The Ψ–Δ Framework: Structural Gaps and Their Dynamics in Multi-Level Systems

From undecidability and NP-hardness to chaos and lossy compression, with implications for modern AI systems

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Abstract

This paper introduces a perspective-indexed structural object for multi-level systems, the Ψ–Δ pair, and uses it to formulate a unified structural criterion for a broad class of persistent "deadlocks" commonly labeled as incompleteness, undecidability, intractability, chaotic unpredictability, and irreducible information loss.

Given a system $\mathcal{L}$ analyzed under a declared perspective A (specification of distinguishable variables, background, and aggregation/compression rules), we define: (i) an intra-level structural frustration index $\Psi(\mathcal{L}, A) \geq 0$ as the infimum of a nonnegative conflict functional over the reachable state space induced by A, quantifying the minimal irreducible tension achievable within that perspective; and (ii) a cross-level structural gap $\Delta_A \geq 0$ as the irreducible residual that remains when lower-level structure is optimally compressed into an upper-level description and then reconstructed under an admissible lifting class. A central structural postulate adopted here is that, whenever one attempts to connect genuinely distinct levels continuously within a single perspective by a single evolution parameter γ_A, the cross-level gap is non-vanishing in the ontological sense ($\Delta_A > 0$), although at any finite resolution one may treat $\Delta_A \approx 0$ as an effective approximation.

To describe persistence under refinement, we promote Ψ and Δ to γ_A-dependent quantities and use the coarse-grained accumulation relation

$$\frac{d\Delta}{d\gamma} = \Psi(\gamma), \qquad \Delta(\gamma) = \Delta(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi(\gamma')\, d\gamma',$$

interpreting intra-level tension as the local density of accumulating cross-level mismatch under structural evolution. On this basis, we formulate a working criterion: a family of problems is intrinsically unsolvable for single-perspective continuous reasoning when, over a *declared admissible family* of perspectives $\mathcal{A}$ at a fixed level specification, $\Psi(\mathcal{L}, A)$ cannot be driven to zero for all $A \in \mathcal{A}$ and the associated gaps accumulate irreversibly across levels.

The paper provides formal definitions, a dynamical extension (HTDS), and cross-domain illustrations. It also specifies a reproducibility-oriented operational protocol (package reporting) together with worked instantiations of the protocol (with full derivations deferred to companion notes; e.g. a one-loop QED endpoint/path normalization check), and introduces a resolution-aware operational notion of *fracture location*—a computable first-detection position along an evolution coordinate under a declared package and resolution regime—so that the Ψ–Δ object can be reused and compared across domains beyond domain-specific content.

Contents

Chapter 1

Introduction: Structural Unification of Fundamental Deadlocks

Across logic, computation, dynamical systems, and information processing, one repeatedly encounters situations in which a system admits many locally valid descriptions but nevertheless resists a single continuous chain of reasoning that connects “parts” to “whole” across levels. Classical examples include Gödel-type incompleteness in formal systems,[8] undecidability and hardness phenomena in computation,[3, 9, 7, 13] long-term unpredictability in nonlinear dynamics,[11, 5, 6] and irreducible loss under compression or coarse-graining. In these settings, the obstruction is not treated here as a temporary lack of ingenuity or data: it appears as a persistent residual generated by attempting to enforce a single-perspective, cross-level continuity principle on a system whose description is intrinsically multi-level.

Most “no-go” results are established *a posteriori* by domain-specific constructions (diagonalization, encoding reductions, adversarial instances, non-uniform sensitivity, and so on). This paper instead asks a prior structural question:

> *Before committing to a particular algorithm, proof technique, or physical model detail, can one state a reusable criterion that diagnoses when a multi-level system will necessarily generate an irreducible deadlock for single-perspective continuous reasoning?*

Terminology and scope (levels, perspectives, and multi-level systems). Throughout this paper, a *perspective* A means a declared representation scheme: it specifies (i) which variables or structural factors are treated as distinguishable, (ii) which background conditions are held fixed, and (iii) which aggregation/compression rules are used to form the representation. A *level* is an abstraction layer induced by such a declared representation together with a declared resolution regime: different levels correspond to different representation/state spaces (e.g. micro/individual descriptions versus macro/global summaries) connected by generally non-invertible maps. A system is called *multi-level* when its analysis necessarily involves at least two such levels linked by an irreversible passage (coarse-graining, completion, identification, or any other genuinely cross-level gluing).

