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Exercise - Corbusier's Modulor

Charles-Édouard Jeanneret-Gris, who was better known as *Le Corbusier* (October 6, 1887 – August 27, 1965), was a Swiss-French architect, designer, painter, urban planner, writer, and one of the pioneers of what is now called modern architecture. He was born in Switzerland and became a French citizen in 1930. His career spanned five decades, with his buildings constructed throughout Europe, India, and the Americas.

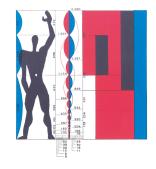


Le Corbusier invented a system for the scale of architectural proportion called *Modulor*. He described it as a "range of harmonious measurements to suit the human scale, universally applicable to architecture and to mechanical things".

One of your friends studies architecture and has to apply Le Corbusiers Modulor in a project where she has to design a tower. Their version of the Modulor is simple: she gets two positive integers i and k and the height of the tower needs to be conguent to i modulo k. Your friend wants to build the tower by stacking up disks of the same radius, but different (integer) heights. She already designed a couple of individual disks. Now she wants to pile up some of these disks (each disk can only be used at most once) to a tower which obeys the rules of the Modulor, that is, its height, which is the sum of its individual disk heights, is congruent to i modulo k. The deadline of the project is coming closer and she asks you to help her deciding whether or not it is even possible to find a suitable *nonempty* subset of the disks which satisfies the rules of the Modulor.



(a) Centre Le Corbusier Zürich



(b) Modulor

Input The first line of the input contains the number t of test cases with $1 \le t \le 100$. Each test case is described as follows:

- It starts with a line that contains three integers n, i, and k, where n is the number of disks $(1 \le n \le 10^3)$ and i and k are the parameters of the Modulor $(0 \le i < k \le 10^3)$.
- The following n lines each describe a disk. They contain one integer h, which is the height of the disk $(1 \le h \le 10^9)$.

Output For every test case, your program should output, on a separate line, whether it is possible to find a nonempty subset of disks such that the sum of their heights is divisible by k. Output yes if it is possible and no otherwise.

Points There are three test sets, worth 100 points in total.

- 1. For the first test set, worth 30 points, you may assume that $n \leq 20$.
- 2. For the second test set, worth 30 points, you may assume that $n \leq 30$.
- 3. For the third test set, worth 40 points, there are no additional assumptions.

Corresponding sample test sets are contained in test i. in/out, for $i \in \{1, 2, 3\}$.

Hint: Put std::ios_base::sync_with_stdio(false) as the first line of the main procedure for faster I/O.

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