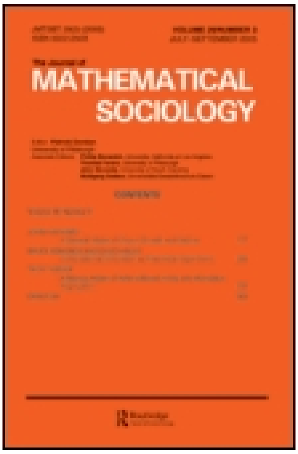

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Spectral Analysis of Social Networks to Identify Periodicity

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SPECTRAL ANALYSIS OF SOCIAL NETWORKS TO IDENTIFY PERIODICITY

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Two key problems in the study of longitudinal networks are determining when to chunk continuous time data into discrete time periods for network analysis and identifying periodicity in the data. In addition, statistical process control applied to longitudinal social network measures can be biased by the effects of relational dependence and periodicity in the data. Thus, the detection of change is often obscured by random noise. Fourier analysis is used to determine statistically significant periodic frequencies in longitudinal network data. Two approaches are then offered: using significant periods as a basis to chunk data for longitudinal network analysis or using the significant periods to filter the longitudinal data. E-mail communication collected at the United States Military Academy is examined.

Keywords: Fourier analysis, longitudinal networks, network dynamics, social network analysis, statistical process control

1. INTRODUCTION

Longitudinal social networks are an important area of study in social network analysis. As Wasserman, Scott, and Carrington (2007) described, "the analysis of

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social networks over time has long been recognized as something of a Holy Grail for network researchers" (p. 6). Doreian and Stokman (1997) described the concept of "network dynamics" as the field of study that assumes an underlying stochastic process that drives network behavior over time. McCulloh and Carley (2008) extended this concept to describe four network dynamic behaviors that a network can exhibit over time. First, a network can remain *stable*. This means that the underlying relationships among agents in a network remain the same, even though there may exist some fluctuation in observed links within the network due to measurement error or weak relationship. It can be analyzed as a static network (McCulloh, Lospinoso, & Carley, 2007; McCulloh & Carley, 2010; Wasserman & Faust, 1994). Next, a network can *evolve.* This occurs when relationships among agents change as a result of agent interaction, exchange of beliefs and ideas, or as agents gain a greater knowledge of the traits and resources of other agents in the network. Network evolution has been explored through multiagent simulation (Doreian & Stokman, 1997; Banks & Carley, 1996; Sanil, Banks, & Carley, 1995; Carley, 1996, 1999). Network evolution has also been explored through Markov chains (Leenders, 1995; Snijders, 1996, 2001, 2007; Snijders & Van Duijn, 1997; Wasserman & Pattison, 1996). A network can exhibit a *shock,* which occurs when some exogenous impact to the network causes relationships to change (McCulloh & Carley, 2008). Finally, a network can experience a *mutation* if a shock initiates evolutionary change (P. Doreian, personal communication, December 2008). Distinguishing between these four different types of network behavior over time is important for understanding the social mechanisms that drive over-time behavior in social groups.

Social network change detection (McCulloh & Carley, 2008) applies statistical process control to graph level measures within a social network to detect statistically significant changes in a network over time. This has been found to be effective in several different data sets ranging from terrorist networks (McCulloh, Webb, & Carley, 2007) to e-mail networks (McCulloh & Carley, 2008; McCulloh, Johnson, Sloan, Graham, & Carley, 2009; McCulloh, Ring, Frantz, & Carley, 2008). Social network change detection estimates the mean and variance of a graph level measure within a longitudinal set of social networks. Sequential observations of the graph level measure are standardized using the estimated mean and variance and then used to calculate some statistic on the network. The test statistic is compared to some decision interval. If the statistic exceeds the decision interval, then the procedure indicates that there may have been a change in the network. The network analysts can use certain change statistics to estimate the point in time when the change most likely occurred. This change may have been evolutionary in nature or may have been caused by some exogenous source such as a shock. Identifying that the change occurred and when the change occurred are the first two steps in understanding the network dynamics affecting empirical data.

One major obstacle to the study of network dynamics is periodicity or over-time dependence in longitudinal network data. For example, if we define a social network link as an agent sending an e-mail to another agent, we have a time-stamped data set. Intuitively, we can imagine that individuals are more likely to e-mail each other at certain times of the day, days of the week, and so forth. If the individuals in the network are students, then their e-mail traffic might follow the school's academic calendar. Seasonal trends in data are common in a variety of other applications as well. When these periodic changes occur in the relationships that define social network links, social network change detection methods are more likely to signal a *false positive*. A false positive occurs when the social network change detection method indicates that a change in the network may have occurred, when in fact there has been no change. To illustrate, assume that we are monitoring the density of the network for change in hourly intervals. The density of the network measured for the interval between 3 a.m. and 4 a.m. might be significantly less than the network measured from 3 p.m. to 4 p.m. because most of the people in the network are asleep and not communicating between 3 a.m. and 4 a.m. This behavior is to be expected, however, and is not desirable for the change detection algorithm to signal a potential change at this point. Rather, it would be ideal to control for this phenomenon by accounting for the time periodicity in the density measure. Only then can real change be identified quickly in a background of noise.

Periodicity can occur in many kinds of longitudinal data. Organizations may experience periodicity as a result of scheduled events, such as a weekly meeting or monthly social event. Social networks collected on college students are likely to have periodicity driven by both the semester schedule and academic year. Even the weather may introduce periodicity in social network data, as people are more or less likely to e-mail or interact face-to-face. At the United States Military Academy, people tend to run outside in warm weather in small groups of two or three. During the winter, people go to the gym, where they are likely to see many people. This causes an increase in face-to-face interaction as people stay inside. In a similar fashion, during the spring and fall, many people participate in interunit sporting events such as soccer or Frisbee football. This can also affect face-to-face interaction and the social network data collected on them.

