32031 Feedback Quiz, 2022/23 S2, Week 09: Cyclic codes

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Recall that a **cyclic** code is a code $C \subseteq \mathbb{F}_q^n$ which is **linear** and **closed under the cyclic shift**:

$$(a_0, a_1, \ldots, a_{n-1}) \in C \quad \Longrightarrow \quad (a_{n-1}, a_0, \ldots, a_{n-2}) \in C.$$

When studying cyclic codes, the key tool is converting vectors to polynomials:

$$(a_0, a_1, \ldots, a_{n-1}) \in \mathbb{F}_q^n \quad \mapsto \quad a_0 + a_1 x + \cdots + a_{n-1} x^{n-1} \in \mathbb{F}_q[x].$$

If a code C is cyclic, it has exactly one **generator polynomial** $g(x)$. This is the monic polynomial of least degree among the code polynomials of C; it is always a factor of $x^n - 1$ in $\mathbb{F}_q[x]$. All the code polynomials are $u(x)g(x)$ where $u(x) \in \mathbb{F}_q[x]$ and $\deg u(x)g(x) < n$. Thus, cyclic codes in $\mathbb{F}_q^n$ are in one-to-one correspondence with monic divisors of $x^n - 1$ in $\mathbb{F}_q[x]$.

You are given that $x^8 - 1 = (x-1)(x+1)(x^2+1)(x^2+x-1)(x^2-x-1)$ in $\mathbb{F}_3[x]$, a factorisation into monic irreducible polynomials. (*Irreducible* means that they cannot be factorised any further.)

Question 1 ♣ What are the possible **dimensions** of cyclic ternary codes of length 8?

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9

Question 2 How many cyclic ternary codes of length 8 are there?

○ 8 ○ 256 ○ 3^5 ○ 32 ○ 5

Question 3 ♣ Select the polynomials which are generator polynomials of cyclic ternary codes of length 8.

○ $x-1$ ○ x^2-1 ○ 1 ○ $1+x-x^2$ ○ x^4+1

An extra question about binary codes — attempt if you have done Q1–3.

Question 4 ♣ Various data transfer protocols such as USB, DECT (cordless phones), Bluetooth etc protect data from errors by using a binary cyclic code with the following properties: **length** $2^{15}-1$, **dimension** 32751, **can detect up to** 3 **bit errors** in a codevector. On the basis of these properties, select the polynomials that <u>could be</u> generator polynomials for such a code.

○ $x^{32751} + x^{32740} + 1$

○ $x^{16} + x^5 + 1$

○ $x^{32751} + x^{15} + x^2 + x$

○ $x^{16} + x^{10} + x^8 + x^7 + x^3 + 1$

○ $x^{16} + x^8 + x^7 + x$

○ $x^{16} + x^{15} + x^2 + 1$

○ $x^{32751} + x^8 + x^7 + 1$