

A3.

- (a) Define the Hamming sphere $S_r(\underline{u})$ with centre \underline{u} and radius r in the vector space \mathbb{F}_q^n .
- (b) Write down a formula for $|S_r(\underline{u})|$, the number of elements in $S_r(\underline{u})$.
- (c) State without proof the Hamming bound for the number M of elements of a code in \mathbb{F}_q^n of minimum distance d .
- (d) Define what is meant by a perfect code.
- (e) Prove that an $[11, 6, 5]_3$ -code is perfect.
- (f) Give an example of a perfect code of minimum distance 9.
- (g) You are given that $C \subseteq D \subseteq \mathbb{F}_q^n$ where $|C| < |D|$ and C is a perfect code. Show that $d(C) > 2d(D)$. You may quote any result from the course without proof.

[15 marks]