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Performance of feedback schemes to suppress nuclear spin fluctuations in quantum dots

by

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Contents

1 Introduction

One possible way of building a qubit is the double dot spin qubit, using the spin state of two electrons to encode data. In Gallium Arsenide (GaAs), the achieved level of control of those qubits is quite good already, which means that different states can be initialized with good fidelities. The problem of GaAs spin qubits is the decreased coherence time due to the hyperfine interaction between the electrons and the nuclei. The spin states of the nuclei fluctuate, which causes the qubits dynamics not to be equal every time before readout. A qubit state cannot be read out with one measurement due to its statistical nature. If the final state of the qubit is not equal to the one before the last measurement, obviously the information of that state cannot be read out. The different effects causing this decoherence are characterized by different coherence times. In order to build a quantum computer, one would need a qubit with a sufficiently long coherence time. If the coherence time was not long enough to be able to perform about 10^4 gate operations per error, normal error correcting algorithms could not decrease the amount of errors so that a quantum computer would work [8]. In order to decrease the fluctuations in the nuclear field, a feedback mechanism was developed that can reduce slow fluctuations by changing the nuclear field at a rate that depends on the nuclear field itself. The coherence time was increased from 14 ns without feedback to about 100 ns [2]. A further optimization of the coherence time can be achieved by a Carr-Purcell-Meiboom-Gill pulse sequence (to about 100 μs) known from NMR experiments. This work's goal is to get a better understanding of the feedback mechanism, to be able to simulate it and to get an idea of its potential to be optimized. In the end there is a proposal of a different feedback mechanism that is based on another mechanism to influence the nuclear field. With this mechanism it is possible to change the nuclear field faster than with the first mechanism, which might increase the feedback's performance and therefore the coherence time. The rate at which a pump-cycle changes the nuclear field is especially important, because the feedback mechanisms use the qubit to flip nuclear spins. That means that a qubit can not be used as such, while a pump-cycle is run.

2 Backround

2.1 Dynamics of a Double Spin Qubit

In double-dot spin qubits there are four possible spin states, one singlet and three triplets. Two of those triplets T_+ ($\uparrow\uparrow$) and T_-($\downarrow\downarrow$) are split off by a Zeeman energy induced by an external magnetic field. The dynamics of the remaining triplet T_0 and the singlet S are the standard dynamics of a two level system visualized by a Bloch sphere. The energy splitting between the T_0 and S state can be varied by choosing the quantum dot's gate voltages accordingly. Those voltages are often characterized by a parameter ε. Figure 2.2 shows the energy of different states as a function of ε. Another possible energy splitting is between the $\uparrow\downarrow$ and $\downarrow\uparrow$ state, where the arrows symbolize a spin state and the position symbolizes a specific quantum dot. This energy splitting can be induced by a gradient in the magnetic field and will cause a sinusoidal precession between the T_0 and S state.

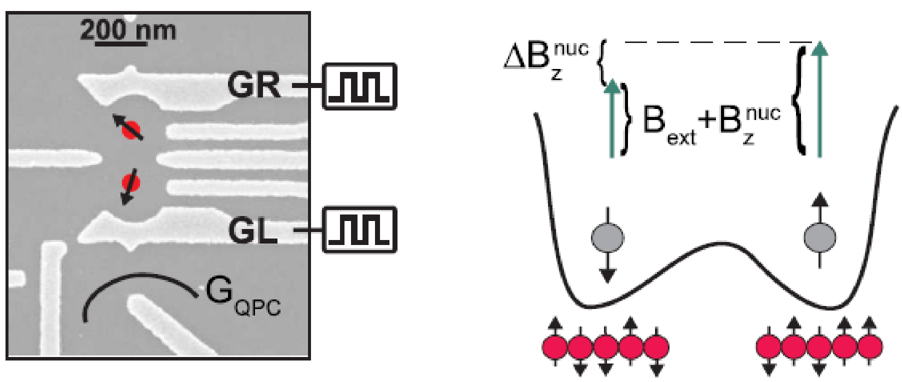


Figure 2.1: Illustration of a double quantum dot [2]

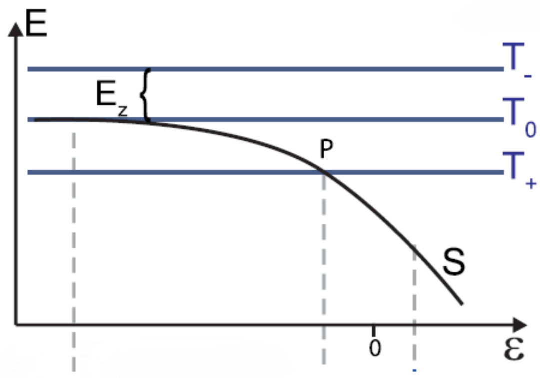


Figure 2.2: ε defines the energies of the different qubit states. The gate voltages have to be calibrated for each experiment. ε therefore gives a more general information about the qubit's dynamics [2].

2.2 Noise

The magnetic field interacting with the electron is not constant due to fluctuations in the nuclear-spin field. Even at low temperatures for example $150\,mK$ the nuclear spins are not statistically correlated. The fluctuations in the spin field characterize the electron as coupled to about 10^6 nuclei [6]. For further simulations of feedback mechanisms, that are used to suppress those fluctuations, they need to be described mathematically. Usually fluctuations of a quantity B are described by a correlator $S_B(\Delta t) = \langle B(t + \Delta t)B(t)\rangle$ (time invariance is already assumed here). There already are theoretical approaches to finding that correlator. It was shown that a spin diffusion model yields the following correlator for the nuclear magnetic field [4]:

$$S_B(\Delta t) \sim \int \mid \psi(k) \mid^4 e^{-D|\Delta t|k^2} d^3k \tag{2.1}$$

ψ is the electron's wave function, or rather an envelope function since it does not completely describe the Coulomb interaction with the nuclei. The diffusion constant D usually is direction dependend and therefore a matrix. In order to characterize the spectral noise it is enough to just take the highest eigenvalue, because this calculation is only going to require the lowest reciprocal value of the diffusion constants for one direction. The spectral noise is:

$$\tilde{S}_B(\omega) \sim \int\int \mid \psi(k) \mid^4 e^{-D|\Delta t|k^2 - i\omega\Delta t} d\Delta t\, d^3k \tag{2.2}$$

$$\tilde{S}_B(\omega) \sim \int \mid \psi(k) \mid^4 \frac{2}{Dk^2(1 + \frac{\omega^2}{D^2k^4})} d^3k \tag{2.3}$$

