则$\{U,V\}$构成$S^n(1)$的一个开覆盖，坐标卡(U,φ)，(V,ψ)的构造与第一题相同，

故$(\xi^1,\cdots,\xi^n)=\varphi(x^1,\cdots,x^n,x^{n+1})=\left(\frac{x^1}{1-x^{n+1}},\cdots,\frac{x^n}{1-x^{n+1}}\right)$

$(\eta^1,\cdots,\eta^n)=\psi(x^1,\cdots,x^n,x^{n+1})=\left(\frac{x^1}{1+x^{n+1}},\cdots,\frac{x^n}{1+x^{n+1}}\right)$

从而$(\eta^1,\cdots,\eta^n)=\psi\circ\varphi^{-1}(\xi^1,\cdots,\xi^n)=\left(\frac{\xi^1}{\sum_{i=1}^n(\xi^i)^2},\cdots,\frac{\xi^n}{\sum_{i=1}^n(\xi^i)^2}\right)$

$(\xi^1,\cdots,\xi^n)=\varphi\circ\psi^{-1}(\eta^1,\cdots,\eta^n)=\left(\frac{\eta^1}{\sum_{i=1}^n(\eta^i)^2},\cdots,\frac{\eta^n}{\sum_{i=1}^n(\eta^i)^2}\right)$

因此(U,φ)与(V,ψ)是C^∞－相容的坐标卡，

且有$(x^1,\cdots,x^n,x^{n+1})=\varphi^{-1}(\xi^1,\cdots,\xi^n)=\left(\frac{2\xi^1}{1+\sum_{i=1}^n(\xi^i)^2},\cdots,\frac{2\xi^n}{1+\sum_{i=1}^n(\xi^i)^2},\frac{1-\sum_{i=1}^n(\xi^i)^2}{1+\sum_{i=1}^n(\xi^i)^2}\right)$

$(x^1,\cdots,x^n,x^{n+1})=\psi^{-1}(\eta^1,\cdots,\eta^n)=\left(\frac{2\eta^1}{1+\sum_{i=1}^n(\eta^i)^2},\cdots,\frac{2\eta^n}{1+\sum_{i=1}^n(\eta^i)^2},\frac{\sum_{i=1}^n(\eta^i)^2-1}{1+\sum_{i=1}^n(\eta^i)^2}\right)$

故有诱导度量$g^+=g^+_{ij}d\xi^i d\xi^j$，$g^-=g^-_{ij}d\eta^i d\eta^j$

其中，$g^+_{ij}=\sum_{\alpha=1}^{n+1}\frac{\partial x^\alpha}{\partial\xi^i}\frac{\partial x^\alpha}{\partial\xi^j}=\frac{4\delta_{ij}}{(1+\sum_{i=1}^n(\xi^i)^2)^2}$，$g^-_{ij}=\sum_{\alpha=1}^{n+1}\frac{\partial x^\alpha}{\partial\eta^i}\frac{\partial x^\alpha}{\partial\eta^j}=\frac{4\delta_{ij}}{(1+\sum_{i=1}^n(\eta^i)^2)^2}$

在$U\cap V$上，$g^+=g^-$，故g^+和g^-给出了$S^n(1)$上的黎曼度量。

17. 仿射联络的定义。

设M是m维光滑流形，M的切丛TM上的一个仿射联络∇是满足下列条件的映射$\nabla:\Gamma(TM)\times\Gamma(TM)\to\Gamma(TM)$。

(1) $\nabla_{Y+fZ}X=\nabla_Y X+f\nabla_Z X$；(2) $\nabla_Y(X+\lambda Z)=\nabla_Y X+\lambda\nabla_Y Z$；(3) $\nabla_Y(fX)=Y(f)X+f\nabla_Y X$。

其中，$X,Y,Z\in\Gamma(TM)$，$f\in C^\infty(M)$，$\lambda\in R$。

18. 仿射联络∇在局部坐标$(U;x^i)$下的表达式。 注：$\Gamma^k_{ij}=\frac{1}{2}g^{kl}\left(\frac{\partial g_{il}}{\partial x^j}+\frac{\partial g_{lj}}{\partial x^i}-\frac{\partial g_{ij}}{\partial x^l}\right)$

设$X=X^i\frac{\partial}{\partial x^i}$，$Y=Y^j\frac{\partial}{\partial x^j}$，$\nabla_{\frac{\partial}{\partial x^i}}\frac{\partial}{\partial x^j}=\Gamma^k_{ji}\frac{\partial}{\partial x^k}$，则

$\nabla_Y X=\nabla_{Y^j\frac{\partial}{\partial x^j}}\left(X^i\frac{\partial}{\partial x^i}\right)=Y^j\left(\frac{\partial X^i}{\partial x^j}\frac{\partial}{\partial x^i}+X^i\Gamma^k_{ij}\frac{\partial}{\partial x^k}\right)=Y^j\left(\frac{\partial X^i}{\partial x^j}+X^k\Gamma^i_{kj}\right)\frac{\partial}{\partial x^i}$

19. 构造与黎曼度量g相容的无挠联络∇（黎曼联络，Levi-Civita联络）

即$\nabla g=0\Longrightarrow(\nabla_X g)(Y,Z)=0$

无挠联络：$[X,Y]=\nabla_X Y-\nabla_Y X$；与度量$g$相容的联络：$X(g(Y,Z))=g(\nabla_X Y,Z)+g(Y,\nabla_X Z)$

注：$(\nabla_X g)(Y,Z)=X(g(Y,Z))-g(\nabla_X Y,Z)-g(Y,\nabla_X Z)$