

Energy-Dependent Tunneling in a Quantum Dot

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We present measurements of the rates for an electron to tunnel on and off a quantum dot, obtained using a quantum point contact charge sensor. The tunnel rates show exponential dependence on drain-source bias and plunger gate voltages. The tunneling process is shown to be elastic, and a model describing tunneling in terms of the dot energy relative to the height of the tunnel barrier quantitatively describes the measurements.

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Gate controlled quantum dots have been used to study a wide variety of physical phenomena, from correlated-electron physics [1], which becomes important when the coupling of the dot to its leads is strong, to the coherence of electron charge and spin states [2], which can be maintained only when the coupling to the leads is weak. Observation of these phenomena is made possible by *in situ* control of the rate of tunneling Γ , coupling the dot to its leads: Γ can be adjusted over more than 10 orders of magnitude by changing the voltages applied to the gates that define the tunnel barriers of the quantum dot. Recently, integrated charge sensors [3,4] have made possible a variety of new investigations of Γ , probing the effect of excited states [5–7], electron number [8], barrier symmetry [9,10], and Coulomb interactions [10,11].

In this Letter, we report exponential sensitivity of the rates for electrons tunneling on (Γ_{on}) and off (Γ_{off}) a lateral GaAs quantum dot as a function of the drain-source bias V_{ds} and plunger gate voltage V_g . The tunnel rates are obtained by performing time resolved measurements of a quantum point contact (QPC) charge sensor adjacent to the dot [12–14] and utilizing pulsed-gate techniques [12,15]. We show that the tunneling is dominated by elastic processes, and that the measured exponential dependence of Γ on V_{ds} and V_g is in excellent quantitative agreement with a model describing tunneling in terms of the dot energy relative to the height of the tunnel barrier.

The device used in this work is fabricated from a AlGaAs/GaAs heterostructure grown by molecular beam epitaxy. The two-dimensional electron gas (2DEG) formed at the AlGaAs/GaAs interface 110 nm below the surface of the heterostructure has a mobility of $6.4 \times 10^5 \text{ cm}^2/\text{Vs}$ and density of $2.2 \times 10^{11} \text{ cm}^{-2}$ [16]. An electron micrograph of the gate geometry is shown in Fig. 1(a) and is similar to that of Ciorga *et al.* [17]. Applying negative voltages to the labeled gates forms a single dot containing one electron coupled to the surrounding 2DEG through two tunnel barriers: one between SG1 and OG (*b1*, connecting the dot to lead 1) and the other between SG2 and OG (*b2*, connecting the dot to lead 2). Lead 2 is grounded, and a

voltage V_{ds} is applied to lead 1. Approximately the same dc voltage V_g is applied to the three plunger gates LP1, PL, and LP2. We have verified that a single, not double, dot is formed. We measure the dot in a dilution refrigerator with a lowest electron temperature of 120 mK.

