32032 Feedback Quiz, 2022/23, Week 03: Linear codes

Open books. 10–15 minutes. Not for credit. To be marked in class.

Recall that a **linear code** is a subspace of the vector space $\mathbb{F}_q^n$.

Question 1 ♣

Given that

$$C = \{0000, 0112, 0221, 2011, 1022, 2120, 1210, \underline{x}, \underline{y}\}$$

is a ternary linear code, fill in the correct responses below.

- C is a $\square$-dimensional vector space over the field $\mathbb{F}_{\square}$.
- A generator matrix of C has $\square$ rows and $\square$ columns.
- The codevectors $\underline{x}$, $\underline{y}$ are (write in either order) $\square$ and $\square$.
- $w(C) = \square$.
- $d(C) = \square$.

Tick true statements:

- ◯ C is a trivial code
- ◯ C is a repetition code
- ◯ C is a zero sum code
- ◯ C is a perfect code
- ◯ C is an MDS code

CORRECTED

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Question 1 ♣
Given that

$$C = \{0000, 0112, 0221, 2011, 1022, 2120, 1210, \underline{x}, \underline{y}\}$$

is a ternary linear code, fill in the correct responses below.

- C is a $\boxed{\mathbf{2}}$-dimensional vector space over the field $\mathbb{F}_{\boxed{\mathbf{3}}}$.

  Explanation: The word "ternary" means that the alphabet is the field $\mathbb{F}_3$. The cardinality of a linear code is $\#C = q^k$ where $q = 3$ and $\#C$ is seen to be 9, so $3^k = 9$ and the dimension k of C is 2.

- A generator matrix of C has $\boxed{\mathbf{2}}$ rows and $\boxed{\mathbf{4}}$ columns.

  Explanation: Rows of a generator matrix of a linear code form a basis of the code as a vector space. The space C is 2-dimensional so there are 2 vectors in the basis, i.e., 2 rows in a generator matrix.
  Each row of a generator matrix is a codevector, so the size of the row (=the number of columns of the matrix) is equal to the number of symbols in a codevector, i.e., the length of the code. There are 4 columns in a generator matrix.

- The codevectors $\underline{x}$, $\underline{y}$ are (write in either order) $\boxed{\mathbf{1101}}$ and $\boxed{\mathbf{2202}}$.

  Explanation: The code C is a vector space, so it is closed under addition of vectors. We need to find a sum of known vectors which is not listed among the 7 known vectors. For example, $0221 + 2011 = 2202$ (remember, addition is modulo 3). Also, $-2202 = 1101$ must be in C.

- $w(C) = \boxed{\mathbf{3}}$.

  Explanation: By inspection, all non-zero codevectors have weight 3 so the minimum weight of a non-zero codevector is 3. This, by definition, is $w(C)$, the weight of the code C.

- $d(C) = \boxed{\mathbf{3}}$.

  Explanation: By a result from lectures, $d(C) = w(C)$ for linear codes.

CORRECTED

Tick true statements:

◯ C is a trivial code

Explanation: No, because C is not the whole space $\mathbb{F}_3^4$. The space $\mathbb{F}_3^4$ contains $3^4 = 81$ vectors but C contains only 9 vectors.

◯ C is a repetition code

Explanation: No, because e.g. the codevector 0112 of C does not consist of a symbol repeated 4 times. Also, repetition codes are 1-dimensional but C is 2-dimensional.

◯ C is a zero sum code

Explanation: No, for example the sum of symbols in the codevector 0112 is $0+1+1+2=1$ and not zero. Also, zero sum codes are $n-1$-dimensional but C is not 3-dimensional.

⊗ C is a perfect code

Explanation: Yes: $n=4$, $d=3$, $q=3$ so $t=[(3-1)/2]=1$ and the Hamming bound is $\#C \le 3^4/\sum_{i=0}^{1}\binom{4}{i}(2-1)^i = 3^4/\big(\binom{4}{0}2^0+\binom{4}{1}2^1\big) = 3^4/\big(1+4\times 2\big) = 9$. The bound is attained because C consists of 9 codevectors.

⊗ C is an MDS code

Explanation: Yes, because the Singleton bound $k \le n-d+1$ becomes $2 \le 4-3+1$ which is attained.