Coupling charge qubits via Raman transitions in circuit QED

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We propose a theoretical scheme to couple superconducting charge qubits (SCQs) via Raman transitions in a circuit QED architecture. Qubit array interacts a quantum data bus generated by a one-dimensional superconducting transmission line resonator (TLR). Based on the Raman transitions, the controllable and selective interqubit couplings mediated by the data bus can be obtained by just addressing the applied gate pulses, which provides the possibility for scaling up to many SCQs.

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I. INTRODUCTION

With potential applications in quantum information processing, superconducting Josephson circuits increasingly attract the attention of researchers [1,2]. Among the various superconducting qubits, charge qubit as an elementary one has its inherent advantages. It can be operated flexibly through external tunable parameters such as gate voltage and biasing flux. At the charge degeneracy point, the charge qubit has a longer dephasing time [3]. This is mainly because qubit eigenbasis (the two lowest energy levels at the "magic point") is less affected by the low-frequency charge noises [4,5].

In superconducting quantum computing, the practical couplings between qubits are crucial issues. Some strategies aimed at controllable coupling charge qubits have been proposed [6,7]. By making use of these coupling schemes, the unwanted decoherence effects of other qubits on the operated ones can be reduced greatly. Additionally, with the assistance of a quantum data bus, selectively coupling any pair of qubits (not necessarily neighbors) in a large qubit array is also an interesting issue [8–10]. Hence, a realizable circuit in which the interqubit couplings can be processed both controllably and selectively is especially desirable in quantum computing.

In recent years, analogous with cavity quantum electrodynamics (QED), a new research field called circuit QED has emerged in superconducting Josephson devices [11–14]. The transmission line resonator (TLR) in circuit QED architecture provides quantized cavity field, superconducting qubit acts as an artificial atom, and the strong coupling between the field and qubit can be realized [15,16]. Very recently, a circuit QED played the role of cavity bus to realize two-qubit logic gates for both transmons [17] and phase qubits [18] experimentally.

In this paper, we theoretically present a simple but feasible scheme to couple superconducting charge qubits (SCQs) via Raman transitions in circuit QED. Many superconducting quantum circuits with similar three-level structures (the two lowest ones act as qubit eigenbasis, and the third one is an ancillary level for Raman transition) interact with a one-dimensional TLR which serves as a quantum cavity bus. In the dispersive limit, through the always-on common cavity mode, the coupling between any selected pair of qubits can be controlled only by the individually tunable gate pulses. Therefore, in the proposed scheme we can obtain both the controllable and selective interqubit coupling, which is highly preferable for implementing scalable quantum information processing.

The paper is organized as follows. In Sec. II, we illustrate the interactions between the external fields and the operated circuits. In Sec. III, we present how to couple qubits via Raman transitions. Finally, a brief discussion and conclusion is given in Sec. IV.

II. INTERACTIONS BETWEEN EXTERNAL FIELDS AND OPERATED CIRCUITS

As in Ref. [19], many operated circuits can be arrayed in a high-Q coplanar resonator with single-mode frequency ω_r, the geometric length of the one-dimensional TLR is L_0, and the distance between the line connected with circuits and the center superconducting line is d (see Fig. 1). As a quantum data bus, TLR interacts with each circuit simultaneously. In a standing-wave cavity field, appropriate boundary condition of TLR makes the electric field zero at the antinodes [12,19], and thus the maximal coupling strengths between systems and cavity mode can be achieved when systems are situated at the antinodes of the magnetic field.

Each circuit in the above architecture consists of a superconducting island with extra Cooper-pairs n, the island is coupled by two symmetric Josephson junctions (each with coupling energy E_J and capacitance C_J) to a segment of a superconducting ring. To adjust the Josephson coupling, an applied magnetic flux Φ_x threads the ring with a small inductance L. Through the gate capacitance C_g, a bias voltage

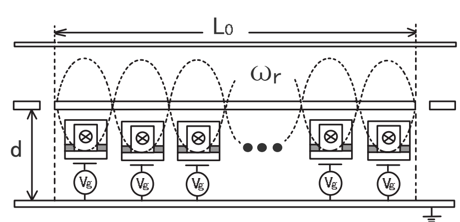

FIG. 1. The schematics of many SCQs placed into an on-chip circuit QED with single-mode frequency ω_r.

