Notice to the Referee. In order to make collusion more difficult, several versions of this take-home exam paper may be prepared, with minor differences between them: e.g., a different generator matrix, different numerical values, etc. Each student will be able to download and attempt only one particular version of the paper.

A1. Let C be the linear code over the field $\mathbb{F}_5 = \{0, 1, 2, 3, 4\}$ generated by the matrix

$$G = \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 1 \\ 0 & 1 & 3 & 1 & 0 & 2 \\ 2 & 0 & 4 & 0 & 0 & 4 \end{bmatrix}.$$

For each of the statements about the code C, given below, determine if the statement is true and briefly justify your answer. Marks will not be given for true/false answers without any justification. **[** *ILO2, basic* **]**

(a) $\dim C = 6$.

Answer. **[** *straightforward exercise similar to examples done in class — 2* **]**

False: $\dim C$ is the number of rows in the generator matrix, which is 3 and not 6.

(b) C is a code of weight 4.

Answer. **[** *straightforward exercise similar to examples done in class — 2* **]**

False: the bottom row $\underline{r}_3$ of G is a codevector of C of weight 3 so $w(C) \le 3$.

(*Note that "$w(C) = 3$" is incorrect as $2\underline{r}_1 - \underline{r}_2$ has weight 2.*)

(c) $d(C^\perp) = 2$.

Answer. **[** *straightforward exercise similar to examples done in class — 2* **]**

False: since the fifth column of G is zero, the dual code $C^\perp$ contains the vector 000010 of weight 1, so $d(C^\perp) = w(C^\perp) = 1$.

(d) C is a cyclic code.

Answer. **[** *straightforward exercise similar to examples done in class — 2* **]**

False: the fifth symbol of every codevector of C is 0. If C were cyclic, then due to cyclic shifts this would imply that all symbols of every codevector are zero, which is manifestly untrue.

(e) $\sum_{\underline{c} \in C} w(\underline{c}) = 600$.

Answer. **[** *straightforward exercise similar to examples done in class — 2* **]**

False: by the Average Weight Equation, the average weight of a codevector of C is $(n-z)(1-q^{-1})$ where $n-z$ is the number of non-zero columns of G and $q = 5$. This gives $5 \times (1 - 1/5) = 4$. The number of codevectors is $5^3 = 125$ so the sum of weights is $125 \times 4 = 500 \ne 600$.

[10 marks]