Universal departure from Johnson-Nyquist relation caused by limited resolution

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We show theoretically that limited resolutions cause a deviation from the Johnson-Nyquist relation. This is demonstrated by examining electronic transport of a resonant level model. We also find that the deviation is described by a universal single-parameter scaling: The deviation prominent at low temperatures or for low-conductance systems disappears exponentially at high temperatures/conductance through an intermediate algebraic decrease, which invalidates a naive account of the resolution effects. Our findings offer an explanation for experimental puzzles in Johnson noise thermometry.

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Fluctuations have been an attracting subject in physics over one hundred years since Einstein's memorial paper on the Brownian motion in 1905 [1]. A recent milestone is the discovery of fluctuation theorems [2, 3], which have extended the well-known fluctuation-dissipation theorem [4, 5] and have established relations applicable for systems far from equilibrium. The Johnson-Nyquist (J-N) relation is an early example of the fluctuation-dissipation theorem, which was observed by Johnson [6, 7], and theoretically accounted for by Nyquist in 1928 [8]. If a dc-bias voltage V is applied to a conducting system, the J-N relation claims that a proportional relation is satisfied between the variance of a fluctuating current, i.e. noise S_0, on the conductor in the thermal equilibrium state, and the conductance, i.e. the averaged current I_0 in the linear response to the applied field as

$$S_0^{(0)} = 2k_B T I_0^{(1)}, \tag{1}$$

where T is the temperature of the conducting electrons, k_B is the Boltzmann constant, and $X^{(n)}$ ($X = S_0$ or I_0 here) expresses the n-th order coefficient in the expansion of X with respect to V, namely $X = \sum_n X^{(n)} V^n$.

In addition to its importance in fundamental physics, the J-N relation also has a practical significance in thermometry [9]. Because the temperature can be determined by measuring only $S_0^{(0)}$ and $I_0^{(1)}$, in particular, the Johnson noise thermometry is exploited in rapidly developing noise measurements of nano devices from which we obtain the useful information about the low-energy excitations in the quantum systems [10–17] and confirm the steady state fluctuation theorem [18–20].

In the Johnson noise thermometry of nanosystems, however, the J-N relation is seemingly violated at very low temperatures: When a sample is placed in a mixing chamber in a dilute refrigerator, the noise temperature of the sample, T_{JN}, determined from the J-N relation is sometimes higher than the temperature of the refrigerator, T_{ref}, when independently measured by a resistance thermometer [16, 20]. The deviation has been recognized already in early 1970s [21], and attributed to a heat leak to the sample in the chamber [16, 21]. In the thermometry, T_{JN} instead of T_{ref} has often been regarded as the intrinsic electronic temperature, though the leakage has not been fully verified yet.

Here, we theoretically propose an alternative possibility for the origin of the deviation: We demonstrate that this basic J-N relation is satisfied only when the experimental resolution is unlimitedly high, which is practically not possible. On the contrary, the J-N relation is apparently violated in the practical experimental conditions with limited resolution inevitable in actual measurements, because the limited resolution makes the departure of the measured noise S from the intrinsic one S_0. This deviation becomes prominent in the region relevant for nanoscience at low temperatures, which is the reason why it has been overlooked or misled for a long time. We will show that the resolution limit affects the noise in a highly nontrivial way which is not corrected by a simple zero point calibration, but the violation is consistent with the experimental results and may account for the difference between T_{JN} and T_{ref}.

In order to understand the resolution effects on the current statistics, we examine the two-point measurement statistics of current proposed by Esposito *et al.* [3]. In this quantum measurement scheme, we estimate the probability distribution of the particle-number change taking place in a part of the system in a measurement time $\mathcal{T}$. We extend the formalism to take into account a limited resolution Δ. Here, Δ is defined by the resolution of the projective measurement of the particle number. Namely, in our model, the particle number N has an uncertainty in the range of $N_0 - \Delta/2 < N < N_0 + \Delta/2$, even if the outcome is N_0. The measured particle numbers at initial ($t = 0$) and final ($t = \mathcal{T}$) time points are assumed to have this limited resolution, which, in actual experiments, corresponds to the resolution in measured current (see Sec. IV of Supplemental Material [22]).

There are two characteristic measurement parameters: One is $\mathcal{T}$ and the other is Δ. This scheme can be described by a positive operator-valued measure [23, 24] [see Eq. (S3) of Supplemental Material [22]]. It is noteworthy that if we take $\Delta = 1$, our method is reduced to that of Esposito *et al.* [3] and the full counting statistics proposed by Levitov and Lesovik, [25]. Namely, at

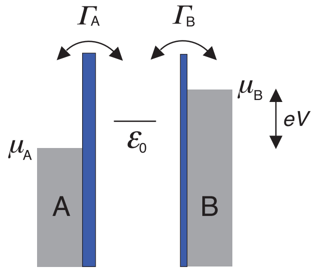

FIG. 1: (color online). Schematic illustration of resonant level model. The level has the energy ε_0 and is coupled to two reservoirs A and B between which the bias voltage V is applied. $\Gamma_{\mathrm{A(B)}}$ reads the characteristic frequency of the electron transfer between the level and the reservoir A(B). $\mu_{\mathrm{A(B)}}$ represents the chemical potential of the reservoir A(B). We introduce $\Gamma \equiv (\Gamma_{\mathrm{A}} + \Gamma_{\mathrm{B}})/2$ and $r \equiv \Gamma_{\mathrm{A}}\Gamma_{\mathrm{B}}/\Gamma^2$ as the characteristic time scale and the degree of asymmetry of the couplings, respectively.

$\Delta = 1$, the device becomes an ideal one with the ability of distinguishing the conducting electrons one by one.

