Warning: this is a collection of past Coding Theory exam papers. Some questions may not be relevant due to differences in the content, the logic and the learning objectives of the course, then and now. I indicated by * below the problems which I think could be attempted by interested students who would like extra practice to revise for the final MATH32031 exam. Use these at your own risk. There are no model solutions; however, if ideas or questions about any of the * -starred problems are posted to Piazza, I will be happy to respond. Y.B.

SECTION A

Answer **ALL** of the four questions

2008

* **A1.**

Let C be the binary linear code with generator matrix [10 marks]

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

(1) Write down all the elements of C.

(2) Calculate the minimum distance $d(C)$ of C.

(3) If this code is used for error detection in a binary symmetric channel with bit error rate $p = 0.1$, calculate $P_{\text{undetect}}(C)$, the probability that an error in a received vector is not detected.

A2.

[10 marks]

(1) Define a Hamming code $Ham(r, q)$.

* (2) Find a parity check matrix for a Hamming code $Ham(3, 2)$.

* (3) Find a generator matrix for a Hamming code $Ham(3, 2)$.

A3.

[10 marks]

(1) Given two codes A and B in $F^{(n)}$, define the code $(A|B)$.

(2) Prove that $d(A|B) = \min\{2d(A), d(B)\}$.

(3) If A and B are both binary linear codes with generator matrices

$$G_A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad G_B = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

find a generator matrix for $(A|B)$.

A4.

[10 marks]

(1) State the Distance Theorem for linear codes. Let C be the linear binary code with generator matrix

$$G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

* (2) Find a generator matrix for C in standard form.

* (3) Find a parity check matrix for C.

* (4) Calculate the minimum distance of C.

2008

SECTION B

Answer **TWO** of the three questions

* **B5.**

Let C be the $\mathbb{F}_3$-linear code with generator matrix [20 marks]

$$G = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

(1) Write down a parity check matrix for C.

(2) Explain why every vector of weight at most 1 is a unique coset leader (you may assume without proof that $d(C) = 3$).

(3) Calculate the table of syndromes.

(4) Decode the vectors $\begin{bmatrix} 0 & 2 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$.

B6.

[20 marks]

(1) For a cyclic code C, considered as an ideal in $R_n = F[x]/(x^n - 1)$, show that there is a unique monic polynomial $g \in C$ of minimum degree such that $C = \bar{g}R_n$.

(2) Show that g divides $x^n - 1$.

(3) For C and g as above, write down a generator matrix G for C in terms of the coefficients of g.

* (4) Given that

$$x^7 - 1 = (x + 1)(x^3 + x + 1)(x^3 + x^2 + 1)$$

over $\mathbb{F}_2$, write down a generating polynomial g and a generator matrix G for a binary cyclic code of length 7 and dimension 3.

B7.

[20 marks]

(1) Define the Reed-Muller code $R(r, m)$.

* (2) State without proof formulas for the distance n, the dimension k and the minimum distance d of the code $R(r, m)$.

(3) Prove that $R(m - r - 1, m)$ is dual to $R(r, m)$ for $0 \leq r \leq m - 1$, (you may quote other results from the course without proof).

* (4) Find the length n and the dimension k of $R(1, 3)$.

* (5) Find a generator matrix for $R(1, 3)$.

* (6) Find a parity check matrix for $R(1, 3)$.

2009

SECTION A

Answer **ALL** of the four questions

A1.

[10 marks]

1. Explain what is meant by:

   (a) a code in $\mathbb{F}_q^{(n)}$,

   (b) the Hamming distance $d(\underline{x}, \underline{y})$ between two vectors $\underline{x}$ and $\underline{y}$ in $\mathbb{F}_q^{(n)}$,

   (c) the minimum distance $d(C)$ of a code C,

   (d) the weight $w(\underline{x})$ of a vector $\underline{x}$,

   (e) a linear code in $\mathbb{F}_q^{(n)}$,

   (f) the weight $w(C)$ of a linear code C.

2. Prove that $d(\underline{x}, \underline{y}) = w(\underline{x} - \underline{y})$ for $\underline{x}, \underline{y} \in \mathbb{F}_q^{(n)}$.

3. Prove that $d(C) = w(C)$ for a linear code C.

A2.

[10 marks]

1. Explain what is meant by the sphere $S_t(\underline{u})$ with centre $\underline{u}$ and radius t in the vector space $\mathbb{F}_q^{(n)}$.

Let C be a linear code over $\mathbb{F}_q$ of length n and distance d; set $t = \left[\frac{d-1}{2}\right]$, the integer part of $\frac{d-1}{2}$.

2. Show that $S_t(\underline{u}) \cap S_t(\underline{v}) = \emptyset$ for all distinct vectors $\underline{u}, \underline{v} \in C$.

3. Deduce that any vector $\underline{u} \in \mathbb{F}_q^{(n)}$ with $w(\underline{u}) \leq t$ is a unique coset leader.

A3.

Let C be the ternary linear code with generator matrix [10 marks]

$$G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

over $\mathbb{F}_3$.

∗ 1. List the elements of C.

∗ 2. Find the minimum distance $d(C)$ of C.

∗ 3. How many errors can be corrected?

∗ 4. How many errors can be detected?

∗ 5. Show that C is linearly equivalent to the ternary code with generator matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}.$$

A4.

Let C be the binary $[6, 3, 3]$-code with parity check matrix [10 marks]

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

∗ 1. Given that 001001 is a coset leader, write down a set of coset leaders.

∗ 2. Calculate the table of syndromes.

∗ 3. Decode 110010 and 101100 using syndrome decoding.

∗ 4. If the code C is transmitted down a binary symmetric channel with symbol error probability 0.01, find $P_{corr}(C)$, the probability that a received vector is decoded correctly.

(You may quote results from the course without proof.)

2009

SECTION B

Answer **TWO** of the three questions

B5.