It is easy to see that the result depends on the envelope function ψ of the electron state. To get an idea of the characteristics of $\tilde{S}_B(\omega)$ rather than an exact parameter it is sufficient to approximate ψ as a Gaussian in all dimensions. Even though in the z-dimension, which is the dimension parallel to the external magnetic field, this will not hold for all calculations done during this project. The parameters σ_z and $\sigma_\perp$ are the parameters defining the width of the Gaussian in the z-direction and orthogonal to the z-direction. Those are different because the electron is in a 2 dimensional electron gas (2DEG).

$$\tilde{S}_B(\omega) \sim \int e^{-(k_\perp^2\sigma_\perp^2 + k_z^2\sigma_z^2)} \frac{2}{Dk^2(1+\frac{\omega^2}{D^2k^4})} d^3k \tag{2.4}$$

$$\approx_{|k|<3max_\mu(1/\sigma_\mu)} \int e^{-(k_\perp^2\sigma_\perp^2 + k_z^2\sigma_z^2)} \frac{2}{Dk^2(1+\frac{\omega^2}{D^2k^4})} d^3k \tag{2.5}$$

It is also known that $\min_\mu(\frac{\sigma_\mu^2}{D}) \approx 1s$ [4]. Thus for $\omega \gg \frac{1}{s}$ the lorentzians can be approximated as $\frac{2Dk^2}{\omega^2}$:

$$\tilde{S}_B(\omega) = \frac{S_0}{\omega^2} \tag{2.6}$$

$$\iff \tilde{S}_{\dot{B}}(\omega) = S_0 \tag{2.7}$$

Of course this does not hold for slow ω but it will be shown that the characteristics of $\tilde{S}_B(\omega)$ for slow ω are not important. Even though $\tilde{S}_B(\omega)$ diverges, the feedback mechanism will reduce those slow fluctuations to a realistic level, which will be shown in section 4.1. S_0 will be taken from an experiment and is about $11\frac{mT^2}{s}$[1] [1].The correlator is also verified by the experiment as shown in figure 2.3[5].

[1] In the paper this constant is given as $\frac{1}{s^3}$ which is the change in the electrons lamor frequency squared.

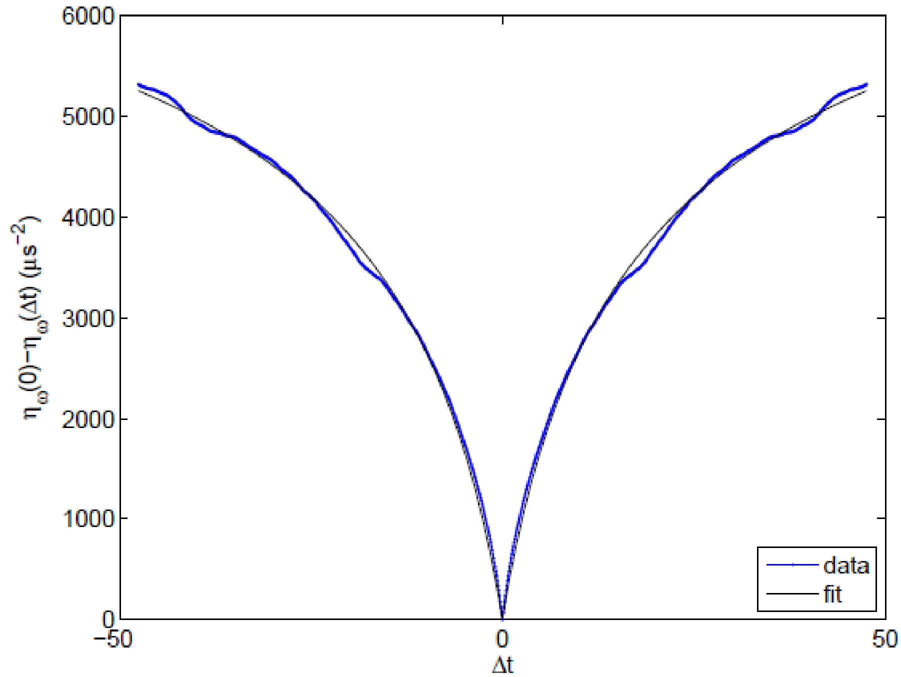


Figure 2: Measured noise correlator and fit to diffusion model.

Figure 2.3: Comparison between the correlator that was theoretically derived and the measured correlator in the time domain[5].

2.3 Coherence Time T_2^*

As mentioned before, a gradient in the magnetic field causes a sinusoidal precession between the S and T_0 state. Fluctuations in the magnetic field gradient therefore cause decoherence. The coherence time of a two level system can easily be calculated for white noise. Here is a model of the spin precession of a two level system with a fluctuating energy splitting [7]:

$$H = \frac{\hbar}{2}(\omega_0 + \beta(t))\sigma_z \tag{2.8}$$

Here $\beta(t)$ is a random number due to fluctuations.

$$\varphi(t) = \int_0^t \beta(t')dt' \tag{2.9}$$

$$\Rightarrow U(t) = \begin{pmatrix} e^{\frac{i}{2}(\omega_0 t+\varphi(t))} & 0 \\ 0 & e^{-\frac{i}{2}(\omega_0 t+\varphi(t))} \end{pmatrix} \tag{2.10}$$

$$\Rightarrow \langle \sigma_x \rangle = cos(\omega_0 t) e^{-\frac{1}{2} < \varphi^2(t) >} \tag{2.11}$$

Here we actually set the inital state of the system to be the eigenstate of $\sigma_x$with the eigenvalue 1. For slow fluctuations on small timescales there is the ensemble average $\langle \varphi(t) \rangle = \langle \beta \rangle t$, hence:

$$\langle \sigma_x \rangle = cos(\omega_0 t) e^{-\frac{1}{2} < \beta^2 > t^2} \tag{2.12}$$

The important timescale characterizing decoherence is:

$$T_2^* = \sqrt{\frac{2}{\langle \beta^2 \rangle}} \tag{2.13}$$

There are more mechanisms causing decoherence, for example faster fluctuations or relaxation, but this is the dominating effect.

