+1/1/60+

32031 Feedback Quiz, 2022/23, Week 05: The dual code

Open-book. 10–15 minutes. Not for credit. To be marked in class.

- The inner product $\underline{x} \cdot \underline{y}$ of vectors $\underline{x}, \underline{y} \in \mathbb{F}_q^n$ is defined as $\underline{x} \cdot \underline{y} = \sum_{i=1}^{n} x_i y_i$.
- If $C \subseteq \mathbb{F}_q^n$ is a linear code, the dual code $C^\perp$ is $\{\underline{v} \in \mathbb{F}_q^n : \underline{v} \cdot \underline{c} = 0 \text{ for all } \underline{c} \in C\}$ (that is: $C^\perp$ consists of all vectors orthogonal to C).
- By a theorem from the course, $\dim C^\perp = n - \dim C$.
- A generator matrix H for $C^\perp$ is called a *check matrix* for C. By a theorem from the course, $C = \{\underline{c} \in \mathbb{F}_q^n : \underline{c}H^T = \underline{0}\}$.
- Recall: ISBN10 = $\{\underline{x} \in \mathbb{F}_{11}^{10} : 1x_1 + 2x_2 + 3x_3 + \cdots + 10x_{10} = 0 \text{ in } \mathbb{F}_{11}\}$.

Question 1 (*warm-up*) The dimension of the ISBN-10 code is:

◯ 0 ◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5 ◯ 6 ◯ 7 ◯ 8 ◯ 9 ◯ 10 ◯ 11

Question 2 You can see from the definition of ISBN10 that the vector $\underline{v} = 123456789X$ belongs to $(\text{ISBN10})^\perp$. Does $\underline{v}$ belong to ISBN10?

◯ Yes ◯ No

Question 3 ♣ From Question 1 you can hopefully conclude that $\dim(\text{ISBN10})^\perp = 1$. Use this fact and the vector $\underline{v}$ from Question 2 to write down a vector $w \in (\text{ISBN10})^\perp$ where the sixth symbol is 5:

$\underline{w} =$ ☐☐☐☐☐**5**☐☐☐☐.

Question 4 ♣ (*correcting an **erasure** in an ISBN*) Recover the missing symbol in an ISBN:

0198 ☐ 38030