[20 marks]

1. State the Distance Theorem for linear codes.
2. Prove the Distance Theorem.
3. Define a Hamming code $Ham(r, q)$.
4. Prove that a Hamming Code has minimum distance 3.

Now let C be a binary code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

* 5. ~~Put G into standard form.~~ **(unclear) Find a generator matrix in standard form for a code linearly equivalent to C.**

* 6. Find a parity check matrix for a code linearly equivalent to C.

* 7. Show that C is a Hamming code.

B6.

[20 marks]

1. For a cyclic code C, considered as an ideal in $R_n = F[x]/(x^n - 1)$, show that there is a unique monic polynomial $g \in C$ of minimum degree such that $C = \bar{g}R_n$.
2. Show that g divides $x^n - 1$.
3. For C and g as above, write down a generator matrix G for C in terms of the coefficients of g.

* 4. Given that
$$x^9 - 1 = (x+1)(x^6 + x^3 + 1)(x^2 + x + 1)$$
over $\mathbb{F}_2$, write down a generating polynomial g and a generator matrix G for a binary cyclic code of length 9 and dimension 6.

2009

B7.

[20 marks]

1. Define the Reed-Muller code $R(r, m)$.

2. State without proof formulas for the length n, the dimension k and the minimum distance d of the code $R(r, m)$.

* 3. Prove that $R(m-1, m)$ is the even weight code.

* 4. Find the length n and the dimension k of $R(1, 3)$.

* 5. Find a generator matrix for $R(1, 3)$.

* 6. Find a parity check matrix for $R(1, 3)$.

(You may quote other results from the course without proof.)

END OF EXAMINATION PAPER

2010

SECTION A

Answer **ALL** of the four questions

A1.

[13 marks]

1. Let C be a linear code. Explain what is meant by:

   (a) the weight, $w(\underline{x})$, of a vector $\underline{x}$;

   (b) the weight, $w(C)$, of C;

   (c) the dual code, $C^{\perp}$;

   (d) a generator matrix for C;

   (e) a parity check matrix for C.

   For the rest of this question suppose that $C \subseteq \mathbb{F}_2^{(n)}$ is a binary linear code.

* 2. Show that the weight of a vector, considered modulo 2, gives a linear function from $\mathbb{F}_2^{(n)}$ to $\mathbb{F}_2$.

   Form a new code $D \subseteq \mathbb{F}_2^{(n+1)}$ by adding $w(c)$ modulo 2 onto the end of each codeword $c \in C$ as the last coordinate.

* 3. Show that every codeword in D has even weight.

* 4. Prove that $w(C) \leq w(D) \leq w(C) + 1$.

* 5. Show that if $w(C)$ is odd then $w(D) = w(C) + 1$ and if $w(C)$ is even then $w(D) = w(C)$.

A2.

[9 marks]

1. Define $\mathbf{P}_{n-1}(\mathbb{F}_q)$, projective $(n-1)$–space over $\mathbb{F}_q$.

2. Show that
$$|\mathbf{P}_{n-1}(\mathbb{F}_q)| = \frac{q^n - 1}{q - 1}.$$

3. Define a Hamming code $\mathrm{Ham}(s, q)$.

4. Find a formula for the length, n, of $\mathrm{Ham}(s, q)$ in terms of s and q.

5. Find a formula for the dimension, k, of $\mathrm{Ham}(s, q)$.

6. Define what is meant by a perfect code.

7. Prove that $\mathrm{Ham}(s, q)$ is a perfect code, (you may assume without proof that $d(\mathrm{Ham}(s, q)) = 3$).

A3.

[9 marks]

1. Define what it means for two linear codes to be linearly equivalent.

* 2. Prove that if two linear codes are linearly equivalent then they have the same minimum distance.

Let C be the ternary, (i.e. over $\mathbb{F}_3$), linear code with generator matrix

$$G = \begin{bmatrix} 2 & 2 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

* 3. Calculate a generator matrix in standard form for a code that is linearly equivalent to C.

* 4. Find the corresponding parity check matrix.

* 5. Calculate the minimum distance $d(C)$ of C.

A4.

[9 marks]

Let C be the binary code with elements

$$C = \begin{cases} 0000001 \\ 1111001 \\ 1100110 \\ 0011110 \end{cases}$$

* 1. Calculate the minimum distance, $d(C)$, of C.

* 2. How many errors can this code detect?

* 3. How many errors can this code correct?

4. Show that C is equivalent to some other code, D say, that contains 0000000. List the elements of D.

5. Show that D is a linear code.

6. If the code D is transmitted down a binary symmetric channel with symbol error rate r, calculate $P_{\text{undetect}}(D)$, the probability that an error in a received vector is not detected (leave your answer as a polynomial in r, without simplifying).

SECTION B

Answer **TWO** of the three questions

B5.

Let C be a linear code with parity check matrix H. [20 marks]

1. Define the syndrome $S(\underline{x})$ of a vector $\underline{x}$.

2. Prove that $S(\underline{x}) = S(\underline{y})$ if and only if $\underline{x}$ and $\underline{y}$ lie in the same coset of C, (you may quote other results from the course without proof).

   Now let C be a binary linear code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

* 3. Find a parity check matrix, H, for C.

* 4. Calculate the minimum distance, $d(C)$, (you may quote any result from the course without proof).

* 5. Find a set of coset leaders and write down a table of syndromes.

* 6. Decode the received vectors:

   (a) 1010000

   (b) 1111111

* 7. Give an example of a codeword that is sent and vector received with two errors that is decoded incorrectly.

* 8. If the code C is transmitted down a binary symmetric channel with symbol error rate r, calculate $P_{\text{corr}}(C)$, the probability that a received vector is decoded correctly (leave your answer as a polynomial in r, without simplifying).

B6.

[20 marks]

1. Define what is meant by an ideal I in a commutative ring R.

2. Define what is meant by a cyclic code in $\mathbb{F}_p^{(n)}$.

3. If we identify $\mathbb{F}_p^{(n)}$ with $R_n = \mathbb{F}_p[x]/(x^n - 1)$ in the usual way, prove that there is a bijection between cyclic codes in $\mathbb{F}_q^{(n)}$ and ideals in R_n.

