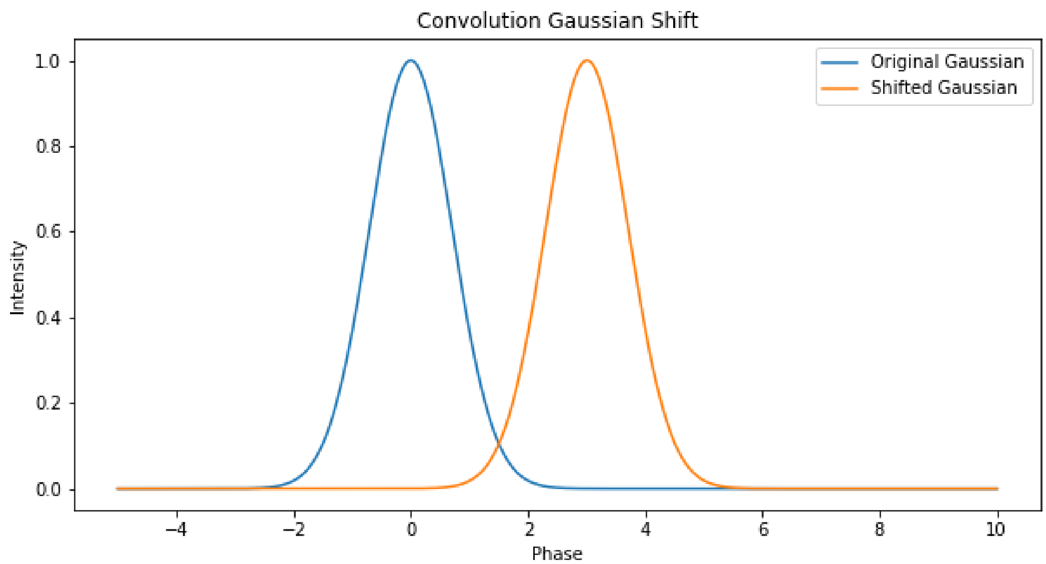
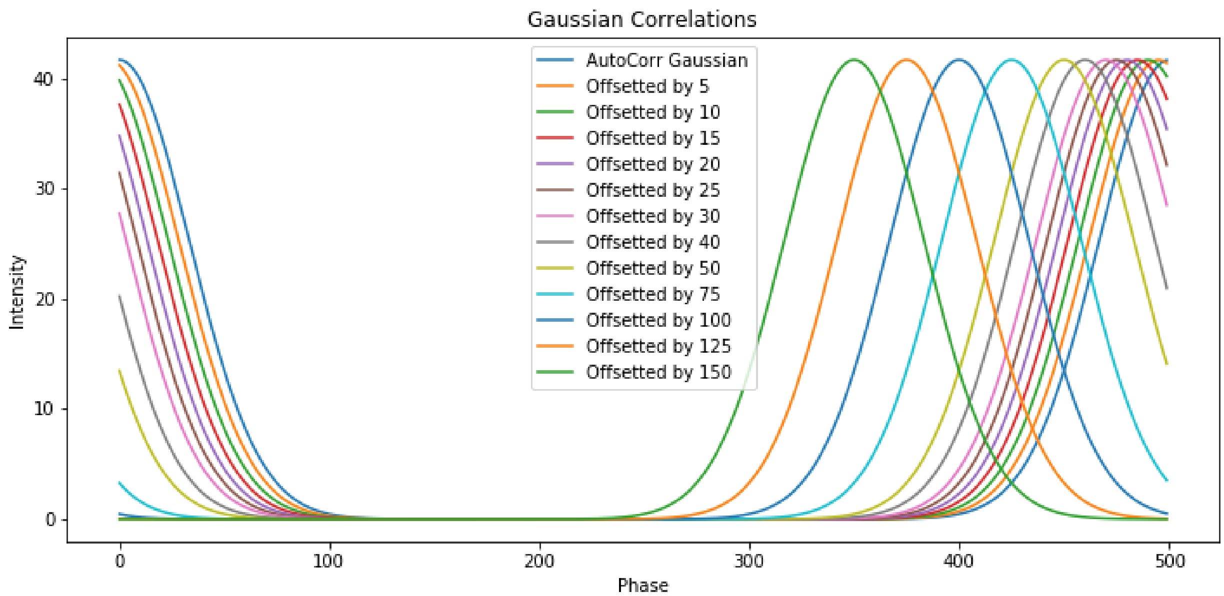
Problem 1)

```python
def dftOffset(fun:np.ndarray, x0:int):

    assert len(fun)%2==0, f"array length {fun.shape} must be even 1-d"
    N = len(fun)

    funft = rfft(fun)
    CTFactor = np.exp(-2j * np.pi * x0/N * np.arange(0, N//2 + 1)) #Got reccomenda
    offsetted = irfft(funft * CTFactor)

    return offsetted
```