Structural fracture law (verbatim statement). The entire framework is organized around the following structural law, stated as a working meta-constraint rather than a theorem:

> We observe that the difficulties above share a deep structure: when a logical system simultaneously contains opposing elements, and these elements produce feedback on the system, if one attempts to connect, across levels, the system's whole (macroscopic determinacy) and the system's internal factors (microscopic behavioral logic) by a continuous-logic principle, then a rupture of judgment is structurally unavoidable. In short:
>
> $$\text{opposing elements} + \text{system self-feedback} + \text{cross-level continuous logic} \Rightarrow \text{rupture of judgment}.$$

In operational terms, the law asserts that once (i) oppositional contributions coexist within a system, (ii) these contributions feed back into the system, and (iii) one insists on a single cross-level continuity principle within a single perspective, the attempt to extend judgments across levels generates an irreducible residual (a cross-level mismatch under the declared regime). The main task of this paper is to turn this statement into a reusable diagnostic framework with explicit objects, explicit admissibility constraints, and resolution-aware semantics.

From a static constraint to an operational framework. The paper proceeds by separating two different sources of obstruction:

- *Intra-level tension:* residual conflict that persists even when one stays within a fixed level and a fixed perspective (no cross-level gluing).
- *Inter-level fracture:* residual mismatch generated by passing between levels (e.g. compressing to a global description and attempting to reconstruct individual structure), even when one allows an admissible class of optimal reconstructions.

This separation is implemented by a perspective-indexed structural object, the Ψ–Δ pair. The object is designed to be *regime-relative*: all statements are evaluated under a declared level specification (level partition), an admissible family of perspectives, and a declared resolution regime. This is not a stylistic choice; it is the minimal requirement for comparability across instantiations.

Contributions and main objects

The contribution of this work is not a retrospective reinterpretation of known results, but the construction of a reusable structural language and a regime-controlled diagnostic criterion. Concretely:

- **Two structural quantities.** For a system $\mathcal{L}$ under a declared perspective A, we define an intra-level structural frustration index $\Psi(\mathcal{L}, A) \geq 0$ as the infimum of a nonnegative conflict functional over the reachable states induced by A, and a cross-level structural gap $\Delta_A \geq 0$ as the irreducible residual remaining after optimal compression and admissible reconstruction across levels.

- **Resolution-aware non-vanishing constraint.** A central structural postulate adopted here is that whenever a *genuine* cross-level continuity attempt is made within a single perspective (i.e. one uses a single evolution parameter to continuously glue distinct levels), the cross-level gap is non-vanishing in the ontological sense ($\Delta_A > 0$). At finite resolution one may adopt $\Delta_A \approx 0$ only as an effective convention, not as an ontological claim.

- **A regime-controlled diagnostic criterion.** Using the Ψ–Δ object, we formulate a working criterion for intrinsic deadlocks of single-perspective continuous reasoning: such deadlocks persist when, over a declared admissible family of perspectives at a fixed level specification, $\Psi(\mathcal{L}, A)$ cannot be driven to zero and the associated fractures Δ_A accumulate irreversibly across level transitions.

- **Dynamical extension (HTDS).** We extend the static picture by promoting Ψ and Δ to evolution-dependent objects along a declared structural parameter γ_A (time, scale, or another regime-controlled evolution coordinate), and by introducing a hierarchical tension dynamical systems (HTDS) description of how fracture is relocated and re-emerges under refinement.

- **Cross-domain instantiations with controlled comparability.** We provide case studies spanning formal incompleteness/undecidability, computational hardness, chaotic dynamics near criticality, and irreducible information loss, with the explicit goal of demonstrating that the same regime-controlled object organizes otherwise disparate benchmark deadlocks. A reproducibility-oriented operational protocol (package reporting) is used to make instantiations comparable rather than viewpoint-dependent.