We apply this method to study the statistics of current through a resonant level ε_0 coupled to two reservoirs A and B between which a bias voltage, V, is applied. The model Hamiltonian is given by

$$\hat{\mathcal{H}}(t) = \hat{H}_0 + \hat{V}(t), \tag{2}$$

where $\hat{H}_0 = \hat{H}_{\mathrm{A}} + \hat{H}_{\mathrm{B}} + \hat{H}_{\mathrm{S}}$, $\hat{V}(t) = \hat{V}_{\mathrm{A}}(t) + \hat{V}_{\mathrm{B}}(t)$, $\hat{H}_{\mathrm{S}} = \varepsilon_0 \hat{d}^\dagger \hat{d}$, $\hat{H}_{\mathrm{X}} = \sum_{x\in\mathrm{X}} \varepsilon_x^{\mathrm{X}} \hat{c}_x^\dagger \hat{c}_x$, $\hat{V}_{\mathrm{X}}(t) = \sum_{x\in\mathrm{X}} (t_{\mathrm{X}}\theta(t)\hat{d}^\dagger \hat{c}_x + \mathrm{H.c.})$ for $\mathrm{X} = \mathrm{A}, \mathrm{B}$. The coupling between the resonant level and the reservoirs are switched on after $t = 0$ because of the step function $\theta(t)$, from which the measurement of the electron counting starts and continues until $t = \mathcal{T}$. $\hat{d}^\dagger$ creates a spinless electron at the resonant level ε_0, while $\hat{c}_{x\in\mathrm{X}}^\dagger$ denotes the creation operator of a spinless electron at the quantum number (typically, wave number) x of $\hat{H}_{\mathrm{X}}$ in the reservoir X=A or B, which is assumed to have a constant density of states ρ_{X} in each reservoir. The resonant level is coupled to the reservoir X with a hybridization t_{X}. The characteristic transport frequency Γ_X is obtained as $\Gamma_{\mathrm{X}} = 2\pi|t_{\mathrm{X}}|^2\rho_{\mathrm{X}}/\hbar$ where $\hbar$ is Planck's constant divided by 2π. We assume that the measurement time, $\mathcal{T}$, is much longer than the characteristic time scale of the particle transport determined by $\Gamma^{-1} \equiv [(\Gamma_{\mathrm{A}} + \Gamma_{\mathrm{B}})/2]^{-1}$. The coupling asymmetry is characterized by $r \equiv \Gamma_{\mathrm{A}}\Gamma_{\mathrm{B}}/\Gamma^2$. This model can be considered as a simple model of a quantum dot coupled to two reservoirs. See also Fig. 1.

By calculating the cumulant generating function of current, we obtain the following current and the noise measured in the presence of a limited resolution (Detailed calculations are presented in Sec. I of Supplemental Material [22]):

$$I = I_0 + \langle \Delta I \rangle_\delta, \tag{3}$$

$$S = S_0 + \langle \Delta S \rangle_\delta, \tag{4}$$

where I_0 and S_0 are the intrinsic current and the intrinsic noise, respectively, which do not depend on $\mathcal{T}$ and Δ. It is found that the limited resolution gives an excess term, $\langle \Delta X \rangle_\delta$, in a measured quantity $X = I$ or S. Note that δ represents the degree of freedom for the initial particle number hidden in the limited resolution of actual measurement, namely $0 \le \delta < \Delta$. Due to the lack of our knowledge on δ, the measured transport quantities should be randomly averaged with $\langle \cdots \rangle_\delta \equiv \int_0^\Delta d\delta/\Delta \cdots$. We will see that the excess current always vanishes on average; $\langle \Delta I \rangle_\delta = 0$, whereas the excess noise satisfies $\langle \Delta S \rangle_\delta \ge 0$. [See Eqs. (S13, S15) of Supplemental Material [22]]. These are important results of this Letter.

Although the above discussion is valid for any V, we

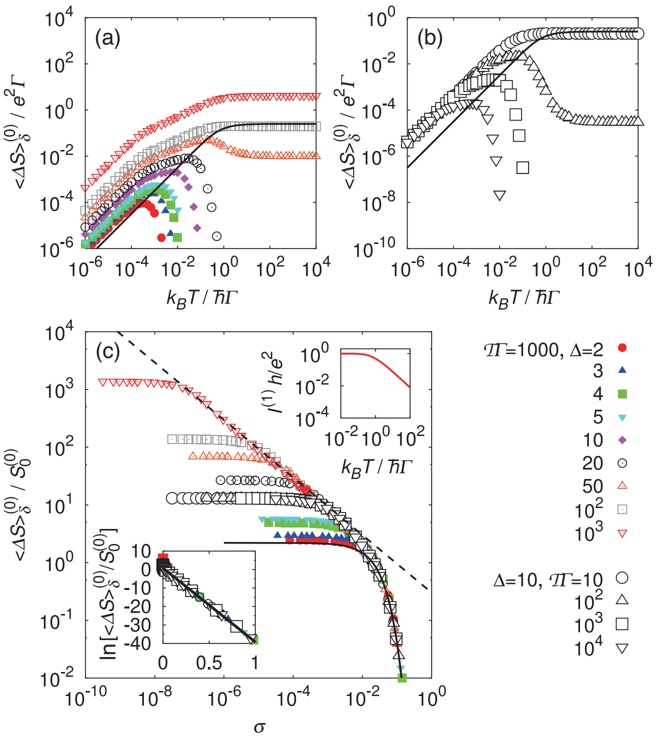

FIG. 2: (color online). (a,b) Excess noise $\langle \Delta S \rangle_\delta^{(0)}$ as a function of temperature for several choices of $(\mathcal{T}, \Delta)$. The solid line indicates the intrinsic noise, $S_0^{(0)}/e^2\Gamma$. The excess noise increases with increasing Δ or decreasing $\mathcal{T}$. (c) Ratio of excess and intrinsic noises $\langle \Delta S \rangle_\delta^{(0)}/S_0^{(0)}$ as a function of $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2$. The black solid line indicates the universal exponential dependence $A\exp[-\gamma\sigma]$, with $\gamma = 40$ and $A = 2.2$. The dashed line represents the inverse of the square root dependence $B/\sqrt{\sigma}$, where $B = 0.3$. The lower inset shows the linear dependence of the logarithm of the ratio on σ. The temperature dependence of conductance is plotted in the upper inset.

