Quantum Computation with Trapped Ions in an Optical Cavity

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Two-qubit logical gates are proposed on the basis of two atoms trapped in a cavity setup and commonly addressed by laser fields. Losses in the interaction by spontaneous transitions are efficiently suppressed by employing adiabatic transitions and the quantum Zeno effect. Dynamical and geometrical conditional phase gates are suggested. This method provides fidelity and a success rate of its gates very close to unity. Hence, it is suitable for performing quantum computation.

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One of the main obstacles in realizing a quantum computer (QC) is decoherence resulting from the coupling of the system with the environment. There are theoretical proposals for models which avoid decoherence [1–4]. For this purpose decoherence-free subspaces (DFS) have been proposed in the literature for performing QC [5–7]. While they are easy to construct in the case of a single qubit, they are more complicated for the case of an externally controlled multipartite system. Their main decoherence channel is the "bus" that couples the different subsystems and is usually strongly perturbed by the environment. In the case of an ion trap the bus is the common vibrational mode which is subject to continuous heating. In the case of cavity QED the bus is a cavity mode which may leak to the environment. Additionally, the cavity couples to an excited state of the atom that shows spontaneous emission. To avoid these phenomena it is most convenient to transfer population by virtually populating the bus [3,4,8–10]. Here we present a model with atoms in an optical cavity that bypasses the decoherence problem with, in principle, arbitrarily large fidelity and success rate.

The system presented here consists of two four-level atoms fixed inside an optical cavity, Fig. 1. This can be achieved by, for example, having trapped ions in a cavity with its axis perpendicular to the ionic chain. It is assumed that the atoms have the lower states, $|0\rangle$, $|1\rangle$, and $|\sigma\rangle$, which could be represented by different hyperfine levels or Zeeman levels, and an excited state $|2\rangle$ coupled individually to each ground state by laser radiation with different polarizations or frequencies. The lasers act *commonly* to both atoms transferring populations via the experimentally robust STIRAP techniques [11], which consist of an adiabatic transfer of population insensitive to most of the experimentally controlled parameters. The atoms interact with each other via the cavity radiation field. Experimentally the proposed scheme could be realized with state-of-the-art technology [12].

Our goal is to perform QC in such a way that coherent evolution can be performed even for high loss rates, κ, of the cavity and relatively large decay rates, Γ, of the excited atomic states. This is achieved by employing an adiabatic procedure that keeps the cavity empty and the excited state of the atoms depopulated. Information is transferred by virtual population of these decohering atomic and cavity states. The entangling adiabatic transfer of population between ground state levels occurs by slowly varying the Rabi frequencies of the lasers in a counterintuitive temporal sequence, similarly to the well-known STIRAP process for Λ systems, but now it is performed in the space spanned by the tensor product states of the two atoms. Its effect is to avoid spontaneous emissions from the atoms by adiabatically eliminating the excited levels. Even though the duration of the two-qubit gate could be increased, the decoherence rate of the qubit states *even during gate performance* is greatly suppressed.

Consider the atomic ground states $|\sigma\rangle$, $|1\rangle$, the excited $|2\rangle$, as well as the Fock states of the cavity denoted by $|n\rangle$ with $n = 0, 1, \dots$. For an empty cavity tuned along the 1–2 transition with equal atom-cavity coupling $g_{(1)} = g_{(2)} = g$ [13] for both atoms the following states span a decoherence-free subspace DFS$_c$ [4] with respect to cavity emissions: $|\sigma\sigma\rangle$, $|\sigma 1\rangle$, $|1\sigma\rangle$, $|11\rangle$, and $|\alpha\rangle = (|12\rangle - |21\rangle)/\sqrt{2}$. These states are annihilated by the atom-cavity interaction Hamiltonian and hence do not populate the cavity. Still the maximally entangling state $|\alpha\rangle$, when populated, may result in atomic emission as it