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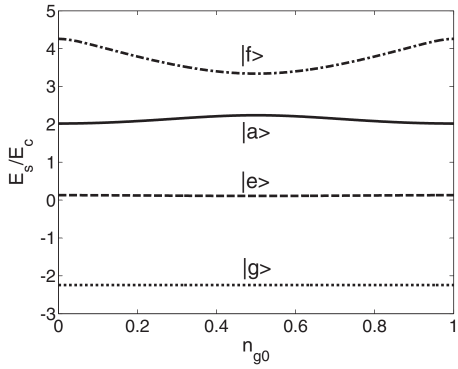

FIG. 2. The first four energy levels E_s of the static system vs gate charges n_{g0} for fixed $\bar{E}_J=3.5E_c$, energies are given in units of the charging energy E_c(=17.5 GHz), $s=g$, e, a, and f. At bias point $n_{g0}=0.5$, the three lower levels $|g\rangle$, $|e\rangle$, and $|a\rangle$ form a three-level subsystem.

$V_g=V_{g0}+\tilde{V}_{g1}$ is applied to the island, where the static component V_{g0} changes the system levels by inducing offset charges, and ac one $\tilde{V}_{g1}$, as a classical microwave pulse, serves for coupling transitions between the energy levels of the system [5,20,21]. The characteristic parameters satisfy $\Delta \gg E_c \sim E_J \gg k_B T$, where the large energy gap Δ suppresses the quasiparticle tunneling, E_c represents the charging energy with the same order of E_J, and T is low temperature.

Without the radiations from quantized and classical fields, the Hamiltonian of the static system is described by [5] $H_0 = E_c(n-n_{g0})^2 - \bar{E}_J \cos\theta$. Here the charging energy is $E_c = 2e^2/C_t$, with $C_t=(C_g+2C_J)$ being the total capacitance of the island, and $n_{g0}=C_g V_{g0}/2e$ represents the gate charges induced by V_{g0}. $\bar{E}_J=2E_J\cos(\phi/2)$ characterizes the effective Josephson energy, where $\phi=2\pi\Phi_t/\Phi_0$ is the total phase difference, $\Phi_t=\Phi_x+LI_s$ is the total flux, with LI_s being the induced flux, and $\Phi_0=h/2e$ stands for the flux quantum. The average phase difference of the two junctions θ is conjugate to the excess number n, i.e., $[\theta,n]=i$. Thus, within the Cooper-pair state representation $\{|n\rangle,|n+1\rangle\}$, the static Hamiltonian can be rewritten as [20]

$$H_0 = \sum_n E_c(n-n_{g0})^2|n\rangle\langle n| - \frac{\bar{E}_J}{2}(|n\rangle\langle n+1| + \text{H.c.}). \quad (1)$$

The first term is the system charging energy, and the second one is the corresponding Josephson coupling.

In the light of Eq. (1), we have the first four levels as a function of gate charge n_{g0} for fixed $\bar{E}_J$, as shown in Fig. 2. Each level state is a superposition of Cooper-pair states. At bias point $n_{g0}=0.5$, the states are denoted by $|g\rangle$, $|e\rangle$, $|a\rangle$, and $|f\rangle$, which have the forms $|s\rangle=\Sigma_n c_{sn}|n\rangle$, with c_{sn} being superposition coefficients, $s=g$, e, a, and f. It is clear that the level $|a\rangle$ is well separated from the fourth one $|f\rangle$, which allows one to effectively choose a three-level subsystem (TLS) consisting of three lower levels, in which the two lowest ones $|g\rangle$ and $|e\rangle$ act as qubit computational basis, and $|a\rangle$ is the ancillary state, respectively [21,22].