Spectral analysis provides a framework to understand periodicity. Spectral analysis is mathematical tool used to analyze functions or signals in the frequency domain as opposed to the time domain. If we look at some measure of a social group over time, we are conducting analysis in the time domain. The frequency domain allows us to investigate how much of the given measure lies within each frequency

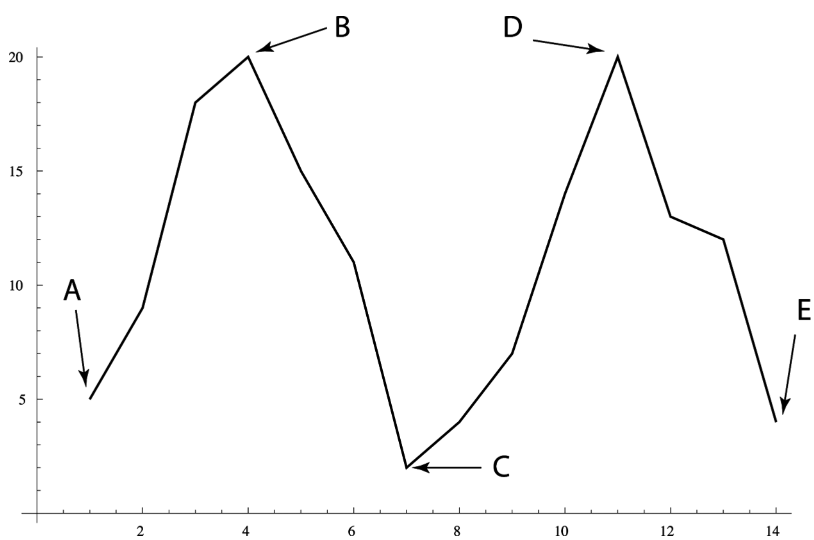

Figure 1 Notional measure in time domain.

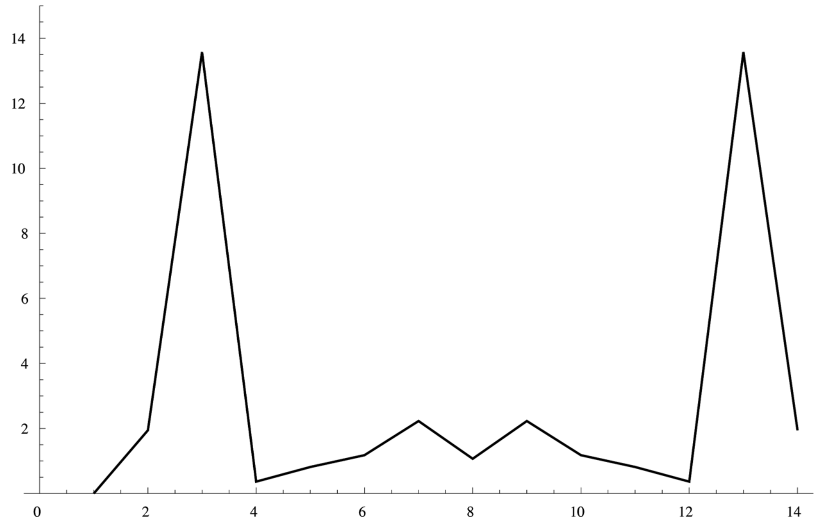

Figure 2 Notional measure in frequency domain.

band over a range of frequencies. For example, Figure 1 shows a notional measure on some notional group in the time domain. The measure is larger at points B and D corresponding to the middle of the week. The measure is smaller at points A, C, and E.

If the signal in Figure 1 is converted to the frequency domain as shown in Figure 2, we can see how much of the measure lies within certain frequency bands. The negative spike in Figure 2 corresponds to 7 days, which is the weekly periodicity in the notional signal. The actual frequency signal only runs to a value of 8 on the x-axis in Figure 2. The frequency domain signal after a value of 8 is a mirror image or harmonic of the actual frequency signal.

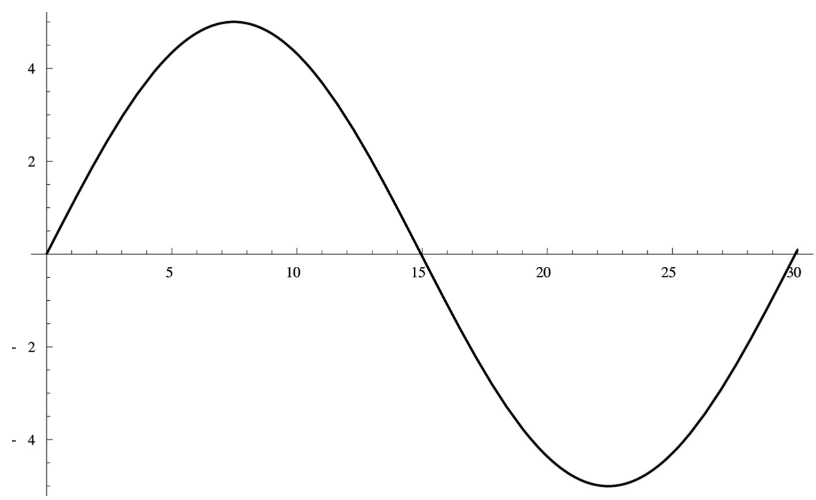

Figure 3 Monthly period.