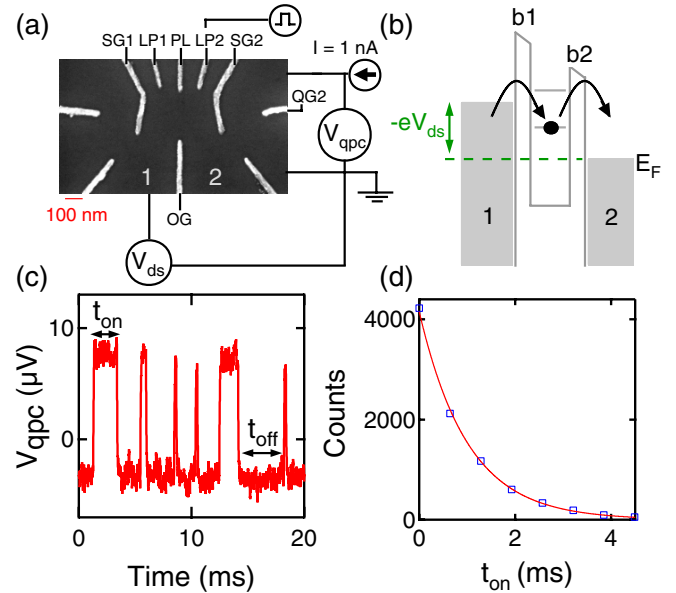


FIG. 1 (color online). (a) Electron micrograph of the gate geometry and schematic of the measurement circuit. Unlabeled gates are grounded. We measure the resistance of the QPC by sourcing a current through the QPC and monitoring the voltage V_{qpc} across the QPC. We have verified that the finite bandwidth of our measurement [24] does not significantly affect the results presented in this work using computer simulations. (b) When a voltage bias V_{ds} is applied across the quantum dot a small current flows and the charge on the dot fluctuates between 0 and 1 as shown in (c). We measure the time intervals t_{on} (t_{off}) that the electron is on (off) the dot using an automated triggering and acquisition system described in Ref. [25]. The offset in the trace is caused by the ac coupling of the voltage preamplifier. (d) Histogram of t_{on} times from data such as in (c). Fitting this histogram yields Γ_{off} as described in the text.

Applying a negative voltage to the gate QG2, we form a QPC between the gates QG2 and SG2. The resistance of the QPC is sensitive to the charge on the dot [3,4], allowing us to measure the number of electrons on the dot in real time [12–14]. If we apply a drain-source bias V_{ds} across the dot [Fig. 1(b)], we observe that the number of electrons on the dot fluctuates between 0 and 1: a typical trace is shown in Fig. 1(c). To measure Γ_{off} , we histogram the times t_{on} that the electron spends on the dot [Fig. 1(d)] and fit to an exponential $Ae^{-\Gamma_{off}t_{on}}$ [14]. We obtain Γ_{on} from the time intervals t_{off} in the same manner. Using this technique we measure Γ_{off} and Γ_{on} as a function of V_{ds} [Fig. 2(a)] for the case in which there is either 0 or 1 electrons on the dot at the position shown in Fig. 2(b). As V_{ds} is made more negative, Γ_{off} is seen to increase exponentially.

To model the tunneling rates, we linearize the exponential in the WKB formula for tunneling through a barrier [18] for a small perturbation δE to the dot energy E and a small deviation δU of the tunnel barrier potential $U(x)$: $\Gamma \sim \Gamma_0 \exp[-(\delta U - \delta E)\kappa]$, where κ and Γ_0 depend on the details of the barrier potential. In a simple capacitor model for the dot we assume a linear dependence of δE on small perturbations δV_g and δV_{ds} about arbitrary but fixed V_{ds} and V_g values: $\delta E = -e\alpha_{dsE}\delta V_{ds} - e\alpha_{gE}\delta V_g$, where α_{dsE} is the ratio of the drain-source capacitance to the total dot capacitance, and α_{gE} is the capacitance ratio for the three plunger gates. Similarly, δU varies linearly as $\delta U = -e\alpha_{dsU}\delta V_{ds} - e\alpha_{gU}\delta V_g$, where α_{dsU} and α_{gU} give the coupling of V_{ds} and V_g to the barrier potential. There will, of course, be different parameters α_{dsU} , α_{gU} , and κ for the two barriers. Note that Γ depends exponentially on $\delta U - \delta E$, and therefore depends exponentially on V_{ds} and V_g : one can show that this holds independent of the particular shape $U(x)$ of the barrier potential, or the shape of the perturbation to the potential induced by the change δV_g or δV_{ds} . Using this linearization, we can write down equations for the V_{ds} dependence of Γ_{off} and Γ_{on} , including Fermi statistics in the leads, assuming elastic tunneling, and considering only a single orbital state of the dot:

$$\Gamma_{off} = \Gamma_{2,0}e^{-\beta_2\delta V_{ds}}[1 - f_2(E)] + \Gamma_{1,0}e^{\beta_1\delta V_{ds}}[1 - f_1(E)] \quad (1)$$

$$\Gamma_{on} = \eta\Gamma_{2,0}e^{-\beta_2\delta V_{ds}}f_2(E) + \eta\Gamma_{1,0}e^{\beta_1\delta V_{ds}}f_1(E). \quad (2)$$