Motivating patterns (why a structural object is needed)

The framework is motivated by a recurring pattern across domains: when one insists on a single continuous cross-level inference chain within a single perspective, the inference must either stop or introduce an explicit re-embedding into a higher description regime.

- **Emergent organization.** In multi-component systems, global organization is often expressible only after one changes the representation regime (coarse-graining, renormalized variables, new macroscopic constraints). The obstruction is not merely practical: it reflects a cross-level residual under the original regime.

- **Self-referential paradoxes.** In self-coupled logical structures, applying a single uniform rule set to an object that is also part of the rule's domain induces a closure that cannot be resolved without changing the regime (e.g. by restricting admissibility or lifting the description).

- **Completion-type identifications (e.g. $0.999\ldots = 1$).** Standard limit identifications reorganize the regime: one introduces a higher-level object and a rule-based identification principle that compresses the residual below the declared resolution of the formal calculus. In this paper's semantics, such identifications are treated as cross-level

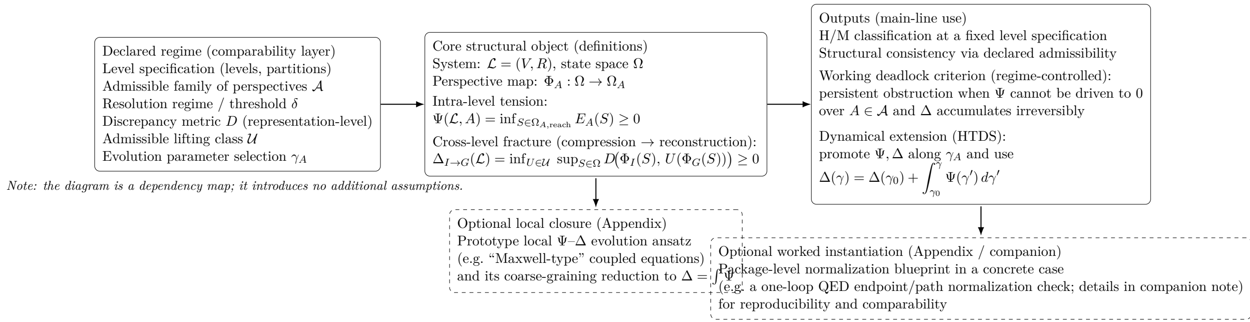

Figure 1.1: Framework at a glance: declared regime → Ψ–Δ object → criterion and dynamics, with optional closure and worked instantiation modules.

> gluings whose operational success depends on the declared regime, while the ontological stance retains a non-vanishing cross-level gap under genuine multi-level connection attempts.

These examples are not used as independent proofs; they serve to motivate why a single structural object should explicitly separate intra-level tension from inter-level fracture.

Roadmap

Chapter 2 formalizes systems, levels, perspectives, admissibility constraints, and resolution-aware semantics, and introduces Ψ, Δ, and the H/M classification as regime-controlled statements. Appendix C provides compact worked instantiation *blueprints* for the operational protocol, including a one-loop QED endpoint/path normalization *outline* (the full QED derivation and protocol are presented in a separate companion paper) and a resolution-aware operational fracture-location diagnostic under a declared package. Later chapters develop the dynamical extension (HTDS and structural evolution along γ) and instantiate the framework across representative domains. The framework is intended to be testable and revisable at the level of declared instantiations (choice of perspectives, conflict functionals, admissible reconstructions, and evolution regimes), while preserving the structural fracture law and the ontological non-vanishing of genuine cross-level gaps.

Chapter 2

Formal Framework: Perspectival Ontology and the Ψ–Δ Structure

2.1 Equality and Structural Equivalence: Necessary Semantic Clarifications

Within traditional calculus, limits, derivatives, and integrals are conventionally written as

$$\lim_{n\to\infty} x_n = L, \qquad \frac{d}{dx} f(x) = g(x), \qquad \int f(x)\,dx = F(x) + C.$$

In the classical framework, these equality signs are usually interpreted as *strict equalities inside the adopted axiomatic system*: once the relevant axioms and definitions are fixed, the expressions above are treated as fully legitimate identities *within that calculus*. In the structural stance of this paper, the semantics of "=" must be separated across layers.