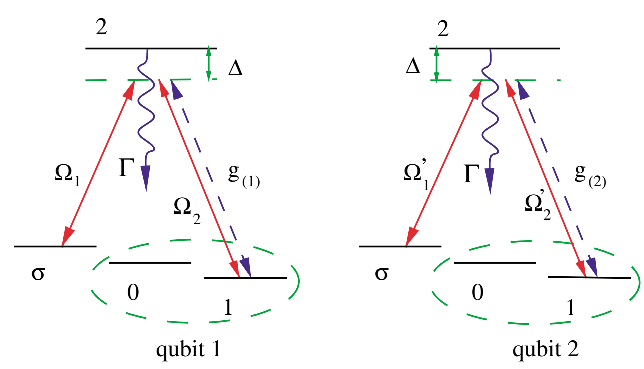

FIG. 1 (color online). Atomic levels of the two atoms and laser and cavity couplings with their detunings. The qubits 1, 2 and the auxiliary states $|\sigma\rangle$ are depicted.

occupies the excited level 2. Hence, the decoherence-free subspace $\mathrm{DFS}_{\mathrm{ca}}$ with respect to both atomic and cavity emissions is spanned only by the states $|\sigma\sigma\rangle$, $|\sigma 1\rangle$, $|1\sigma\rangle$, and $|11\rangle$ which are all ground states. In addition, the ground state $|0\rangle$ is decoherence-free, but it does not participate in the presented evolution. Together with the state $|1\rangle$ they span the computational space. First, we shall review the mechanism for suppressing the effect of κ during the performance of a gate, and then we shall show how to suppress the effect of Γ.

By observing at frequent time intervals that no photons have leaked from the cavity, a conditional evolution is constructed. Emission of a photon corresponds to a quantum jump [14], and the evolution is described here within this framework (quantum jump approach [15]), where the system evolves according to a non-Hermitian Hamiltonian due to its coupling with the environment. The combination of a strong κ and the detector forces the system to remain in the $\mathrm{DFS}_{\mathrm{c}}$ by a mechanism called *environment-induced quantum Zeno effect* [16,17]. Combined with the adiabatic procedure described above, it has the result that weak laser couplings between the ground and excited atomic levels do not move the system out of the initially populated $\mathrm{DFS}_{\mathrm{c}}$. In other words, no population of the cavity occurs for a long time interval.

In particular, we shall apply common laser addressing to the two atoms with a π-phase difference in their Rabi frequencies. Consider a laser tuned between an auxiliary ground state $|\sigma\rangle$ and $|2\rangle$ and a second one tuned between $|1\rangle$ and $|2\rangle$. The conditional Hamiltonian that describes the evolution of the system is given by

$$H_{\mathrm{cond}} = g\hbar \sum_{i=1}^{2} (|2\rangle_{ii}\langle 1|b + \mathrm{H.c.}) + \frac{1}{2}\hbar(\Omega_1|\sigma\rangle_{11}\langle 2| + \Omega_1'|\sigma\rangle_{22}\langle 2| + \Omega_2|1\rangle_{11}\langle 2| + \Omega_2'|1\rangle_{22}\langle 2| + \mathrm{H.c.}) - \frac{\mathrm{i}\kappa}{2}\hbar b^\dagger b - \left(\Delta + \frac{\mathrm{i}\Gamma}{2}\right)\hbar \sum_{i=1}^{2} |2\rangle_{ii}\langle 2|, \tag{1}$$

where b is the annihilation operator of the cavity mode, Δ is the cavity and laser detuning, and the subscript i on the states denotes different atoms. For different values of the atomic spontaneous emission rate, Γ, we may reconfigure the detuning Δ and amplitudes Ω_i in order to optimize the fidelity of the gates and their success rate.

Consider initially the effect of κ and small Ω's on the evolution of the system. Similarly to Beige *et al.* [4,18], Ω_i is such that it performs a transition slow enough to keep the state of the system inside the $\mathrm{DFS}_{\mathrm{c}}$, i.e.,

$$|R| \ll \frac{g^2}{\kappa} \quad \text{and} \quad \kappa, \tag{2}$$

where R is the state transition rate from the $\mathrm{DFS}_{\mathrm{c}}$. For this configuration the effective Hamiltonian (see [17]) is given by $H_{\mathrm{eff}} = \mathbf{P}_{\mathrm{DFS_c}} H_{\mathrm{laser}} \mathbf{P}_{\mathrm{DFS_c}}$, where $\mathbf{P}_{\mathrm{DFS_c}}$ is the projector in the cavity decoherence-free space, while H_{laser} is the laser part of the Hamiltonian (1). With the laser amplitudes tuned as $-\Omega_1 = \Omega_1' = \Omega$, $-\Omega_2 = \Omega_2' = \bar{\Omega}/\sqrt{2}$, and on the basis $|A\rangle \equiv (|\sigma 1\rangle + |1\sigma\rangle)/\sqrt{2}$, $|11\rangle$, and $|\alpha\rangle$, the effective Hamiltonian becomes

$$H_{\mathrm{eff}} = -\hbar\Delta|\alpha\rangle\langle\alpha| + \frac{\hbar}{2}(\Omega|\alpha\rangle\langle A| + \bar{\Omega}|\alpha\rangle\langle 11| + \mathrm{H.c.}). \tag{3}$$