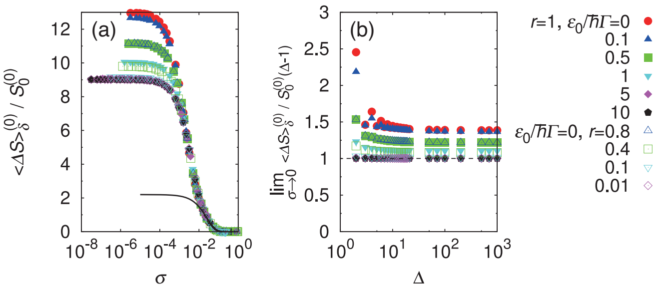

FIG. 3: (color online). (a) Ratio of excess and intrinsic noises $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}$ as a function of $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2$ for several choices of (ε_0, r). $\mathcal{T} = 1000$ and $\Delta = 10$ is used for calculation. The black solid line indicates the universal exponential dependence $A\exp[-\gamma\sigma]$, with $\gamma = 40$ and $A = 2.2$. (b) Saturation value of the ratio of noises. The dashed line represents the lower bound of the saturation value.

here focus on the zero-bias noise $S^{(0)}$ and the linear conductance $I^{(1)}$ to discuss the resolution effects on the J-N relation. First, we show the $(\mathcal{T}, \Delta)$-dependence of the excess noise at zero bias voltage, $\langle \Delta S\rangle_\delta^{(0)}$ for $\varepsilon_0 = 0$ and $r = 1$. Figure 2(a) shows $\langle \Delta S\rangle_\delta^{(0)}$ as a function of temperature T for several choices of Δ. For small-Δ ($\Delta < 50$), $\langle \Delta S\rangle_\delta^{(0)}$ is negligible at temperatures larger than the characteristic energy scale, $k_BT > \hbar\Gamma$, because it is much smaller than the intrinsic noise, $S_0^{(0)}$. $\langle \Delta S\rangle_\delta^{(0)}$ becomes more important in the measured noise, $S^{(0)}$ for $k_BT < \hbar\Gamma$ where $\langle \Delta S\rangle_\delta^{(0)}$ shows a peak at a finite temperature while $S_0^{(0)}$ decreases proportionally to T because of the saturated conductance at e^2/h [see upper inset of Fig. 2(c)]. At sufficiently low temperatures, $\langle \Delta S\rangle_\delta^{(0)}$ eventually becomes much larger than $S_0^{(0)}$, which leads to a difficulty in measuring the intrinsic noise in experiments. With an increase in Δ, $\langle \Delta S\rangle_\delta^{(0)}$ is enhanced, and becomes pronounced even at high temperatures $k_BT > \hbar\Gamma$. We can ignore $\langle \Delta S\rangle_\delta^{(0)}$ or easily absorb it as a zero point calibration at $k_BT \gg \hbar\Gamma$ because it saturates to temperature-independent constants. While, $\langle \Delta S\rangle_\delta^{(0)}$ shows a complicated temperature dependence at temperatures lower than the characteristic temperature of the system. The measurement time $\mathcal{T}$ also affects $\langle \Delta S\rangle_\delta^{(0)}$ as seen in Fig. 2(b) where $\langle \Delta S\rangle_\delta^{(0)}$ is suppressed with increase in $\mathcal{T}$. Note that $\langle \Delta S\rangle_\delta^{(0)}$ is independent of $\mathcal{T}$ at sufficiently low temperatures.

While the J-N relation is satisfied between intrinsic noise and intrinsic current, S_0 and I_0, it can be violated between measured noise and measured current, S and I. To investigate the resolution effects on the J-N relation in more detail, we calculate the ratio of excess and intrinsic noises which characterizes the deviation from the J-N relation,

$$S^{(0)}/2k_BTI^{(1)} - 1 = \langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)} \geq 0. \tag{5}$$

Figure 2(c) illustrates the ratio $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}$ as a function of a single non-dimensional parameter $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2 = 2k_BTI^{(1)}\mathcal{T}/(e\Delta)^2$ for several choices of $(\mathcal{T}, \Delta)$. All the curves saturate at low σ and decreases with increasing σ, which means that the deviation from the J-N relation becomes serious only at low temperatures and for low conductance. Furthermore, we find that the ratio becomes unity at $\sigma \approx 0.01$ and shows a strong decay for $\sigma > 0.01$ and the deviation from the J-N relation becomes quickly negligible in the actual experiment with increasing σ. For instance, the ratio becomes less than 10^{-17} for $\sigma > 1$: σ offers a universal criterion for the necessity of the correction to the J-N relation.

Furthermore, we find a scaling behavior as a function of σ both for different values of $\mathcal{T}$ with a fixed value of Δ and for different choices of Δ, where all the curves collapse into a single one at $\sigma > 0.01$, described by an exponential decay $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)} = A\exp[-\gamma\sigma]$ with A and γ being constants. A universal single-parameter scaling is satisfied irrespective of $\Gamma, \mathcal{T}$ and Δ. Below $\sigma \sim 0.01$, there exists a clear intermediate region of another universal single-parameter scaling that follows an algebraic behavior described by $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)} = B/\sqrt{\sigma}$ with $B = 0.3$, which has wider region at larger Δ. On the other hand, the saturated value of the ratio at low σ is not universal but roughly scaled commonly by Δ. The saturation of the ratio $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}$ occurs roughly at the crossing of Δ and $B/\sqrt{\sigma}$ as $\sigma \approx 0.1/\Delta^2$.

The single-parameter scaling at $\sigma > 0.01$ is realized at high temperatures in general and both of $\langle \Delta S\rangle_\delta^{(0)}$ and $S_0^{(0)}$ saturate in this region [see Fig. 2(a,b)]. We note that $\langle \Delta S\rangle_\delta^{(0)}$ becomes independent of temperature in accordance with the fact that the conductance becomes $\propto 1/T$ in the incoherent region.

