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32031 Feedback Quiz, 2022/23, Week 07: The check matrix and the dual code

Open-book. 10–15 minutes. Not for credit. To be marked in class.

Also at `https://is.gd/math32031`

Recall that "*H is a check matrix for C*" means the same as "*H is a generator matrix for* $C^\perp$".

Question 1 ♣ Select all statements which are true for *all* linear codes C. If false, think of a counterexample:

- ◯ For all matrices H, if H is a check matrix for C, then $\underline{c}H^T = \underline{0}$ for all $\underline{c} \in C$
- ◯ For all matrices H, if $\underline{c}H^T = \underline{0}$ for all $\underline{c} \in C$, then H is a check matrix for C
- ◯ For all matrices H, if H is a check matrix of C, then H is of the form $[-A^T | I_{n-k}]$ for some matrix A

Now consider the ternary linear code C generated by the matrix $G = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$.

Question 2 ♣ Find an example of a check matrix H for the code C:

$$H = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}.$$

Bring H to standard form to obtain H':

$$H' = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}.$$

Calculate the following matrix products:

$$GH^T = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}, \qquad GG^T = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}.$$

Now select all the statements and explanations that you agree with.

- ◯ The fact that $GH^T = 0$ tells us that C is self-orthogonal (and self-dual, because $n = 2k$)
- ◯ The fact that $GG^T = 0$ tells us that C is self-orthogonal (and self-dual, because $n = 2k$)
- ◯ Since the check matrix H found above is not equal to G, the code C is not self-dual
- ◯ H' is also a check marix for G, and $H' = G$ which tells us that $C^\perp = C$

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