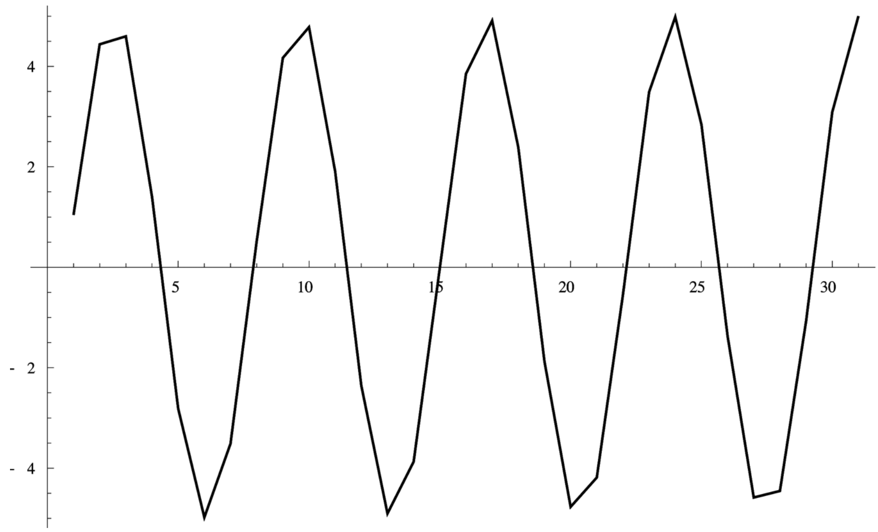

Figure 4 Weekly period.

The frequency domain representation of a signal also includes the phase shift that must be applied to a summation of sine functions to reconstruct the original over-time signal. In other words, we can combine daily, weekly, monthly, semester, and annual periodicity to recover the expected signal over time due to periodicity. For example, Figures 3–5 represent monthly, weekly, and subweekly periodicities. If these signals are added together, meaning that the observed social network exhibits all three of these periodic behaviors, the resulting signal is shown in Figure 6.

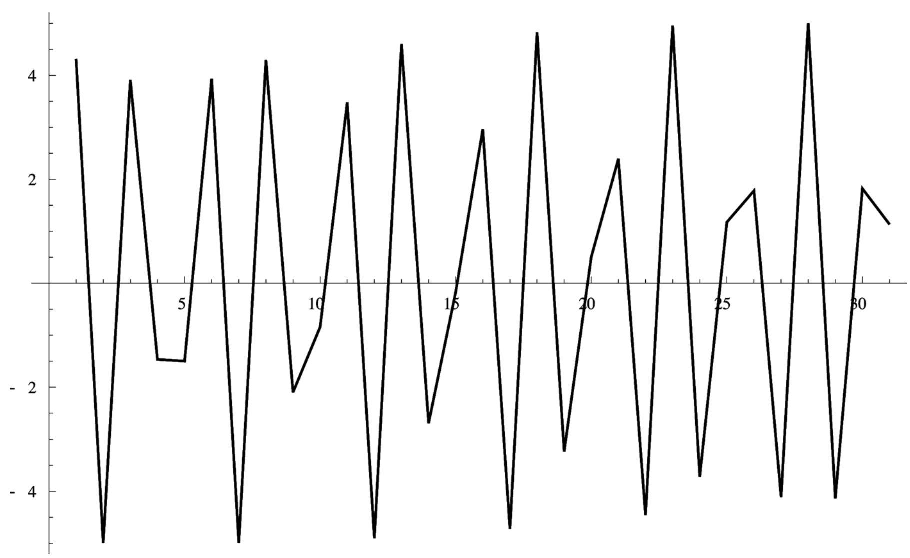

Figure 5 Subweekly period.

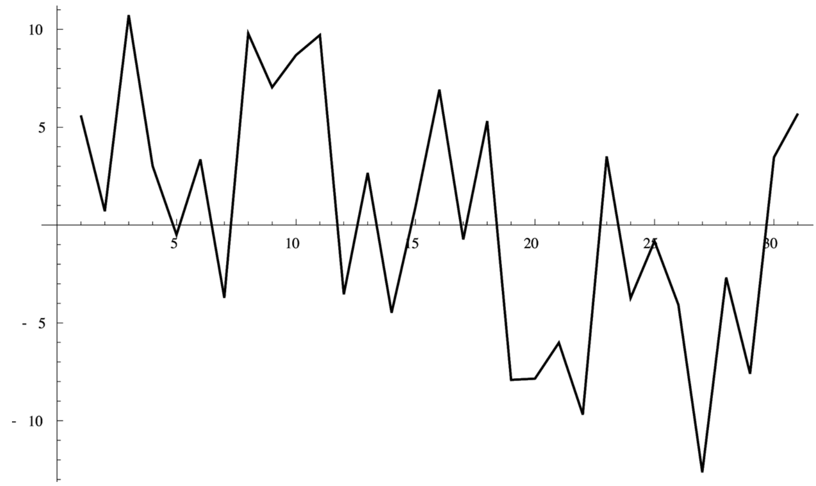

Figure 6 Sum of the signal in Figures 3–5.

If the periodicity in the signal shown in Figure 6 is not accounted for, it appears that there may be a change in behavior around time period 20, where the signal is negatively spiked. In reality, this behavior is caused by periodicity. If we transform the signal to the frequency domain as shown in Figure 7, we can see the weekly periodicity at point B and the subweekly periodicity at point A.

We propose that spectral analysis applied to social network measures over time will identify periodicity in the network. We will transform an over-time network measure from the time domain to the frequency domain using Fourier analysis.