The J-N relation between S and I is apparently violated by the limited resolution. Nonetheless, we find a scaling behavior in the deviation from the J-N relation. The scaling behavior is further confirmed by changing the values of ε_0 and r. Though both $\langle \Delta S\rangle_\delta^{(0)}$ and $S_0^{(0)}$ depend on ε_0 and r (see Sec. II of Supplemental Material [22]), the universality of the exponential decay for $\sigma > 0.01$ is again satisfied in Fig. 3(a). Furthermore, the saturation value of $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}$ at low σ stays at the order of Δ, although the values quantitatively depend on the system parameters. As a result, the saturation value is dependent strongly on Δ and weakly on ε_0 and r, and is independent of $\mathcal{T}$. The saturation region is characterized by $\sigma \ll 0.1/\Delta^2$ as discussed above. Figure 3(b) shows the Δ dependence of the saturation value of $\langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}$ at sufficiently low σ. We find the lower bound of the saturation value: If scaled by $\Delta - 1$, the value stays always larger than unity. Hence, $S^{(0)}$ is always larger than $S_0^{(0)}\Delta$ in the limit of $\sigma \to 0$.

The measured and intrinsic current noises are plotted

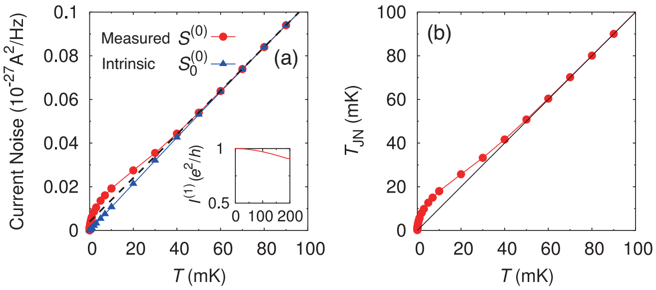

FIG. 4: (color online). (a) Current noise as a function of temperature T. The parameters are $\hbar\Gamma/k_B = 1$K, $\varepsilon_0 = 0$, $r = 1$, $\mathcal{T} = 1\mu$s, and $\Delta = 130$. The inset shows the conductance as a function of T. The dashed line indicates the fitted line for the measured noise from 50mK to 100mK, $aT + b$. $a = 1.00$ ($10^{-27}\,$A^{2}Hz^{-1}K^{-1}) is slightly smaller than the expected value for the intrinsic noise at low temperatures, $2k_Be^2/h \simeq 1.07$ ($10^{-27}\,$A^{2}Hz^{-1}K^{-1}). $b = 3.93$ ($10^{-30}\,$A^{2}Hz^{-1}). (b) Noise temperature $T_{\rm JN}$ plotted versus T. The parameters are the same as those in (a). The solid line shows $T_{\rm JN} = T$. The deviation of noise temperature from the intrinsic one is pronounced below $T = 50$mK.

versus the temperature in a realistic typical example with $\hbar\Gamma/k_B = 1$K, $\varepsilon_0 = 0$, $r = 1$, $\mathcal{T} = 1\mu$s and $\Delta = 130$ as shown in Fig. 4(a). Note that $\mathcal{T} = 1\mu$s and $\Delta = 130$ are estimated from an actual available current amplifier on the market (see Sec. IV of Supplemental Material [22]). While the measured noise takes nearly the same value as the intrinsic one at temperatures higher than 50mK, it deviates and makes a hump at lower temperatures. Before the hump, the measured noise shows a clear linear dependence on temperature from 50mK to 100mK where the conductance hardly changes and takes a constant $\simeq e^2/h$ as seen in the inset. The fitted line has a finite positive offset at $T = 0$. These features are qualitatively consistent with the experiment [16].

Because the noise temperature, $T_{\rm JN}$, is explicitly written as

$$T_{\rm JN} = S^{(0)}/2k_BI^{(1)} = T(1 + \langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}), \qquad (6)$$

it is always larger than the thermodynamic temperature, T. In Fig. 4(b), we show $T_{\rm JN}$ as a function of T for the same parameters in Fig. 4(a). The disagreement of $T_{\rm JN}$ with T appears for temperatures much lower than the characteristic temperature of the system, which is also consistent with the experiments [16, 20, 21]. This fact indicates that the intrinsic temperature may not be obtained in the Johnson noise thermometry at very low temperatures even if $T_{\rm JN} \simeq T_{\rm ref}$ holds at higher temperatures.

Here we summarize our findings as schematic in Fig. 5, where the universal departure from the J-N relation is characterized by the single parameter σ. The scaling behavior is ascertainable in experiments because it can be characterized only by measured quantities. In this Letter, we have focused on the J-N relation within the linear response. Even for the nonlinear regime, similar puzzles of the deviation from the fluctuation theorem [20] and the discrepancy of the shot noise between theory and experiment are known [17]. The resolution effects may also give us a clue to resolve them as discussed in Sec. III of Supplemental Material [22]. More generally, our results propose the necessity of amending naive accounts of resolution effects in widespread instruments based on the fluctuation-dissipation theorem such as nuclear magnetic resonance, X-ray scattering, neutron scattering, and photoemission. We anticipate our study to be a starting point for more sophisticated protocols to evaluate the intrinsic fluctuations and correlations from the limited information available in actual measurements.

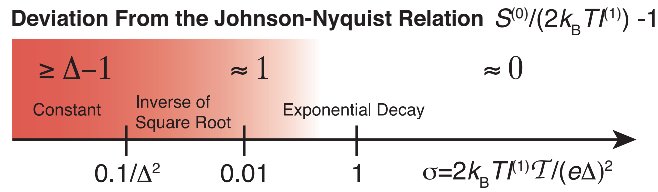

FIG. 5: (color online). Schematic illustration of deviation from Johnson-Nyquist relation for $\Delta > 1$. The scaling of the deviation form the Johnson-Nyquist relation, $S^{(0)}/2k_BTI^{(1)} - 1 = \langle \Delta S\rangle_\delta^{(0)}/S_0^{(0)}$, appears as a function of $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2 = 2k_BTI^{(1)}\mathcal{T}/(e\Delta)^2$. While the deviation is ignorable for $\sigma > 1$, it is exponentially increased with decreasing σ until $\sigma \approx 0.01$, where it reaches unity. Then, it is saturated to the value of the order of Δ (not less than $\Delta - 1$) for $\sigma \ll 0.1/\Delta^2$, following after the inverse square root dependence which exists roughly for $\Delta > 4$.

