c) Use your table of syndromes to determine, as a function of p, the probability $P_{\mathsf{corr}}(C)$ that the received vector is decoded back to $\underline{c}$.

(d) Let $P_{\mathsf{undetect}}(C)$ be the probability of an undetected error occurring when $\underline{c}$ is transmitted. Without working out $P_{\mathsf{undetect}}(C)$, give a reason why $P_{\mathsf{undetect}}(C) \leq 1 - P_{\mathsf{corr}}(C)$ for all p.

[15 marks]

A3. This question is about the code $C = \{(c_1, c_2, \ldots, c_{12}) \in \mathbb{F}_{13}^{12} : \sum_{i=1}^{12} i c_i = 0 \text{ in } \mathbb{F}_{13}\}$. You may assume that C is a subspace of the vector space $\mathbb{F}_{13}^{12}$, hence a linear code over the field $\mathbb{F}_{13} = \{0, 1, 2, \ldots, 12\}$.

(a) What is the dimension of C? Justify your answer briefly.

(b) Prove that the code C detects a single symbol error.

Given a vector $\underline{v} = (v_1, v_2, \ldots, v_{12})$, denote by $\underline{v}_{\mathsf{backwards}}$ the vector $(v_{12}, \ldots, v_2, v_1)$.

(c) Show that if $\underline{v} \in C$, then $\underline{v}_{\mathsf{backwards}} \in C$.

(d) Find the number of codevectors $\underline{v}$ of C such that $\underline{v} \neq \underline{v}_{\mathsf{backwards}}$.

[15 marks]