Problem 2)

(a) Since the convolution of two gaussians is a gaussian and they're even functions, we the correlation matches the convolution. Subsequently, the correlation of a gaussian with an offsetted version of itself is the convolution of a gaussian then offsetted by negative that amount. A gaussian's correlation to itself is a gaussian.



(b)

```
def correlation(u,v):

    return irfft(rfft(u) * np.conj(rfft(v)))
    #This function will throw back the correlation of (u,v)

def offsettedAutoCorrelation(u, offset):

        #Correlating some fun with its shifted self
    u_offset = np.roll(u,offset)
    return correlation(u, u_offset)

```

Problem 3)

```python

def wraplessCorrelation(u,v):

        #Question of Padding: pad u and v with zeroes to maintain the center of convolution
    zeroes = np.zeros(len(v))

        #To avoid circulant conditions, padding input with zeroes should be sufficent.
    u_Padding = np.hstack((u, zeroes))
    v_Padding = np.hstack((v, zeroes))

        #Does the correlation of (u,v) but without the circulant, as it can be problematic...
    return correlation(u_Padding, v_Padding)

```

Q4)

(a) let's demonstrate that

$$\sum_{x=0}^{N-1} \exp[-2\pi i k x/N] = \frac{1-\exp[-2\pi i k]}{1-\exp[-2\pi i k/N]}$$

$$\hookrightarrow \left(1-\exp[-2\pi i k/N]\right) \sum_{x=0}^{N-1} \exp[-2\pi i k x/N] = 1-\exp[-2\pi i k] =$$

$$= \sum_{x=0}^{N-1} \exp[-2\pi i k x/N] - \sum_{x=0}^{N-1} \exp[-2\pi i k(x+1)/N]$$

Every term except $x = \begin{cases} 0 \\ N-1 \end{cases}$ cancel. This results in:

$\exp[0] = 1$ and $\exp[-2\pi i k \frac{N}{N}] = e^{-2\pi i k}$

$$\Rightarrow \left(1-e^{-2\pi i k/N}\right) \cdot \sum_{x=0}^{N-1} \exp[-2\pi i k x/N] = 1-e^{-2\pi i k}$$

(b) let's show that this approaches N as k approaches zero, and it is zero for any integer K (not a multiple of N)... {L'Hôpital's}

As $k \to 0$, $$\lim_{k\to 0} \frac{1-\exp[-2\pi i k]}{1-\exp[-2\pi i k/N]} = \lim_{k\to 0} \frac{\frac{d}{dk}\left(1-\exp[-2\pi i k]\right)}{\frac{d}{dk}\left(1-\exp[-2\pi i k/N]\right)}$$

$$= \lim_{k\to 0} \left(\frac{2\pi i}{2\pi i/N}\right)\left(\frac{\exp[-2\pi i k]}{\exp[-2\pi i k/N]}\right) = N$$

$\rightarrow$ It must be $=$ zero for non-multiples of N as $(K \% N) \neq 0$

(c)

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \rightsquigarrow \text{let's scale this arbitrarily:}$$