Note that the states $|\alpha\rangle$ and $|A\rangle$ do not belong in the two-qubit computational space spanned by $|ij\rangle$ for $i, j = 0, 1$. Two of the eigenvalues of this Hamiltonian, $E_\pm = -(\Delta \pm \sqrt{|\Omega|^2 + |\bar{\Omega}|^2 + \Delta^2})/2$, have eigenvectors that occupy the antisymmetric state $|\alpha\rangle$, while the third eigenvalue, $E_0 = 0$, corresponds to the eigenvector $|D\rangle \equiv (-\bar{\Omega}, \Omega, 0)/\sqrt{|\Omega|^2 + |\bar{\Omega}|^2}$. The latter is the only one with zero component on the $|\alpha\rangle$ state and hence on the excited state $|2\rangle$. As a consequence, adiabatic transfer of population can occur between states $|11\rangle$ and $|A\rangle$ by slowly varying the laser amplitudes Ω and $\bar{\Omega}$ in such a way that the population remains on the third eigenstate without ever populating the decaying level 2.

This adiabatic passage employed to eliminate the excited level 2 is enhanced by the Zeno effect [17] applied to the combination of strong Γ and detectors that observe emitted photons from the atoms. The projection to the atomic *and* cavity decoherence-free subspace, $\mathrm{DFS}_{\mathrm{ca}}$, has as a result that an initial population of the eigenstate with $E_0 = 0$ remains there throughout the adiabatic procedure while the two other eigenstates are projected out. Note that if state $|\sigma\rangle$ was not employed, but instead we used state $|0\rangle$, then the state $|A\rangle$, constructed now out of $|0\rangle$ and $|1\rangle$, could be populated initially, and it would not be possible to perform the adiabatic transfer of population. For amplitudes satisfying $|\Omega|/|\bar{\Omega}| = \tan(\theta/2)$ and with phase difference $\phi = \phi_1 - \phi_2$, the eigenstate corresponding to the zero eigenvalue takes the form of the dark state of the system $|D\rangle = \cos\frac{\theta}{2}|11\rangle - \sin\frac{\theta}{2} e^{\mathrm{i}\phi}|A\rangle$. In Fig. 2 the θ transition from zero to π is depicted where $\Omega_{\max}/\sqrt{2} = \bar{\Omega}_{\max} = 0.018g$, $\Gamma = \kappa = 0.1g$, and $\Delta = 0.02g$. The simulation is performed with the evolution dictated by the conditional Hamiltonian (1). The probability of no photon emission from the cavity, or success rate, is $P_0 = 0.852$, while the fidelity is $F = 0.999$. Along this evolution the antisymmetric state $|\alpha\rangle$ does not get populated. Such a procedure resembles the STIRAP process, which produces population transfer between ground states of one atom, but now the transfer is between states of two atoms.

It is possible to optimize the fidelity, F, and the probability for no photon emission, P_0, of this transition

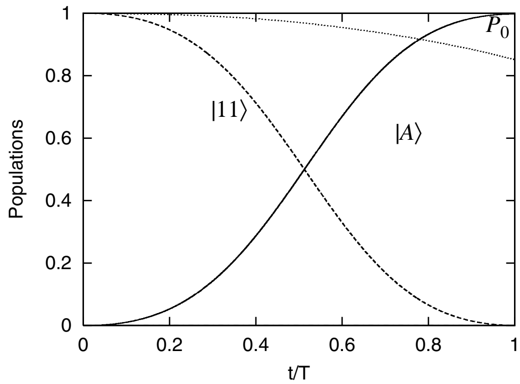

FIG. 2. Adiabatic evolution for $\Gamma = 0.1g$ and $\kappa = 0.1g$. The population of the state $|11\rangle$ is completely transferred to the state $|A\rangle$ with success rate $P_0 = 0.852$ and fidelity $F = 0.999$. For this transition the overall time of the adiabatic procedure, which is performed with linear ramp for the laser pulse, is $T = 5 \times 10^4/g$.

with respect to different values of the detuning Δ and the maximum Rabi frequency $\Omega_{\max}$. In the following simulations we vary both Δ and $\Omega_{\max}$ for values of the spontaneous atomic emission Γ and leakage of the cavity κ that are within the relatively strong coupling regime $g^2 = 10^2\kappa\Gamma$, which is improved by orders of magnitude compared to previous works.