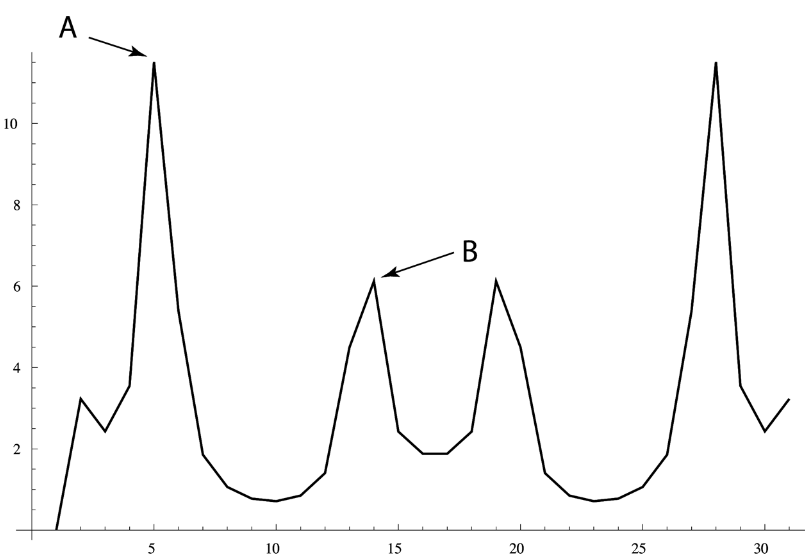

Figure 7 Transformation of Figure 6 to the frequency domain.

We will then identify significant periodicity in the over-time network and present two methods for handling the periodicity. This newly proposed method will be demonstrated on real-world data sets as well as simulated data sets.

Handling periodicity is very important. For social scientists to gain insight into the evolution of social networks, they must be able to distinguish among shock, evolutionary change, and typical periodic behavior. We will present a method for identifying and removing the periodic behavior of a signal so that change detection can be performed more accurately.

2. BACKGROUND

Networks can be described by a number of different measures. Measures can be defined for individual nodes or for the network as a whole. We will restrict our attention to network level measures, but there is no reason that the methodology presented could not be applied to node level measures as well. Common network level measures include density, the number of nodes in the network, and the average path length. In addition, node level measures such as betweenness, closeness, and eigenvector centrality can be averaged over all nodes in a network to create network level measures. For more information on social network measures, both graph level and node level, the reader is referred to Wasserman and Faust (1994).

Measures may fluctuate in a periodic fashion over time. As agents in a network change their relationships to other agents based on seasonal trends, these fluctuations may be noticed in the network measures of those relationships. For example, during the workweek, one might expect more e-mail communication within an office than during the weekend. This could be observed by a greater network density (percentage of possible relationships) during the week than during the weekend. The social network measures therefore provide a measure of the group as a whole.

Spectral analysis can be used to detect periodicity within social network measures over time. Periodicity in the social network measure provides some insight into the periodicity of the underlying social organization. Spectral analysis can be used to either filter out periodicity in overtime measures or provide insight into how data should be aggregated to best represent a social group.

Spectral analysis is a mathematical process of converting a function or series from the time domain into the frequency domain. A function or signal can be converted from the time domain to the frequency domains with a transformation. A common transformation is the Fourier transform, which decomposes a signal into a sum of sine waves having different phase shifts and amplitudes. The Fourier transform is given by

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-iz\pi ft}dt.$$

A convenient property of the Fourier transform is that the inverse of the Fourier transform is also a Fourier transform. This property makes it convenient to convert back and forth between the time and frequency domains. We will use this property to convert a signal from the time domain to the frequency domain, identify significant frequencies, and convert those frequencies back into the time domain to provide an understanding of the periodicity inherent in longitudinal social network measures.

3. DATA

The approaches for handling periodicity in network data are demonstrated on a longitudinal data set of e-mail traffic collected at the United States Military Academy at West Point, New York. This data set was collected in part to demonstrate longitudinal network analysis. The participants were 25 undergraduate cadets at the United States Military Academy serving in military leadership positions in one of four cadet regiments. All participants volunteered to allow us to monitor the header information of their e-mail traffic for the Fall 2008 semester. This study was approved for ethics by the West Point Institutional Review Board. The e-mail header information was used to create social networks by assigning a directed link from node i to node j if node i sent node j an e-mail sometime during the designated time period. This unique data set allowed us to investigate the periodicity of the data for many hourly networks or a few monthly networks. In addition, we were able to interview the participants to investigate potential causes of periodicity in the e-mail communication networks.

While the West Point cadet data are sufficient to demonstrate spectral analysis of networks, we use a simulated periodic signal to demonstrate the importance of spectral analysis for change detection. The simulated data consists of a simulated sine wave representing some measure of interest, where a change in the mean of the wave is introduced at a known point in time. Random uniform error between 0 and the amplitude of the sine wave is added to the signal. The accuracy of the CUSUM change point identification against a background of noise is then compared between whether spectral analysis is applied or not.

4. METHOD

The spectral analysis approach proposed in this article consists of five steps to determine the significant periodicity and then suggests two methods of handling the periodicity in the data.[1] We list these steps here and demonstrate them on the West Point Cadet data in the next section.

4.1. Step 1: Plot the Measure of Interest

This first step is to determine network measures of interest. These can be network level measures or node level measures. In this article we have restricted our attention to network level measures. For the purpose of demonstration, we will use the average betweenness of nodes in the network as a network level measure. Another issue in this step is the number and length of time periods. In this example, we investigate daily networks with the hope of determining weekly or monthly periodicity. We could measure hourly networks or even networks corresponding to each second of the day. Intuitively, smaller time periods will result in sparser networks. Some amount of judgment will be required by the analyst to select an

[1]These methods have been made available as part of the over-time analysis report in *ORA, http://www.casos.cs.cmu.edu/projects/ora.

aggregation level where most of the nodes in the network are connected, but every node is not necessarily connected to every other node.