   Given that, over $\mathbb{F}_2$,

$$x^7 - 1 = (x - 1)(x^3 + x + 1)(x^3 + x^2 + 1)$$

   as a product of irreducible polynomials:

2010

$\ast$ 4. How many different binary cyclic codes of length 7 are there?

$\ast$ 5. List the possible dimensions for a binary cyclic code of length 7.

$\ast$ 6. Write down a generator polynomial and a parity check polynomial for a binary cyclic code of length 7 and dimension 4.

$\ast$ 7. Write down a generator matrix and a parity check matrix for a binary cyclic code of length 7 and dimension 4.

$\ast$ 8. Show that the code for which you have just found the matrices is a Hamming code $\mathrm{Ham}(s, q)$, stating the values of s and q.

B7.

[20 marks]

1. Define what is meant by a Boolean function on $V^m = \mathbb{F}_2^{(m)}$.

2. Prove that every Boolean function on V^m can be expressed as a polynomial in the coordinate functions $v_1, \ldots, v_m$.

3. Define the Reed-Muller code $R(r, m)$ in terms of Boolean functions.

$\ast$ 4. Find a generator matrix for $R(2, 2)$ and identify the rows that appear in the generator matrices for $R(1, 2)$ and $R(0, 2)$.

$\ast$ 5. Find a generator matrix and a parity check matrix for $R(1, 3)$ (you may quote any result from the course without proof).

END OF EXAMINATION PAPER

2011

SECTION A

Answer **ALL** of the five questions

A1.

[6 marks]

Define:

1. a code C in $\mathbb{F}_q^{(n)}$,
2. the Hamming distance $d(\underline{x}, \underline{y})$ between two vectors $\underline{x}$ and $\underline{y}$ in $\mathbb{F}_q^{(n)}$,
3. the minimum distance $d(C)$ of a code C,
4. the weight $w(\underline{x})$ of a vector $\underline{x}$,
5. an $\mathbb{F}_q$-linear code C in $\mathbb{F}_q^{(n)}$,
6. the weight $w(C)$ of a linear code C.

A2.

[9 marks]

1. Define what it means for two codes to be equivalent.
2. Show that any (non-empty) code is equivalent to one that contains the vector $111\cdots111$ (all coordinates 1).
3. Define what it means for two linear codes to be linearly equivalent.

* 4. Give an example to show that a linear code is not always linearly equivalent to one that contains the vector $111\cdots111$.

A3.

[7 marks]

Let C be the binary code with generator matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

* 1. List all the elements of C.

* 2. Calculate the minimum distance $d(C)$.

* 3. How many errors can the code detect?

* 4. How many errors can the code correct?

* 5. If the code C is transmitted down a binary symmetric channel with symbol error rate r, calculate $P_{\text{undetect}}(C)$, the probability that the received vector contains an error that is not detected (leave your answer as a polynomial in r).

2011

A4.

[9 marks]

1. Define the sphere $S_t(\underline{u})$ with centre $\underline{u}$ and radius t in the vector space $\mathbb{F}_q^{(n)}$.

* 2. Write down a formula for $|S_t(\underline{u})|$, the number of vectors in $S_t(\underline{u})$.

3. Prove this formula.

4. State the sphere-packing bound for the number of elements M of a code of minimum distance d in $\mathbb{F}_q^{(n)}$.

* 5. Use the sphere-packing bound to find an upper bound on the number of elements of an $\mathbb{F}_3$-code of length 10 and minimum distance 6 (you may leave your answer as a fraction).

A5.

[9 marks]

1. Define the dual code $C^\perp$ of a linear code $C \subseteq \mathbb{F}_q^{(n)}$.

* 2. If C is linear code, state a formula for the dimension of $C^\perp$.

3. Prove that $C^{\perp\perp} = C$.

Let C be the linear $\mathbb{F}_5$-code with generator matrix

$$G = \begin{bmatrix} 0 & 1 & 3 & 3 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

* 4. Find a generator matrix in standard form for a code D that is linearly equivalent to C.

* 5. Find a generator matrix for a code that is linearly equivalent to $C^\perp$.

2011

SECTION B

Answer **TWO** of the three questions

B6.

[20 marks]

1. State the Distance Theorem for linear codes.

2. Prove the Distance Theorem.

3. Define a Hamming code Ham(s, q).

4. Show that the minimum distance of a Hamming code is 3.

* 5. Write down a parity check matrix and a generator matrix for a Ham$(3, 2)$ code.

* 6. Write down the coset leaders (you may quote any result from the course without proof).

* 7. For each of the received vectors:

   (a) 0000111,

   (b) 1111111,

   calculate its syndrome and decode it.

* 8. Give an example of a codeword that is sent and a vector received with two errors that is decoded incorrectly.

* 9. If the code C is transmitted down a binary symmetric channel with symbol error rate r, calculate $P_{\text{corr}}(C)$, the probability that a received vector is decoded correctly (leave your answer as a polynomial in r, without simplifying).

2011

B7.

[20 marks]

1. Let C be a cyclic code and consider it as an ideal in $R_n = \mathbb{F}_p[x]/(x^n - 1)$ in the usual way. Prove that there is a unique monic polynomial $g \in \mathbb{F}_p[x]$ of minimum degree such that $C = \bar{g}R_n$.

2. Prove that g divides $x^n - 1$.

* 3. Write down an expression for the dimension of C in terms of g and n.

Given that, over $\mathbb{F}_3$,

$$x^8 - 1 = (x^5 + x^4 + x^3 - x^2 + 1)(x^3 - x^2 - 1) :$$

* 4. Write down a generator polynomial and a check polynomial for a ternary cyclic code of length 8 and dimension 5.

* 5. Write down a generator matrix and a parity check matrix for this code.