In our scheme, the reasons for choosing the TLS at the magic point ($n_{g0}=0.5$) are as follows. First, since the qubit eigenbasis at the magic point is immune to the first-order fluctuations of the gate charges, we should keep the advantage to fight against the dephasing effects as much as possible. Additionally, the selection rules due to the parity symmetry of energy levels do not impede the desired transitions induced by the external fields [20,23]. As will be discussed below, the quantized cavity field causes the coupling between $|g\rangle$ and $|a\rangle$, and the transition $|e\rangle \leftrightarrow |a\rangle$ is obtained via classical microwave field.

The Hamiltonian of the single-mode quantum TLR is described by $H_r=\hbar\omega_r(a^\dagger a+1/2)$, where $a^\dagger(a)$ is the creation (annihilation) operator of the cavity field. The cavity field and the kth circuit is coupled inductively, whose coupling energy can be given by [24] $H_{rs}=\frac{1}{L_k}(\Phi_t^{(k)}-\Phi_x^{(k)})\Phi_r$, where L_k is the inductance bounded by the kth superconducting ring, $\Phi_t^{(k)}$ and $\Phi_x^{(k)}$ are the total and external applied flux threading the ring, respectively, and Φ_r is the quantized flux generated by the TLR [19], $\Phi_r=\frac{S_k}{d}\sqrt{\hbar l_k\omega_r/L_0}(a+a^\dagger)$, where S_k is the enclosed area of the ring, with l_k being the inductance per unit length.

In the kth TLS, the transition frequency $\omega_{ag}^{(k)}=(E_a^{(k)}-E_g^{(k)})/\hbar$ is nearly matched with the cavity field frequency ω_r, while the other ones $\omega_{ae}^{(k)}=(E_a^{(k)}-E_e^{(k)})/\hbar$ and $\omega_{eg}^{(k)}=(E_e^{(k)}-E_g^{(k)})/\hbar$ are largely detuned from ω_r. Then the only transition between $|g\rangle^{(k)}$ and $|a\rangle^{(k)}$ can be realized under the radiation from the quantized field ω_r, where the coupling strength reads

$$\lambda_{ga}^{(k)} = \frac{S_k}{L_k d}\sqrt{\hbar l_k\omega_r/L_0}\langle g|\Phi_t^{(k)}|a\rangle^{(k)}. \quad (2)$$

On the other hand, the transition $|e\rangle^{(k)} \leftrightarrow |a\rangle^{(k)}$ can be induced by applying ac gate voltage $\tilde{V}_{g1}^{(k)}=V_{g1}^{(k)}\cos(\omega_k t)$, where $V_{g1}^{(k)}$ is the small amplitude satisfying $C_g V_{g1}^{(k)}/2e \ll 1/2$, and ω_k is the ac frequency. Since the gate pulse diagonally couples the charge states, the interaction Hamiltonian between the microwave field and the TLS becomes [20,21] $H_1^{(k)}=-2E_c\Sigma_n(n-n_{g0})\tilde{n}_k|n\rangle^{(k)}\langle n|$, where $\tilde{n}_k=n_k\cos(\omega_k t)$, with $n_k=C_g V_{g1}^{(k)}/2e$ being the ac amplitude. Here we have ignored the fast oscillating term $\tilde{n}_k^2$ under the rotating wave approximation (RWA). Under the microwave radiation, the transition between levels $|e\rangle^{(k)}$ and $|a\rangle^{(k)}$ can be characterized by the interaction matrix element

$$t_{ea}^{(k)} = \langle e|H_1^{(k)}|a\rangle^{(k)}. \quad (3)$$

In this case where the ac frequency ω_k is comparatively matched with the transition frequency $\omega_{ae}^{(k)}$, the microwave pulse $\tilde{n}_k$ aimed at inducing $|e\rangle^{(k)} \leftrightarrow |a\rangle^{(k)}$ will not cause the other transitions $|g\rangle^{(k)} \leftrightarrow |e\rangle^{(k)}(|a\rangle^{(k)})$ nearly.