4.2. Step 2: Discreet Fourier Transform

The second step is to transform the network measure of interest from the time domain to the frequency domain. Since the network measures correspond to discrete time periods and the measure is not continuous, the Fourier transformation cannot be applied directly. A discrete version of the Fourier transform is used. The discrete version is given as

$$X(f) = \sum_{k=0}^{N-1} x(k)e - \frac{i2\pi fk}{N} \quad f = 0, 1, \ldots, N-1.$$

Henceforth, when describing the Fourier transform, we mean the discreet version. This operation is standard in many mathematical software packages such as MATLAB and Mathematica. It is also available in the Organizational Risk Analyzer (ORA) social network analysis software.

4.3. Step 3: Determine Normal Frequencies

The third step is to determine the normal range of frequencies for the signal. The Fourier coefficients of the transformation are estimated by the sum of independent random variables. The mean of the coefficients approaches the normal distribution as the sampling size (N) tends towards infinity in accordance with the central limit theorem. Therefore, we may assume that the frequencies of the transformed signal approximate a normal distribution. In fitting a normal distribution to the frequencies, we will be able to determine statistically anomalous or significant frequencies.

4.4. Step 4: Identify Significant Frequencies

This step requires that the analyst determine a confidence level for detecting periodicity. The 95% confidence level is approximately equal to ± 2 standard deviations from the mean frequency. Therefore, all frequencies within two standard deviations from the mean are set to equal 0. This creates a new discrete signal in the frequency domain of only statistically significant signals.

4.5. Step 5: Identify Significant Periods

Recall that the Fourier transform has an inverse given by

$$X(k) = \sum_{k=0}^{N-1} x(f)e - \frac{i2\pi fk}{N} \quad k = 0, 1, \ldots, N-1.$$

Therefore, the Fourier transform is applied to the discrete signal in Step 4 to determine the significant periodicity.

At this point the analyst has two options for handling the periodicity in the data. The simplest method is to aggregate over the period. For example, the analyst may find weekly periodicity. People may have different email behavior on the weekend than they do during the weekday. The analyst could then aggregate over the daily networks to create weekly networks. Then the weekly periodicity would be controlled within each weekly network. If the network becomes too dense by establishing a link between nodes for a single weekly e-mail, the analyst is free to require more than one e-mail message to define a link.

The analyst can also choose to keep using the daily networks but control for the periodicity. The discrete signal in Step 5 is really the expected value of the chosen social network measure from Step 1 for each point in time. The analyst can create a filtered network measure by taking the difference between the original signal from Step 1 and the signal from Step 5. This new signal is then a filtered signal that can improve the performance of social network change detection.

This second approach for handling periodicity is investigated through simulation. A periodic signal is simulated in Mathematica, a mathematical software environment. The signal is shifted at a particular point in time. Uniform random noise is added to the signal where the range of error is equal to the amplitude of the signal. The CUSUM change detection algorithm is applied to the periodic signal as well as a signal filtered in the manner described above. The change point identification of the CUSUM applied to each signal is compared.

5. RESULTS

The West Point cadet data average betweenness is displayed in Figure 8 for a 1-month period during the Fall 2008 semester. If an analyst were just looking at this data, it may appear that the average betweenness is unusually high around Day 15. There also appears to be moderately high values around Day 8 and Day 22.

The Fourier transform is applied to the average betweenness scores, transforming these values from the time domain to the frequency domain. A plot of

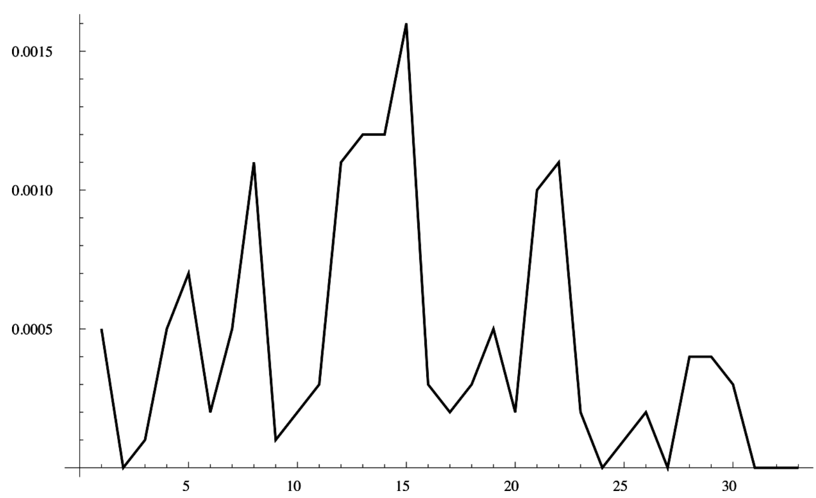

Figure 8 Cadet data average betweenness.

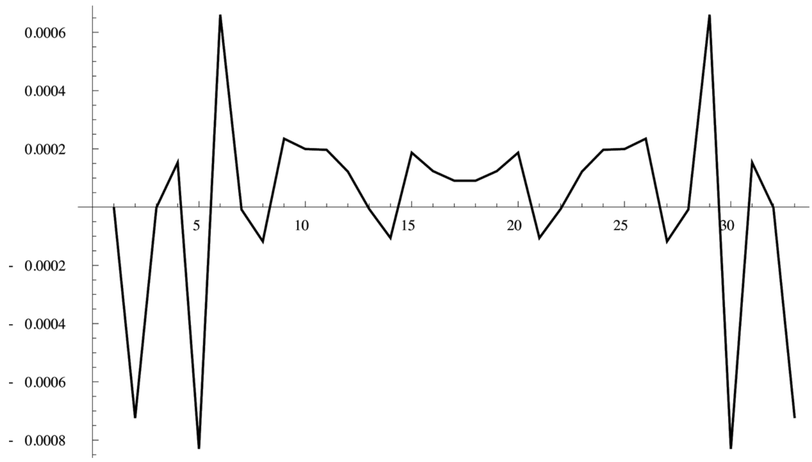

Figure 9 Fourier transform of average betweenness.

the transformed values is shown in Figure 9. It appears that there may exist significant periodicity in the over-time measure.