Thus we obtain Fig. 3 where the maximum fidelity is given for $\Omega = 0.0145g$ and Δ less than $0.024g$. For these ranges of Δ and $\Omega_{\max}$ the corresponding value of the probability for no photon emission is given in Fig. 4 which is within acceptable limits. In general, strong lasers lead to population of the excited state $|2\rangle$, which either emits photons with rate Γ or it populates the cavity that leads to losses of its photons with rate κ. This case is out of the regime (2) where the projection in $\mathrm{DFS_c}$ holds and leads to a decreasing of F and P_0 for increasing laser amplitudes. With a close inspection we observe that the maximum success rate increases when Δ takes a nonzero but small value. For large values of the detuning the fidelity drops down as the eigenvalues of H_{eff} come closer and the adiabatic procedure fails. This causes unwanted population of the antisymmetric state $|\alpha\rangle$ that spontaneously emits with rate Γ. For strong lasers the success rate improves slightly for larger Δ as expected, but the fidelity decreases as now the evolution is unfaithfully described by the eigenvector $|D\rangle$. Fidelity of the gates differing from unity by 10^{-3} or less is necessary for performing quantum computation, while the relatively low success rate can be compensated by having detectors measuring possible emitted photons. In that case the whole computation has to be started from the beginning. Alternatively, a teleportation scheme can be employed as in [19] where the successful quantum gates may be teleported to perform the whole algorithm.

There is the possibility of realizing dynamical and geometrical gates between the two atoms. Consider the following evolution. Take θ from zero to π, then apply a 2π pulse to transform the state $|\sigma\rangle$ to $-|\sigma\rangle$ for both atoms and then take θ back to zero. Then the state $|11\rangle$ acquires an overall minus sign, while the rest of the computational states $|00\rangle$, $|01\rangle$, and $|10\rangle$ remain unchanged. This is a conditional phase shift, $\mathrm{diag}(1, 1, 1, e^{i\varphi})$ with $\varphi = \pi$. As an additional application holonomic gates [20] can be constructed in the same fashion as in the ion trap model proposed by Duan *et al.* [21]. By continuously changing the variables θ and ϕ starting from $\theta = 0$ one can perform a cyclic adiabatic evolution on the (θ, ϕ) plane described by a loop C. At the end of this evolution the state $|11\rangle$ acquires a Berry phase, and the overall gate is given by the holonomy $\Gamma(C) = \exp(i|11\rangle\langle 11|\varphi_{\mathrm{Berry}})$, where $\varphi_{\mathrm{Berry}} = \int \sin\theta d\theta d\phi$ and the integration runs over the surface the loop C encloses. The evolution has the form of a conditional phase-shift gate. This model has the experimental advantages presented in [21], and in

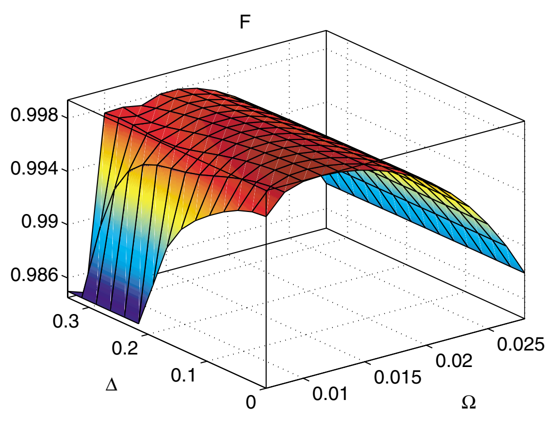

FIG. 3 (color online). The fidelity for the adiabatic transfer of population for $\Gamma = 0.1g$ and $k = 0.1g$. We observe maximum at $\Omega = 0.0145g$ and $\Delta \approx 0.024g$ with fidelity $F = 0.999$.

