32031 Feedback Quiz, 2022/23, Week 11: General Revision II

Open books. 10–15 minutes. Not for credit. To be marked in class.

Question 1 ♣ Select trivial codes:

- ◯ $\mathrm{Ham}(q,q)$
- ◯ A binary linear code C such that $C \supseteq E_4$ and $C \ni 1101$
- ◯ $\mathrm{Rep}(7,\mathbb{F}_2)^\perp$
- ◯ A ternary code with generator matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$
- ◯ The cyclic code in $\mathbb{F}_q^n$ with generator polynomial $x^n - 1$

Question 2 ♣ Select self-dual codes:

- ◯ $\mathrm{Ham}(2,3)$
- ◯ $\mathrm{Ham}(3,3)$
- ◯ $\mathrm{Ham}(4,3)$
- ◯ $\mathrm{Ham}(6,3)$
- ◯ G_{23}

Question 3 The code $C \subseteq \mathbb{F}_3^6$ is cyclic with generator polynomial $g(x) = (x-1)^3$. What is the check polynomial of C?

- ◯ 1
- ◯ $(x-1)(x+1)^2$
- ◯ $(x-1)^2(x+1)$
- ◯ $(x-1)^3$
- ◯ $(x+1)^3$

CORRECTED

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Question 1 ♣ Select trivial codes:

○ $\mathrm{Ham}(q,q)$

Explanation: **False: Hamming codes are never trivial — for example, they have minimum distance 3 not 1.**

⊗ A binary linear code C such that $C \supseteq E_4$ and $C \ni 1101$

Explanation: **True. C is of length 4 so $\#C \le 2^4 = 16$. C contains E_4, so $\#C \ge \#E_4 = 8$. Since $C \ni 1101$, a vector of odd weight, $\#C > \#E_4$. Since C is binary linear, $\#C$ is a power of 2. The next power of 2 after 8 is 16, so $\#C \ge 16$. We conclude that $\#C = 16$ and so $C = \mathbb{F}_2^4$.**

○ $\mathrm{Rep}(7,\mathbb{F}_2)^\perp$

Explanation: **False: the dual code of Rep$(n,2)$ is the even weight code E_n, not the trivial code.**

⊗ A ternary code with generator matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Explanation: **True: this code has length 2 and dimension 2 hence is the trivial code $\mathbb{F}_3^2$. By the way, the matrix has linearly independent rows, hence is indeed a generator matrix.**

○ The cyclic code in $\mathbb{F}_q^n$ with generator polynomial $x^n - 1$

Explanation: **False: by definition, this is the null code $\{0\} \subset \mathbb{F}_q^n$, not the trivial code $\mathbb{F}_q^n$. The generator polynomial of the trivial code is $g(x) = 1$.**

Question 2 ♣ Select self-dual codes:

⊗ $\mathrm{Ham}(2,3)$

Explanation: **True: take a check matrix, e.g., $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. Its rows are self-orthogonal and orthogonal to each other: $1012 \cdot 1201 = 1012 \cdot 1012 = 1201 \cdot 1201 = 0$ in $\mathbb{F}_3$. Hence Ham$(2,3)$ is self-orthogonal, and because $n = 2k$, it is self-dual**

○ $\mathrm{Ham}(3,3)$

Explanation: $n = (3^3-1)/(3-1) = 13$**, odd length, cannot be self-dual**

◯ $\mathrm{Ham}(4,3)$

Explanation: $n = (3^4 - 1)/(3-1) = 40$**,** $k = n - r = 40 - 4 = 36$**, clearly** $n \neq 2k$ **so not self-dual**

◯ $\mathrm{Ham}(6,3)$

Explanation: $n = (3^6 - 1)/(3-1) = 364$**,** $k = n - r = 358$**, clearly** $n \neq 2k$ **so not self-dual**

◯ G_{23}

Explanation: **This is the binary Golay code of length** 23**. The length is odd, so the code is not self-dual. *Aside*: the extended binary Golay code** G_{24} **is self-dual.**

Question 3 The code $C \subseteq \mathbb{F}_3^6$ is cyclic with generator polynomial $g(x) = (x-1)^3$. What is the check polynomial of C?

◯ 1

◯ $(x-1)(x+1)^2$

◯ $(x-1)^2(x+1)$

◯ $(x-1)^3$

⊗ $(x+1)^3$

Explanation: $(x-1)^3 = x^3 - 3x^2 + 3x - 1 = x^3 - 1$ **over** $\mathbb{F}_3$ **and** $x^6 - 1 = (x^3-1)(x^3+1)$**. So the check polynomial of** C **is** $x^3 + 1 = (x+1)^3$**.**