32032 Feedback Quiz, 2022/23, Week 02: Parameters. Bounds Open books. 10–15 minutes. Not for credit. To be marked in class.

Recall that all $(n, M, d)_q$ -codes satisfy the following bound on M : $M \le q^n / \left(\sum_{i=0}^r \binom{n}{i} (q-1)^i\right)$,
called the Hamming bound . Here $t = [(d-1)/2]$. Codes which attain this bound are <i>perfect</i> .
Furthermore, all $[n,k,d]_q$ -codes satisfy $k \le n-d+1$, the Singleton bound . Codes which attain this bound are called <i>MDS codes</i> . You will have to use these two bounds in the questions below.
Question 1 \clubsuit This question is about the binary repetition code of length 7, defined as $Rep(7, \mathbb{F}_2) = \{0000000, 11111111\}$. Fill in the blanks:
$\operatorname{Rep}(7,\mathbb{F}_2)$ is a $\left(\square, \square, \square \right)$ code and a $\left[\square, \square, \square \right]$ code which can detect up to \square bit errors and can correct up to \square bit errors per codeword. Its rate is \square .
You now need to use the formulas for the Hamming bound and the Singleton bound and do a calculation to decide if the following are true:
The binary repetition code of length 7 is perfect
The binary repetition code of length 7 is an MDS code
Question 2 What is the upper bound on the information dimension k of a binary linear code of length 31 and minimum distance 3 dictated by the Singleton bound ?
O 22 O 23 O 24 O 25 O 26 O 27 O 28 O 29 O 30 O 31
Question 3 What is the upper bound on the information dimension k of a binary code of length 31 and minimum distance 3 dictated by the Hamming bound ? (<i>Careful</i> : dimension is k , not M)
O 22 O 23 O 24 O 25 O 26 O 27 O 28 O 29 O 30 O 31
Question 4 Assuming that you answered Questions 2 and 3 correctly, what conclusion can you make from the answers?
A binary code of length 31 and minimum distance 3 cannot be perfect.
A binary code of length 31 and minimum distance 3 cannot be MDS.

CORRECTED

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Recall that all $(n, M, d)_q$ -codes satisfy the following bound on $M: M \leq q^n / \left(\sum_{i=0}^{r} \binom{n}{i} (q-1)^i\right)$, called the **Hamming bound**. Here t = [(d-1)/2]. Codes which attain this bound are *perfect*.

Furthermore, all $[n, k, d]_q$ -codes satisfy $k \le n - d + 1$, the **Singleton bound**. Codes which attain this bound are called *MDS codes*. You will have to use these two bounds in the questions below.

This question is about the binary repetition code of length 7, defined as Ouestion 1 & $Rep(7, \mathbb{F}_2) = \{0000000, 1111111\}$. Fill in the blanks:

 $\operatorname{Rep}(7,\mathbb{F}_2)$ is a $\left(\boxed{7},\boxed{2},\boxed{7} \right)_{\boxed{2}}$ code and a $\left[\boxed{7},\boxed{1},\boxed{7} \right]_{\boxed{2}}$ code which can detect up to $|\mathbf{6}|$ bit errors and can correct up to $|\mathbf{3}|$ bit errors per codeword. Its rate is $|\mathbf{1/7}|$.

You now need to use the formulas for the Hamming bound and the Singleton bound and do a calculation to decide if the following are true:

- The binary repetition code of length 7 is perfect
- The binary repetition code of length 7 is an MDS code

Explanation: From the definition, n = 7, M = 2 and q = 2 (binary), so $k = \log_2 M = 1$. The minimum distance is d = 7, by inspection. This code can detect up to d - 1 = 6 errors and can correct up to $t = \left[\frac{d-1}{2}\right] = 3$ errors. The rate is R = k/n = 1/7. Calculation: $t = \left[(7-1)/2\right] = 3$ so the denominator of the Hamming bound is

$$\binom{7}{0}(1)^0 + \binom{7}{1}(1)^1 + \binom{7}{2}(1)^2 + \binom{7}{3}(1)^3 = 1 + 7 + \frac{7 \times 6}{1 \times 2} + \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 1 + 7 + 21 + 35 = 64.$$

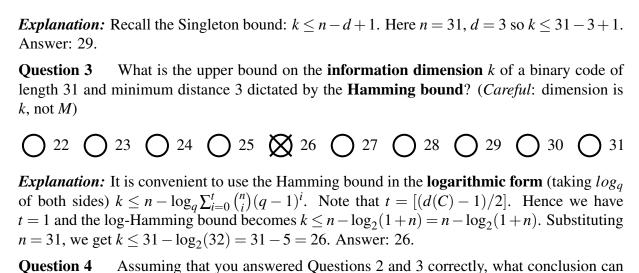
The Hamming bound says $M \le 2^7/64 = 2^7/2^6 = 2$. Since M = 2 for this code, the bound is attained and the code is perfect.

k = 1 = 7 - 7 + 1 so the Singleton bound is attained and the code is MDS.

Question 2 What is the upper bound on the information dimension k of a binary linear code of length 31 and minimum distance 3 dictated by the Singleton bound?

 $\bigcirc 22 \bigcirc 23 \bigcirc 24 \bigcirc 25 \bigcirc 26 \bigcirc 27 \bigcirc 28 \bigotimes 29 \bigcirc 30 \bigcirc 31$

CORRECTED



you make from the answers?

A binary code of length 31 and minimum distance 3 cannot be perfect.

A binary code of length 31 and minimum distance 3 cannot be MDS.

Explanation: The Hamming bound says that every such code will have information dimension at most 26. Hence, attaining the Singleton bound is impossible because that would need k = 29 which is prohibited by the Hamming bound. As a matter of fact, there are perfect $[31,26,3]_2$ codes which will be seen later in the course.