A normal distribution is fit to the discrete frequency signal and values within two standard deviations of the mean are set equal to zero. Figure 10 shows the significant frequencies.

The significant frequencies are transformed back into the time domain. This is known as taking an inverse transform of the signal. The resulting plot in the time domain can be interpreted as the significant periodicity in the measure, since only the significant frequencies were transformed back into the time domain. The significant frequencies are plotted in the frequency domain. The significant periodicity, on the other hand, is plotted in the time domain. Figure 11 displays a plot of the significant periodicity in the average betweenness signal.

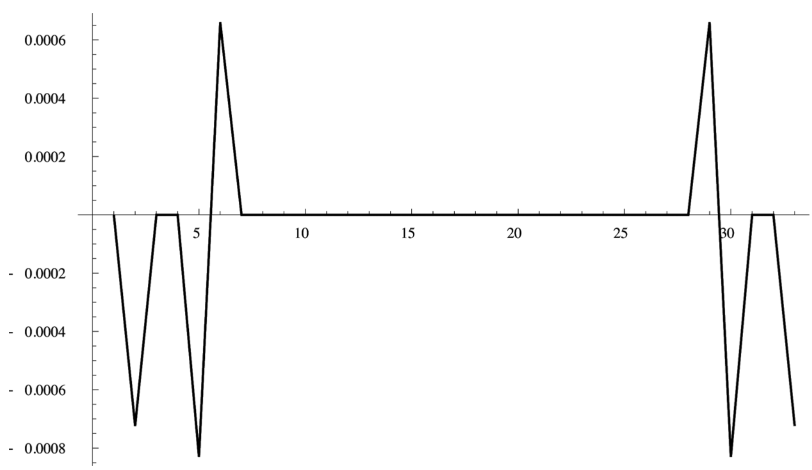

Figure 10 Significant frequencies in cadet data.

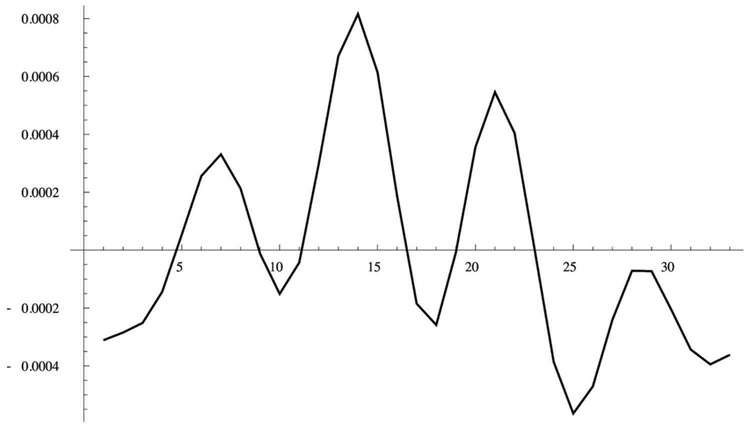

Figure 11 Significant periodicity in cadet data.

It can be seen in Figure 11 that there is a spike in significant periodicity corresponding to Days 7, 14, 21, and 28. This is perfect weekly periodicity. An interview with the regimental commander of the participants in the study revealed that the participants have a weekly meeting every Sunday. During this meeting, important information is given to the group regarding events and activities for the week. In addition, subordinate leaders are required to account for the whereabouts of all of the cadets within their subordinate units and report the information up the chain of command. This process of sending information up and down the chain of command will significantly affect the average betweenness of the network on Sundays. Failing to account for this behavior may in turn affect an analyst's ability to detect real organizational change within this group.

At this point, an analyst can choose to monitor weekly networks, or continue to monitor daily networks and filter out some of the periodicity. Figure 12 shows a

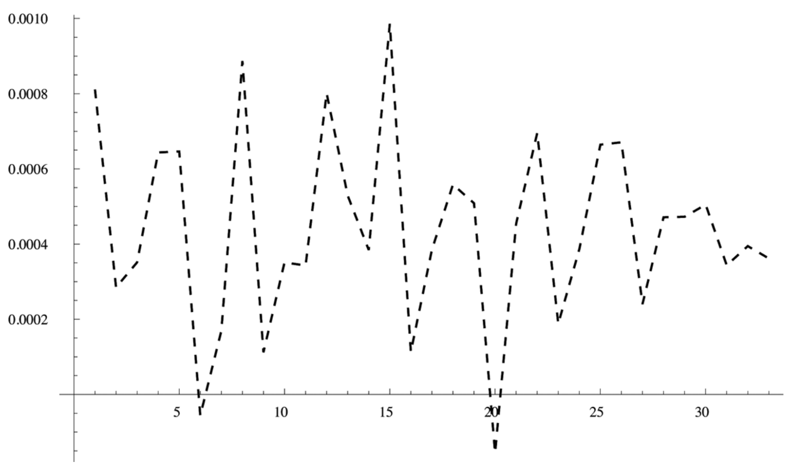

Figure 12 Filtered plot of average betweenness in cadet data.

