Review Week 04

2022-10-17

* **Reminder: coursework 1** TUE WEEK 05 11am → upload Wed W05 11am

√ **Brief response to the results of Week 3 anonymous feedback survey**

Standard array decoding: worked example (review task from the video lecture)

- **Explain why the matrix *G* below is a generator matrix of some binary linear code *C*.**
- **Is it easy to find:**
  - n = # columns of G
  - $k = \dim C$ = # rows of G
  - $d(C) = w(C)$? difficult
- **Encode the message 101 to a codevector c of *C*.**

$$G = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] = [I_3 \mid \cdots]$$

in standard form

ENCODE(101) = [1 0 1] G = [1 0 1 0 1]

- **Construct a standard array for *C*.** Rem $\underline{y} \in \mathbb{F}_2^5$: coset of $\underline{y}$ is $\underline{y} + C$

DECODE($\underline{y}$) = $\underline{y}$ − COSET_LEADER($\underline{y} + C$)

a vector of lowest weight in $\underline{y} + C$

Coset of $\underline{0} = C$ ← by THE WAY, $w(C) = 2$

00000	10011 ← DECODE($\underline{y}$)	01001	00110	11010	10101	01111	11100
10000 ←	00011 = $\underline{y}$	11001	10110	01010	00101	11111	01100
01000	11011	00001	01110	10010	11101	00111	10100
00100	10111	01101	00010	11110	10001	01011	11000

4 rows × 8 cols = 32 vectors in St. Array ($32 = \#\mathbb{F}_2^5$)

- **Give examples where c is transmitted, one bit error occurs and is corrected/not corrected by the decoder.** $\underline{c} = 00000$ $\underline{y} = 10000$ $DECODE(\underline{y}) = \underline{c}$
- **Assuming BSC(p), calculate $P_{undetect}$ and P_{corr}.** $\underline{z} = 00001$ $DECODE(\underline{z}) = 01001 \neq \underline{c}$
- **Decide if C offers a significant improvement in error detection and correction compared to sending unencoded messages.**

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$P_{corr}(C) = \sum_{i=0}^{n} \alpha_i (1-p)^{n-i} p^i$$

α_i = # cosets where the coset leader is of weight i

$$\simeq (1-p)^5 + 3p(1-p)^4 \approx$$

$\alpha_0 = 1$

$\alpha_1 = 3$

$$1 - 5p + 3p + O(p^2)$$

$$\simeq 1 - 2p$$

00000	10011	01001	00110