32031 Feedback Quiz, 2022/23, Week 01: basic notions, Hamming distance

Open books. 10 minutes. Not for credit. To be marked in class.

Recall:

- the **Hamming distance** $d(\underline{x},\underline{y})$ between two words $\underline{x}=(x_1,\ldots,x_n)$ and $\underline{y}=(y_1,\ldots,y_n)$ is defined as the number of positions i where $x_i \neq y_i$;

Question 1
Find $d(111101,\ 010121)$. The alphabet is $F=\{0,1,2\}$.

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7

Question 2 ♣ (*In this question, you will need to apply the definition of the Hamming distance, d, and use reasoning skills.*)
You are given that the word $\underline{x}\in\mathbb{F}_2^6$ is such that $d(111100,\ \underline{x})=2$. How many bits in $\underline{x}$ can be "1"? Select all possible answers (there is more than one).

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7

Question 3 ♣ (*We will soon see that in most cases, error-correcting codes use alphabets which are finite fields. This question tests your knowledge of an important fact about finite fields.*)
You are given that the alphabet F is a finite field. Which numbers in the following list are possible values for $q=\#F$?

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10

CORRECTED

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Recall:

- the **Hamming distance** $d(\underline{x}, \underline{y})$ between two words $\underline{x} = (x_1, \ldots, x_n)$ and $\underline{y} = (y_1, \ldots, y_n)$ is defined as the number of positions i where $x_i \neq y_i$;

Question 1
Find $d(111101,\ 010121)$. The alphabet is $F = \{0, 1, 2\}$.

○ 0 ○ 1 ○ 2 ⊗ 3 ○ 4 ○ 5 ○ 6 ○ 7

Explanation: The words 111101 and 010121 differ in the first position ($1 \neq 0$), the third position ($1 \neq 0$) and the fifth position ($0 \neq 2$). Hence, by definition, the Hamming distance between these words is 3.

Question 2 ♣ (*In this question, you will need to apply the definition of the Hamming distance, d, and use reasoning skills.*)
You are given that the word $\underline{x} \in \mathbb{F}_2^6$ is such that $d(111100,\ \underline{x}) = 2$. How many bits in $\underline{x}$ can be "1"? Select all possible answers (there is more than one).

○ 0 ○ 1 ⊗ 2 ○ 3 ⊗ 4 ○ 5 ⊗ 6 ○ 7

Explanation: Note that 111100 has four bits equal to 1 and that $\underline{x}$ is obtained by "flipping" (inverting) two bits in 111100. This can be made in one of the following three ways:

- change 1,1 to 0,0: then $\underline{x}$ has two 1s and four 0s;
- change 1,0 to 0,1: then $\underline{x}$ still has four 1s and two 0s;
- change 0,0 to 1,1: then $\underline{x}$ has six 1s and no zeros.

Question 3 ♣ (*We will soon see that in most cases, error-correcting codes use alphabets which are finite fields. This question tests your knowledge of an important fact about finite fields.*)
You are given that the alphabet F is a finite field. Which numbers in the following list are possible values for $q = \#F$?

○ 1 ⊗ 2 ⊗ 3 ⊗ 4 ⊗ 5 ○ 6 ⊗ 7 ⊗ 8 ⊗ 9 ○ 10

Explanation: Recall from linear algebra that the cardinality of a finite field is always a power of a prime number.