An Interpolation Formula for T_1 for EWJN from a Cylindrical Antenna

For magnetic noise that couples directly to the electron spin, we can always write the relaxation rate as

$$\frac{1}{T_1} = \frac{1}{\mathcal{L}} \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2},$$

as long as the skin depth δ is large compared with the dimensions of the metallic elements of the device and with the distance of the qubit from the metallic objects. Also, this formula assumes that $k_B T$ is small compared with the qubit level separation $\hbar\omega_0$. $1/\mathcal{L}$ is a geometric factor that depends only on the geometry and has the dimensions of inverse length. $\mathcal{L}$ does not depend on temperature but it does depend on the direction of the external field. Thus we see that if the conditions on δ and T are satisfied then since the level separation ω_0 is proportional to B we expect $1/T_1$ to be proportional to B. Calculating the slope of the $1/T_1$ vs. B curve thus reduces to calculating the geometric factor $1/\mathcal{L}$. $1/\mathcal{L}$ can vary over many orders of magnitude for parameters found in common devices, since it falls off rapidly with the distance from the qubit to the electrodes.

The situation of interest is a spin qubit at a distance d in the x-direction from the axis of an infinite metallic cylinder of radius a with its axis along the z-direction. Unfortunately, we do not have a simple formula valid for all d and a. We do have that for $d >> a$

$$\begin{aligned}
\frac{1}{T_{1x}^{cyl}} &= \frac{75\pi^2 a^4}{1024\ d^5} \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2} \\
\frac{1}{T_{1y}^{cyl}} &= \frac{273\pi^2 a^4}{1024\ d^5} \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2} \\
\frac{1}{T_{1z}^{cyl}} &= \frac{147\pi^2 a^4}{512\ d^5} \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2}.
\end{aligned}$$

A simple way to address the issue of the relative magnitude of d and a is to note that when $d \to 0$ then we are back to the half-space case. Hence we can try an interpolation formula, treating T_1^{cyl} as the right form for $d >> a$ and T_1^{hs} as the right form for $d < a$. We have

$$\frac{1}{T_{1z}^{hs}} = \frac{\pi \mu_B^2 \sigma \omega_0}{2\hbar d c^2}$$

$$\frac{1}{T_{1x}^{hs}} = \frac{1}{T_{1y}^{hs}} = \frac{3\pi \mu_B^2 \sigma \omega_0}{4\hbar d c^2}$$

The simplest interpolation formula is

$$\frac{1}{T_{1i}} = \frac{1}{T_1^{hs} + T_1^{cyl}}$$

and in detail this gives:

$$\frac{1}{T_{1x}} = \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2 d} \frac{\frac{75\pi^2 a^4}{1024\ d^4} \frac{3\pi}{4}}{\frac{75\pi^2 a^4}{1024\ d^4} + \frac{3\pi}{4}}$$
$$\frac{1}{T_{1y}} = \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2 d} \frac{\frac{273\pi^2 a^4}{1024\ d^4} \frac{3\pi}{4}}{\frac{273\pi^2 a^4}{1024\ d^4} + \frac{3\pi}{4}}$$
$$\frac{1}{T_{1z}} = \frac{\mu_B^2 \sigma \omega_0}{\hbar c^2 d} \frac{\frac{147\pi^2 a^4}{512\ d^4} \frac{\pi}{2}}{\frac{147\pi^2 a^4}{512\ d^4} + \frac{\pi}{2}}.$$

We use these formulas to construct a table.

$d(nm)$	$1/T_{1x}\ \ (s^{-1})$	$1/T_{1y}\ \ (s^{-1})$	$1/T_{1z}\ \ (s^{-1})$
20	52.4	55.5	37.3
50	5.3	12.0	9.7
100	0.21	0.74	0.77
200	0.0068	0.024	0.026

TABLE

Relaxation times for qubits near a cylinder of radius $a = 50$ nm. The z-axis is the long axis of the cylinder and the x-axis points prependicularly from the cylinder to the qubit. Thus, for example, $1/T_{1x}$ is the rate when the external field points along the line that joins the qubit to the cylinder. The values used are for the device and the field strength in Muhonen *et al.* for which we have $d = 20$ nm, $\sigma = 1.4 \times 10^8 S/m$ (SI) $= 1.3 \times 10^{18}/s$ (cgs), and operating frequency $\omega_0 = 2.6 \times 10^{11}/s$ at $B = 1.5T$. For these parameters $\delta = c/\sqrt{2\pi\sigma\omega} \approx 200\ nm > a$.