* 6. Find the minimum distance of this code.

* 7. Are either of the vectors 11000000 or 11102000 in this code?

* 8. The repetition code in $\mathbb{F}_p^{(n)}$ is always a cyclic code. Write down a generator matrix and a check polynomial for the repetition code.

B8.

[20 marks]

1. Given two codes C_1 and C_2 in $F^{(n)}$, define the code $|C_1|C_2|$.

2. Prove that $d(|C_1|C_2|) = \min\{2d(C_1), d(C_2)\}$.

3. Define the rth order binary Reed-Muller code $R(r, m)$ in terms of Boolean functions.

4. Show that $R(r + 1, m + 1) = |R(r + 1, m)|R(r, m)|$.

* 5. Find a generator matrix and a parity check matrix and the distance for the code $R(2, 3)$ (you may quote any result from the course without proof).

* 6. Both $R(0, m)$ and $R(m - 1, m)$ are well-known codes with their own names. What are these names (or give a simple description of these codes)?

END OF EXAMINATION PAPER

2012

SECTION A

Answer **ALL** of the five questions

A1.

[9 marks]

Let C be a linear code in $\mathbb{F}_q^{(n)}$.

1. Define what is meant by a generator matrix for C.
2. Define what is meant by the dual code $C^\perp$ to C.
3. Define what is meant by a parity check matrix for C.
4. Show that if G is a generator matrix for C then $C^\perp = \{\underline{x} \in \mathbb{F}_q^{(n)} \mid \underline{x}G^T = 0\}$.
5. Deduce that if H is a parity check matrix for C then $C = \{\underline{x} \in \mathbb{F}_q^{(n)} \mid \underline{x}H^T = 0\}$ (you may assume without proof that $C^{\perp\perp} = C$).

∗ **A2.**

[7 marks]

Let C be a code in $\mathbb{F}_q^{(n)}$ with minimum distance $d(C) \geq 2$. Let D be the code in $\mathbb{F}_q^{(n-1)}$ obtained from C by deleting the last coordinate.

1. Show that $d(C) - 1 \leq d(D) \leq d(C)$.
2. Show that C and D both have the same number of elements.
3. Use this to show that a code in $\mathbb{F}_2^{(3)}$ with minimum distance 2 cannot have more than 4 elements.

∗ **A3.**

Let C be a binary linear code with generator matrix [7 marks]

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

1. What is the length of C? What is its dimension?
2. List the elements of C.
3. Calculate the minimum distance of C.
4. Calculate $P_{\text{undetect}}(C)$, the probability that an error is undetected when a codeword is transmitted down a binary symmetric channel with symbol error rate r.

2012

A4.

[10 marks]

1. Define projective $(n-1)$-space, $\mathbb{P}_{n-1}(\mathbb{F}_q)$.

2. Define a Hamming code $\mathrm{Ham}(s,q)$.

3. Write down, without proof, formulas in terms of s and q for the length n, the dimension k and the minimum distance d of $\mathrm{Ham}(s,q)$.

4. Define what is meant by a perfect code.

5. Prove that $\mathrm{Ham}(s,q)$ is perfect.

* 6. Write down a parity check matrix and a generator matrix for a $\mathrm{Ham}(2,5)$ code.

A5.

[7 marks]

1. State the Distance Theorem for linear codes.

   Let C be the $\mathbb{F}_3$-linear code with generator matrix

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 & 1 \end{bmatrix}$$

* 2. Find a generator matrix in standard form for a code D that is linearly equivalent to C.

* 3. Write down a parity check matrix for D.

* 4. Find the minimum distance $d(D)$.

* 5. Find the minimum distance $d(C)$.

2012

SECTION B

Answer **TWO** of the three questions

B6.

Let H be the parity check matrix of a linear code C. [20 marks]

1. Define the syndrome $S(\underline{x})$ of a vector $\underline{x}$.

2. Define the coset $\underline{x}+C$ of C.

3. Prove that $S(\underline{x}) = S(\underline{y})$ if and only if $\underline{x}$ and $\underline{y}$ are both in the same coset of C.

4. What is a coset leader? What is a unique coset leader?

5. Prove that syndrome decoding decodes a vector correctly if and only if the error is a coset leader.

6. If the minimum distance of C is d, show that every vector $\underline{x}$ with $w(\underline{x}) < \frac{d}{2}$ is a unique coset leader.

   Now let C be a binary linear code with parity check matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

∗ 7. Find a set of coset leaders and write down a table of syndromes.

∗ 8. For each of the received vectors 110000 and 110100, calculate its syndrome and decode it.

∗ 9. Give an example of a codeword that is sent and a vector received with two errors that is decoded incorrectly.

∗ 10. If the code C is transmitted down a binary symmetric channel with symbol error rate r, calculate $P_{\text{corr}}(C)$, the probability that a received vector is decoded correctly.

2012

B7.

[20 marks]

1. Define what is meant by a cyclic code.

2. Prove that the dual of a cyclic code is also cyclic.

3. Define an ideal I in a commutative ring R.

4. Identify $\mathbb{F}_q^{(n)}$ with $R_n = \mathbb{F}_p[x]/(x^n - 1)$ by sending $(a_0, \dots, a_{n-1})$ to $\sum_{i=0}^{n-1} a_i x^i$. Prove that this induces a bijection between cyclic codes in $\mathbb{F}_q^{(n)}$ and ideals in R_n.

Given that, over $\mathbb{F}_2$,

$$x^9 - 1 = (x+1)(x^2+x+1)(x^6+x^3+1),$$

is a factorisation into ireducible polynomials,

* 5. What are the possible dimensions for a binary cyclic code of length 9?

* 6. Write down a generator polynomial and a check polynomial for a binary cyclic code of length 9 and dimension 3.

* 7. Write down a generator matrix and a parity check matrix for this code.

* 8. Find the minimum distance of this code.

* 9. Are either of the vectors 111111111 or 111000000 in this code?