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Supplemental Material

I. DETAILED CALCULATIONS OF CURRENT STATISTICS BASED ON TWO-POINT MEASUREMENT

The details of method used in the Letter are given in this section. We calculate the two-point measurement statistics under limited resolutions of current through a resonant level coupled to two reservoirs, A and B as is illustrated in Fig. 1 of the Letter.

Before the current measurement, it is assumed that the resonant level is disconnected for $t \leq 0$ from two reservoirs, A and B, which are in the isolated thermal equilibrium states with the different chemical potentials, μ_A and μ_B, respectively. Then, the density matrix at $t = 0$ is given by $\hat{\rho}(0) = \frac{\exp[-\beta(\hat{H}_\mathrm{A}-\mu_\mathrm{A}\hat{N}_\mathrm{A})]}{\mathrm{Tr}\left[\exp[-\beta(\hat{H}_\mathrm{A}-\mu_\mathrm{A}\hat{N}_\mathrm{A})]\right]} \otimes \frac{\exp[-\beta(\hat{H}_\mathrm{B}-\mu_\mathrm{B}\hat{N}_\mathrm{B})]}{\mathrm{Tr}\left[\exp[-\beta(\hat{H}_\mathrm{B}-\mu_\mathrm{B}\hat{N}_\mathrm{B})]\right]} \otimes \hat{\rho}^0_\mathrm{S}$, where $\hat{N}_\mathrm{X}$ is the total number operator of the reservoir X, $\beta \equiv 1/k_B T$ is the inverse temperature of the system, and $\hat{\rho}^0_\mathrm{S}$ is the initial density matrix of the resonant level. Since the reservoir A is isolated for $t \leq 0$, the particle number of the reservoir A takes a constant, N^0_A, which is the initial particle number of the reservoir A at $t = 0$: $\hat{\rho}(0)\hat{N}_\mathrm{A} = N^0_\mathrm{A}\hat{\rho}(0)$. The chemical potentials of reservoirs have the different values, $\mu_B = eV$ and $\mu_A = 0$, because of the applied bias voltage V between the reservoirs.

We observe the particle current as the net change of the particle number in the reservoir A by using a quantum measurement with the limited resolution Δ described by measurement operators $\{\hat{M}_k(\mathcal{T},\Delta)\}$, where

$$\hat{M}_k(\mathcal{T},\Delta) \equiv \sum_l \hat{P}_{l+k}\hat{U}(\mathcal{T},0)\hat{P}_l, \tag{S1}$$

and

$$\hat{P}_k \equiv \int_{\chi_k-\frac{\Delta}{2}}^{\chi_k+\frac{\Delta}{2}} dx\delta(x-\hat{N}_A). \tag{S2}$$

Here, $\chi_k \equiv \chi_0 + k\Delta$, and χ_0 expresses the zero point deviation of the particle measurement, and $\hat{U}(t,t') \equiv \check{T}\exp\left[-\frac{i}{\hbar}\int_{t'}^{t}\hat{\mathcal{H}}(t_1)dt_1\right]$ reads the time-evolution operator. $\hat{P}_k$ is a projection operator satisfying $\hat{P}_k\hat{P}_l = \delta_{k,l}\hat{P}_k$. Δ represents the uncertainty in the outcome of the particle number measurement of the reservoir A. Namely, the particle number N that would be obtained under an unlimited resolution is assumed to be in the range $N_0 - \Delta/2 < N < N_0 + \Delta/2$, where N_0 is the outcome of the particle measurement with the limited resolution. Here, $\hat{M}_k(\mathcal{T},\Delta)$ consists of two projective measurements with the limited resolution before and after the unitary time evolution.

$\mathcal{P}(k;\mathcal{T},\Delta)$ is defined by the probability that the change in the particle number in the reservoir A during the measurement time $\mathcal{T}$ is equal to $k\Delta$. With using the measurement operators, $\mathcal{P}(k;\mathcal{T},\Delta)$ is described by

$$\mathcal{P}(k;\mathcal{T},\Delta) = \mathrm{Tr}[\hat{D}_k(\mathcal{T},\Delta)\hat{\rho}(0)], \tag{S3}$$

where $\hat{D}_k(\mathcal{T},\Delta) \equiv \hat{M}^\dagger_k(\mathcal{T},\Delta)\hat{M}_k(\mathcal{T},\Delta)$ is a positive operator-valued measure [1, 2] which satisfies $\sum_k \hat{D}_k(\mathcal{T},\Delta) = \hat{1}$. Note that this two-point measurement statistics with the limited resolution is an extension of that proposed by Esposito *et al.* in Ref. 3. The probability in Eq. (S3) is indeed equal to that given by Esposito *et al.* when we take $\Delta = 1$. Although, in this paper, we consider the particle flow with the two-point measurement statistics with a limited resolution, this quantum measurement scheme is applicable for the flow of other physical quantities such as heat current.

To proceed the calculation, we consider the characteristic function of the probability defined by $\mathcal{M}(\lambda;\mathcal{T},\Delta) \equiv \sum_k \exp[i\lambda k]\mathcal{P}(k;\mathcal{T},\Delta)$. With some calculations, the characteristic function is written as

$$\mathcal{M}(\lambda;\mathcal{T},\Delta) = \sum_{m=-\infty}^{\infty} \mathrm{sinc}(\frac{\lambda+2\pi m}{2}) \exp[i2\pi m\frac{\delta}{\Delta}]\mathcal{M}_0(\frac{\lambda+2\pi m}{\Delta},\mathcal{T}) \tag{S4}$$

where

$$\mathcal{M}_0(\lambda;\mathcal{T}) \equiv \mathrm{Tr}[\hat{U}^{\dagger}(\mathcal{T},0;-\frac{\lambda}{2})\hat{U}(\mathcal{T},0;\frac{\lambda}{2})\hat{\rho}(0)], \tag{S5}$$

$$\delta \equiv N_{\mathrm{A}}^0 - \chi_0 \bmod \Delta \quad (0 \le \delta < \Delta). \tag{S6}$$

$\hat{U}(t,t';\lambda) \equiv \breve{T}\exp\left[-\frac{i}{\hbar}\int_{t'}^{t}\hat{\mathcal{H}}(t_1;\lambda)dt_1\right]$ is the modified time evolution operator with the counting field λ where $\hat{\mathcal{H}}(t;\lambda) \equiv \exp[i\lambda\hat{N}_{\mathrm{A}}]\hat{\mathcal{H}}(t)\exp[-i\lambda\hat{N}_{\mathrm{A}}]$, and $\mathrm{sinc}(x) \equiv \sin(x)/x$. Note that in the above calculation, we ignore a constant factor of $\mathcal{M}(\lambda;\mathcal{T},\Delta)$ which does not affect our final results.