III. COUPLING QUBITS VIA RAMAN TRANSITIONS

Using these devices shown in Fig. 1, we present in detail how to obtain the interqubit couplings by means of a common data bus and individually tunable voltage pulses. In any pair of selected TLSs A and B (see Fig. 3), besides the common radiation from the cavity field ω_r, we apply two micro-

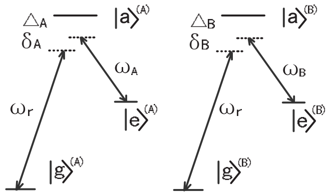

FIG. 3. In an arbitrary pair of TLSs, the common cavity field ω_r transits $|g\rangle^{(k)} \leftrightarrow |a\rangle^{(k)}$, and the couplings between $|e\rangle^{(k)}$ and $|a\rangle^{(k)}$ are induced by microwave pulses $\tilde{n}_k$ with frequencies ω_k. Here Δ_k and δ_k are the detunings, $k=A,B$.

wave pulses at frequencies $\omega_{A,B}$ to address circuits A and B separately [25,26]. When microwave frequencies ω_k satisfy the detunings $\Delta_k=\omega_{ae}^{(k)}-\omega_k$ and $\delta_k=\omega_{eg}^{(k)}+\omega_k-\omega_r$, Raman processes mediated by ancillary states $|a\rangle^{(k)}$ in TLSs $k(=A,B)$ occur individually. Here Δ_k are large enough to make the populations of $|a\rangle^{(k)}$ negligible. As a result, the Hamiltonian describing Raman processes in two-TLS AB is given by [26,27]

$$H^{(AB)} = 2\hbar\omega_r a^\dagger a + \frac{\hbar\omega_{eg}^{(k)}}{2}\sum_{k=A,B}\sigma_z^{(k)} + \hbar\sum_{k=A,B}\Omega_k(a^\dagger\sigma_-^{(k)}e^{-i\omega_k t} + \text{H.c.}). \tag{4}$$

$\sigma_z^{(k)}$ are Pauli operators with $\sigma_z^{(k)}|g\rangle^{(k)}=-|g\rangle^{(k)}$ and $\sigma_z^{(k)}|e\rangle^{(k)}=|e\rangle^{(k)}$, $\sigma_-^{(k)}=|g\rangle^{(k)}\langle e|$, and $\sigma_+^{(k)}=|e\rangle^{(k)}\langle g|$ are defined as the inversion operators. Note that the energy-nonconserving terms consisting of $a^\dagger\sigma_+^{(k)}$ and $a\sigma_-^{(k)}$ have been dropped under the RWA. Here the Rabi frequencies satisfy

$$\Omega_k = \lambda_{ga}^{(k)}t_{ea}^{(k)}[1/\Delta_k + 1/(\Delta_k+\delta_k)]/2. \tag{5}$$

When the detunings δ_k are large enough compared with Ω_k and the cavity linewidth, but much smaller than $\omega_{eg}^{(k)}$, the near two-photon resonance between qubits A and B can be described by the effective Hamiltonian [27]

$$H_{eff}^{(AB)} = \frac{\hbar\Omega_{AB}}{2}(\sigma_-^A\sigma_+^B + \sigma_+^A\sigma_-^B), \tag{6}$$

where the Rabi frequency takes the form

$$\left(\frac{\Omega_{AB}}{2}\right)^2 = \left|\frac{\langle egm|H_i|ggm+1\rangle\langle ggm+1|H_i|gem\rangle}{\delta} + \frac{\langle egm|H_i|eem-1\rangle\langle eem-1|H_i|gem\rangle}{-\delta}\right|^2 = \left(\frac{\Omega_A\Omega_B}{\delta}\right)^2. \tag{7}$$

Here H_i is the Hamiltonian of Eq. (4) in the rotating frame with regard to $\omega_r a^\dagger a + \omega_{eg}^{(k)}/2\Sigma_{k=A,B}\sigma_z^{(k)}$. And $|\cdots\rangle$ are the product states of qubit states and photon Fock states. For simplicity, we have chosen $\omega_A=\omega_B$ and $\delta_A=\delta_B=\delta$ in Eq. (7). Therefore, two qubits are coupled via intermediate states $|ggm+1\rangle$ and $|eem-1\rangle$, which are only virtually excited due to the large detuning δ. In fact, the intermediate state-assisted coupling mechanism is the same as that often used in trapped ion systems [10,28].