3 T-S Feedback

There was a feedback cycle introduced[2] that consists of two parts. This work will focus more on what they do rather than how exactly they work, which is not completely understood.

3.1 S Pump Cycle

The S-pumping cycle starts by initializing an S state. Then ε is swept through the degeneracy point P where the energy of S and T_+ are equal. While sweeping through P, the electron in the spin down state can flip to spin up, which also requires the spin-state of one nucleus to change. The electron can flip because the energy of the system would not be changed while ε is in the degeneracy point. It is unknown where the spin-up and where the spin-down electron is. This pumping should have the same effect on both dots, assuming the dots are identical. However this pump cycle does change the nuclear field gradient as shown in the experiment[2]. One possible explanation is to assume different dot sizes which would result in different pumping rates, which means different speeds of changing the magnetic field in the two dots.

In order to make this pump cycle a feedback loop, there is the possibility to change ε so that T_0 and S have the same energy after initializing the S state and letting it evolve there for a time that will be called τ. Depending on the magnetic field gradient the electron can be in the S state as well as in the T_0 state after this precession time. If ε is then sweept through P, the spin flip probability and therefore the pumping rate is proportional to the S probability. So the pump rate depends on the current magnetic field gradient, which makes the S pump cycle a feedback cycle. Figure 3.1 shows the S pulse and the complete feedback pulse.

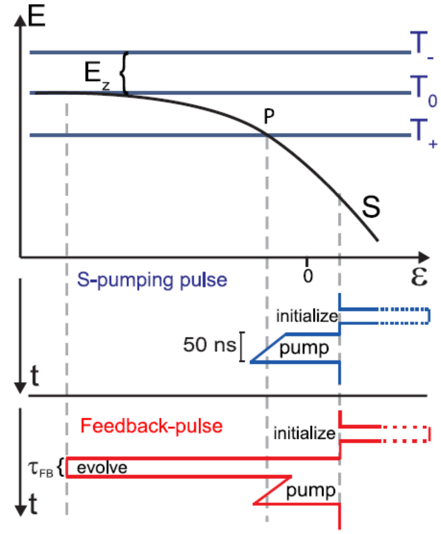


Figure 3.1: [2]

The pump rate dependence on the magnetic field gradient B and the precession time τ is given as:

$$\dot{B} = P_+ cos^2(\frac{g\mu_B B}{2\hbar}\tau) \tag{3.1}$$

Here g is the Landé factor and P_+ is the maximum achievable pumprate. Since the pumping mechanism is not completely understood the maximum pumprate will be taken from an experiment rather than trying to calculate it.

3.2 T_+ Pumpcycle

The T_+-pump cycle works in a similar way as the S-cycle just with the initial and final spin-state switched. It starts by initialising a T_+ state. Sweeping through P will then cause spin flips in the other direction, causing a nuclear polarisation in the other direction. The energy diagram in section 3.1 shows that there is no equivalent feedback mechanism for the T_+ cycle. That implies the pump rate due to this cycle is:

$$\dot{B}(B,\tau) = \dot{B} := P_- \tag{3.2}$$

The whole feedback cycle will be designed to have stationary points where the Pumprate is zero with a negative slope $\frac{\partial \dot{B}}{\partial B}$. The pumpcycle in section 3.1 does not fulfill both requirements in one point. A combination of both cycles will solve that problem.

3.3 Technical Data and Feedback

In typical experiments each of those cycles takes about 250ns. In the case of the feedback cycle we have to add the precession time to this. The precession time will often be neglectible, however if τ is long enough P_+ changes with τ but all other equations are left unchanged.

$$P_+ = P_+(\tau) = \frac{P_+(\tau = 0) \cdot 250ns}{250ns + \tau} \tag{3.3}$$

The pumprates achieved by those cycles are about $P_+ = 40\frac{mT}{s}$ and $P_- = 20\frac{mT}{s}$ [5]. Since this is experimental data it is not something that we can just assume for any experimental setup, but this is the order of magnitude used in most simulations done during this project. A combined cycle that results from repeated switching between the S and T_+ cycle yields the following pump curve:

$$\dot{B} = \frac{n_+}{2n} P_+ (1 + cos(\frac{g\mu_B B}{\hbar}\tau)) - \frac{n_-}{n} P_- \tag{3.4}$$

Here $n = n_+ + n_-$, n_+ is the number of S cycles and n_- is the number of T_+ cycles. It was assumed that an average pump rate can be used for all calculations instead of discretely simulating every pulse. In section 5 it will be shown that this is possible on far bigger timescales than 10^{-7}s, which is quite intuitive considering the magnitude of the pump rates. In the following n_+ and n_- will be chosen so that $n_+ P_+ = 2n_- P_-$ to achieve a symmetry in the pump curve[2].

[2] Simulations that are not included in this work showed that this optimizes the coherence time achieved by the pump cycle. It will be shown that the slope of the pump curve in the stationary point should be maximized.

4 Statistical Dynamics of the Nuclear field

4.1 Spectral Noise with Feedback

The coherence time is changed by a feedback mechanism. There are two ways to calculate the variance of the magnetic field if a feedback mechanism is running. This yields T_2^*.

$$\dot{B}(t) = F(B(t)) + \eta(t) \tag{4.1}$$

Here $F(B)$ is the pump curve following from the feedback mechanism and $\eta(t)$ is due to noise. This is a Langevin equation that will help to find the variance of the magnetic field gradient, which leads to T_2^*. In order to be able to fourier transform this equation $F(B)$ is approximated as a first order Taylor polynomial around a stationary point[3] in $F(B)$, which of course will give bad results if the magnetic field leaves the area where this approximation's error is small. Then there is the substitution $B(t) = B'(t) + \Delta B$ so that the equation becomes an inhomogenous linear differential equation:

$$\Rightarrow \dot{B}'(t) \approx -\alpha B'(t) + \eta(t). \tag{4.2}$$

A Fourier-transformation yields:

$$\Rightarrow \langle \mid \tilde{B}'(f) \mid^2 \rangle \approx \frac{1}{1 + \frac{\alpha^2}{4\pi^2 f^2}} \tilde{S}_B(f) \tag{4.3}$$

$$\Rightarrow \langle \mid \tilde{B}'(f) \mid^2 \rangle \approx \frac{1}{4\pi^2 f^2 + \alpha^2} S_0. \tag{4.4}$$

