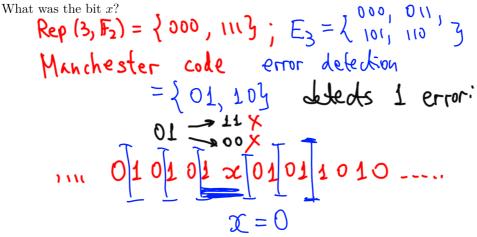
Chapter 1

Exercises (answers at end)

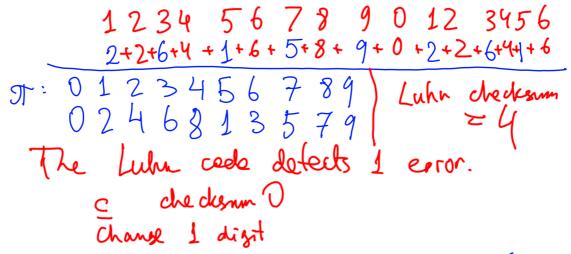
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Exercise 1.1. The Manchester code was first used in the Manchester Mark 1 computer at the University of Manchester in 1949 and is still used in low-speed data transfer: e.g. TV remote sending signals via infrared. This binary code consists of two codewords: 10 and 01. The codeword 10 is interpreted by the recipient as the message 0, and 01 is understood to mean 1; whereas the received word 00 or 11 indicates a detected error.

The following error-free fragment of a bit stream encoded by Manchester code (that is: the stream is a sequence of codewords) had been intercepted: $\dots 010101x01011010\dots$



Exercise 1.2. Consider the alphabet $\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The Luhn checksum of a word $x_1x_2 \dots x_{16} \in (\mathbb{Z}_{10})^{16}$ is $\pi(x_1) + x_2 + \pi(x_3) + x_4 + \pi(x_5) + \dots + x_{16} \mod 10$, viewed as an element of \mathbb{Z}_{10} . Here $\pi \colon \mathbb{Z}_{10} \to \mathbb{Z}_{10}$ is defined by the rule " $\pi(a)$ is the sum of digits of 2a". The Luhn code consists of all words in $(\mathbb{Z}_{10})^{16}$ whose Luhn checksum is 0.



- (i) Write down all values of π and check that π is a permutation of the alphabet \mathbb{Z}_{10} .
- (ii) Find the total number of codewords of the Luhn code.
- (iii) Prove that a single digit error is detected by the Luhn code.
- (iv) Look at your 16-digit debit/credit card numbers. Are they codewords of the Luhn code? If you have a card with a number which is **not** a codeword of the Luhn code, can you bring it to the tutorial? Thanks!

Exercise 1.3. (based on a question from a past exam. Not done in the tutorial.). Alice transmitted the same binary word of length 6 to Bob three times, but Bob received three different words: 101010, 011100, 110001. Engineer Clara told Bob that at most two bit