Being described by the forward and backward time-evolutions obeying the different modified Hamiltonians, $\hat{\mathcal{H}}(t;\pm\frac{\lambda}{2})$, $\mathcal{M}_0(\lambda;\mathcal{T})$ is adequately evaluated with using the Keldysh Green's function method [4, 5]. Assuming that the measurement time, $\mathcal{T}$, is much longer than the characteristic time scale of the particle transport determined by $\Gamma^{-1} \equiv [(\Gamma_{\mathrm{A}}+\Gamma_{\mathrm{B}})/2]^{-1}$, we evaluate $\mathcal{M}_0(\lambda;\mathcal{T})$ as $\mathcal{M}_0(\lambda;\mathcal{T}) \approx \exp\Big[\mathcal{T}\mathcal{C}_0(\lambda)\Big]$ where

$$\mathcal{C}_0(\lambda) \equiv \int_{-\infty}^{\infty}\frac{d\varepsilon}{h}\ln\Bigg[1+T(\varepsilon)\Big[(\exp[i\lambda]-1)[1-f_{\mathrm{A}}(\varepsilon)]f_{\mathrm{B}}(\varepsilon) + (\exp[-i\lambda]-1)f_{\mathrm{A}}(\varepsilon)[1-f_{\mathrm{B}}(\varepsilon)]\Big]\Bigg] \tag{S7}$$

is the cumulant generating function of current obtained with the Levitov-Lesovik formula [3, 6]. Here $T(\varepsilon) \equiv \frac{r}{(\varepsilon-\varepsilon_0)^2/(\hbar\Gamma)^2+1}$ reads the transmission probability of the system, $r \equiv \Gamma_A\Gamma_B/\Gamma^2$ represents the asymmetry in couplings, and $f_{\mathrm{X}}(\varepsilon) \equiv \big[\exp[[\beta(\varepsilon-\mu_{\mathrm{X}})]+1\big]^{-1}$ is the Fermi-Dirac distribution function for the reservoir X. We then obtain the following asymptotic form of the characteristic function:

$$\mathcal{M}(\lambda;\mathcal{T},\Delta) = \sum_{m=-\infty}^{\infty} \mathrm{sinc}(\frac{\lambda+2\pi m}{2})\exp[i2\pi m\frac{\delta}{\Delta}] \times \exp\Big[\mathcal{T}\mathcal{C}_0(\frac{\lambda+2\pi m}{\Delta})\Big] \quad (\mathcal{T}\Gamma \gg 1). \tag{S8}$$

In Eq. (S8), $\mathcal{M}(\lambda;\mathcal{T},\Delta)$ depends on δ, which means that we can in principle distinguish each specific initial state with the ideal resolution. The distinction, however, blurs in actual experiments. To take into account the actual resolution limit, we take a simple average over δ for $\ln\mathcal{M}(\lambda;\mathcal{T},\Delta)$ as

$$\langle\cdots\rangle_\delta \equiv \int_0^{\Delta}\frac{d\delta}{\Delta}\cdots. \tag{S9}$$

This is an analogy of the random average in quenched random systems.

Accordingly, the cumulant generating function of the particle current in the long time measurement is given by

$$\mathcal{C}_I(\lambda;\mathcal{T},\Delta) = \frac{\partial\langle\ln\mathcal{M}(\lambda;\mathcal{T},\Delta)\rangle_\delta}{\partial\mathcal{T}}, \tag{S10}$$

Note that in the case of $\Delta=1$, the cumulant generating function in Eq. (S10) is identical to that obtained in the previous study, $\mathcal{C}_I(\lambda;\mathcal{T},1)=\mathcal{C}_0(\lambda)$ [3].

Here, we focus on the averaged current I and the noise S measured by the above measurement scheme. By differentiating the cumulant generating function $\mathcal{C}_I(\lambda;\mathcal{T},\Delta)$ in terms of λ, we evaluate I and S as

$$I = e\Delta\frac{\partial\mathcal{C}_I(\lambda,\mathcal{T},\Delta)}{\partial(i\lambda)}\Big|_{\lambda=0} = I_0 + \langle\Delta I\rangle_\delta, \tag{S11}$$

$$S = e^2\Delta^2\frac{\partial^2\mathcal{C}_I(\lambda,\mathcal{T},\Delta)}{\partial(i\lambda)^2}\Big|_{\lambda=0} = S_0 + \langle\Delta S\rangle_\delta, \tag{S12}$$

where $I_0 \equiv e\frac{\partial\mathcal{C}_0(\lambda)}{\partial(i\lambda)}\Big|_{\lambda=0}$ and $S_0 \equiv e^2\frac{\partial^2\mathcal{C}_0(\lambda)}{\partial(i\lambda)^2}\Big|_{\lambda=0}$ are the intrinsic current and the intrinsic noise. I_0 and S_0 are determined only by the parameters of the system. The excess terms, attributed to the limited resolution measurement, can be evaluated as

$$\langle\Delta I\rangle_\delta = 0, \tag{S13}$$

and

$$\langle\Delta S\rangle_\delta = -\frac{e^2\Delta^2}{2\pi^2}\sum_{m\ge1}\frac{\exp\big[\mathcal{T}\mathcal{C}_0^{\mathrm{sym}}(\frac{2\pi m}{\Delta})\big]\mathcal{C}_0^{\mathrm{sym}}(\frac{2\pi m}{\Delta})}{m^2}. \tag{S14}$$

Here we define $\mathcal{C}_0^{\mathrm{sym}}(\lambda) \equiv \mathcal{C}_0(\lambda)+\mathcal{C}_0(-\lambda)$. Equation (S13) agrees with the naive intuition. Note that $\langle\Delta S\rangle_\delta$ depends on the measurement parameters, $\mathcal{T}$ and Δ, as well as the parameters of the system. From this result, it is found that the limited resolution does not affect the average of the current, which means our measurement scheme is unbiased. In addition, we find

$$\langle\Delta S\rangle_\delta \ge 0 \tag{S15}$$

because $\mathcal{C}_0^{\mathrm{sym}}(\lambda) \le 0$.

