MATH10101, for supervision in week 10. Congruences

Q18. (*warm-up*) Let a, b be integers, $d = \gcd(a, b)$, $d \neq 0$.

(i) Prove that the integers a/d and b/d are coprime.

(⋆)(ii) Write down an example of a and b where a/d is not coprime to b and a is not coprime to b/d.

Q19. Let a, b, q, m be integers, $q, m > 0$. Prove: $qa \equiv qb \mod qm \implies a \equiv b \mod m$.

Q20. Jean was asked to solve the congruence $12x \equiv 7 \mod 17$. Jean wrote:

$12x \equiv 7 \mod 17$
Add 17
$12x \equiv 24 \mod 17$
Divide by 12
$x \equiv 2 \mod 17$

Is this is a well-written argument? Is the answer correct? Is the method valid? Write down a better solution.

Q21. Solve the following congruences for x. Your answer should be expressed as a congruence in the original modulus, 777, and given as remainder(s) mod 777.

(⋆)i) $199x \equiv -6 \mod 777$;

(⋆)ii) $6x \equiv 3 \mod 777$;

(⋆)iii) $77x \equiv 2 \mod 777$;

iv) $6x \equiv 0 \mod 777$;

v) $10101x \equiv 0 \mod 777$.

Q22. i) Find a multiplicative inverse of 5 modulo 47.

ii) Solve the congruences: a) $5x \equiv 2 \mod 47$, b) $25x \equiv 3 \mod 47$, c) $19x \equiv 20 \mod 47$.

Q23. Find the least non-negative integer x satisfying $x \equiv 4 \mod 11$ and $x \equiv 3 \mod 13$.

(⋆)**Q24**. Find the remainders of 20^{19} and 19^{19} when divided by 13. Show that $20^{19} + 19^{19}$ is divisible by 13.