The variance can now be calculated using $\langle B'^2 \rangle = S_{B'}(0) = \int \tilde{S}_{B'}(f) df$ and $\tilde{S_{B'}}(f) = \langle \mid \tilde{B}'(f) \mid^2 \rangle$:

$$\Rightarrow \langle B'^2 \rangle \approx \int_{-\infty}^{\infty} \frac{S_0}{4\pi^2 f^2 + \alpha^2} df \tag{4.5}$$

$$\Rightarrow \langle B'^2 \rangle \approx \frac{S_0}{2\alpha} \tag{4.6}$$

4.2 Statistical Model

4.2.1 Fokker-Planck Equation

A more general and more detailed way of modelling the dynamics of the magnetic field is to introduce B as a statistical value and using a Fokker-Planck

[3] A stationary point in phase space goes through zero with a negative slope.

equation[4] to describe its dynamics. The exact equation can be derived from the Langevin equation:

$$\dot{B(t)} = F(B(t)) + \eta(t). \tag{4.7}$$

Since $\eta(t)$ is white noise, as seen in equation 2.7:

$$\Rightarrow \dot{p}(B,t) = -\frac{\partial}{\partial B}(F(B)p(B,t)) + \frac{S_0}{2}\frac{\partial^2}{\partial B^2}p(B,t). \tag{4.8}$$

Here $p(B,t)$ is the probability distribution of B at time t. The parameters can be found by using the Langevin equation to calculate $\lim_{\Delta t \to 0}(\frac{\langle (B(t+\Delta t)-B(t))^n \rangle}{\Delta t})\mid_{B(t)=B}$. As always in Gaussian processes only the first two parameters do not necessarily reduce to zero. From here it is simple to derive the probability current $J(B,t)$ and the development of the variance $V_{B'}$.

$$J(B,t) = \frac{\partial}{\partial t}\int_B^\infty p(B',t)dB' \tag{4.9}$$

$$\Rightarrow J(B,t) = F(B)p(B,t) - \frac{S_0}{2}\frac{\partial}{\partial B}p(B,t) \tag{4.10}$$

Here it is useful to substitute $B = B' + \Delta B$ so that $\langle B' \rangle = 0$

$$\frac{\partial}{\partial t}V_{B'}(t) = \frac{\partial}{\partial t}\int_{-\infty}^{\infty} p(B',t)B'^2 dB' \tag{4.11}$$

$$\Rightarrow \frac{\partial}{\partial t}V_{B'}(t) = \int_{-\infty}^{\infty} 2B'p(B',t)F(B'+\Delta B)dB' + S_0 \tag{4.12}$$

In the case of no feedback this reduces to S_0.

4.2.2 Steady States

With the Fokker-Planck equation it is now possible to find stationary probability distributions.

$$\frac{\partial}{\partial t}p(B,t) = 0 \tag{4.13}$$

$$\Rightarrow -F(B)p(B) + \frac{S_0}{2}\frac{\partial}{\partial B}p(B) = 0 \tag{4.14}$$

A simple integral alone would not yield equation 4.14 without a possible constant on the right side. But since this is supposed to be a steady state the probability current has to be zero.

$$\Rightarrow p(B) = A \cdot e^{\int^{\infty} \frac{2}{S_0}F(B')dB'} \tag{4.15}$$

[4]The Fokker-Planck equation is derived by using the master equation and describes continuus statistical processes. In many cases, like here, discrete problems with very small steps are approximated to be continuous.

In order to compare this model's result with the result from section 4.1 one approximates $F(B)$ as done in section 4.1,hence:

$$p(B) = \sqrt{\frac{\alpha}{\pi S_0}} e^{-\frac{\alpha}{S_0} B'^2} \tag{4.16}$$

This is a Gaussian with the variance $< B'^2 >= \frac{S_0}{2\alpha}$ equivalent to equation 4.6.

Right now it might seem problematic that the variance of the magnetic field gradient in GaAs without external feedback pumping would diverge and result in $T_2^* = 0$. In reality however the magnetic field gradient should approach very small values, which is a small feedback already. Assuming that simple relaxation is the dominating effect here, this built-in feedback should in fact be described by a linear pump curve. In a GaAs double dot spin qubit without feedback the coherence time was measured to be 14ns [2], which can easily be converted into a variance of the magnetic field gradient (Var $\approx 33mT^2 \Rightarrow \sigma_B \approx 5.7\,mT$) by using equation 2.13 and transforming the phase aquiring rate into a magnetic field. With equation 4.6 this yields the constant α, so that the description of the built-in feedback is complete. In the following, this built-in feedback will often be neglected, because it is very small compared to the one induced from the outside (α is about $\frac{1}{24s}$). It will only significantly change the characteristics of $p(B)$ for very large $\mid B \mid$.

5 Cycles and Stability of States

5.1 Instability

The $S{-}T_+$ pumpcycle can induce more than one stationary point. Equation 4.15 implies that there is a local maximum around those points in the steady state. To keep the variance low it would be best if there was just one stationary point. This cannot easily be achieved by this pumpcycle[5], but the pumpcycle can make the diffusion of the probability distribution between stationary points slow enough to be negligible in the experiment. Figure 5.1 visualizes this diffusion for a rather unstable pump cycle.