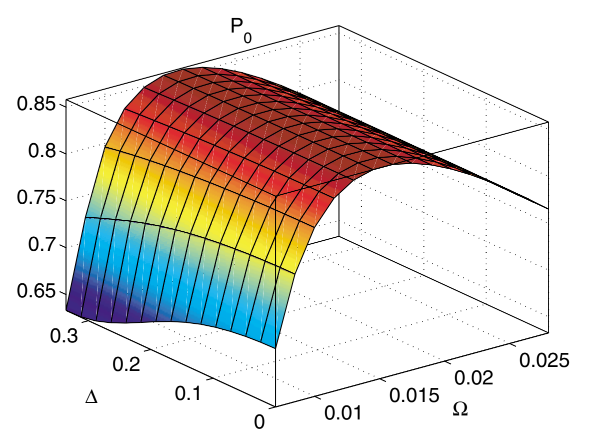

FIG. 4 (color online). The probability for no photon emissions for the adiabatic transfer with $\Gamma = 0.1g$ and $k = 0.1g$. We observe maximum for $\Omega = 0.0145g$ and any Δ with $P_0 = 0.858$. This coincides with the range the fidelity is also maximum.

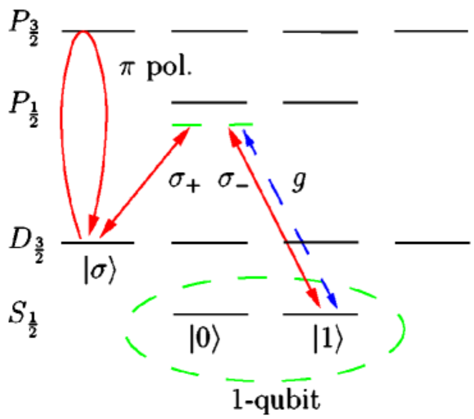

FIG. 5 (color online). Application of the proposed scheme to the calcium ion Ca^+.

addition *no* cooling of the trapped ions' modes is required beyond Doppler cooling.

This proposal can be implemented within the ion trap and cavity experiments performed, e.g., in Garching [12]. There, trapped calcium ions, Ca^+, are placed inside a cavity and can be addressed with laser fields. The application of our proposal to the internal levels of Ca^+ is presented in Fig. 5. The 2π laser pulse acts on the level σ of both atoms to perform the transformation $|\sigma\rangle \rightarrow -|\sigma\rangle$. It is a π polarized beam tuned on resonance, and it is activated when the laser σ_+ is turned off at the end of the adiabatic procedure. The states $|0\rangle$ and $|1\rangle$ are the ones which encode the qubit. Level σ is the auxiliary level that enables one to perform a conditional transfer of population without affecting the qubit state $|0\rangle$, and it is emptied by the end of the manipulation. Taking for Ca^+ the rate $\Gamma = 3g$ and a cavity with $\kappa = 0.01g$ where $g/(2\pi) = 6 \sim 7$ MHz, we obtain from the simulation for $\Omega_{max}/(3\sqrt{2}) = \tilde{\Omega}_{max} = 0.035g$, $\Delta = 0.3g$ and $T = 10^5/g$ fidelity $F = 0.998$ and success rate $P_0 = 0.815$.

In this proposal we describe two-qubit conditional phase-shift gates for ions trapped in a cavity. Together with one-qubit rotations they consist of a universal set of gates. The cavity and the atomic spontaneous emission rates are considered here to be close to the atom-cavity coupling. An adiabatic transition between states of the two atoms is performed in such a way that the cavity mode and the excited atomic levels are virtually populated, thus avoiding the problem of their decoherence. This population transfer resembles the STIRAP procedure; hence it enjoys the experimental advantage of the final state being independent of the exact intermediate value of the amplitude of the laser beams. By observing at frequent time intervals for emitted photons from the atom and the cavity, the adiabatic transition is assisted by the Zeno effect guaranteeing that the population will remain in the decoherence-free subspace by projecting out the excited atomic and cavity states. It has been shown here that the success rate and the fidelity of the gates are close to unity, allowing construction of a system for quantum computation in the presence of decoherence. Gates are constructed dynamically as well as geometrically in order to take advantage of the additional fault-tolerant features of geometrical quantum computation [22].

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