Formal layer (inside an H-calculus). Inside a given formal system (an H-calculus in the sense of this work), the symbol "=" is treated as an *inference symbol*:

"indistinguishable under the axioms and inference rules of the calculus."

At this layer, "=" is an operational rule rather than a declaration of ontological identity. As long as no contradiction is derivable within the calculus, one may write equalities such as $\lim_{n\to\infty} x_n = L$ or $F'(x) = f(x)$ in the standard way.

Structural layer (ontological stance with $\Delta^* > 0$). Once an incompressible meta-gap $\Delta^* > 0$ is acknowledged for genuine cross-level constructions, the interpretation changes: equalities produced via limiting, completion, or cross-level gluing must be read as *structural equivalence at a declared resolution*, rather than as gap-free identity. We denote this by $\approx_\delta$, where δ is the resolution threshold at which the equivalence is declared.

- Within the formal layer, "=" is used as a strict calculational rule inside the chosen H-calculus.

- When the discussion involves real structures, cross-level gluing, or the meta-gap Δ^*, each "=" produced by such constructions is to be interpreted as a shorthand for an equivalence of the form

$$\approx_\delta \quad (\text{"indistinguishable under resolution } \delta\text{"}),$$

  with δ stated explicitly whenever needed.

From this standpoint, limit and completion constructions do *not* literally eliminate all cross-level gaps between discrete rule systems and completed infinite objects; rather, they implement a canonical *structural identification* at an adopted axiomatic or resolution scale. The step from "local rules in a finite process" to "a completed infinite object" is treated here as a structural leap that carries a positive cross-level component in the presence of genuine layering.

It is worth emphasizing that even under the Dedekind cut construction, "a real number" is produced by identifying an abstract cut of the rationals with a single object at the axiomatic level. This differs from computable reals (e.g. π), which at least admit an explicit finite procedure generating increasingly fine approximations (though never exhaustively). For a generic cut, the Dedekind construction supplies no effective finite-information generation rule; it assigns an abstract name to an infinite partition. In the present semantics, such naming operations are interpreted as cross-level structural gluings, not as ontological confirmations of a gap-free "pre-existing point."

Within this semantic framework, the roles of Ψ and Δ are delineated as follows:

- $\Psi(\mathcal{L}, A)$ quantifies *intra-level* coordinability under a fixed perspective A. Regimes with $\Psi = 0$ indicate that, at that level and declared resolution, no internal rule conflict is manifested.

- Δ quantifies *inter-level* irreversibility: it records residual discrepancy arising from cross-level compression and the best-allowed reconstruction. Under an ontological stance accepting $\Delta^* > 0$, strictly gap-free inter-level identity is not taken as a default expectation for genuine cross-level relations; operationally, one may only compress Δ below a declared threshold and treat it as "effectively negligible" at that resolution.

Accordingly, "gap-free" statements in what follows are always to be read in the operational sense:

$$(\Delta \approx 0) \text{ at a declared resolution} \quad \text{rather than} \quad (\Delta = 0) \text{ as an ontological claim.}$$

This convention forms the default semantic background for equality and approximation symbols throughout the paper.

Relation to the classical ε-language (resolution versus meta-fracture). In the classical ε-style semantics, $\varepsilon > 0$ is a tunable tolerance that may be chosen arbitrarily small, and conclusions are stated relative to that choice. In the present framework, the role analogous to such a tolerance is played by the *declared resolution threshold* δ (more generally, the declared resolution regime in the reported package). Accordingly, statements of the form "$\Delta \approx 0$" are always to be read operationally as "$\Delta < \delta$" under the declared regime.

The meta-fracture $\Delta^* > 0$, by contrast, is not a tolerance parameter. It is an irreducible structural lower bound for *genuine* cross-level gluing: whenever a true cross-level connection is attempted within a single perspective by a single evolution parameter, the resulting fracture satisfies $\Delta \geq \Delta^*$ in the ontological sense. Therefore, "$\Delta \approx 0$" is never an ontological claim; it is only a resolution-dependent convention, and it becomes unavailable once the declared resolution is finer than the intrinsic floor (i.e. when $\delta < \Delta^*$).