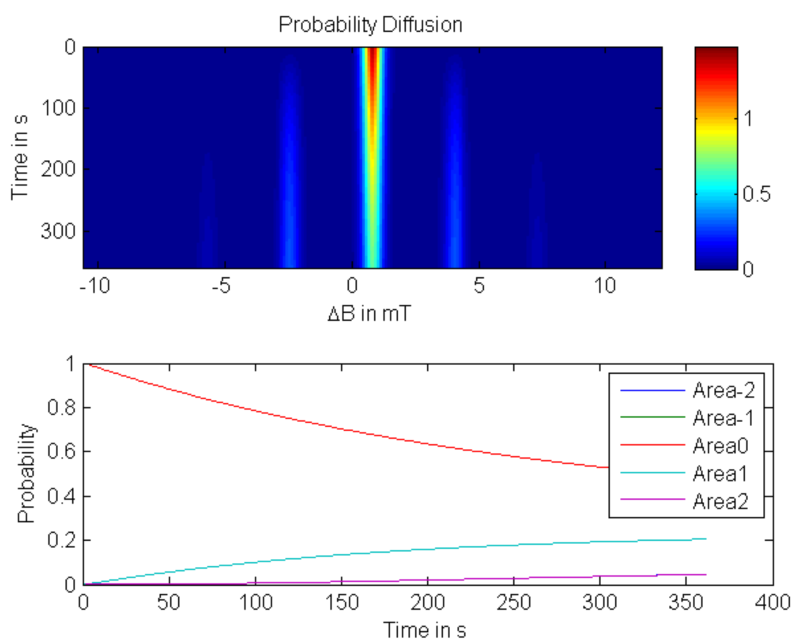


Figure 5.1: The color-plot shows the developement of the probability distribution. There are maxima building up at the stationary points. The plot shows the total probability in the area around the five visible stationary points. Each area is as wide as one period of the pump curve. Since the problem is symmetric, some graphs are identical.

The following simulations will calculate the theoretical probability current that is flowing out of the area in which the magnetic field gradient was ini-

[5]With fast precession time sweeps it is actually possible to get only one major stationary point, but the simulated cycles did not achieve good coherence times and therefore are not included in this work. They might be handy for initialising the magnetic field gradient at a value close to zero.

tialized. In figure 5.1 it is shown that those currents are not constant. Since the experiment has to be paused to reinitialise the magnetic field gradient as soon as it leaves the target area the currents in the beginning will be the focus of those simulations. There also were attempts to find a model approximating those probability currents based on the steady states' probability density on the border between two areas, but thos models did not agree with the simulation. Figures 5.2,5.3and5.4 show the simulation's results.

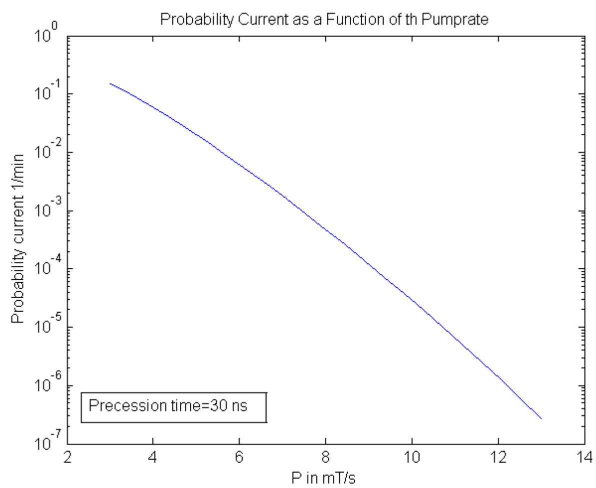
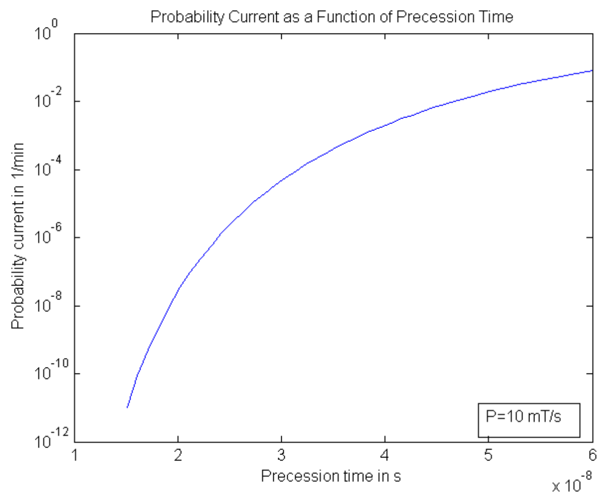


Figure 5.2: P is the amplitude of the pump curve. Using the notation from 3.3 $P := P_- = \frac{1}{2}P_+$



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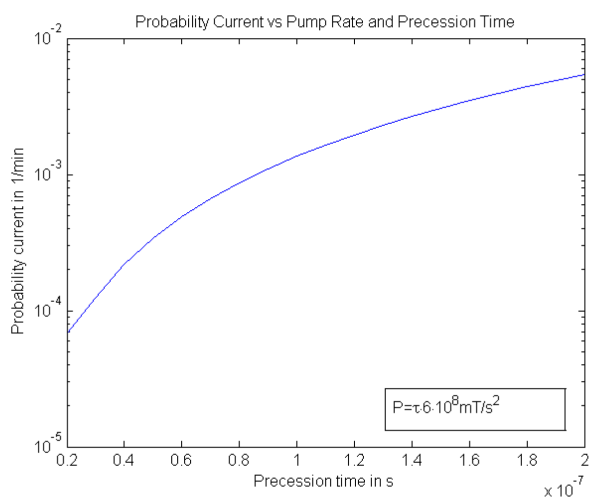

Figure 5.4: P is the amplitude of the pump curve. Using the notation from 3.3 $P := P_- = \frac{1}{2}P_+$

Since any change in the pump curve amplitude P or precession time τ changes the probability current's order of magnitude much more than changing both and keeping the factor between them fixed, it is a valid assumption that this factor would be almost fixed if either P or τ was changed and the other parameter was chosen to keep the probability current fixed. This will be helpful to quickly optimize pump cycles.

5.2 Coherence Time

The variance of B and therefore the coherence time can be estimated by the linear approximation resulting in equation 4.6. This approximation works because those cycles that provide stable enough probability distributions are very similar to Gaussians, because they are well localized around the stationary points (this follows from the factor between P and τ). Of course the limits $\lim_{B\to\infty} p(B)$ differ by orders of magnitude, but both tend to zero quickly enough so that there is no significant change in the variance. Equation 4.6 yields:

$$T_2^* \sim \sqrt{P \cdot \tau} \tag{5.1}$$

Section 5 explains that the stability of the states requires optimized pump cycles to fulfill:

$$const \approx \frac{P}{\tau}. \tag{5.2}$$

With those two equations it is possible to estimate the gain of coherence time if higher pump rates could be achieved. It is important to remember that this estimation is specific to the pump curve from section 3.3. Other feedback cycles, for example one that has a steady state with a low variance might not need to compromise the coherence time in order to achieve a stable probability distribution. The Equations 5.1 and 5.2 yield:

$$T_2^* \approx \frac{P}{\sqrt{const}} \tag{5.3}$$

5.3 Pump Cycles combined with Measurements

The feedback cycle uses the electrons in the double quantum dot to flip nuclear spins. While the pump-cycle is running, there is no way to use the those quantum dots as a qubit. That means that the pump-cycle cannot run without pause. If the timescales of the measuring time, i.e. the time when the feedback cannot run, are short enough, it is possible to average the pump rate for the whole experiment. Obviously during the measuring time this rate is zero. There are two important timescales. One is the timescale how long the measuring time can last, so the probability distribution is still stable enough. There is one possibility to approximate a stability condition by calculating the change in the variance using equation 4.12 and comparing it to the width of one period of the pump curve taken from equation 3.4. The measuring time will be called T. The stability condition then is:

$$S_0 \cdot T \ll \left(\frac{\pi\hbar}{g\mu_B\tau}\right)^2 \tag{5.4}$$

This approximation is based on the idea that the probability distribution is similar to a Gaussian with changing variance. If too much of probability diffuses out of the area around one stationary point during the measuring time the probability current will be high. This can also be tested in a simulation. Figure 5.5 shows the results of that simulation.

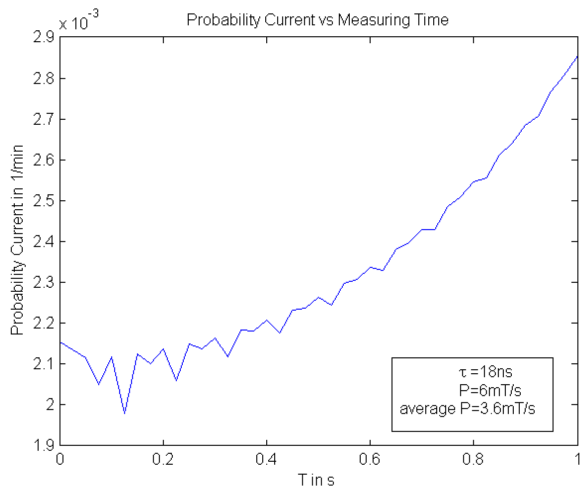

Figure 5.5: The pumping time is 1.5 times as long the measuring time. At T=0 there is the probability current of a pure pump cycle with the average pumprate. The oscillations in the plot are due to the discrete time steps in the simulation, which do not always keep the factor between the measuring and the pumping time perfectly fixed.

Long measuring times yield another problem though. Since T_2^* depends on the variance of B the coherence time can significantly decrease while measuring. That means that there are two different variances, one in the beginning, one in the end of the measuring time, that yield different coherence times. The measured coherence time should be in-between those two values. In order to efficiently use the potential of the feedback those two values should not differ too much. Put into an equation and using equation 4.6 in the second step that means:

$$S_0 T < \mathrm{Var}_0 \approx \frac{S_0}{2\alpha} \tag{5.5}$$

Where Var_0 is the variance of the magnetic field gradient in the beginng of the measurement. Var_0 can be approximated to be between the variance resulting from the averaged cycle and the variance resulting from the pumpcycle neglecting the measuring time (but usually closer to the averaged one).

5.4 Comparison with the Experiment

To check whether the simulation's results agree with reality, the simulation was used to reproduce the results from an actual experiment. The experiment's pump cycle was built by a feedback pulse with a pump rate of up to 40 mT/s and a T_+ pulse with up to 20 mT/s. Both pulses were used 0.03 s and afterwards the measuring time was 0.04 s, which gives the total cycle a length of 0.1 s. After ten cycles there were another 0.05 s without pumping, for technical reasons. The magnetic field was observed to be unstable if the precession time was much higher than 30 ns. This resulted in a coherence time of about 90 ns[5]. Averaging the pumprates over a complete cycle yields a pump curve amplitude $P \approx 6\,\mathrm{mT/s}$. Figure 5.6 shows the simulated probability currents for this experiment.

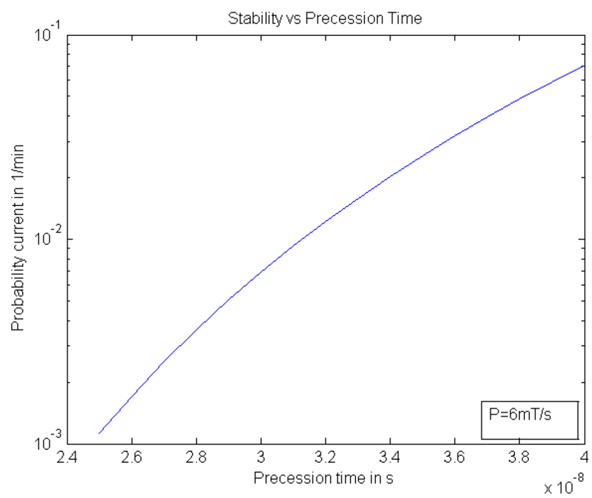

Figure 5.6: In an experiment the magnetic field gradient should be stable for about an hour, that means that the probability current should not be much higher than one percent per minute.

The simulated maximum precession time is $31\,ns$ to $32\,ns$, if the magnetic field is supposed to be stable for about an hour. Using equation 2.13 and 4.6[6] it is posible to calculate the coherence time:

$$T_2^* = \sqrt{\frac{16\tau P\hbar}{S_0 g\mu_B}} \tag{5.6}$$

This yields $T_2^* = 88\,ns$. Considering that the mistake in the pump rate measurement (especially P_+) can easily be a couple of percent, this agrees with the experiment quite well. The precession time agrees with the experiment as well, even though it is suprising that it is longer than in the experiment, while the calculated coherence time is shorter.

[6]This variance is in $\mathrm{m}T^2$. To calculate T_2^* this variance has to be transformed into $\frac{1}{\mathrm{time}^2}$ by using the factor $\frac{g\mu_B}{2\hbar}$ to be used in equation 2.13.

6 Electric Dipole Spin Resonance

This section is going to be about electric dipole spin resonance or short EDSR. It can also change the nuclear magnetization. Therefore it is another possible mechanism for building a feedback cycle. EDSR might achieve higher pump rates than the S and T_+-cycles introduced before.