Here $\beta_{1,2} = \kappa_{1,2}|\alpha_{dsU_{1,2}} - \alpha_{dsE}|$, $E = -e\alpha_{dsE}V_{ds} - e\alpha_{gE}\Delta V_g$ is the energy of the one-electron state relative to the Fermi energy E_F , and ΔV_g is the shift in V_g from the 0 to 1 electron transition. f_2 and f_1 are the Fermi functions of the two leads $f_2(E) = f(E)$, $f_1(E) = f(E + eV_{ds})$, and η is the ratio between the tunnel rate onto and off of the dot for a given lead when the one-electron state is aligned with the Fermi level in that lead. We expect that $\eta = 2$ because of spin degeneracy [11,19], and use this value in the calculations below.

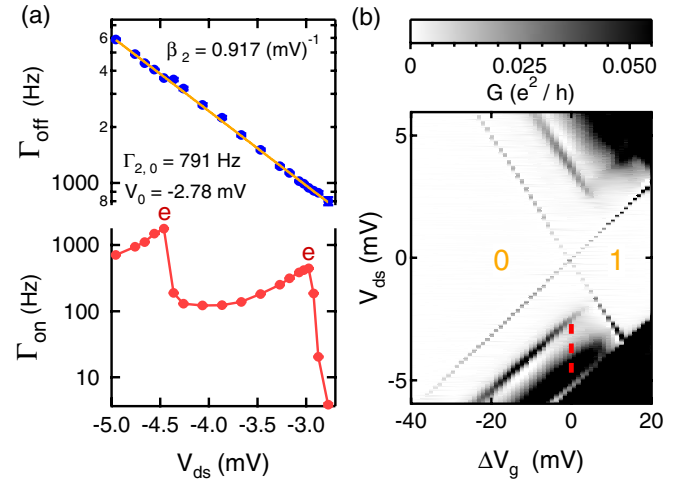


FIG. 2 (color online). (a) Γ_{on} and Γ_{off} as a function of V_{ds} for large negative V_{ds} . The solid line in the upper panel is based on a theoretical fit to the data discussed in the text. (b) Differential conductance vs V_{ds} and V_g , showing the 0 to 1 electron transition. The tunnel rates for this case are made large enough so that the differential conductance can be measured using standard transport techniques. The data shown in (a) are taken at the position of the dashed line.

To understand Fig. 2(a), we note that whether Γ increases or decreases with V_{ds} depends on whether lead 1 is better coupled to the barrier or the dot, that is, whether α_{dsU} or α_{dsE} is larger. Since $b1$ is closer to lead 1 than the dot, and $b2$ is farther from lead 1 than the dot, it follows that $\alpha_{dsU_1} > \alpha_{dsE} > \alpha_{dsU_2}$ [see Fig. 1(b)] [20]. Therefore, tunneling through $b1$ ($b2$) increases (decreases) exponentially with increasing V_{ds} . This is reflected in the signs of the exponentials appearing in Eq. (1) and (2).

The solid line through the Γ_{off} data in Fig. 2(a) is a fit to Eq. (1), which for large negative bias reduces to $\Gamma_{2,0}e^{-\beta_2\delta V_{ds}}$. The rate Γ_{on} is shown as a function of V_{ds} in the lower panel of Fig. 2(a). At the two points marked “e” in the figure Γ_{on} increases rapidly as the Fermi energy in lead 1 is aligned with an excited orbital state of the dot [7], giving the 1st and 2nd excited state energies to be 1.9 and 2.9 meV, respectively, above the ground state. The higher-energy states are better coupled to the leads and thus Γ_{on} rises rapidly when these states become available. These energies can also be measured, with larger tunneling rates through $b1$ and $b2$, using standard transport techniques [21] [Fig. 2(b)], and the results are consistent. We note that Γ_{off} does not show any special features at these points: this is because the electron decays rapidly out of the excited orbital states and subsequently tunnels off the dot from the ground state [7]. We therefore continue to use Eq. (1) when there are multiple orbital states in the transport window.

