SECTION B

Answer **TWO** of the three questions in this section (40 marks in total).

If more than TWO questions from this section are attempted, then credit will be given for the best TWO answers.

B4.

(a) Define the Hamming sphere $S_r(\underline{u})$ with centre $\underline{u}$ and radius r in the vector space $\mathbb{F}_q^n$. Write down a formula for the number of elements in $S_r(\underline{u})$.

(b) Let $C \subseteq \mathbb{F}_q^n$ be a linear code, $t = \left[\frac{d(C)-1}{2}\right]$, $\underline{a} \in S_t(\underline{0})$. Prove that $\underline{a}$ is the only coset leader of the coset $\underline{a} + C$.

(c) State without proof the Hamming bound for the number M of elements of a code in $\mathbb{F}_q^n$ of minimum distance d.

(d) Define what is meant by a perfect code.

(e) Show: if $C \subseteq \mathbb{F}_q^n$ is a perfect linear code, $t = \left[\frac{d(C)-1}{2}\right]$, then every coset leader belongs to $S_t(\underline{0})$.

(f) You are given that q is a prime and that C is a perfect linear $[n, k, d]_q$-code with weight enumerator $W_C(x, y) = Ay^n + Bx^2y^{n-2} + nx^3y^{n-3} + nx^4y^{n-4} + x^n$. Find n, k, d, q, A and B. Justify your answer briefly. You may use any facts from the course without giving a proof.

[20 marks]

B5. Let $C \subseteq \mathbb{F}_q^n$ be a linear code.

(a) Explain what is meant by the inner product $\underline{x} \cdot \underline{y}$ of vectors $\underline{x}, \underline{y} \in \mathbb{F}_q^n$ and by the dual code $C^\perp$.

(b) Prove that if G is a generator matrix for C, then $C^\perp = \{\underline{v} \in \mathbb{F}_q^n \mid \underline{v}G^T = \underline{0}\}$. (Any facts from linear algebra may be used without particular comment.)

(c) What is meant by saying that H is a check matrix of $C \subseteq \mathbb{F}_q^n$? State the number of rows and the number of columns of H, given that $\dim C = k$.

(d) State without proof the Distance Theorem for linear codes.

(e) Write down an example of a check matrix of a ternary linear code C such that C consists of 27 codevectors and $d(C) = 3$.

(f) Let $C \neq \{\underline{0}\}$ be a ternary linear code such that $C \subseteq C^\perp$. Prove that $d(C)$ is a multiple of 3. You may use any results from the course without giving a proof.

[20 marks]