6.1 Theory

The general idea of EDSR is to use an oscillating electric field to induce Rabi oscillations in the (single) electron state. Of course there is no direct coupling between the electric field and the electron spin. But there already is a theory explaining the effect by the electron's oscillations due to the electric field and the spatial fluctuations of the nuclear field[3]. Even though the estimation of the Rabi frequency resulting from that theory is off by a factor of about six compared to the experiment, this theory might provide a good idea of the basic mechanisms to explain the effect. So here are the most important ideas and equations from that theory.

As mentioned before, in EDSR there is an electric field that causes the electron to oscillate. To induce Rabi oscillations the electric field $\tilde{E}$ needs to oscillate at a frequency that is close to the Lamor frequency of the electron. For external magnetic fields of a couple of hundred mT, the Lamor frequency is on a GHz scale. Assuming that the electron in the quantum dot can be approximated by a harmonic oscillator[7], the changes in the wavefunction will be assumed to be adiabatic because the oscillator frequency will be much higher than the Lamor frequency of the electron. So the wave function is completely solved except for the spin information. Neglecting the spin this yields:

$$\psi(x,t) = \sqrt{\frac{m\omega_0}{\pi d\hbar}} e^{-\frac{1}{2}\frac{m\omega}{\hbar}((x_1 - \frac{e\tilde{E}(t)}{m\omega_0^2})^2 + x_2^2)} \theta(x_3)\Theta(d - x_3) \tag{6.1}$$

Here in x_3 direction ψ was approximated by heaviside functions, because the exact form in this direction does not significantly change the results and as mentioned before the electron is in a 2DEG with the thickness d. The Hamiltonian describing the hyperfine interaction then is:

$$H = A \sum_j \delta(r - r_j)(I_j \cdot S) \tag{6.2}$$

I_j and r_j are the polarization and the position of the nuclei. Hence:

$$J_\pm = \frac{eA}{m\omega_0^2} \sum_j \mid \psi(r_j) \mid^2 I_j^\pm \tag{6.3}$$

Since the electron is oscillating at a frequency close to its Lamor frequency the nuclear field orthogonal to the external magnetic field should also oscillate at

[7] Of course this is just an envelope function. The wavefunction can differ close the nuclei.

that frequency and induce Rabi oscillations. This yields a probability distribution of the Rabi frequency Ω:

$$p(\Omega) = \frac{2\Omega}{\Omega_R^2} e^{-\frac{\Omega^2}{\Omega_R^2}} \tag{6.4}$$

$$\Omega_R = \frac{e\tilde{E}A}{\hbar^2\omega_0}\sqrt{\frac{I(I+1)n_0}{8\pi d}} \tag{6.5}$$

GaAs nuclei are spin 3/2 particles so $I = 3/2$ and n_0 is the density of the nuclei. However equation 6.5 does not yield results that agree with the experiment [3]. In the simulations that are done to develop a feedback based on EDSR, Ω_R will be based on the experimantal value, which is smaller than the theoretical value. The total spin flip probability during a precession time τ can be calculated to be[8]:

$$p_{Flip}(\delta\omega, \Omega_R) = \int_0^\infty \frac{2\Omega^3}{\Omega_R^2(\Omega^2 + (\delta\omega/2)^2)} \sin^2(\sqrt{\Omega^2 + (\delta\omega/2)^2}\tau) e^{-\frac{\Omega^2}{\Omega_R^2}} d\Omega \tag{6.6}$$

Where $\delta\omega$ is the difference between the electron's Lamor frequency and the electric field frequency. In order to simulate a possible pump curve similar to equation 3.4, this has to be converted to a function $F(B) = \dot{B}$.

$$E_{\text{Hyperfine}} := g\mu_B B_{\text{nuclear}} \cdot S = A \sum_j \mid \psi_j \mid^2 (I_j \cdot S) \tag{6.7}$$

$$\Rightarrow \dot{B}_{\text{nuclear}} = \frac{A}{g\mu_B} \sum_j \mid \psi_j \mid^2 \dot{I}_j = \pm\frac{1}{T} \cdot \frac{A}{g\mu_B} \sum_j \mid \psi_j \mid^2 p_j^{Flip} \tag{6.8}$$

Here T is the duration of one cycle and p_j^{Flip} is the chance of the nucleus j to flip during this cycle. The sign in equation 6.8 depends on the initial state of the electron.

$$p_j^{Flip} = \frac{\mid \psi_j \mid^2}{\sum_j \mid \psi_j \mid^2} p_{Flip}(\delta\omega, \Omega_R) \tag{6.9}$$

$$\Rightarrow \dot{B}_{nuclear} = \pm\frac{1}{T} \cdot \frac{A}{g\mu_B} \frac{\sum_j \mid \psi_j \mid^4}{\sum_j \mid \psi_j \mid^2} p_{Flip}(\delta\omega, \Omega_R) \tag{6.10}$$

The nuclei are close enough together so the sums can be approximated as integrals, which yields:

$$\Rightarrow \dot{B}_{nuclear} = \pm\frac{1}{T} \cdot \frac{A}{g\mu_B} \frac{m\omega_0}{2\pi\hbar d} p_{Flip}(\delta\omega, \Omega_R) \tag{6.11}$$

[8] Here the spin flip probability due to Rabi oscillations was averaged for all Rabi frequencies.

6.2 Possible EDSR Feedback Cycle

This section is about an actual suggestion of how to use EDSR in a feedback mechanism. It was mentioned in section 6.1 that EDSR is a mechanism that works on one single quantum dot. If there was a mechanism to control the magnetic field in each single dot this mechanism could also control the magnetic field gradient.

If τ is chosen to be small enough $\dot{B}(\delta\omega, \Omega_R)$ can look similar to a Lorentzian[9]. The maximum of this curve can be assigned to any value of B by setting the frequency of the electric field to the lamor frequency of that value, $\frac{\omega}{2\pi} = \frac{g\mu_B B}{2\pi\hbar}$. Here B obviously is the compelete magnetic field, which means the external plus the nuclear field.

A feedback pulse needs a stationary point. This stationary point can be achieved by switching between two pulses, one pulse with an initial electron state that will cause the EDSR mechanism to increase the magnetic field and an electric field frequency that is slightly lower than the Lamor frequency of the target value of B and one pulse with an initial electron state that will cause the EDSR mechanism to decrease the magnetic field and an electric field frequency that is slightly higher than the Lamor frequency of the target value of B. To initialize those electron states, the double quantum dot can be used. Here singlets or triplets can be initialized and then be changed to $\uparrow\downarrow$ or $\downarrow\uparrow$ states adiabatically. It depends on the sign of the magnetic field gradient which final state each initial state will develop into. If the feedback keeps the magnetic field gradient stable at one value, this is not a problem. Only getting the feedback started might be some additional work.