B8.

[20 marks]

1. Define what is meant by a Boolean function on $V^m = \mathbb{F}_2^{(m)}$.

2. Prove that every Boolean function on V^m can be expressed as a polynomial in the coordinate functions $v_1, \dots, v_m$.

3. Define the rth order binary Reed-Muller code $R(r, m)$ in terms of Boolean functions.

* 4. Find a generator matrix and a parity check matrix and the minimum distance for the code $R(2, 3)$ (you may quote any result from the course without proof).

* 5. Consider the codes $R(r, 9)$ for $0 \le r \le 9$. Which values of r will give you the following (you may quote any result from the course without proof):

   (a) the repetition code,

   (b) the even weight code of all vectors of even weight,

   (c) a self-dual code (i.e. a code that is equal to its dual),

   (d) a code of dimension 130,

   (e) a code with minimum distance 16.

END OF EXAMINATION PAPER

2013

SECTION A

Answer **ALL** questions in this section (40 marks in total)

A1.

(a) Explain what is meant by:

1. a code C in $\mathbb{F}_q^n$;
2. a linear code in $\mathbb{F}_q^n$;
3. the weight $w(\underline{y})$ of a vector $\underline{y} \in \mathbb{F}_q^n$;
4. the weight $w(C)$ of a linear code C in $\mathbb{F}_q^n$;
5. the trivial code in $\mathbb{F}_q^n$.

* (b) Explain why the weight of the trivial code is 1.

* (c) Give an example of a non-trivial linear code of weight 1 in $\mathbb{F}_2^3$.

[10 marks]

A2. The linear code $C \subseteq \mathbb{F}_3^3$ is given by

$$C = \{(x_1, x_2, x_3) \mid x_1, x_2, x_3 \in \mathbb{F}_3,\ x_1 + 2x_2 + x_3 = 0 \text{ in } \mathbb{F}_3\}.$$

* (a) List all the codewords of C.

* (b) Write down a generator matrix for C in standard form.

* (c) Calculate the minimum distance $d(C)$.

* (d) How many errors can the code detect?

* (e) How many errors can the code correct?

(f) What is meant by a coset leader of a coset $\underline{y} + C$ where $\underline{y} \in \mathbb{F}_3^3$?

* (g) Give an example of a coset $\underline{y} + C$ which has more than one coset leader.

[10 marks]

A3.

(a) Define the Hamming sphere $S_t(\underline{u})$ with centre $\underline{u}$ and radius t in the vector space $\mathbb{F}_q^n$.

* (b) Write down a formula for $|S_t(\underline{u})|$, the number of elements in $S_t(\underline{u})$.

(c) State the Hamming bound for the number of codewords M of a code of minimum distance d in $\mathbb{F}_q^n$.

(d) What is meant by saying that a code $C \subseteq \mathbb{F}_q^n$ of minimum distance d is perfect?

* (e) Prove:

1. The sphere $S_{10}(\underline{0})$ in $\mathbb{F}_3^{2013}$ consists of an odd number of elements.
2. Any perfect code in $\mathbb{F}_3^{2013}$ consists of an odd number of codewords.

[12 marks]

* **A4.** Consider the following binary code of length 6: $C = \{000111, 110001, 011100\}$.

(a) Is C a linear code? Give a reason for your answer.

(b) Find $d(C)$.

(c) Show that there does not exist a vector $\underline{y} \in \mathbb{F}_2^6$ such that $d(\underline{y}, \underline{c}) = 1$ for all $\underline{c} \in C$.

(d) Find a vector $\underline{z} \in \mathbb{F}_2^6$ such that $d(\underline{z}, \underline{c}) = 2$ for all $\underline{c} \in C$.

[8 marks]

2013

SECTION B

Answer **TWO** of the three questions in this section (40 marks in total).

If more than TWO questions from this section are attempted, then credit will be given for the best TWO answers.

B5.

(a) Define the minimum distance, $d(C)$, of a code $C \subseteq \mathbb{F}_q^n$.

(b) Prove that a code $C \subseteq \mathbb{F}_q^n$ contains no more that $q^{n+1-d(C)}$ elements.

We say that $C \subseteq \mathbb{F}_q^n$ is an MDS code if $|C| = q^{n+1-d(C)}$.

(c) Define the Hamming code $\mathrm{Ham}(s, q)$.

(d) Write down expressions for the length, n, and the dimension, k, of $\mathrm{Ham}(s, q)$ in terms of s, q.

* (e) Let q be given. Describe all values of s such that $\mathrm{Ham}(s, q)$ is an MDS code. You may quote any result from the course without proof.

* (f) Write down a generator matrix for $\mathrm{Ham}(3, 2)$ in standard form.

* (g) Find $\max\{d(\underline{x}, \underline{y}) : \underline{x}, \underline{y} \in \mathrm{Ham}(3, 2)\}$, that is, the *maximum* distance between two codewords in $\mathrm{Ham}(3, 2)$. Justify your answer.

[20 marks]

B6. Let $C \subseteq \mathbb{F}_q^n$ be a linear code of dimension k.

(a) Define the dual code $C^\perp$ and state the formula for $\dim C^\perp$.

* (b) Assume that $q = 2$ and the binary linear code C is self-dual, that is, $C^\perp = C$.

(i) Show that n is even.

(ii) Show that every codeword in C has even weight.

(iii) Show that the vector $11\ldots1$ of weight n belongs to C.

* (c) Show that the binary code D with generator matrix $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ is self-dual.

* (d) Work out a standard array for the code D.

* (e) Assume that the code D is transmitted down a binary symmetric channel with bit error rate r. Let $P_{\mathrm{corr}}(D)$ denote the probability that a received vector is decoded correctly. Show that $P_{\mathrm{corr}}(D) = (1 - r)^2$.

* (f) Find a self-dual code $E \subseteq \mathbb{F}_2^4$ such that $E \neq D$. Show that E is linearly equivalent to D.

