Exercises to Chapter 8

Exercise 8.1. Deduce from the Plotkin bound that every binary linear code C of length n with $d(C) > \frac{2}{3}n$ is one-dimensional.

Plotkin bound: if C is a binary code such that $\boxed{d(C) > \frac{n}{2}}$ then $\#C \leq \frac{d}{d - n/2}$ $(d = d(C))$

Hamming bd: $\#C \leq \frac{q^n}{\Sigma \ldots}$ Singleton: $\#C \leq q^{n-d+1}$

Assume $d > \frac{2}{3}n$. Because $\frac{2}{3}n > \frac{n}{2}$, the Plotkin bound applies:

$$\#C \leq \frac{d}{d - n/2} = 1 + \frac{n/2}{d - n/2} = 1 + \frac{n \times \frac{1}{2}}{d - n \times \frac{1}{2}} = 1 + \frac{1/2}{\frac{d}{n} - \frac{1}{2}} <$$

$$1 + \frac{1/2}{\frac{2}{3} - \frac{1}{2}} = 1 + \frac{1/2}{1/6} = 4$$ so $\dim C = \log_2 \#C < 2$

Exercise 8.2. Let $\widehat{\mathcal{H}}$ be the extended Hamming $[8, 4, 4]_2$-code $\widehat{\mathrm{Ham}}(3, 2)$. Recall from Ex.7.3 that $\widehat{\mathcal{H}}$ is a self-dual code with weight enumerator $W_{\widehat{\mathcal{H}}}(x, y) = x^8 + 14x^4y^4 + y^8$. Verify directly that $W_{\widehat{\mathcal{H}}}(x, y) = (\#\widehat{\mathcal{H}})^{-1} W_{\widehat{\mathcal{H}}}(x + y, x - y)$.

(*Of course, this must be true by the MacWilliams identity, given that* $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}^{\perp}$.)

$\widehat{\mathcal{H}}$ = code generated by

$\mathcal{H}$ is a $[7,4,3]_2$-code

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$H = [\underbrace{-A^T}_{} | I_3]$

4 columns
1 1 1 0
1 1 0 1
1 0 1 1

generator for $\widehat{\mathcal{H}}$

$\widehat{\mathcal{H}}$ is self-orthogonal (and $n = 2k \Rightarrow$ self-dual) because $GG^T = 0$.

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ybazlov@yucca:~$
ybazlov@yucca:~$
ybazlov@yucca:~$ sage
┌────────────────────────────────────────────────────────────────────┐
│ SageMath version 9.7, Release Date: 2022-09-19                     │
│ Using Python 3.10.5. Type "help()" for help.                       │
└────────────────────────────────────────────────────────────────────┘
sage: f(x,y)=x^8+14*x^4*y^4+y^8
sage: f(x+y,x-y)
(x + y)^8 + 14*(x + y)^4*(x - y)^4 + (x - y)^8
sage: expand(f(x+y,x-y))
16*x^8 + 224*x^4*y^4 + 16*y^8
sage:
~
```

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