

Chapter 12

MATH32031 Coding Theory: end-of-semester revision 2022

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See suggested answers and hints at end

This list is not guaranteed to cover all possible topics that may arise in the exam. Your questions and suggestions are welcome; please post them to the Piazza discussion board or contact the lecturer during the revision period.

Suggested revision format: ask yourself ***Can I...*** followed by a question below. In case of difficulty/lack of confidence, revise the relevant part of the course material. Brief comments on a suggested approach are available below for most questions. Questions marked (*) are more challenging: they have not been covered in the course but follow from lecture material or exercises.

12.1 General codes

- find the Hamming distance between two words
- find the minimum distance of a code with a small number of codewords
- given parameters $(n, M, d)_q$ of a code C , find $[n, k, d]_q$ and the rate R
- given a code C as a list of codewords, decode a received word \underline{y}
- write down the parameters of a trivial code, of a repetition code
- given the minimum distance d of a code, write down the number of errors (per codeword) that the code can detect/correct
- write down the probability that i errors occur in a binary word of length n sent via $\text{BSC}(p)$

12.2 Bounds

...write down:

- the Hamming bound for q -ary codes of length n and minimum distance d
- the Singleton bound?

...calculate:

- the Hamming and Singleton bounds for a code with given parameters — and use these to check if the code is perfect/MDS?

...give an example of:

- perfect codes of minimum distance 1, 3, 5, 7, 9, ...

12.3 Linear codes I

...write down:

- the parameters of E_n (with explanation)?
- the parameters of ISBN-10 (with explanation)?
- the weight enumerators of the trivial code, the repetition code, the code E_n ?
- the special values of the weight enumerator: $W_C(0,0)$, $W_C(1,0)$, $W_C(1,1)$?

12.4 Linear codes II: encoding and decoding

- given a generator matrix G of a code C , encode a message vector \underline{u} . What is the number of rows/columns of G ? What must be the length of \underline{u} ? What do you get as the output of the encoder?

...calculate:

- a generator matrix in standard form for a given code?
- all the codevectors, and the weight enumerator of the linear code, if a generator matrix is given?
- $P_{\text{undetected}}(C)$? (what do you need to know to find it? for what codes and channels?)
- $P_{\text{correct}}(C)$? (what do you need to know to find it? for what codes and channels?)

12.5 Dual codes

...calculate:

- the inner product of two vectors?
- a check matrix of a given code? (what data do you need?)
- the dual code of the trivial/repetition/even weight/ISBN-10 code?
- the length and dimension of C^\perp if C has length n , dimension k ?

...check:

- whether a given code is self-orthogonal? self-dual? (what data do you need?)
- calculate the syndrome of a vector? (what data do you need?)

- check if a given vector belongs to the code?
- construct a table of syndromes, and decode a received vector using your table?
- use the Average Weight Equation?

12.6 Hamming codes and simplex codes

...write down:

- the parameters of $\text{Ham}(r, q)$?
- the weight of any non-zero codevector and the parameters of $\Sigma(r, q)$?
- the weight enumerator of $\Sigma(r, q)$?

...construct:

- a check matrix for $\text{Ham}(r, q)$ (q is a prime)? A generator matrix?
- given a check matrix for a Hamming code, decode a received vector?

12.7 Cyclic codes

- write the given vector in \mathbb{F}_q^n as a polynomial, and a polynomial as a vector?
- given a (small) cyclic code C , find the generator polynomial of C ?
- carry out long division of polynomials?

...calculate:

- the dimension of a cyclic code with a given generator polynomial?
- the check polynomial of a given cyclic code? (what do you need to know?)
- generator polynomials, check polynomials, generator matrices, check matrices of all possible cyclic codes in \mathbb{F}_q^n ? (what do you need to know?)

12.8 Classification of perfect codes

- write down the parameters of the Golay codes, and prove that the codes are perfect?
- use the Classification Theorem for perfect codes where q is a prime power?

12.9 Reed-Muller codes

...write down:

- the parameters of $R(r, m)$?