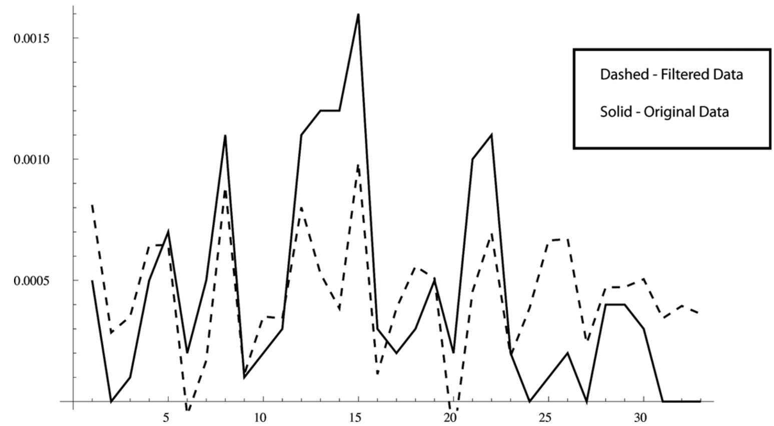

Figure 13 Original and filtered plots of average betweenness.

filtered signal in the time domain. Taking the original signal found in Figure 8 and subtracting the periodicity found in Figure 11 for each time period obtained this signal. In effect, the new figure shown in Figure 12, displays the deviation from what is expected in the signal due to the time of week.

Figure 13 shows the original and filtered signals together. It can be seen that the extreme values of average betweenness detected in our first observation of the network do not appear as extreme in the filtered signal. Therefore, the filtered signal is less likely to cause a false alarm in change detection.

To further illustrate the importance of accounting for periodicity, we turn our attention to an extreme case. Figure 14 displays a sine wave, where a change in the mean of the signal occurs at Time Period 40. In addition to the periodicity, noise is added to the signal in the form of uniform random error with a range equal to the

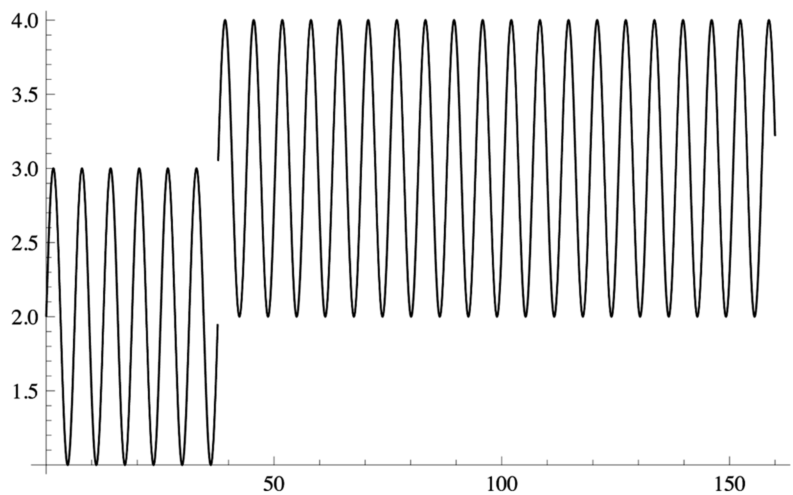

Figure 14 Sine wave with change at Time 40.

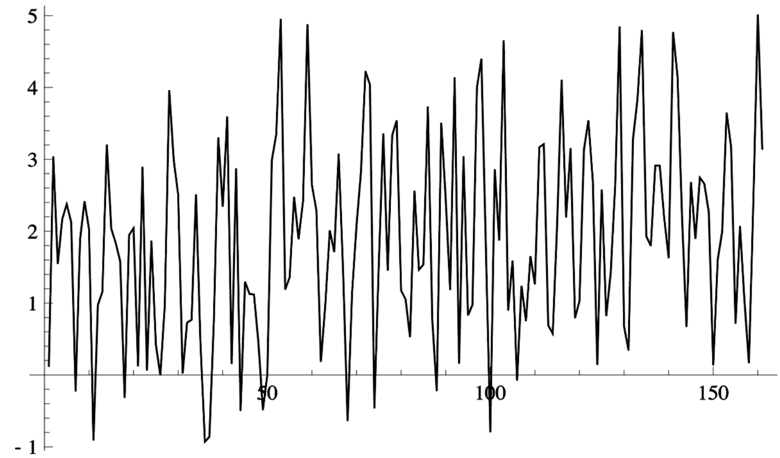

Figure 15 Sine wave with random error and change at Time 40.

amplitude of the sine wave. A random instance of this signal is displayed in Figure 15. It can be seen that identifying the change at Time Period 40 may be difficult with the combination of periodicity and noise.

The CUSUM change detection algorithm is applied to the noisy signal in Figure 15. Figure 16 shows a plot of the CUSUM statistic. The CUSUM statistic can be powerful in illuminating subtle change in a background of noise. It also appears that the algorithm may have signaled false alarms around Time Points 10 and 30. It is not clear that there is a good solid indication of change until after Time Point 50.

The filtering approach can be extremely useful in improving the performance of the change detection approach. Figure 17 shows a plot of the CUSUM statistic on the

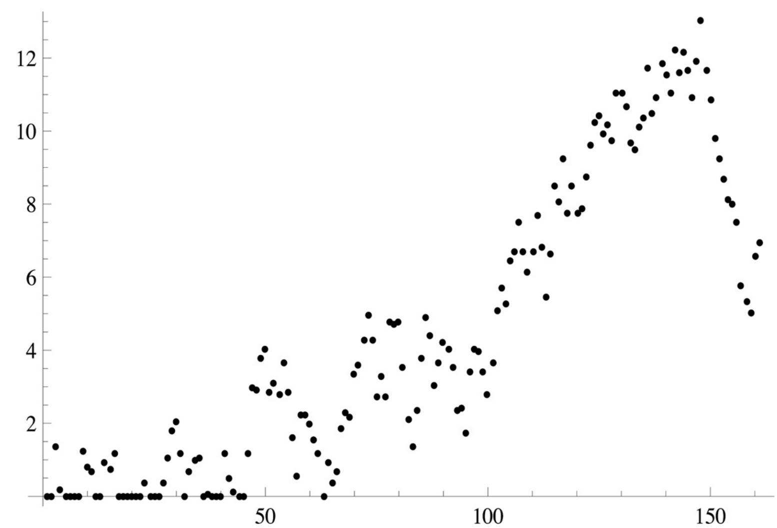

Figure 16 CUSUM statistic applied to noisy sine wave.

