32031 Feedback Quiz, 2022/23, Week 10: General Revision I

Open books. 10–15 minutes. Not for credit. To be marked in class.

Question 1 ♣ Let C be the binary linear code with generator matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. Fill in the correct responses below.

- List the codevectors of C: $C = \{$ ☐ $\}$
- C is a (☐ , ☐ , ☐)☐ code
- C is a [☐ , ☐ , ☐]☐ code
- The dimension of the dual code $C^\perp$ is ☐
- The weight enumerator of C is $W_C(x,y) =$ ☐
- If C is transmitted down BSC(p) then the probability of an undetected error is

  $P_{\text{undetect}} =$ ☐

  where the most significant term (the term with the lowest power of p) is ☐

Tick true statements:

- ◯ C is a self-orthogonal code
- ◯ C is a self-dual code
- ◯ C is an MDS code
- ◯ C is a perfect code
- ◯ C is a Hamming code
  - If true, what are r and q in Ham(r,q)? $r =$ ☐ $q =$ ☐
- ◯ C is a simplex code
  - If true, what are r and q in $\Sigma(r,q)$? $r =$ ☐ $q =$ ☐
- ◯ C is the even weight code E_n
- ◯ C is a cyclic code
  - If true, what are the generator polynomial and the check polynomial?

    $g(x) =$ ☐ $h(x) =$ ☐

CORRECTED

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Question 1 ♣ Let C be the binary linear code with generator matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. Fill in the correct responses below.

- List the codevectors of C: $C = \{$ $\boxed{\mathbf{0000\,,\,0101\,,\,1010\,,\,1111}}$ $\}$

  Explanation: The zero vector, the top row of G, the sum of the rows of G, the bottom row of G

- C is a $(\boxed{\mathbf{4}}\,,\ \boxed{\mathbf{4}}\,,\ \boxed{\mathbf{2}})_{\boxed{\mathbf{2}}}$ code

  Explanation: The length of C is 4 which is the number of columns in the generator matrix. The dimension of C is 2 which is the number of rows in the generator matrix. $C = \{0000, 0101, 1010, 1111\}$ which shows that $d(C) = w(C) = 2$. C is binary so $q = 2$.

- C is a $[\boxed{\mathbf{4}}\,,\ \boxed{\mathbf{2}}\,,\ \boxed{\mathbf{2}}]_{\boxed{\mathbf{2}}}$ code

  Explanation: Due to the square brackets, the middle parameter is the dimension, which is 2, the number of rows in the generator matrix.

- The dimension of the dual code $C^{\perp}$ is $\boxed{\mathbf{2}}$

  Explanation: $\dim C^{\perp} = n - \dim C = 4 - 2 = 2$

- The weight enumerator of C is $W_C(x,y) = \boxed{x^4 + 2x^2y^2 + y^4}$

  Explanation: The length of the code is $n = 4$, the codevector 0000 of weight 0 contributes x^4, the codevectors 0101 and 1010 of weight 2 each contribute x^2y^2, and the codevector 1111 of weight 4 contributes y^4.

- If C is transmitted down BSC(p) then the probability of an undetected error is

  $P_{\text{undetect}} = \boxed{2(1-p)^2p^2 + p^4}$

  where the most significant term (the term with the lowest power of p) is $\boxed{2p^2}$

  Explanation: To obtain P_{undetect}, delete the x^4 term from $W_C(x,y)$ then substitute $x = 1 - p$, $y = p$. Clearly the term with the lowest power of p when P_{undetect} is expanded will be $2p^2$.

CORRECTED

Tick true statements:

☒ C is a self-orthogonal code

Explanation: Yes, the inner product of each codevector with itself and of each pair of codevectors is $0 \in \mathbb{F}_2$. Alternatively, check that $GG^T = \mathbf{0}$ where G is the generator matrix

☒ C is a self-dual code

Explanation: Yes, C is self-orthogonal and also $k = \frac{1}{2}n$ because $k = 2$ and $n = 4$

○ C is an MDS code

Explanation: No, $k \neq n - d + 1$ as $2 \neq 4 - 2 + 1$

○ C is a perfect code

Explanation: No, a code with even d cannot be perfect; here $d = 2$

○ C is a Hamming code

– If true, what are r and q in $\mathrm{Ham}(r,q)$? $r = \boxed{}$ $q = \boxed{}$

Explanation: Hamming codes are perfect, C is not

○ C is a simplex code

– If true, what are r and q in $\Sigma(r,q)$? $r = \boxed{}$ $q = \boxed{}$

Explanation: All non-zero codevectors of $\Sigma(r,q)$ are of the same weight, but C has vectors of weight 2 and 4

○ C is the even weight code E_n

Explanation: Not all vectors of even weight are in C, for example $1100 \notin C$

☒ C is a cyclic code

– If true, what are the generator polynomial and the check polynomial?

$g(x) = \boxed{1 + x^2}$ $h(x) = \boxed{1 + x^2}$

Explanation: We are given that C is linear, and it is easy to see that C is closed under the cyclic shift. The generator polynomial is the monic polynomial of least degree in C, this is the polynomial $1 + x^2$ which corresponds to 1010. The check polynomial is $h(x) = \frac{x^4-1}{1+x^2} = \frac{x^4-1}{x^2-1} = x^2 + 1$.