SUPPLEMENTARY INFORMATION for
"Robust electric dipole transition at microwave frequencies for nuclear spin qubits in silicon",
by G. Tosi *et.al.*

S1. Electron gyromagnetic ratio

Even though it does not have a substantial effect of the physics presented in this paper, in the simulations of Fig. 2 we have considered, for completeness, that the electron gyromagnetic ratio is different when the electorn is at the donor or at the Si/SiO_2 interface. Therefore, the electron Zeeman Hamiltonian depends on the electron position via the charge qubit σ_z operator:

$$\mathcal{H}_{B_0}^{\mathrm{orb}} = \gamma_e B_0 \left[1 + \left(\frac{1+\sigma_z}{2}\right)\Delta_\gamma\right] S_z - \gamma_n B_0 I_z, \tag{S1}$$

where Δ_γ is the relative change in the gyromagnetic ratio between donor and interface, which can be as large as 0.7% [S1].

S2. Level hybridization

In the complex level diagram of Fig. 2, different coupling terms hybridize two-level sub-systems. Below we show the explicit form of the hybridization coefficients and energy shifts, easily obtained by matrix diagonalization.

The hybrid flip-flop-charge coefficients are:

$$\beta = \phi/\sqrt{\phi^2+1}, \quad \phi = \left(\delta_{\mathrm{so}} + \sqrt{{\delta_{\mathrm{so}}}^2 + 4{g_{\mathrm{so}}}^2}\right)/2g_{\mathrm{so}} \tag{S2}$$

$$\alpha = \theta/\sqrt{\theta^2+1}, \quad \theta = \left(\delta_{\mathrm{so}} - \sqrt{{\delta_{\mathrm{so}}}^2 + 4{g_{\mathrm{so}}}^2}\right)/2g_{\mathrm{so}} \tag{S3}$$

with a corresponding energy shift:

$$D_{\mathrm{so}} = \delta_{\mathrm{so}}\left[\sqrt{1+\left(2g_{\mathrm{so}}/\delta_{\mathrm{so}}\right)^2} - 1\right]/2 \tag{S4}$$

The hybrid up-down electron coefficients are:

$$\alpha_1 = \theta_1/\sqrt{{\theta_1}^2+1}, \quad \theta_1 = \left(\delta_1 - \sqrt{{\delta_1}^2 + (2\beta g_B)^2}\right)/2\beta g_B \tag{S5}$$

$$\beta_1 = \phi_1/\sqrt{{\phi_1}^2+1}, \quad \phi_1 = \left(\delta_1 + \sqrt{{\delta_1}^2 + (2\beta g_B)^2}\right)/2\beta g_B \tag{S6}$$

$$\alpha_2 = \theta_2/\sqrt{{\theta_2}^2+1}, \quad \theta_2 = \left(\delta_2 - \sqrt{{\delta_2}^2 + (2\alpha g_B)^2}\right)/2\alpha g_B \tag{S7}$$

$$\beta_2 = \phi_2/\sqrt{{\phi_2}^2+1}, \quad \phi_2 = \left(\delta_2 + \sqrt{{\delta_2}^2 + (2\alpha g_B)^2}\right)/2\alpha g_B \tag{S8}$$

$$\alpha_3 = \theta_3/\sqrt{{\theta_3}^2+1}, \quad \theta_3 = \left(\delta_3 - \sqrt{{\delta_3}^2 + 4{g_B}^2}\right)/2g_B \tag{S9}$$

$$\beta_3 = \phi_3/\sqrt{{\phi_3}^2+1}, \quad \phi_3 = \left(\delta_3 + \sqrt{{\delta_3}^2 + 4{g_B}^2}\right)/2g_B \tag{S10}$$

where the terms $\delta_1, \delta_2, \delta_3$ are the energy splittings between pairs of Hamiltonian eigenstates in the limit $B_{\mathrm{ac}} \to 0$, as indicated in Fig. 2c.

Finally, the AC Stark-shift responsible for the creation of the first-order 'clock transition' is given by:

$$D_{\mathrm{drive}}(E_z) = \sum_{i=1,2,3} \frac{\delta_i}{2}\left(\sqrt{1+\left(\frac{2g_i}{\delta_i}\right)^2}-1\right), \tag{S11a}$$

$$g_1 = \beta g_B, \quad g_2 = -\alpha g_B, \quad g_3 = g_B. \tag{S11b}$$

[S1] R. Rahman *et al.*, *Phys. Rev. B* **80**, 1553301 (2009)