Reading protocol (semantic defaults used after this section). From this point onward, all statements involving Ψ, Δ, and H/M are to be read as *regime-relative* statements: they are evaluated under a declared level specification (level partition), an admissible family of perspectives, and a declared resolution regime. In particular, any equality sign "=" that arises from limits, completions, or cross-level identifications is understood as structural equivalence at a stated (or implicit) resolution threshold (i.e. a shorthand for an $\approx$-type relation), unless the discussion is explicitly restricted to purely internal manipulations inside an H-calculus.

Whenever numerical values or comparisons of Ψ or Δ are reported, the relevant operational ingredients (at minimum the admissible family, the resolution regime, and the chosen discrepancy/reconstruction conventions) are part of the statement. Finally, the H/M label is always understood *within* a fixed regime: refining resolution or changing admissibility may change what becomes visible, while invariance is required only across admissible re-expressions at the same fixed level specification.

Levels and structural dimension (a derived reading). Throughout this paper, *levels* are primary: a level is specified operationally by a declared representation/perspective scheme together with its admissible family and resolution regime. There is nonetheless a natural connection to a broadened notion of *dimension*: once a concrete model provides a way to quantify the effective degrees of freedom (or descriptive complexity) of a level—for instance via an explicit parameterization, an embedding/realization, or an information-theoretic proxy at the declared resolution—one may use such a quantity as a *derived* numerical summary of that level. In this sense, "dimensional uplift" is a possible geometric/quantitative re-expression of certain level transitions, rather than an additional primitive notion of the framework. Unless explicitly specified in a concrete instantiation, the term "dimension" is used only in this structural/effective sense and is not assumed to coincide with the narrow Euclidean coordinate dimension.

Terminology note ("ontological"). Throughout this paper, the adjective "ontological" is used as a technical label for *resolution-independent structural claims* (i.e. claims that do not disappear under admissible re-encodings within a declared regime), rather than as an appeal to philosophical ontology.

Minimal notation (10 symbols, high-confusion set). For quick reference, we list only the symbols that are most likely to be misread on first contact. All meanings are regime-

Symbol	Meaning (as used in this paper)
A	Perspective / encoding specification (declares representable variables, admissibility, and aggregation/compression rules).
Ω_A	Representation/state space induced by the perspective A.
$\Psi(\mathcal{L}, A)$	Intra-level structural frustration index under fixed A (best-achievable intra-level coordinability proxy).
$\Delta_{A_I \to A_G}(\mathcal{L})$	View/perspective-switch fracture: irreducible residual under optimal admissible lifting when compressing $A_I \to A_G$ and reconstructing.
$\Delta(\gamma)$	Accumulated fracture along evolution parameter γ (coarse-grained scalar), with $\Delta(\gamma) = \Delta(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi(\gamma')\, d\gamma'$.
Δ_{dyn}	Dynamical fracture lower bound (HTDS): $\Delta_{\text{dyn}} := \inf_{\mu \in \mathcal{M}_T} \int \Psi \, d\mu$ for a system $T : X \to X$.
Δ^*	Meta-fracture lower bound postulate: $\Delta^* > 0$ in the intended ontological reading for genuine cross-level gluing.
δ	Resolution threshold (operational negligibility cutoff); statements like $\Delta \approx 0$ are read as $\Delta < \delta$ under the declared regime.
$\approx_\delta$	Structural equivalence "indistinguishable under resolution δ" (resolution-aware reading of cross-level equalities).
$\delta_{\text{Feig}},\ \alpha_{\text{Feig}}$	Classical Feigenbaum constants (notation reserved to avoid collision with the resolution symbol δ).

Table 2.1: Minimal notation quick reference (restricted to 10 symbols).

relative to a declared operational package and resolution threshold.

2.2 Hierarchical Modeling of Logical Systems and Perspectival Relativity

Whenever a logical system attempts, via continuous logic within a single perspective, to establish a seamless cross-level connection between a "global" description and "individual" factors, the structural rupture principle implies that a cross-level discontinuity is unavoidable. To formalize this phenomenon, the present chapter introduces definitions of *system*, *level*, and *perspective* in a manner consistent with the perspectival ontology stance.