[20 marks]

B7. Let p be a prime.

(a) Define a cyclic code in $\mathbb{F}_p^n$.

(b) Define what is meant by an ideal I in a commutative ring R.

(c) Prove that if vectors in $\mathbb{F}_p^n$ are identified with elements of $R_n = \mathbb{F}_p[x]/(x^n - 1)$ in the usual way, a cyclic code C becomes an ideal of R_n.

You are given that there is a factorisation

$$x^9 - 1 = (x^2 + x + 1)(x^7 + x^6 + x^4 + x^3 + x + 1) \qquad \text{over } \mathbb{F}_2.$$

* (d) Write down a generator polynomial and a check polynomial for a binary cyclic code C of length 9 and dimension 7.

* (e) Write down a parity check matrix for C.

(f) State without proof the Distance Theorem for linear codes.

* (g) Use the Distance Theorem to compute $d(C)$.

* (h) Find an example of a binary linear code D such that $\dim D = \dim C$, $d(D) = d(C)$, but the length of D is less than the length of C. Give a well-known name or a simple description of D.

[20 marks]

END OF EXAMINATION PAPER

2014

SECTION A

Answer **ALL** questions in this section (40 marks in total)

A1.

(a) Let C be a linear code in $\mathbb{F}_q^n$. Explain what is meant by:

1. the weight $w(\underline{x})$ of a vector $\underline{x}$;
2. the weight $w(C)$ of C;
3. a generator matrix of C;
4. the inner product $\underline{x} \cdot \underline{y}$ of vectors $\underline{x}$, $\underline{y}$;
5. the dual code $C^{\perp}$.

∗ (b) If H is a matrix with n columns over $\mathbb{F}_q$, prove that $C = \{\underline{x} \in \mathbb{F}_q^n \mid \underline{x}H^T = \underline{0}\}$ is a linear code.

∗ (c) Let E_5 denote the binary even weight code of length 5. Write down a generator matrix of E_5.

∗ (d) Write down a generator matrix of $(E_5)^{\perp}$. Identify the code $(E_5)^{\perp}$ by its well-known name.

[10 marks]

A2.

(a) State the Distance Theorem for linear codes.

Let C be the binary linear code with parity check matrix $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$.

∗ (b) State the length and the dimension of C.

∗ (c) Using the Distance Theorem, find the minimum distance $d(C)$ of C.

∗ (d) How many errors can the code detect?

∗ (e) How many errors can the code correct?

∗ (f) Find three vectors of weight 1 with pairwise distinct syndromes.

∗ (g) Using (f), write down a set of coset leaders. Calculate the table of syndromes, and decode the vectors 000101 and 111000 using syndrome decoding.

∗ (h) If the code C is transmitted down a binary symmetric channel with bit error rate r, write down a formula for $P_{\text{corr}}(C)$, the probability that a received vector is decoded correctly.

[15 marks]

2014

A3.

(a) Define projective $(n-1)$-space, $\mathbb{P}_{n-1}(\mathbb{F}_q)$.

(b) Define a Hamming code $\mathrm{Ham}(s,q)$.

* (c) Write down, without proof, formulas for the length n, dimension k and minimum distance d of $\mathrm{Ham}(s,q)$.

* (d) Let C be a $\mathrm{Ham}(2,3)$ code. In the following, you may quote any result from the course without proof:

1. Write down a generator matrix for C.
2. Show that C is a self-dual code, that is, $C^{\perp} = C$.
3. Show that $\mathrm{Ham}(s,q)$ is not a self-dual code if $s > 2$.

[15 marks]

2014

SECTION B

Answer **TWO** of the three questions in this section (40 marks in total).

If more than TWO questions from this section are attempted, then credit will be given for the best TWO answers.

B4.

(a) Define the Hamming sphere $S_t(\underline{u})$ with centre $\underline{u}$ and radius t in the vector space $\mathbb{F}_q^n$.

(b) Write down a formula for $|S_t(\underline{u})|$, the number of elements in $S_t(\underline{u})$.

* (c) Let C be a code in $\mathbb{F}_q^n$. Show that if $t < \frac{d(C)}{2}$, $\underline{u}, \underline{v} \in C$, $\underline{u} \neq \underline{v}$, then $S_t(\underline{u}) \cap S_t(\underline{v}) = \varnothing$.

(d) State and prove the Hamming bound for the number M of elements of a code in $\mathbb{F}_q^n$ of minimum distance d.

* (e) Show that a k-dimensional linear code in $\mathbb{F}_2^n$ of minimum distance 3 satisfies $k \leq n - \log_2(n+1)$.

(f) Define what is meant by a perfect code.

* (g) State without proof for which pairs (q, d) where q is a prime there exists a perfect q-ary code of minimum distance d. Name a perfect code corresponding to each pair.

[20 marks]

B5. Let p be a prime.

(a) Let a cyclic code $C \subseteq \mathbb{F}_p^n$ be considered as an ideal of $R_n = \mathbb{F}_p[x]/(x^n - 1)$ in the usual way. Show that there is a unique monic polynomial g of minimum degree such that $C = \overline{g}R_n$.

(b) For C and g as above, write down a generator matrix for C in terms of the coefficients of g.

You are given that, in $\mathbb{F}_3[x]$,

$$x^8 - 1 = (x^5 - x^4 + x - 1)(x^3 + x^2 + x + 1).$$

* (c) Write down a generator polynomial and a check polynomial for a ternary cyclic code D of length 8 and dimension 5.

In the rest of the question, D refers to the code that you obtained in part (c).

* (d) Let $a, b, c \in \mathbb{F}_3$ be such that the vector $ab00000c$ is a codeword of D. Show that $a = b = c = 0$.

* (e) Does D contain codewords of weight 2? If so, write down a codeword of D of weight 2.

[20 marks]

B6.

(a) Let C_1, C_2 be linear codes in $\mathbb{F}_q^n$.