In terms of Eqs. (5) and (7), the coupling between qubits A and B can be switched on and off effectively by adjusting Ω_A and/or Ω_B, which depend on the gate pulse determined parameters $t_{ea}^{(A)}$ and $t_{ea}^{(B)}$, respectively. Through the paper we assume that the cavity-induced coupling $\lambda_{ga}^{(k)}$ are stationary while the couplings caused by classical microwaves are controlled. The coupling will exist when both $\tilde{n}_A$ and $\tilde{n}_B$ are turned on; otherwise, the coupling vanish when any one of pulses is absent. Since the gate pulse is individually applied to each circuit, we can selectively address the operated SCQs. As a consequence, based on the cavity bus, the controllable and selective coupling between any pair of qubits can be easily processed only by adjusting the gate pulses.

The above Hamiltonian (6) describes the swap of two-qubit states (SWAP) through virtual excited intermediate states. In the two-qubit computational basis $\{|gg\rangle,|ge\rangle,|eg\rangle,|ee\rangle\}$, the corresponding gate operation can be written as

$$U_{2q} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\Omega_{AB}t}{2} & -i\sin\frac{\Omega_{AB}t}{2} & 0 \\ 0 & -i\sin\frac{\Omega_{AB}t}{2} & \cos\frac{\Omega_{AB}t}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Obviously, a $\sqrt{i\,\text{SWAP}}$ logical gate will be generated when the evolution time $t=\pi/2\Omega_{AB}$. One nontrivial controlled gate along with two noncommutable single-qubit gates are sufficient for universal quantum computing [11,16].

With the experimentally accessible parameters, we now analyze the coupling strengths between qubits in the proposed architecture. In detail, for microwave pulse $\tilde{n}_A$ with amplitude $n_A=0.04$, it can cause the effective coupling strength between $|e\rangle^{(A)}$ and $|a\rangle^{(A)}t_{ea}^{(A)}=0.039E_c$. And the cavity-induced coupling strength $\lambda_{ga}^{(A)}=0.08$ GHz has been obtained experimentally [11]. In the case where $\Delta_A=1.0$ GHz and $\delta_A=0.25$ GHz, we have $\Omega_A=49.3$ MHz. If $\Omega_A\approx\Omega_B$ and $\delta_A=\delta_B$, the effective Rabi frequency will be $\Omega_{AB}\approx 19.4$ MHz. With the relaxation time 10 μs at the optimal point [3], the two-qubit gate operations can be completed at least $\sim 10^2$ times, and thus simple quantum information processing can be demonstrated sufficiently.

Since the anharmonicity of charge qubit is positive [13], the microwave field at frequency ω_k has the possibility of inducing the transition $|g\rangle^{(k)}\leftrightarrow|e\rangle^{(k)}$. Now we numerically consider the Rabi population between $|g\rangle^{(k)}$ and $|e\rangle^{(k)}$. Due to the large detuning ($\omega_{eg}^{(k)}-\omega_k=4.875$ GHz), the microwave pulse will cause the small amplitude of the Rabi population. By solving the time-dependent Schrödinger equation, we find the inversion possibility between $|g\rangle^{(k)}$ and $|e\rangle^{(k)}$ not more than 5%. In addition, Raman transition can also occur above $|a\rangle^{(k)}$ level. In this case, since the fourth level is far away from level $|a\rangle^{(k)}$ (the transition frequency between them equals 19.23 GHz), the microwave field at frequency $\omega_k'(=\omega_{ae}^{(k)}+\Delta_k)=38.29$ GHz will cause negligibly the transition between $|a\rangle^{(k)}$ level and the fourth one. Therefore, the gate pulse desired to couple $|e\rangle^{(k)} \leftrightarrow |a\rangle^{(k)}$ only causes small Rabi population between other levels, which contributes to enhance the gate fidelity [29].

