1 Goal

We would like to show that at each step the algorithm returns a correct result and at the end it returns a complete result.

1.1 Strategy

beginning *nth step* *final*

1.1.1 *beginning*

fff

1.1.2 nth step

```
- Move the terms to below to definitions
- Define beachline as an inductive, draws the similarity between its
definition and balanced brackets

   - Write how to move from a beachline to show the voronoi diagram
up to the beachline is correct
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We divide the plane into three regions :

1. The **cliff**: Everything (*sites, edges, cells*) in this region is complete and it doesn't affect anything below this line. This regions is characterized as anything doesn't belong to the.

   Proof:

   (handwavy) every site in this region must be enclosed by a cell since it disappeared from the beach-line by circle event(s). Then from triangle inequality, argue that nothing beyond its cell bleong to the site's region.

2. **Beach**: The region between the sweep-line and the beach-line.

3. **offshore** Below the sweep-line: This region has no affect on lies above it.

   Proof:

   Anything above the beach-line is closer to regions *2* and *3* than any site below the sweep-line.

4. **Events**

   - *Site event* :

     Adding a new site will affect the *offshore* and the *beach*

   - *Circle event:*

     - Definition : when three sites are equidistant to each other
     - It happens at the top of the circle

- When adding a new site p, it's enough to check the three consecutive arcs where p is the right-most or the left-most.

  Proof: Other circle events has been discovered before and when there are other circle events involves p but the arcs are not consecutive the they are necessarily has a lower priority, thus they will be discovered when an arc is removed and become consecutive.

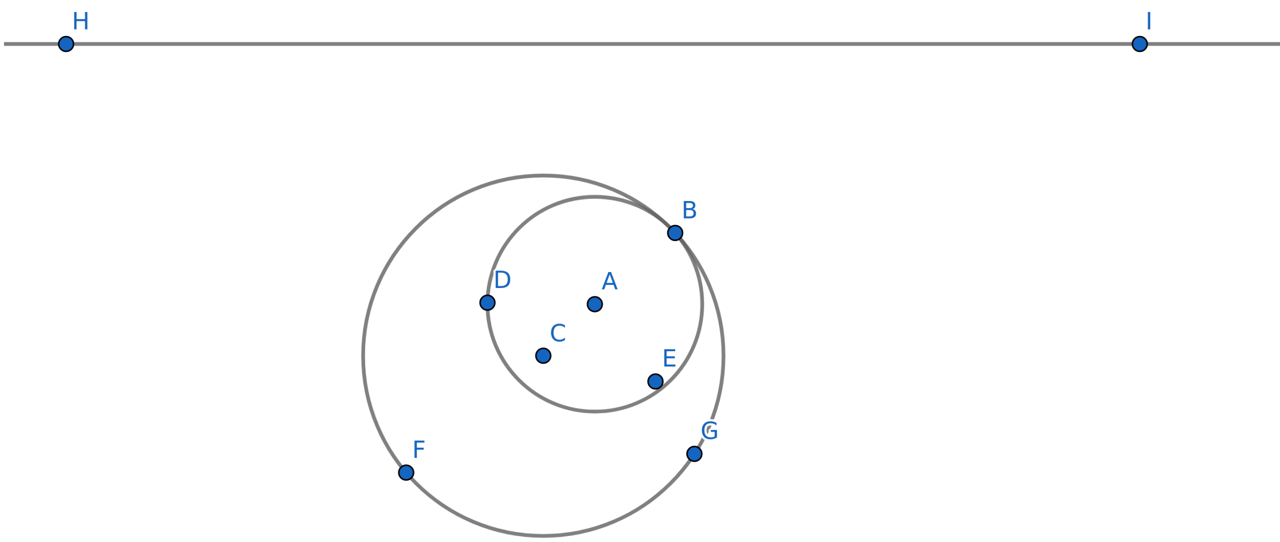

Figure 1: circle events

- moving the *sweepline*

  How the three components will be affected

  * no event *event* is encountered: Update the intersection points and edges
  * *site event* update intersection points:
  * *circle event*

- overriding events

1.1.3 Final

2 Definitions

2.1 Cells

For any site, if the collection of edges that has the site as one of the focal points form a cycle. Also, each end of those edges are equidistant from three sites then this cell is convex and a valid Voronoi decomposition.

Proof: This cell is formed by an intersection of half-planes thus it's convex. Also, it's valid since if a, b are equidistant from p, q then the whole segment $[a, b]$ is equidistant from p, q.

3 Functions

3.1 Parabola Functions

3.1.1 Intersection of two arcs

Let p_1, p_2 be two arcs and their intersections are a_1, b_1 then as the sweepline moves there will be new intersections $(a_1, b_1), (a_2, b_2), \ldots (a_k, b_k)$ the line segments formed by a_is lies on the same line similarly b_is satisfy the same property.

The main method is to show that a solution belongs to two curves which boils down to algebraic manipulations.

3.2 Lines, Circles, and distances

4 Events

We assume that the algorithm has processed n events. We would like to check if we have given a correct decomposition then preforming a site event or a circle event will preserve the correctness of the decomposition up to the cliff.

4.1 Bound on events length

If the beach-line is of length after a site event then at most there will be $n - 2$ circle events. Thus, the queue length can't exceed n^2 where is the number of sites. Tightening this bound has no almost effect on the performance as it sole role to tell Coq the main function's recursion will terminate.