1. Define the code $|C_1|C_2|$.
2. Prove that $d(|C_1|C_2|) \geq \min\{2d(C_1), d(C_2)\}$.

(b) Define the rth order Reed-Muller code $R(r, m)$ in terms of Boolean functions.

(c) Explain why $R(r+1, m+1) = |R(r+1, m)|R(r, m)|$.

* (d) Give an example of a Reed-Muller code $R(r, m)$ which has relative distance $\delta = 0.5$.

* (e) You are given that C is some linear code in $\mathbb{F}_2^n$ which has relative distance $\delta > 0.5$ and contains the codeword $11\ldots1$ (where all n bits are 1). Prove that $\dim C = 1$.

[20 marks]

END OF EXAMINATION PAPER

SECTION A

Answer **ALL** questions in this section (40 marks in total)

A1.

(a) Explain what is meant by:

1. a linear code $C \subseteq \mathbb{F}_q^n$;
2. the weight $w(\underline{x})$ of a vector $\underline{x}$;
3. the weight $w(C)$ of a linear code C;
4. the inner product $\underline{x} \cdot \underline{y}$ of vectors $\underline{x}$, $\underline{y}$;
5. the dual code $C^\perp$.

* (b) Assume that C is a one-dimensional linear code in $\mathbb{F}_3^7$.

i. How many rows and how many columns are there in a standard array for C?

ii. How many codewords are there in the dual code $C^\perp$?

iii. Prove that $w(C^\perp) \geq w(C) - 5$.

[10 marks]

* **A2.** The linear code $C \subseteq \mathbb{F}_2^5$ is given by $C = \{(x_1, x_2, x_3, x_4, x_5) \mid x_1+x_2+x_3 = 0,\ x_4+x_5 = 0 \text{ in } \mathbb{F}_2\}$.

(a) Write down a parity check matrix for C.

(b) Write down a generator matrix for C.

(c) List all the codewords of C.

(d) Find the minimum distance $d(C)$ of C.

(e) How many bit errors can the code detect?

(f) How many bit errors can the code correct?

(g) Write down the weight distribution function $A(x)$ of the code C.

(h) If the code C is transmitted down a binary symmetric channel with bit error rate r, calculate $P_{\text{undetect}}(C)$, the probability that the received vector contains an error that is not detected (leave your answer as a polynomial in r).

(i) Write down a vector $\underline{y} \in \mathbb{F}_2^5$ of weight 4 such that $\underline{y}$ has more than one nearest neighbour in C.

[15 marks]

2015

A3.

(a) Define the Hamming sphere $S_r(\underline{\mathrm{u}})$ with centre $\underline{\mathrm{u}}$ and radius r in the vector space $\mathbb{F}_q^n$.

(b) Write down a formula for $|S_r(\underline{\mathrm{u}})|$, the number of elements in $S_r(\underline{\mathrm{u}})$.

(c) State without proof the Hamming bound for the number M of elements of a code in $\mathbb{F}_q^n$ of minimum distance d.

(d) Define what is meant by a perfect code.

(e) Prove that an $[11, 6, 5]_3$-code is perfect.

* (f) Give an example of a perfect code of minimum distance 9.

* (g) You are given that $C \subseteq D \subseteq \mathbb{F}_q^n$ where $|C| < |D|$ and C is a perfect code. Show that $d(C) > 2d(D)$. You may quote any result from the course without proof.

[15 marks]

2015

SECTION B

Answer **TWO** of the three questions in this section (40 marks in total).

If more than TWO questions from this section are attempted, then credit will be given for the best TWO answers.

B4.

(a) Let C be a linear code in $\mathbb{F}_q^n$. Define what is meant by saying that H is a check matrix for C.

(b) Explain how to find a check matrix H for C, given a generator matrix G for C in standard form.

(c) State and prove the Distance Theorem for linear codes. (You may assume basic facts about check matrices without particular comment.)

Now let C be the linear 5-ary code with check matrix $\begin{bmatrix} 1 & 4 & 2 & 2 \\ 3 & 3 & 4 & 1 \end{bmatrix}$.

* (d) What are the length and the dimension of C?

* (e) Find the minimum distance of C.

* (f) Show that C is self-dual, that is, $C^\perp = C$.

* (g) Prove that for every even n there exists a self-dual linear code in $\mathbb{F}_5^n$.

[20 marks]

B5.

(a) Define projective $(n-1)$-space, $\mathbb{P}_{n-1}(\mathbb{F}_q)$.

(b) Define a Hamming code $\mathrm{Ham}(s,q)$.

(c) State and prove formulas for the length n and dimension k of $\mathrm{Ham}(s,q)$.

* (d) Write down a check matrix for a $\mathrm{Ham}(2,7)$ code.

From now on, let C be the $\mathrm{Ham}(2,7)$ code with the check matrix you obtained in (d).

* (e) Does the code C attain the Singleton bound? Justify your answer briefly.

* (f) Does the code C contain the vector $11\ldots1$ (all symbols 1)? Justify your answer briefly.

* (g) Prove that every *binary* Hamming code $\mathrm{Ham}(s,2)$ contains the vector $11\ldots1$ (all bits 1).

[20 marks]

2015

B6. Let p be a prime.

(a) Define what is meant by a cyclic code in $\mathbb{F}_p^n$.

(b) Define what is meant by an ideal I in a commutative ring R.

(c) Prove that if vectors in $\mathbb{F}_p^n$ are identified with elements of $R_n = \mathbb{F}_p[x]/(x^n - 1)$ in the usual way, a cyclic code C becomes an ideal of R_n.

You are given that there is a factorisation

$$x^7 + 1 = (x+1)(x^3+x+1)(x^3+x^2+1) \qquad \text{over } \mathbb{F}_2.$$

* (d) Write down a generator polynomial and a generator matrix for a binary cyclic code C of length 7 and dimension 3.

* (e) Now write down a generator polynomial and a generator matrix for a binary cyclic code D of length 7 and dimension 3 such that $D \neq C$.