In the case of $\Delta=1$, since $\mathcal{C}_0^{\mathrm{sym}}(2\pi m)=0$, the excess noise obviously disappears in accordance with our expectation. On the other hand, for large Δ, Eq. (S14) is evaluated as

$$\langle\Delta S\rangle_\delta \approx \frac{e^2\Delta}{2\pi^2}\int_0^{\infty}\sigma(x,\mathcal{T})dx \tag{S16}$$

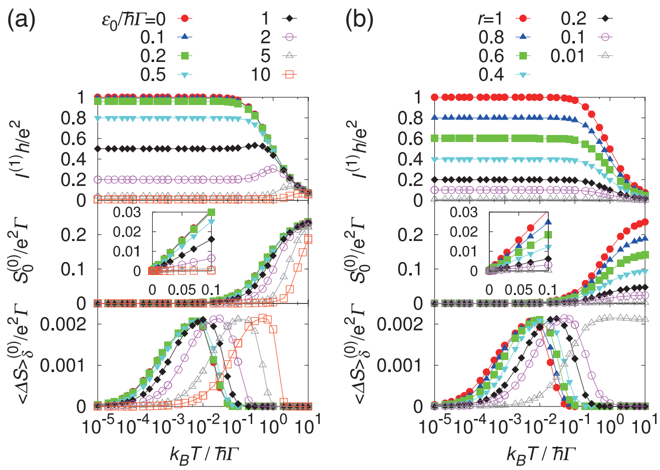

FIG. S1: Conductance $I^{(1)}$, intrinsic noise $S_0^{(0)}$, and excess noise $\langle \Delta S\rangle_\delta^{(0)}$ as a function of temperature T. The measurement parameters are fixed at $\mathcal{T}\Gamma = 1000$ and $\Delta = 10$. $r = 1$ and several choices of ε_0 are used in (a), and $\varepsilon_0 = 0$ and several choices of r are used in (b). The insets show the enlarged plots of $S_0^{(0)}/e^2\Gamma$.

where

$$\sigma(x, \mathcal{T}) \equiv -\frac{\exp\left[\mathcal{T}\mathcal{C}_0^{\rm sym}(2\pi x)\right]\mathcal{C}_0^{\rm sym}(2\pi x)}{x^2}. \tag{S17}$$

Since $\sigma(x, \mathcal{T})$ is independent of Δ, the excess noise scales linearly with large Δ.

II. ε_0 AND r DEPENDENCES

Figure S1 shows the conductance $I^{(1)}$, the intrinsic noise $S_0^{(0)}$, and the excess noise $\langle \Delta S\rangle_\delta^{(0)}$ as a function of the temperature for the several choices of ε_0 and r. It is seen that all these transport quantities are strongly dependent on ε_0 and r. Since $I^{(1)}$ and $S_0^{(0)}$ are only determined by the system parameters, the characteristic temperature of those quantities is given by $\hbar\Gamma/k_B$. For $k_BT/\hbar\Gamma \ll 1$, $I^{(1)}$ takes a constant value and $S_0^{(0)}$ shows a simple linear dependence on T expected from the J-N relation. While, $\langle \Delta S\rangle_\delta^{(0)}$ shows a strong temperature dependence even for $k_BT/\hbar\Gamma \ll 1$ because it also depends on the measurement parameters, $\mathcal{T}$ and Δ. Though both $\langle \Delta S\rangle_\delta^{(0)}$ and $S_0^{(0)}$ show a variety of curves for several choices of ε_0 and r, the universal exponential decay at high $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2$ is again confirmed as in the Letter.

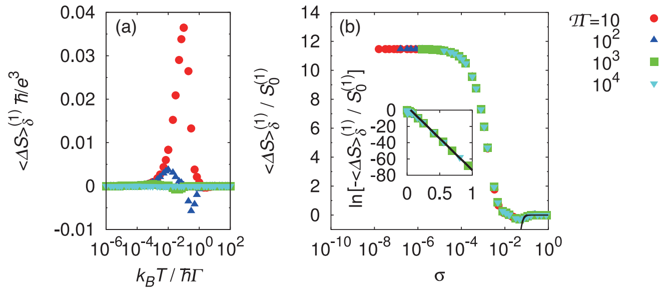

FIG. S2: First order excess noise in voltage and deviation from next order proportional relation beyond the J-N relation. (a) $\langle \Delta S\rangle_\delta^{(1)}$ as a function of temperature for $\Delta = 10$, $\varepsilon_0/\hbar\Gamma = 1$, $r = 1$ and for several choices of $\mathcal{T}$. (b) $\langle \Delta S\rangle_\delta^{(1)}/S_0^{(1)}$ as a function of $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2$. The parameters are the same as in (a). All the curves collapse into the single scaling curve. The exponential dependence, $-A_1\exp[-\gamma_1\sigma]$, is represented by the solid line, with $\gamma_1 = 78$ and $A_1 = 73$. The inset shows the logarithm of $-\langle \Delta S\rangle_\delta^{(1)}/S_0^{(1)}$.

III. VIOLATION OF STEADY-STATE FLUCTUATION THEOREM BY LIMITED RESOLUTION

The cumulant generating function observed with an ideal resolution, $\mathcal{C}_0(\lambda)$, has a nontrivial symmetry of

$$\mathcal{C}_0(\lambda) = \mathcal{C}_0(iA - \lambda), \tag{S18}$$

where $A = e\beta V$, which is equivalent to the steady-state fluctuation theorem (SSFT) as discussed in refs. 7 and 8. The symmetry gives nontrivial relations between the cumulants, such as

$$S_0^{(0)} = 2k_BTI_0^{(1)}, \tag{S19}$$

$$S_0^{(1)} = k_BTI_0^{(2)}, \tag{S20}$$

where $X^{(n)}$ is the coefficient of X by expanding V, [7, 8]

$$X = X^{(0)} + X^{(1)}V + X^{(2)}V^2 + \cdots. \tag{S21}$$

The first equation is the J-N relation between S_0 and I_0. The second one expresses a nontrivial proportional relation beyond the J-N relation.

