

const 0r = f  $\Rightarrow x \Rightarrow x$ ;

const 1r = f  $\Rightarrow x \Rightarrow f(0r, 0r(f)(x))$ ;

const 2r = f  $\Rightarrow x \Rightarrow f(1r, 1r(f)(x))$ ;

const 3r = f  $\Rightarrow x \Rightarrow f(2r, 2r(f)(x))$ ;

const to\_int = r  $\Rightarrow r(\text{iter-count}, x) \Rightarrow x+1(0)$ ;

const inc-r = r  $\Rightarrow f \Rightarrow x \Rightarrow f(r, r(f)(x))$ ;

const add-r = (r1, r2)  $\Rightarrow$  ??

to\_int (add-r (2r, 3r));

How to approach?

Less straightforward, but lets look for signs.

First, can we make use of something that we have already defined?

Yes! inc-r

How to make use of inc-r?

Increment one of the repeaters, by "the other repeater"  
function value  
amount of times  
 the other repeater is called as function

$\Rightarrow r1(f)(r2)$

f is function that increments repeater  
 I still need to follow specification

How?

(iter-count, x)  $\Rightarrow$  inc-r(x);

$\Rightarrow r1(\text{iter-count}, x) \Rightarrow \text{inc-r}(x)(r2)$

$\Rightarrow$  const add-r = (r1, r2)  $\Rightarrow r1(\text{iter-count}, x) \Rightarrow \text{inc-r}(x)(r2)$ ;

Now can you tell how this representation of repeaters repeat functions?

1. Find out how repeaters work

E.g. 3r:

$f \Rightarrow x \Rightarrow f(2r, 2r(f)(x))$

function that accepts a function, returns a function that accepts a value.

how to call this function?

function f is a function that accepts 2 arguments.

Without looking at what f is, let's call it the function 3r first with a random function  $F_x$  & random value 0

$\Rightarrow 3r(F_x)(0)$

$\Rightarrow F_x(2r, 2r(F_x)(0))$  this expression same as above!

$\Rightarrow F_x(2r, F_x(1r, 1r(F_x)(0)))$

$\Rightarrow F_x(2r, F_x(1r, F_x(0r, 0r(F_x)(0))))$

$\Rightarrow F_x(2r, F_x(1r, F_x(0r, 0)))$

At this point, what does this repeating pattern say about the representation of repeaters?

$\rightarrow$  all have to follow same structure

How to carry on?

First a  $F_x$ : (iter-count, x)  $\Rightarrow x+1$ ;

$\Rightarrow G(2r, G(1r, G(0r, 0)))$

1  
2  
3

How to increment repeater then?

$\Rightarrow G(3r, G(2r, G(1r, G(0r, 0))))$

$G(3r, 3r(G)(0))$

$f \Rightarrow x \Rightarrow f(r, r(f)(x))$

representation of a repeater