IV. DISCUSSION AND CONCLUSION

In the dispersive limit, namely $\Delta_k' = \omega_r - \omega_{eg}^{(k)} \gg \lambda_{ge}^{(k)}$ ($k = A, B$), the common cavity field can directly induce the coupling between qubits A and B, not through the above Raman processes, where $\lambda_{ge}^{(k)}$ is the cavity-induced coupling between $|g\rangle^{(k)}$ and $|e\rangle^{(k)}$. In this case, the interaction Hamiltonian with the similar form as Eq. (6) [16,17]

$$H_d^{(AB)} = \hbar J_{AB}(\sigma_-^A \sigma_+^B + \sigma_+^A \sigma_-^B),$$

J_{AB} is the exchange interaction strength, which is a result of virtual exchange of photons with the cavity. The coupling between states $|egm\rangle$ and $|gem\rangle$ realized via two traversed paths with intermediate states $|ggm+1\rangle$ and $|eem-1\rangle$. The two intermediate states cause the identical strength $J_{AB} = \lambda_{ge}^{(A)} \lambda_{ge}^{(B)} (1/\Delta_A' + 1/\Delta_B')/2$, with $\lambda_{ge}^{(A)}$ being the coupling strengths of transitions $|g\rangle^{(A)}|m\rangle \leftrightarrow |e\rangle^{(A)}|m-1\rangle$ and $|e\rangle^{(A)}|m\rangle \leftrightarrow |g\rangle^{(A)}|m+1\rangle$, and $\lambda_{ge}^{(B)}$ the corresponding ones of $|g\rangle^{(B)}|m\rangle \leftrightarrow |e\rangle^{(B)}|m-1\rangle$ and $|e\rangle^{(B)}|m\rangle \leftrightarrow |g\rangle^{(B)}|m+1\rangle$. In our scheme, we have the same order of $\lambda_{ge}^{(A)} = \lambda_{ge}^{(B)}$ ($=0.07$ GHz) as that of $\lambda_{ga}^{(A)}$, and very large detunings $\Delta_A' = \Delta_B' = 36.04$ GHz, then $J_{AB} = 0.13\ \text{MHz} \ll \Omega_{AB}$. Hence, we can safely neglect the effect of cavity-induced coupling J_{AB} on Ω_{AB}.

Our proposed scheme likely possesses the following advantages. (1) Since the operated circuits are placed inside the circuit QED architecture, qubits can be prevented from the external noises strongly and the spontaneous emissions are inhibited effectively. So, the decoherence-protected cavity is helpful to couple many SCQs. (2) By setting the static bias points as magic points, the effects of the first-order fluctuations of the gate charges on the qubit eigenbasis are removed, which contribute to enhance the dephasing time. (3) The macroscopic circuit qubits may be selectively addressed by the controlled parameters. The interqubit couplings can be easily controlled and selected just by the gate pulses. (4) Through the exchange of virtual rather than real photons, the interqubit couplings make the quantum operations insensitive to cavity-induced loss [30]. Consequently, cavity field acting as a quantum bus can couple any pair of distant qubits and coherently transfer quantum information.

However, the random fluctuations of control parameters may be major noise sources, which reduce the system coherence time. If the gate operation time ($\sim 1/\Omega_{AB}$) is nearly stationary, the available times of the two-qubit gate will be decreased. So, how to remove the effects of random noises is a critical issue. In addition, with the limitation of the geometric size and radiation wavelength of the cavity, the number of addressed qubits is generally finite. In the considered standing-wave cavity, the operated circuits are located at the antinodes of the microwave. When the direct interactions between qubits can be safely neglected [25,31], for the chosen cavity mode $\lambda_r = 3.89$ mm, we can process about 13 qubits along the cavity with the length of $L_0 = 27$ mm [32].

In conclusion, we present a theoretical scheme to couple charge qubits that are placed into a common circuit QED architecture. By making use of circuit QED as a cavity bus, controllable and selective coupling can be obtained only through individually tuning the gate pulses applied to SCQs, and then the couplings between any pair of qubits can be effectively switchable. The interqubit coupling strengths are completely available to conditional gate operations and quantum leakages caused by gate pulses can be neglected. Further analysis shows the cavity-induced direct couplings between qubits are very small. We hope this work is helpful to experimentally couple SCQs for quantum information processing.

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