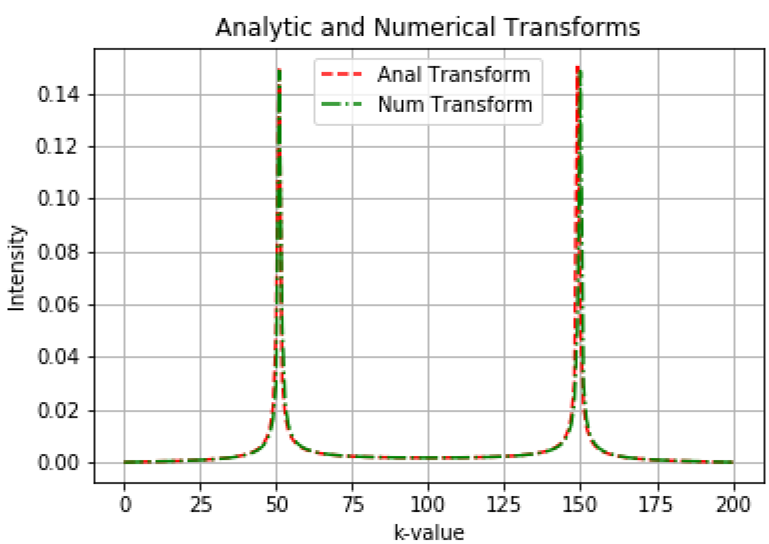
$$2i\sin\left(\frac{2\pi k' x}{N}\right) = e^{2\pi i k' x/N} - e^{-2\pi i k' x/N}$$

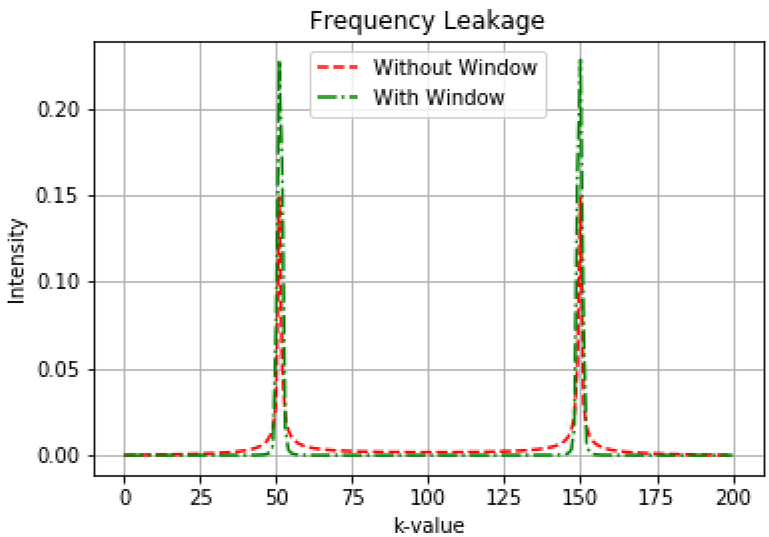
$$\hookrightarrow \text{DFT} \Rightarrow F(k) = \sum_{x=0}^{N-1} \exp\left[\frac{-2\pi i k x}{N}\right] \cdot \left(\exp\left[\frac{2\pi i k' x}{N}\right] - \exp\left[-\frac{2\pi i k' x}{N}\right]\right)$$

$$= \sum_{x=0}^{N-1} \exp\left[\frac{2\pi i (k'-k)x}{N}\right] + \sum_{x=0}^{N-1} \exp\left[-\frac{2\pi i (k'+k)x}{N}\right]$$

$$= \frac{1 - \exp[2\pi i (k'-k)]}{1 - \exp\left[\frac{2\pi i (k'-k)}{N}\right]} + \frac{1 - \exp[-2\pi i (k'+k)]}{1 - \exp\left[-\frac{2\pi i (k'+k)}{N}\right]}$$

Now, we may compare the analytic and numerical estimations of our transforms...





Clearly Less leaking with window (sharper peaks falling to zero more quickly in k-space).

Q4 AND 5 WORK IN PROGRESS
(informed TA I would be slightly late)