To check the potential of this feedback figure 6.1 shows the results of simulating the nuclear magnetic field of one quantum dot[10]. The initialization times of the electron are about 200 ns [5]. The pump curve was also calculated numerically. All parameters used were taken from Ref [3].

$\hbar\omega_0$	$\tilde{E}$	d	n_0	An_0	Ω_R
1 meV	$3 \cdot 10^3$ V/m	5 nm	$4 \cdot 10^{28}\mathrm{m}^{-3}$	90 μeV	$1.8 \cdot 10^6$ 1/s

[9] If τ is chosen to be large the Lorentzian is an envelope to the sine term

[10] Of course this is still assuming that the initialisation of the electron state is done by using the double dot.

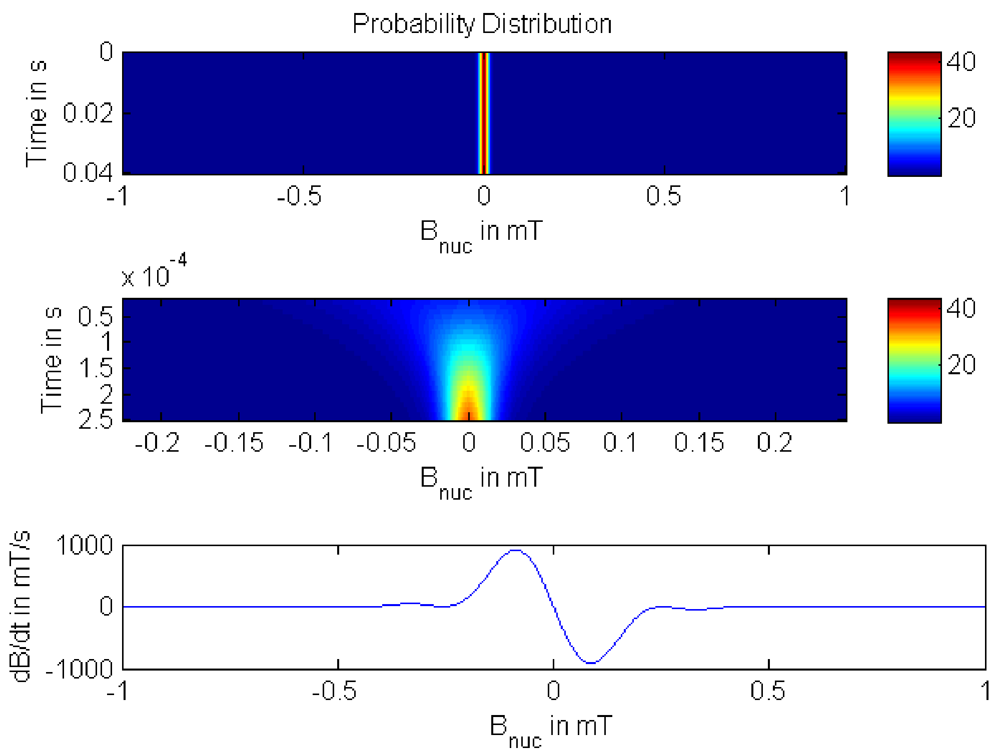

Figure 6.1: Figure: The first two color-plots show the development of the probability distribution. The third plot shows the pump curve. The simulated timescales are very short because of the required computation time. In picture two it is visible, that the final state develops from a state with far larger variance. The magnetic field given on the x-achsis is only the nuclear field. The precession time $\tau = 500\,ns$ and the electric field frequencies are about 6.4 MHz different and equally far away from the Lamor frequency resulting from the external magnetic field, which is why the nuclear magnetic field tends to zero. Different parameters were tried out and those achieved the best coherence time.

The variance of the final probability distribution is $8.5 \cdot 10^{-5}\mathrm{m}T^2$. Considering that the variance of the magnetic field gradient is twice as much as the variance of B in one dot, those simulations combined with equation 2.13 imply that coherence times of $6 \cdot 10^{-6}s$ are possible. Estimating the achievable coherence time if a Carr-Purcel-Meiboom-Gill pulse sequence was additionally used is difficult, because during the coherence time the variance in the magnetic field would easily increase by one order of magnitude[11]. But the simulation also

[11] Even during coherence times that were already experimentally achieved.

yields quite a few problems. First of all it was not possible to simulate those cycles on time scales long enough to learn more about the stability of these states, because the required computation time would be very large. Theoretically this is not necessary because the steady state's probability distribution seems to be very concentrated in one maximum. Equation 4.15 implies a very simple estimation to show that. $\frac{\int_{-\infty}^{0} F(B)dB}{S_0} \approx 22$, which shows that the probabilty outside the range of B where the pump cycle significantly effects B is very low. Approximations based on steady states including spin relaxation predict far less than one percent probability that the probability maximum in figure 6.1 does not include. This value might change if the the target value of the magnetic field is moved far away from zero. It helps that this cycle does not seem to create more than one stationary point. So the stability of states might not even be an issue, because steady states should be good enough. The second even bigger problem is that it is not known how far the theory used to explain EDSR can be trusted to calculate the spin flip probability. The calculated Ω_R does not agree with the experiment. It is not clear why the theory does not work in that regard. One possible idea is that the pumping itself might change the probability distribution of the Rabi frequencies.

7 Conclusion

The statisctial model of the magnetic field gradient succesfully describes the experiment and gave some insight on how much the coherence time of a GaAs double spin qubit can be increased if the pump rates achieved in the experiment could be raised. It also yields the possibility optimize the parameters of a pump cycle much faster than trying them out in an actual experiment. In the end a feedback cycle based on EDSR was proposed. If that cycle achieves the same results in the experiment as in the simulation this could result in significantly increasing T_2^*. This cycle might also solve the stability issues the $T_+ - S$ feedback cycle had.

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I declare that this document has been composed by myself and describes my own work, unless otherwise acknowledged in the text. It has not been accepted in any previous application for a degree. All sources of information have been specifically acknowledged in all conscience.

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