The cumulant generating function obtained with a limited resolution $C_I(\lambda; \mathcal{T}, \Delta)$, however, has no such a symmetry, which means the violation of the SSFT. Similarly to the case of the J-N relation, the nontrivial relation (S20) between the higher order coefficients does not exactly hold if we replace S_0 and I_0 with S and I, respectively. We, however, find a similar scaling behavior even beyond the linear response. Let us discuss the deviation in the next order term,

$$S^{(1)}/k_BTI^{(2)} - 1 = \langle \Delta S\rangle_\delta^{(1)}/S_0^{(1)}. \tag{S22}$$

First, we see the bare value of $\langle \Delta S\rangle_\delta^{(1)}$ in Fig. S2(a) as a function of the temperature for several choices of $\mathcal{T}$. Similarly to $\langle \Delta S\rangle_\delta^{(0)}$, the value is crucially dependent on T and $\mathcal{T}$, and its maximum absolute value decreases with increasing $\mathcal{T}$. Note that $\langle \Delta S\rangle_\delta^{(1)}$ can have a negative value, whereas $\langle \Delta S\rangle_\delta^{(0)}$ must be positive because of the positivity of $\langle \Delta S\rangle_\delta$. Similarly to the case of $\langle \Delta S\rangle_\delta^{(0)}$, $\langle \Delta S\rangle_\delta^{(1)}/S_0^{(1)}$ collapses into the single scaling form as a function of $\sigma \equiv S_0^{(0)}\mathcal{T}/(e\Delta)^2$ as seen in Fig. S2(b), which reaches zero at high temperatures and saturates at a constant value at low temperatures. We also find an exponential decay of the deviation as $\langle \Delta S\rangle_\delta^{(1)}/S_0^{(1)} = -A_1 \exp[-\gamma_1\sigma]$ with $\gamma_1 = 78$ and $A_1 = 73$. Therefore, $\sigma < 1$ again gives the criterion for the detection of the deviation. The present result $S^{(1)} > k_B T I^{(2)}$ at sufficiently low σ caused by the limited resolution may basically account for the experimental observation in the nonlinear response in Ref. 9. It is an intriguing future issue to pursue the possibility of recovering the symmetry by extending Eq. (S18), after considering the information obtained in the measurement with a resolution. This shares a view common to the generalized Jarzynski equality [10].

IV. ESTIMATION OF MEASUREMENT PARAMETERS

In this section, we estimate realistic and presently accessible measurement parameters, $\mathcal{T}$ and Δ, from an available measurement device. In our two-point measurement model, the current is obtained by measuring the net transferred particle number within the measurement time, $\mathcal{T}$. Although the averaged current is precisely measurable for any choice of the parameters as discussed above, the rigorous value is obtained only when the average is given from the measurement performed infinitely many times. When we consider the case of a single measurement, however, the measurement parameters should give a limit of available information about the current.

If the current is fluctuating with a frequency f, the detectability of the current must be crucially dependent on the measurement time $\mathcal{T}$. For $2f > \mathcal{T}^{-1}$, we hardly obtain the signal from the single measurement because the net transferred particle number within $\mathcal{T}$ is almost zero in our model. Therefore, we estimate the measurement time from the maximum detectable frequency in the actual single measurement, $f_{\max}$, as $\mathcal{T} = (2f_{\max})^{-1}$. In addition, the amplitude of the sinusoidal current with a frequency, $f_{\max}$, is important for the detectability. Δ specifies the detectable difference of the particle numbers at the initial and final states in the two-point measurement. If the net change of the number is less than Δ, we have no meaningful signal in the single measurement. Hence, the minimum amplitude of the detectable sinusoidal current $I_{\min}$, with the frequency of $f_{\max}$ in the single measurement may give the estimation of Δ as $\Delta = \int_0^{\mathcal{T}} I_{\min}\sin(2\pi f_{\max} t)dt/e = I_{\min}/e\pi f_{\max}$.

In the actual measurement of current through a mesoscopic device, the signal of current is enhanced via an amplifier because it is too weak to be directly measured with normal ammeters. The amplifiers have two significant parameters: The maximum detectable frequency of the amplifier, $f_{\rm amp}$, and the input current noise, i_n, which has the dimension of $\mathrm{A}/\sqrt{\mathrm{Hz}}$. Since the precision of the current measurement is limited mainly by the amplifier, we connect our model parameters with those of the amplifier. Since the maximum frequency of the detectable current, $f_{\max}$, is supposed to be given by $f_{\rm amp}$, the measurement time is given by

$$\mathcal{T} = (2f_{\rm amp})^{-1}. \tag{S23}$$

The input current noise of the amplifier limits the amplitude of the detectable current. To obtain meaningful information in a single measurement, the input sinusoidal current with a frequency of $f_{\rm amp}$ must have the amplitude larger than $i_n\sqrt{f_{\rm amp}}$, which leads to $I_{\min} = i_n\sqrt{f_{\rm amp}}$. Hence, we estimate Δ as

$$\Delta = i_n/e\pi\sqrt{f_{\rm amp}}. \tag{S24}$$

More concretely, we estimate $\mathcal{T}$ and Δ from the amplifier of CA-554F2 manufactured by NF Corporation in Japan. CA-554F2 is one of the best amplifiers on the market, which has $f_{\rm amp} = 500\mathrm{KHz}$ and $i_n = 45\mathrm{fA}/\sqrt{\mathrm{Hz}}$. Substituting these parameters into Eq. (S23) and Eq. (S24), we obtain $\mathcal{T} \simeq 1\mu\mathrm{s}$ and $\Delta \simeq 130$. This should, however, be regarded as a rough estimate. To investigate the precise correspondence of the measurement parameters between theory and experiment, we further need more quantitative estimate of the resolution together with the data on the deviation of the J-N relation and its extensions in the nonlinear response.

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