In the regions between the points marked “e,” Γ_{on} is seen to decrease exponentially as V_{ds} is made more negative, as expected from Eq. (2). Note that this decrease in Γ_{on} , with increasingly negative V_{ds} , occurs even though the

number of electrons that could tunnel onto the dot inelastically from lead 1 is growing, presenting strong evidence that the tunneling is predominantly elastic, dominated by states very close to the dot energy E . There is, however, an apparent flattening of Γ_{on} above the extrapolated exponential decrease near $V_{\text{ds}} = -4$ mV, close to the second excited orbital state. We find that this line shape is consistent with broadening of the second excited state by a Lorentzian of full width at half maximum $\gamma \sim 10$ μeV . Calculated line shapes are shown for the broadening of the first excited state in Fig. 3 and are discussed below.

If a square barrier is assumed, one can compute an effective barrier height U_2 and width w_2 for b_2 from the fit in Fig. 2(a). For a square barrier $\Gamma = f_0 e^{-2w_2 \sqrt{2m^*(U_2 - E)/\hbar^2}}$ [18]. Linearizing the square root in the exponential and estimating $\alpha_{\text{ds}E} - \alpha_{\text{ds}U_2} \sim \alpha_{\text{ds}E}$ and $f_0 \sim E_{\ell s}/h \sim 1$ THz, where $E_{\ell s}$ is the level spacing of the dot, we obtain $w_2 \approx 130$ nm and $U_2 - E_F \approx 5$ meV at $V_{\text{ds}} = V_0$. These values are only logarithmically sensitive to f_0 and thus depend very little on our estimate of this parameter. The voltages we apply to the gates to create the tunnel barriers are the same order of magnitude as the voltages at which the 2DEG depletes and thus it is reasonable that $U_2 - E_F$ is found to be comparable to the Fermi energy ($E_F \approx 7.7$ meV). The value for w_2 is also reasonable given the dimensions of our gate pattern and heterostructure.

We next examine, in Fig. 3, the dependence of Γ_{on} and Γ_{off} on V_{ds} for both positive and negative V_{ds} . The data are taken with V_g adjusted so that E is $\sim kT$ away from the 0 to 1 electron transition at $V_{\text{ds}} = 0$. The solid lines in Fig. 3 are calculated from Eq. (1) and (2) and are in good agreement with the data. We note that the value of β_1 is very close to the value of β_2 . If the height and width of b_1 and b_2 are

comparable one can show that $\beta_1 \sim \frac{\alpha_{\text{ds}U_1} - \alpha_{\text{ds}E}}{\alpha_{\text{ds}E} - \alpha_{\text{ds}U_2}} \beta_2$, and it is therefore expected that $\beta_1 \sim \beta_2$.

At $V_{\text{ds}} \approx -3$ mV in Fig. 3, the first excited orbital state is aligned with the Fermi energy in lead 1. For slightly less negative values of V_{ds} , where the excited state is just above the Fermi energy, Γ_{on} appears enhanced above the solid line calculated from Eq. (2). The dashed line in Fig. 3 includes broadening of the first excited orbital state by a Lorentzian of full width at half maximum $\gamma = 13$ μeV , corresponding to a lifetime broadening of $\tau_{e1} = 50 \pm 30$ ps. Emission of acoustic phonons can lead to excited state lifetimes ~ 100 ps, but for our device parameters we expect much slower relaxation from this mechanism [15,22,23]. Γ_{on} also deviates from the solid curve for $V_{\text{ds}} \geq 2$ mV: this deviation may be caused by broadening of the first excited state as well. These deviations could also be caused by inelastic processes, which might begin to contribute significantly to Γ_{on} when the electron energy is sufficiently far below the Fermi energy.