* (f) Prove: the codes C and D that you obtained are linearly equivalent.

[20 marks]

END OF EXAMINATION PAPER

2016

SECTION A

Answer **ALL** questions in this section (40 marks in total)

A1. (a) Explain what is meant by:

1. the Hamming distance $d(\underline{x}, \underline{y})$ between two vectors $\underline{x}, \underline{y} \in \mathbb{F}_q^n$;
2. the weight $w(\underline{x})$ of a vector $\underline{x} \in \mathbb{F}_q^n$;
3. the minimum distance $d(C)$ of a code $C \subseteq \mathbb{F}_q^n$;
4. a linear code $C \subseteq \mathbb{F}_q^n$;
5. the weight $w(C)$ of a linear code C.

(b) Explain why $d(\underline{x}, \underline{y}) = w(\underline{x} - \underline{y})$ for all $\underline{x}$, $\underline{y} \in \mathbb{F}_q^n$.

(c) Prove that $d(C) = w(C)$ for a linear code C.

(d) Define E_n, the binary even weight code of length n. State without proof the number of codewords of E_n and the minimum distance of E_n.

[15 marks]

A2. (a) What is meant by a generator matrix of a linear code $C \subseteq \mathbb{F}_q^n$?

You are given that C is a binary linear code with generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$.

∗ (b) State the length and the dimension of C.

∗ (c) List all the elements of C.

∗ (d) Find the minimum distance of C.

∗ (e) How many bit errors can the code detect?

∗ (f) How many bit errors can the code correct?

∗ (g) Determine whether the code C attains the Singleton bound.

∗ (h) If the code C is transmitted down a binary symmetric channel with bit error rate r, calculate $P_{\text{undetect}}(C)$, the probability that the received vector contains an error that is not detected (leave your answer as a polynomial in r).

[15 marks]

A3. (a) Define projective $(n-1)$-space, $\mathbb{P}_{n-1}(\mathbb{F}_q)$.

(b) Define a Hamming code $\text{Ham}(s, q)$.

∗ (c)
1. Express (without proof) the length n, the dimension k and the minimum distance d of $\text{Ham}(s, q)$ in terms of s and q.
2. Show that if k is odd, then n is also odd.

[10 marks]

2016

SECTION B

Answer **TWO** of the three questions in this section (40 marks in total).

If more than TWO questions from this section are attempted, then credit will be given for the best TWO answers.

B4.

(a) Define the Hamming sphere $S_r(\underline{u})$ with centre $\underline{u}$ and radius r in the vector space $\mathbb{F}_q^n$. Write down a formula for the number of elements in $S_r(\underline{u})$.

(b) Let $C \subseteq \mathbb{F}_q^n$ be a linear code, $t = \left[\frac{d(C)-1}{2}\right]$, $\underline{a} \in S_t(\underline{0})$. Prove that $\underline{a}$ is the only coset leader of the coset $\underline{a} + C$.

(c) State without proof the Hamming bound for the number M of elements of a code in $\mathbb{F}_q^n$ of minimum distance d.

(d) Define what is meant by a perfect code.

(e) Show: if $C \subseteq \mathbb{F}_q^n$ is a perfect linear code, $t = \left[\frac{d(C)-1}{2}\right]$, then every coset leader belongs to $S_t(\underline{0})$.

∗ (f) You are given that q is a prime and that C is a perfect linear $[n,k,d]_q$-code with weight enumerator $W_C(x,y) = \underline{Ay^n + Bx^2y^{n-2} + nx^3y^{n-3} + nx^4y^{n-4} + x^n}$. Find n, k, d, q, A and B. Justify your answer briefly. You may use any facts from the course without giving a proof.

swap x and y

[20 marks]

B5. Let $C \subseteq \mathbb{F}_q^n$ be a linear code.

(a) Explain what is meant by the inner product $\underline{x} \cdot \underline{y}$ of vectors $\underline{x}, \underline{y} \in \mathbb{F}_q^n$ and by the dual code $C^\perp$.

(b) Prove that if G is a generator matrix for C, then $C^\perp = \{\underline{v} \in \mathbb{F}_q^n \mid \underline{v}G^T = \underline{0}\}$. (Any facts from linear algebra may be used without particular comment.)

(c) What is meant by saying that H is a check matrix of $C \subseteq \mathbb{F}_q^n$? State the number of rows and the number of columns of H, given that $\dim C = k$.

(d) State without proof the Distance Theorem for linear codes.

∗ (e) Write down an example of a check matrix of a ternary linear code C such that C consists of 27 codevectors and $d(C) = 3$.

∗ (f) Let $C \neq \{\underline{0}\}$ be a ternary linear code such that $C \subseteq C^\perp$. Prove that $d(C)$ is a multiple of 3. You may use any results from the course without giving a proof.

[20 marks]

B6. Let p be a prime.

(a) Let a cyclic code $C \subseteq \mathbb{F}_p^n$ be considered as an ideal of $R_n = \mathbb{F}_p[x]/(x^n - 1)$ in the usual way. Show that there is a unique monic polynomial g of minimum degree such that $C = \overline{g}R_n$.

(b) For C and g as above, write down a generator matrix for C in terms of the coefficients of g. How is the dimension of C related to the degree of g?

You are given that there is a factorisation

$$x^9 - 1 = (x^2 + x + 1)(x^7 + x^6 + x^4 + x^3 + x + 1) \qquad \text{over } \mathbb{F}_2.$$

* (c) Use the above factorisation to find the generator polynomial and the check polynomial of a binary cyclic code D of length 9 and dimension 2.

* (d) Find the weight $w(D)$ of D. Justify your answer. Write down a codevector of D of weight $w(D)$.

* (e) Find the weight $w(D^{\perp})$ of the dual code $D^{\perp}$. Write down a codevector of $D^{\perp}$ of weight $w(D^{\perp})$.

[20 marks]

END OF EXAMINATION PAPER