ASSIGNMENT 5 SOLUTIONS

Note: Uploaded slightly late due to technical issues with RAM overloading on MCMC high-step count computations.

Q1)

Using some preset parameters return a Chi2 of 15268 for 2501 degrees of freedom. As our fitted Chi2 is way too far from the DOF, this fit is unacceptable with a survival of zero…

$H_0 = 69,\ \Omega_b h^2 = 0.22, \Omega_c h^2 = 0.12,\ \tau = 0.06,\ A_s = 2.1e - 9,\ n_s = 0.95$

Using the second set of parameters shown above, we extrapolate a value of 3272 for the Chi2 with a survival of 10e-24... This new fit remains unacceptable roughly 10+ sigmas away from it!

The mean is 2501 ± 71 which agrees with the predicted $n \pm \sqrt{2n}$ value where n is the DOF.

Q2)

For this question here, we just apply the LM many times. As such, we may minimize to find a reasonable Chi2 value… A damping factor gets lower and lower as we approach our new committed Chi2 target value. And as such, the parameters are recalculated along each step to find reach a new set of optimized values.

NEW OPTIMIZED FIT:

$H_0 = 68.0 \pm 1.18$

$\Omega_b h^2 = 2.24e - 2 \pm 2.32e - 4$

$\Omega_c h^2 = 1.19e - 1 \pm 2.65e - 3$

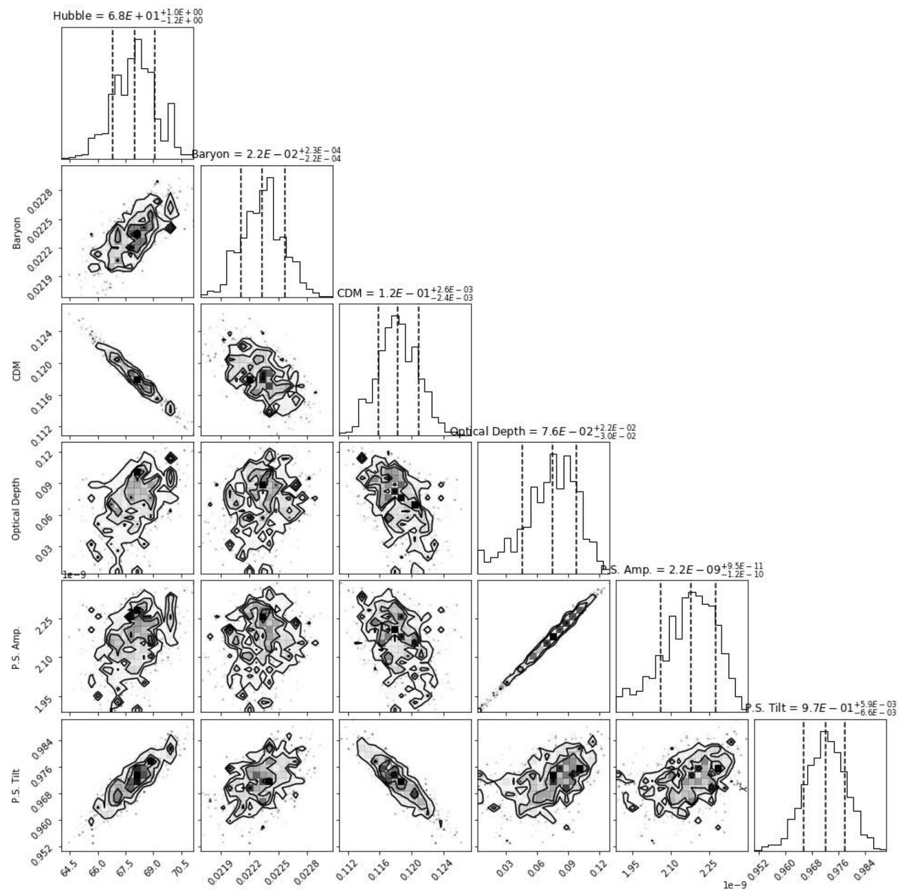
$\tau = 5.82e - 2 \pm 3.66e - 2$

$A_s = 2.11e - 9 \pm 1.47e - 10$

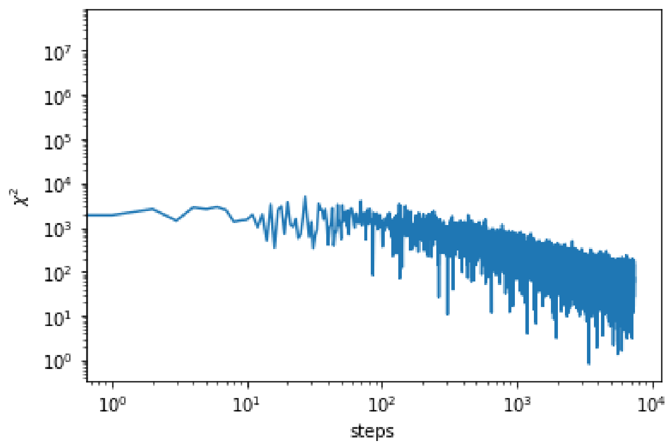
$n_s = 9.7e - 1 \pm 6.40e - 3$

We store the optimized values and cov-matrix for later use in MCMC in text files and later chain files.

Q3) We here implement numerical derivates. Applying the curvature matrix from before. As discussed with other classmates, learning how to code the Levenberg-marquardt from their examples, I differentiated along what seems to be six directions in three dimensions. A damping process was introduced. Subsequently, we save the optimized results on SSD for later usage. We here run continuous chains using previous data. Fusing these results together over roughly 3000 MCMC steps, we obtain the following Corner plot:

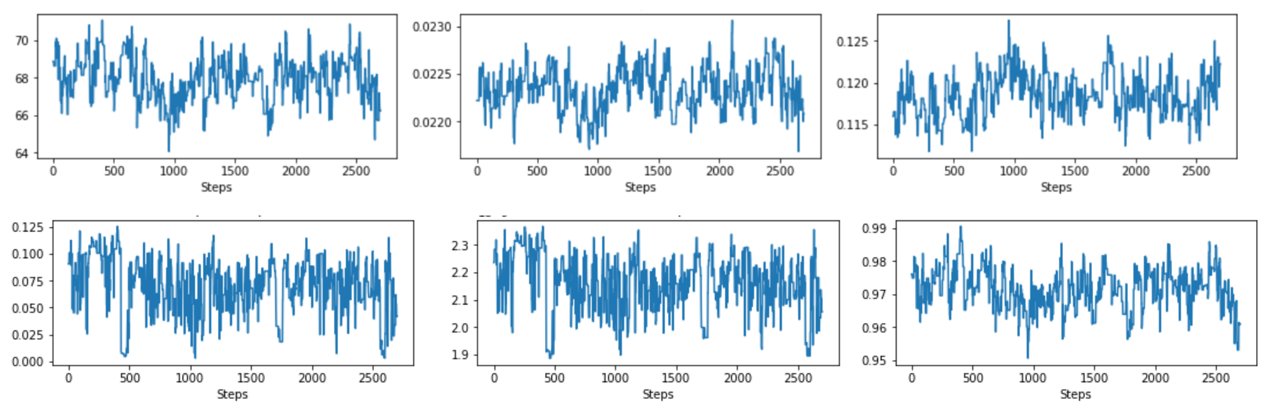


The chain seems to be doing quite well. No odd patterns have emerged necessarily. Looking at the FFT Power Spectrum, we note that chain convergence is evident. And the corner plot presents interesting correlations to indicate further convergence.