Level specification (what counts as "the same level"). In this framework, a "level" is not an a priori metaphysical stratum. It is specified operationally by a *level specification*: a fixed level partition together with its admissibility and resolution constraints. Concretely, fixing a level means fixing (i) which degrees of freedom are treated as individual versus global at that stage, (ii) the admissible family of level-respecting perspectives (re-expressions that do not change the partition), and (iii) a resolution regime under which indistinguishability is declared. A *multi-level system* is then a system equipped with at least two such specifications linked by coarse-graining/encoding maps, where irreversibility across specifications is precisely what gives rise to inter-level gaps quantified by Δ.

Traditional accounts often treat "whole" and "individual" as a priori, absolutely fixed levels. This work adopts a different stance: "whole" and "individual" are relational notions that are jointly determined by a perspective (what is made visible) together with an encoding/coarse-graining scheme (how visibility is implemented).

Core principle: perspective determines ontology. Within the present framework, the ontology relevant to inference is treated as perspective-dependent. Whether an object counts as a "whole," which constraints are regarded as internal, and what kinds of computability boundaries arise are all properties *relative to* a chosen perspective

$$A = (F, P).$$

Here, F denotes the family of structural factors made visible, and P denotes the representation/coarse-graining rule that implements the level of description.

More concretely, once an object S is fixed for discussion, it is temporarily treated as a unit. Under a perspective A, if attention is restricted to S itself and external entities are excluded, then S is treated as a "whole" at that level. Once interactions with external objects are included, a larger object S' is induced, within which the original S becomes an "individual" component. Thus "whole" versus "individual" is not an intrinsic label of S, but a boundary choice determined by the adopted perspective.

Neglecting this relativity leads to conceptual confusion: many apparent structural deadlocks reduce to a failure to specify perspectives and their embeddings. For this reason, the relativity of perspective is treated as a primary principle in the present framework.

Example: choice of perspective in statistical averaging

Consider a simple system of N individuals with micro-states

$$(x_1, \ldots, x_N).$$

Under a fine-grained perspective A_{micro}, let

$$F_{\text{micro}} = \{x_1, \ldots, x_N\},$$

and let P_{micro} encode states at a resolution distinguishing individual differences. The "system state" appears as the full vector $(x_1, \ldots, x_N)$.

Under a coarser perspective A_{macro}, let the only visible factor be the average

$$x = \frac{1}{N}\sum_{i=1}^{N} x_i, \qquad F_{\text{macro}} = \{x\},$$

and let P_{macro} encode only x while suppressing differences among the x_i. Formally, the passage from micro to macro is a many-to-one map

$$\pi : (x_1, \ldots, x_N) \mapsto x,$$

which is structurally irreversible: distinct micro-configurations may share the same macro-state. This illustrates two general points:

1. A perspective is an explicit choice of what is visible and how it is encoded, not a secondary description attached to a fixed ontology.

2. Coarse-graining may yield smoothness at the macro-level, but it does not resolve individual differences; it suppresses them via an irreversible map.

Definition 2.1 (Logical system). A logical system is represented as a pair

$$\mathcal{L} = (V, R),$$

where $V = \{v_1, \ldots, v_n\}$ is the set of variables and R is a collection of local constraint rules on V. Each constraint $r \in R$ involves only finitely many variables and can be viewed as a relation or predicate on those variables.

A *system state* is an assignment of values to all variables in V. The set of all states forms the fine-grained state space Ω. The system is not assumed to always reside in an ideal state satisfying all constraints; states may violate part of R in order to capture structural conflict and gap phenomena.

Definition 2.2 (Perspective maps and representation spaces). Given a logical system $\mathcal{L}$ with fine-grained state space Ω, each perspective

$$A = (F, P)$$

induces a representation space Ω_A and a map

$$\Phi_A : \Omega \to \Omega_A,$$

where $\Phi_A(S)$ is the representation of S under A.

For two perspectives A_1, A_2, if there exists a bijection

$$U_{A_2 \leftarrow A_1} : \Omega_{A_1} \to \Omega_{A_2}$$

such that for all $S \in \Omega$,

$$\Phi_{A_2}(S) = U_{A_2 \leftarrow A_1}\big(\Phi_{A_1}(S)\big),$$

then the system is *perspective-independent* between A_1 and A_2.