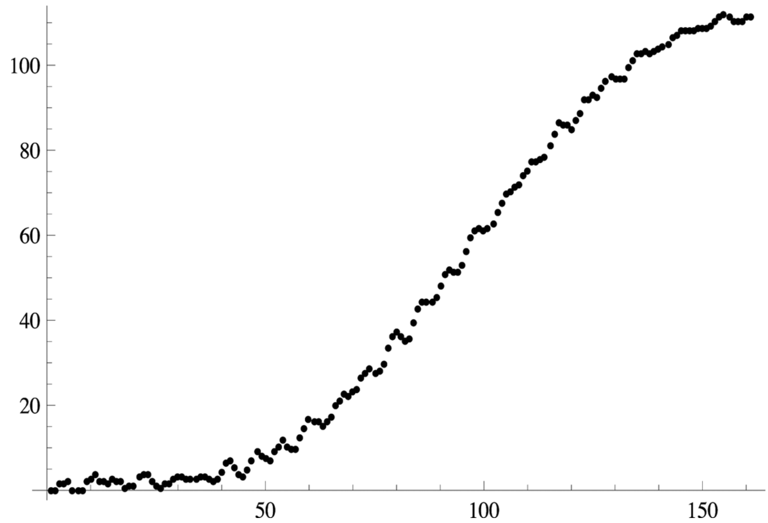

Figure 17 CUSUM statistic applied to filtered signal.

same signal as Figures 15 and 16, where the signal was first filtered for periodicity using the steps outlined above. It can be seen in Figure 17 that the signal may more accurately identify the correct change point in the signal and is less prone to false signal.

The simulation was repeated with four different levels of uniform random noise. The level of random noise was set as a percentage of the amplitude of the sine wave at 30%, 50%, 67%, and 100%. The change occurred at time 40, and the size of the change was the amplitude. The average time to detect the change was compared across the four levels of noise. For each simulation run, the CUSUM was applied to both the original signal and the filtered signal. A pair-wise t test for the time to detect change was conducted between the original and filtered signals for 100 independently seeded instances of the noisy sine wave. The null hypothesis was that there was no difference between detection performance between the original and filtered signals. The p values for this null hypothesis are 0.05, 0.04, 0.72, and 0.88, respectively, for noise levels of 30%, 50%, 67%, and 100% of amplitude. The p values for the error that was less than or equal to 50% of amplitude are significant, indicating that the filtering improves the time to detect a change. The p values for the error that was greater than 50% of the amplitude are not significant, meaning we have no reason to reject the null hypothesis that filtering does not improve change detection.

This behavior in performance appears reasonable. If the periodicity in the over-time measure is greater than the level of observation error, then filtering the signal is likely to improve change detection performance. If, on the other hand, the level of error in the observed over-time measure is greater than the periodicity, then spikes in error may appear as a significant frequency, which may adversely bias the change detection algorithm. It is possible that if the error is much greater than periodicity, the spectral analysis may even mask true change. Future work should investigate the impacts of spectral analysis on change detection performance.

6. CONCLUSION

Periodicity is an important issue in the longitudinal analysis of social networks. Intuitively, peoples' observable relationships may change with the time of day, week, month, year, and so forth. Accurate modeling of social network relations therefore requires a way to account for and control for this periodicity. This issue is especially important for any longitudinal analysis.

Fourier analysis can detect periodicity and provide insight to control for its effect. The success of this approach has been demonstrated on both real-world and simulated data sets. More research is needed to investigate how observation error and organizational dynamics might affect the periodicity. It is expected that if the random error in the signal is much higher than the amplitude, the filtering techniques proposed here might not be effective. Likewise, if there is very little error, filtering may be unnecessary. For most longitudinal analysis, however, we propose that applying the approach laid out in this article may detect significant periodicity and therefore improve the performance of change detection.

The spectral analysis has only been investigated for filtering and detecting trigonometric cycles in an over-time signal. It is conceivable that some forms of periodicity may not follow a trigonometric cycle. For example, major holidays in the United States are likely to affect communication patterns between individuals; however, they do not occur on the calendar with regular trigonometric frequency. In addition, changes in relations may taper off suddenly as in the case of an organization that has a prescribed start and stop time to the workday. In this situation, a sine wave may not appropriately capture the periodic behavior of the group. More research into wavelets that consider different periodic signals is warranted. While the same general approach laid out here may apply, the choice of transformation may differ.

The success of spectral analysis will be related to the number of available time periods with network data. This approach requires continuous data with many time periods. This type of data may be difficult to obtain. In some cases the number of longitudinal networks may be already aggregated over some period of time. We recommend that a prospective analyst apply this approach when looking at longitudinal data, but be aware of the potential problems when investigating fewer than 10 longitudinal networks.

Spectral analysis of longitudinal network measures appears to be a powerful technique for understanding periodicity in over-time data. While an entire special issue of a journal could be devoted to this topic, we have shown how it can be effective on one real-world data set. We have further demonstrated how spectral analysis can improve the performance of the CUSUM algorithm using a simulated noisy sine wave. In addition to the change detection performance implications, this approach also leads to interesting insights into organizational behavior. The spectral analysis of the West Point cadet data, for example, revealed the organization's weekly meeting time. Whether used for change detection or simply organizational insight, spectral analysis represents a major contribution to the study of longitudinal network data.

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