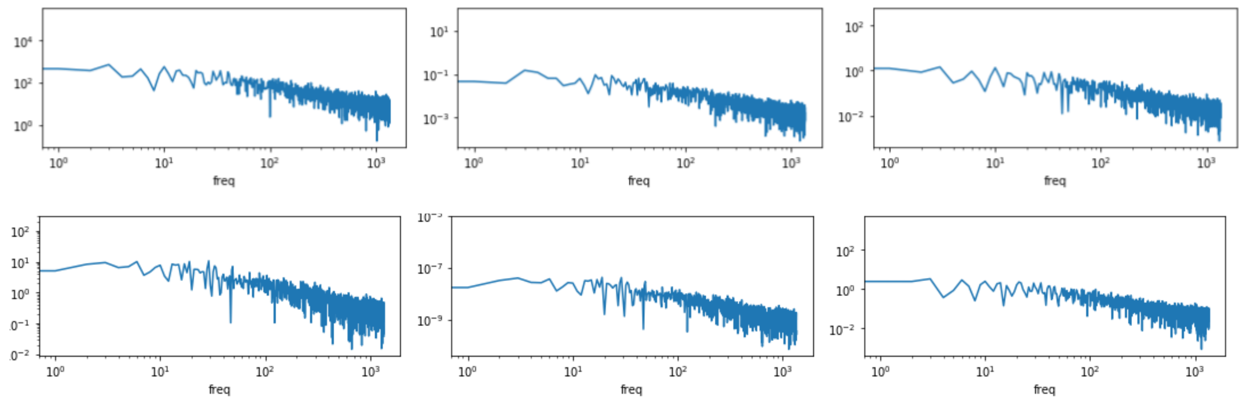


Again, this chain does seem to converge… Corner plots are shown below with minor skewing, which was not observed in the previous corner plots. This may also be due to the substantially higher step count (which could have revealed skewing that was not visible previously). Other classmates did observe similar results.

Here are the parameter plots for Hubble, Baryon, CDM, Optical Depth, Amp, and Tilt from left to right…



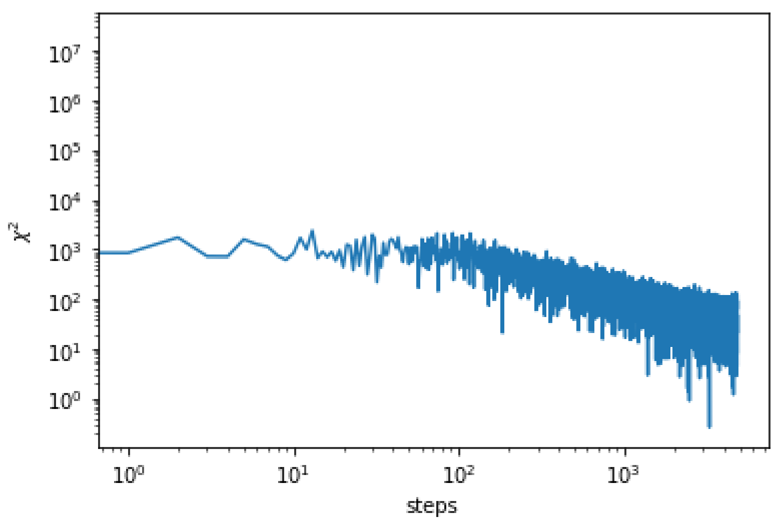
FFT Spectra for Hubble, Baryon, CDM, Optical Depth, Amp, and Tilt from left to right…