We now turn to the dependence of the tunnel rates on the plunger gate voltage V_g (see Fig. 4). For these measurements, the barriers are tuned so that the tunneling through b_1 is negligible. In the region near $\Delta V_g = 0$ (where $E \sim kT$), the electron hops on and off the dot spontaneously because there are both electrons and holes in the lead at these energies [Fig. 4(b)]. In this region we measure the rates Γ_{on} and Γ_{off} in the same way as described above. In the inset to Fig. 4(a) we compare the probabilities that the electron is on and off the dot $p_{\text{on}} = \Gamma_{\text{on}}/(\Gamma_{\text{on}} + \Gamma_{\text{off}})$ and $p_{\text{off}} = \Gamma_{\text{off}}/(\Gamma_{\text{on}} + \Gamma_{\text{off}})$ to $f(E, \eta) = \frac{1}{1 + \frac{1}{\eta} e^{E/kT}}$ and $1 - f(E, \eta)$, respectively, with $T = 120$ mK [7].

A pulsed technique [see Fig. 4(c)] is used to measure Γ_{off} when ΔV_g is made sufficiently negative that thermally assisted electron tunneling ceases. We begin with the electron energy, determined by ΔV_g , well above E_F . We then apply a positive voltage pulse to the gate LP2 to bring the one-electron energy level near E_F , so that an electron can hop onto the dot. A short time after the pulse the electron will hop off the dot because it is above E_F . We record the time at which this occurs, Δt , measured relative to the end of the gate pulse. This process is repeated and we make a histogram of the number of tunnel-off events versus Δt ; an exponential fit to this histogram yields Γ_{off} . An analogous technique is used to measure Γ_{on} when the one-electron state is well below E_F .

To understand the V_g dependence of Γ , we note that $\delta U_2 = -e\alpha_{gU_2}\delta V_g$, and $\alpha_{gU_2} < \alpha_{gE}$ because the three plunger gates are closer to the dot than they are to b_2 . The dependence of the tunnel rate through b_2 on V_g is thus very similar to its dependence on V_{ds} (Fig. 3). Starting at the far left of Fig. 4(a), we see that Γ_{off} decreases exponentially as ΔV_g is made less negative: this is analogous to the Γ_{off} data shown in the top panel of Fig. 2(a) and 3 (for $V_{\text{ds}} < 0$, where the electron tunnels off through b_2). Γ_{off} decreases rapidly at the 0 to 1 electron transition as the

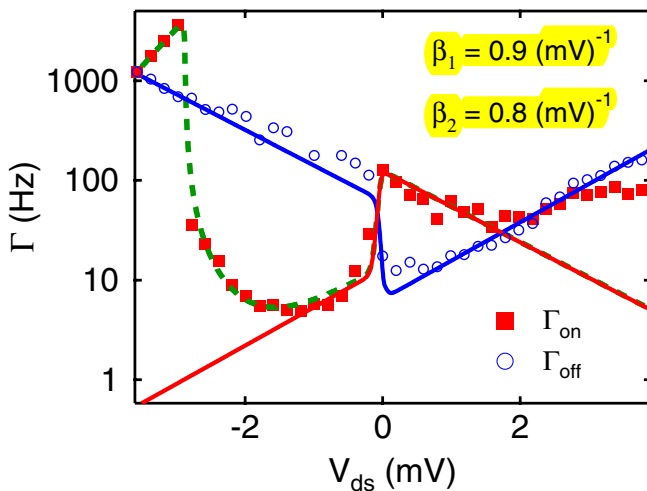


FIG. 3 (color online). Γ_{on} (■) and Γ_{off} (○) as a function of V_{ds} . The solid and dashed curves are calculations described in the text. The step features near $V_{\text{ds}} = 0$ result from the Fermi distribution.

