Chapter 12

MATH32031 Coding Theory: end-of-semester revision problems 2022

Version 2022-12-14. To accessible online version of this chapter

We illustrate some of the revision topics by examples of questions form past assessment papers. A selection of these questions will be discussed in the Week 12 tutorial on Thursday 15 December 2022 at 10am in AT G.205.

12.1 General codes

From the 2013 Coding Theory exam paper — medium difficulty:

A4. Consider the following binary code of length 6: $C = \{000111, 110001, 011100\}$.

(a) Is C a linear code? Give a reason for your answer.

(b) Find $d(C)$.

(c) Show that there does not exist a vector $\underline{y} \in \mathbb{F}_2^6$ such that $d(\underline{y}, \underline{c}) = 1$ for all $\underline{c} \in C$.

12.2 Bounds

From the 2013 Coding Theory exam paper, question A3 — medium difficulty:

(e) Prove:

1. The sphere $S_{10}(\underline{0})$ in $\mathbb{F}_3^{2013}$ consists of an odd number of elements.
2. Any perfect code in $\mathbb{F}_3^{2013}$ consists of an odd number of codewords.

Challenging: A3g from the 2015 Coding Theory exam paper, also used in coursework in later years:

(g) You are given that $C \subseteq D \subseteq \mathbb{F}_q^n$ where $|C| < |D|$ and C is a perfect code. $d(C) > 2d(D)$. You may quote any result from the course without proof.

12.3 Linear codes I

From 2019/20 coursework: fairly difficult

D. An interesting weight enumerator [20 marks] Show that there is no linear code over $\mathbb{F}_8$ with weight enumerator $x^9 + 16x^5y^4 + 16x^4y^5 + 256y^9$. Does a linear code with such weight enumerator exist over any other field? Justify your answer.

12.4 Linear codes II: encoding and decoding

12.5 Dual codes

From the 2020/21 exam: medium difficulty

A1. Let C be the linear code over the field $\mathbb{F}_5 = \{0, 1, 2, 3, 4\}$ generated by the matrix

$$G = \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 1 \\ 0 & 1 & 3 & 1 & 0 & 2 \\ 2 & 0 & 4 & 0 & 0 & 4 \end{bmatrix}.$$

For each of the statements about the code C, given below, determine if the statement is true and justify your answer. Marks will not be given for true/false answers without any justification.

(c) $d(C^\perp) = 2$.

(e) $\sum_{\underline{c} \in C} w(\underline{c}) = 600$.

12.6 Hamming codes and simplex codes

From the 2013 exam, question B6 — medium difficulty:

(e) Let q be given. Describe all values of s such that $\mathrm{Ham}(s, q)$ is an MDS code. You may quote any result from the course without proof.

(f) Write down a generator matrix for $\mathrm{Ham}(3, 2)$ in standard form.

(g) Find $\max\{d(\underline{x}, \underline{y}) : \underline{x}, \underline{y} \in \mathrm{Ham}(3, 2)\}$, that is, the *maximum* distance between two codewords in $\mathrm{Ham}(3, 2)$. Justify your answer.

12.7 Cyclic codes

From the 2011 exam, question B7 — medium difficulty:

Given that, over $\mathbb{F}_3$,

$$x^8 - 1 = (x^5 + x^4 + x^3 - x^2 + 1)(x^3 - x^2 - 1) :$$

4. Write down a generator polynomial and a check polynomial for a ternary cyclic code of length 8 and dimension 5.
5. Write down a generator matrix and a parity check matrix for this code.
6. Find the minimum distance of this code.
7. Are either of the vectors 11000000 or 11102000 in this code?
8. The repetition code in $\mathbb{F}_p^{(n)}$ is always a cyclic code. Write down a generator matrix and a check polynomial for the repetition code.

12.8 Classification of perfect codes

From the 2016 exam, question B4 — difficult:

(f) Given that C is a perfect linear $[n, k, d]_q$-code with weight enumerator $W_C(x, y) = Ax^n + Bx^{n-2}y^2 + nx^{n-3}y^3 + nx^{n-4}y^4 + y^n$, find n, k, d, q, A and B.

12.9 Reed-Muller codes