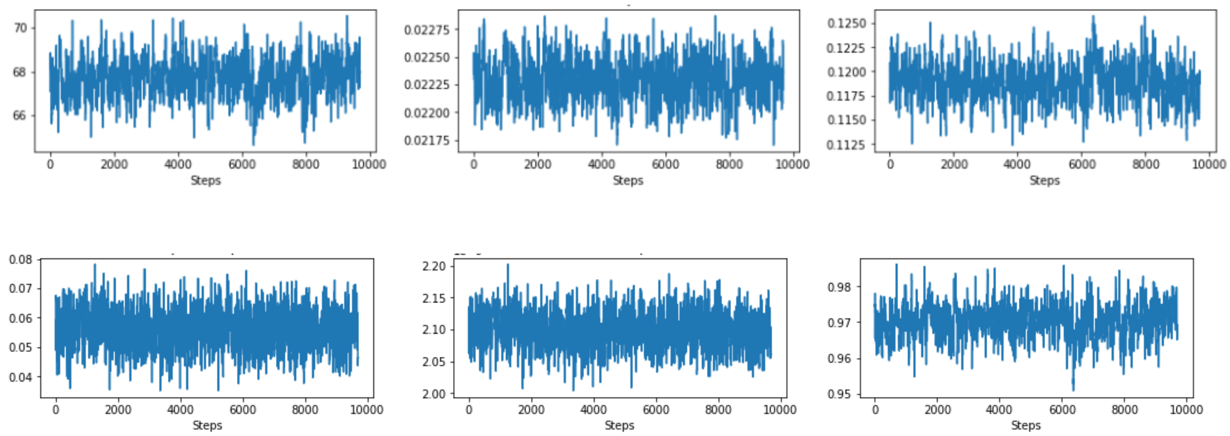
Q4)

Now, we do an MCMC with data solely taken from the covariance importance sampling rather than raw data. This time, running over 10,000 steps.

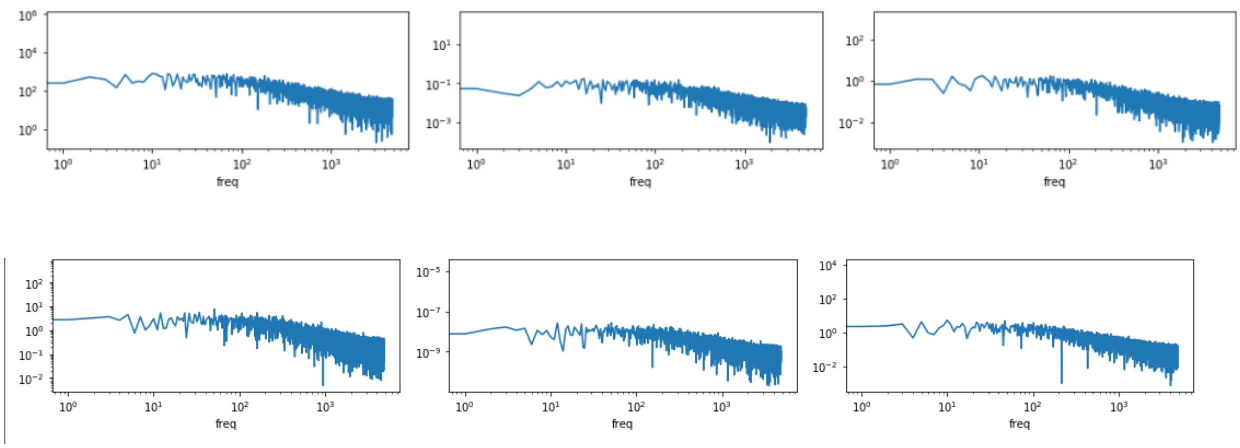
Below is the FFT Power Spectrum:



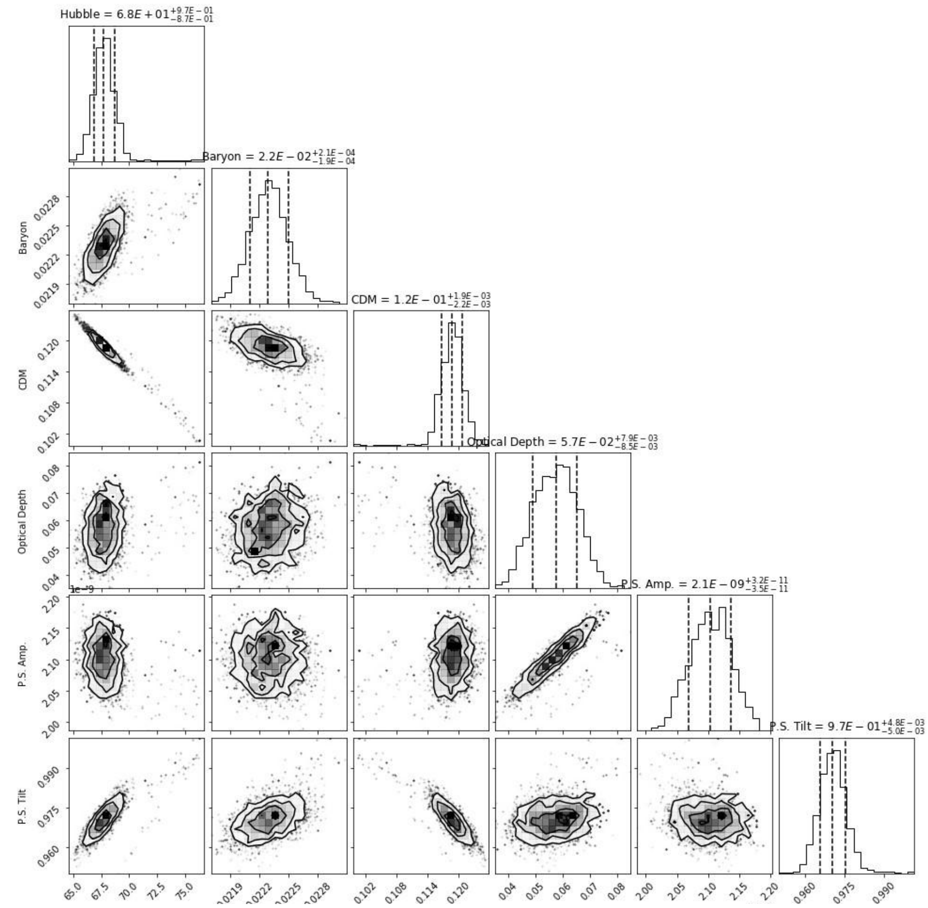
Here are the **parameter plots** for *Hubble, Baryon, CDM, Optical Depth, Amp, and Tilt* from left to right…



FFT Spectra for *Hubble, Baryon, CDM, Optical Depth, Amp, and Tilt* from left to right…



Below are the corner plots.



The **newly acquired MCMC parameters** (optimized from the importance sampling) are

$H_0 = 67.8 \pm 1.25$

$\Omega_b h^2 = 2.23e - 2 \pm 1.99e - 4$

$\Omega_c h^2 = 1.19e - 1 \pm 2.69e - 3$

$\tau = 5.73e - 2 \pm 8.01e - 2$

$A_s = 2.10e - 9 \pm 3.25e - 10$

$n_s = 9.71e - 1 \pm 5.51e - 3$

Whereas, the **importance sampling** was:

$H_0 = 68.0 \pm 9.55e - 1$

$\Omega_b h^2 = 2.24e - 2 \pm 2.43e - 4$

$\Omega_c h^2 = 1.18e - 1 \pm 2.14e - 3$

$\tau = 5.76e - 2 \pm 1.27e - 2$

$A_s = 2.10e - 9 \pm 5.31e - 11$

$n_s = 9.71e - 1 \pm 5.17e - 3$

Clearly, these do not completely match in comparison. This is not as expected. Quite odd, some type of overshooting. They do agree, however, within a few sigma intervals with roughly similar errors. An exact replica should not be fully expected either.

Based on recommendations from other classmates, weighing the tau factor is critical here in getting accurate results. Not sure if this is fully visible in this computation and ran out of time to further investigate the root of the skewing and other factors.