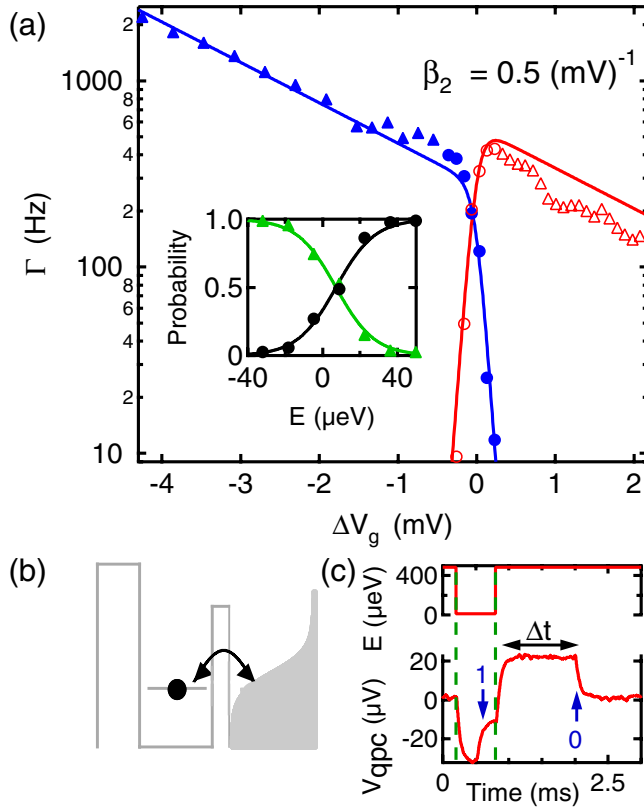


FIG. 4 (color online). (a) Γ_{on} and Γ_{off} as a function of ΔV_g . Closed (open) circles are Γ_{off} (Γ_{on}) measured by observing spontaneous hopping caused by thermal broadening in the leads as depicted in (b). Closed (open) triangles are Γ_{off} (Γ_{on}) measured using a pulsed-gate technique. Solid lines are calculations as described in the text. (Inset) p_{on} (triangles) and p_{off} (circles) compared to $f(E, \eta)$ and $1 - f(E, \eta)$ (solid lines), respectively, as described in the text. (b) Sketch of spontaneous hopping caused by thermal broadening in the leads. (c) Pulsed technique used to measure Γ_{off} . The top panel shows the pulsed modulation of the one-electron state energy E . The bottom panel shows a sample time trace. The dashed vertical lines indicate when the gate pulse begins and ends: the QPC responds to the gate pulse because of direct capacitive coupling to LP2. When the electron energy level is brought near E_F an electron tunnels on the device (indicated by a 1). When the electron level is brought back above E_F the electron tunnels off the device (indicated by a 0).

ground state is brought below E_F . Continuing into the 1 electron valley we see that Γ_{on} decreases as ΔV_g is made more positive: this is analogous to the Γ_{on} data shown in Fig. 3 (for $V_{\text{ds}} > 0$ where the electron tunnels onto the dot through $b2$). This decrease in Γ_{on} is further evidence that the tunneling is elastic because as the one-electron state is brought farther in energy below E_F there are more electrons that could tunnel onto the dot inelastically at energies closer to the top of the barrier. Γ_{on} , however, decreases because elastic tunneling onto the dot happens at a lower energy relative to the barrier height.

We model the data shown in Fig. 4(a) using the same equations as for the drain-source dependence [Eq. (1) and (2)] except with $\Gamma_{1,0}$ set to zero and δV_{ds} replaced by ΔV_g

in the exponents. The solid lines in Fig. 4(a) are calculated from Eq. (1) and (2) with these adjustments and describe the data well. Here again we take $\eta = 2$, although the fit is improved with a smaller value for η [11]. We note that the value for β_2 obtained from the ΔV_g dependence is smaller than the value obtained from the δV_{ds} dependence: this is expected because $\alpha_{gE} < \alpha_{\text{ds}E}$.

Summarizing, we find that the dependence of Γ_{on} and Γ_{off} on V_{ds} and V_g is well described by elastic tunneling at a rate set by the difference between the electron energy and the barrier height.

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