A3.

(a) Define the Hamming sphere $S_t(\underline{u})$ with centre $\underline{u}$ and radius t in the vector space $\mathbb{F}_q^n$.

(b) Write down a formula for $|S_t(\underline{u})|$, the number of elements in $S_t(\underline{u})$.

(c) State the Hamming bound for the number of codewords M of a code of minimum distance d in $\mathbb{F}_q^n$.

(d) What is meant by saying that a code $C \subseteq \mathbb{F}_q^n$ of minimum distance d is perfect?

(e) Prove:

1. The sphere $S_{10}(\underline{0})$ in $\mathbb{F}_3^{2013}$ consists of an odd number of elements.
2. Any perfect code in $\mathbb{F}_3^{2013}$ consists of an odd number of codewords.

[12 marks]

A4. Consider the following binary code of length 6: $C = \{000111, 110001, 011100\}$.

(a) Is C a linear code? Give a reason for your answer.

(b) Find $d(C)$.

(c) Show that there does not exist a vector $\underline{y} \in \mathbb{F}_2^6$ such that $d(\underline{y}, \underline{c}) = 1$ for all $\underline{c} \in C$.

(d) Find a vector $\underline{z} \in \mathbb{F}_2^6$ such that $d(\underline{z}, \underline{c}) = 2$ for all $\underline{c} \in C$.

[8 marks]