Review Week 05

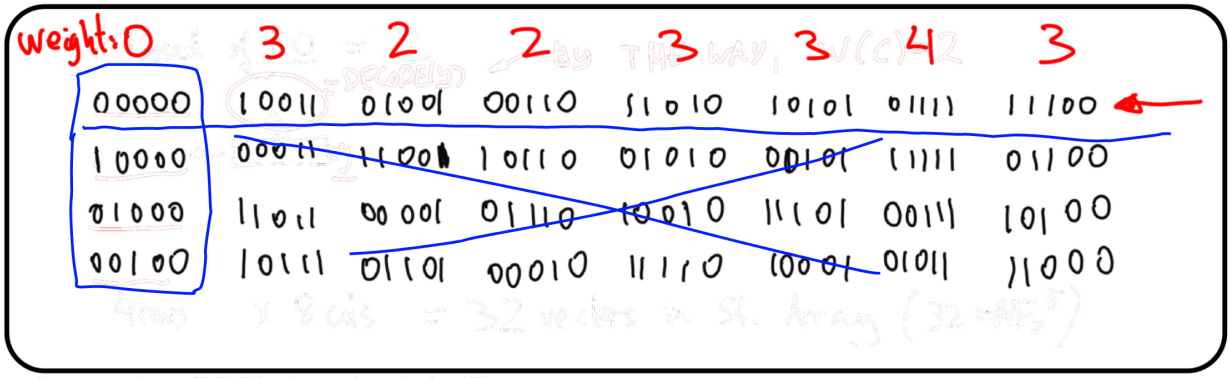
2022-10-24

ANNOUNCEMENT: CAREERS FAIR Wed. 26 Oct

Reminder: coursework 1 TOMORROW Tuesday 25-Oct 11^{am}

Weight enumerator and $P_{undetect}$

Recall the code with standard array



- **Assuming BSC(p), calculate $P_{undetect}$.**
- **Decide if C offers a significant improvement in error detection compared to sending unencoded messages.**

Recall the definition of the weight enumerator :

(polynomial, each monomial has total degree n)

$$W_C(x,y) = \sum_{\underline{v}\in C} x^{n-w(\underline{v})} y^{w(\underline{v})} = A_0 x^n + A_1 x^{n-1} y + \cdots + A_n y^n$$

where $A_i = \#\{\underline{v} \in C : w(\underline{v}) = i\}$

$A_i \in \mathbb{Z}_{\geq 0}$, $i = 0, 1, \ldots, n$; $A_0 = 1$

In the above example, $W_C(x,y) = x^5 + 2x^3y^2 + 4x^2y^3 + xy^4$

$P_{undetect}(C) = W_C(1-p, p) - (1-p)^n$

10011 prob. $= p^3(1-p)^2$

contain p^2 $\Rightarrow$ $P_{undetect} \approx 2p^2$

The inner product. The dual code

- **The inner product $\underline{x} \cdot \underline{y}$ of vectors $\underline{x}, \underline{y} \in \mathbb{F}_q^n$ is defined as $\underline{x} \cdot \underline{y} = \sum_{i=1}^{n} x_i y_i$.**
- **If $C \subseteq \mathbb{F}_q^n$ is a linear code, the dual code $C^\perp$ is $\{\underline{v} \in \mathbb{F}_q^n : \underline{v} \cdot \underline{c} = 0 \text{ for all } \underline{c} \in C\}$ (that is: $C^\perp$ consists of all vectors orthogonal to C).**
- **Result from the lectures: $\dim C^\perp = n - k$ where $k = \dim C$.**

The check matrix H. The syndrome of a vector. The use of H for error detection

- **A generator matrix H for $C^\perp$ is called a check matrix for C.**
- **Fix H. Given a vector $\underline{y} \in \mathbb{F}_q^n$, the vector $S(\underline{y}) = yH^T$ of size $n - k$ is the *syndrome* of $\underline{y}$. By the same theorem, $C = \{\underline{c} \in \mathbb{F}_q^n : S(\underline{c}) = \underline{0}\}$.**

- **This allows the receiver to detect errors.**

The use of H for error correction - syndrome decoding.

A2. Let C be the binary linear code with parity check matrix $H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$.

(a) Construct a table of syndromes for C.

(b) Use your table of syndromes to decode the received vectors 11110 and 10011.

$\underline{a}$ (coset leader)	$S(\underline{a})$
00000	000
00001	101
00010	100
00100	011
01000	010
10000	001
00011	110
01010	111
00110	

$q = 2^3$ rows

$S(\underline{y}) = S(\underline{z}) \Leftrightarrow \text{coset}(\underline{y}) = \text{coset}(\underline{z})$

Look for a vector of least weight whose syndrome is not yet in the table

DECODE (11110) = 11100

$S(11110) = 100 \Rightarrow \underline{a} = 00010$