Chapter 3 Exercises

Exercise 3.1. Write down a generator matrix for the repetition code Rep$(5, \mathbb{F}_7)$. =

$\{00000, 11111, 22222, 33333, 44444, 55555, 66666\}$

$G = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \end{bmatrix}$ $\quad k = \underset{= 1}{\dim}\, \text{Rep}(5, \mathbb{F}_7) = \log_7 7$

Exercise 3.2 (important — you need to know the ISBN-10 code for the exam). Consider the field $\mathbb{F}_{11} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X\}$ of integers modulo 11; by convention, X means ten.

The **ISBN-10 checksum** of a word $x_1 x_2 \dots x_{10}$ in $\mathbb{F}_{11}^{10}$ is

$$1x_1 + 2x_2 + \cdots + 10x_{10} = \sum_{i=1}^{10} i x_i \in \mathbb{F}_{11}.$$

The **ISBN-10** code, which was used to give unique IDs to books until it was superseded by ISBN-13, consists of all vectors in $\mathbb{F}_{11}^{10}$ which have zero checksum. It is a linear code (the set of solutions to a homogeneous linear equation is a vector space).

Show that the code detects a single error. Show that the code detects a transposition error (when two adjacent digits are swapped in a codeword, it is no longer a codeword). Show that the code has $d = 2$ hence is not perfect.

$d(C) = 2. \iff w(C) = 2$

① $w(C) \le 2$: $C \ni 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1$, weight 2

② $w(C) \geq 2 \Leftarrow w(C) \neq 1$: if $w(\underline{v}) = 1$, $\underline{v} = 0\,0\,0 \dots 0 \underset{i}{x} 0 \dots 0$

Checksum of $\underline{v}$ = $\overset{\neq 0}{i} \cdot \overset{\neq 0}{x}$

Exercise 3.3 (an exam style question)**.** Let C be the ternary linear code generated by $G = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 1 & 1 \end{bmatrix}$. (Reminder: *ternary* means that the alphabet is $\mathbb{F}_3$.)

(a) List all the codevectors of C. Find $d(C)$ by inspection. Deduce that C is a perfect code. Does C attain the Singleton bound?

(b) Find a generator matrix of C in standard form. $\leftarrow$ $\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \end{bmatrix}$

$[00]\, G = [0000]$

$[01]\, G = \cancel{[0121]}\ [2011]$

$*\ [02]\, G = \cancel{[0212]}\ [1022]$

$*\ [10]\, G = [0121]$

$[20]\, G = [0212]$

$[12] = [1110]$