If no such bijection exists, the system exhibits *perspective-dependence*. Equivalently, if there exist $S, S' \in \Omega$ such that

$$\Phi_{A_1}(S) = \Phi_{A_1}(S') \quad \text{but} \quad \Phi_{A_2}(S) \neq \Phi_{A_2}(S'),$$

then two states indistinguishable under A_1 become distinguishable under A_2.

Reachability convention. When dynamics or admissibility constraints are present, the fine-grained states that are actually attainable will be denoted by

$$\Omega^{\text{reach}} \subseteq \Omega.$$

This avoids overloading Ω_A (which is reserved for representation spaces).

Definition 2.3 (Ψ: intra-level structural conflict measure). Fix a logical system $\mathcal{L} = (V, R)$ with fine-grained state space Ω, and fix a perspective $A = (F, P)$ inducing $\Phi_A : \Omega \to \Omega_A$. Let $\Omega^{\text{reach}} \subseteq \Omega$ be the reachable set under the declared dynamics/constraints. Let $E_A : \Omega \to \mathbb{R}_{\geq 0}$ be a *conflict functional* under perspective A.

The structural frustration index in perspective A is defined as

$$\Psi(\mathcal{L}, A) = \inf_{S \in \Omega^{\text{reach}}} E_A(S).$$

Examples of admissible conflict functionals include:

- discrete constraint systems: $E_A(S)$ = number (or weighted number) of violated local constraints visible under A;
- physical systems: $E_A(S)$ = energy above the lowest accessible energy at the declared level;
- statistical systems: $E_A(S)$ = divergence from an "ideal" distribution (e.g. Kullback–Leibler divergence).

If $\Psi(\mathcal{L}, A) = 0$, then under this perspective (and at the declared resolution) the system admits reachable states in which internal constraints are fully coordinated at that level. If $\Psi(\mathcal{L}, A) > 0$, then irreducible intra-level conflict remains under A.

Definition 2.4 (Δ: inter-level gap between global and individual levels). The structural gap Δ quantifies irreversible discrepancy generated by aggregating an individual-level description into a global-level description and then attempting reconstruction.

Fix a logical system $\mathcal{L}$ with fine-grained state space Ω and reachable set $\Omega^{\text{reach}} \subseteq \Omega$. Consider two perspectives:

- individual-level perspective $A_I = (F_I, P_I)$ with map $\Phi_I : \Omega \to \Omega_I$;
- global-level perspective $A_G = (F_G, P_G)$ with map $\Phi_G : \Omega \to \Omega_G$.

Let $\mathcal{U}$ be a declared admissible class of *lifting* (reconstruction) operators, where each element

$$u \in \mathcal{U} \qquad \text{satisfies} \qquad u : \Omega_G \to \Omega_I.$$

Let

$$D : \Omega_I \times \Omega_I \to \mathbb{R}_{\geq 0}$$

be a discrepancy measure on the individual-level representation space (e.g. divergence, loss, or information discrepancy), interpreted under a declared resolution scale.

The inter-level structural gap is defined as

$$\Delta_{I \to G}(\mathcal{L}) = \inf_{u \in \mathcal{U}} \sup_{S \in \Omega^{\text{reach}}} D\big(\Phi_I(S),\, u(\Phi_G(S))\big).$$

Intuitively, $\Delta_{I \to G}(\mathcal{L})$ is the worst-case residual discrepancy that remains even after selecting the best admissible reconstruction u.

Operational interpretation. If $\Delta_{I\to G}(\mathcal{L}) > 0$, then within the declared admissible regime $(D, \mathcal{U})$ the global description cannot reconstruct the individual description beyond that residual discrepancy. If $\Delta_{I\to G}(\mathcal{L}) = 0$, then within the declared regime there exists an admissible u^* such that the discrepancy vanishes on all reachable states:

$$D\big(\Phi_I(S), u^*(\Phi_G(S))\big) = 0 \quad \forall S \in \Omega^{\mathrm{reach}}.$$

In the semantics of this paper, such a statement is read as *gap-invisible at the declared resolution and admissible regime.* It does not, by itself, assert a global ontological "gap-free identity" when the two levels are genuinely distinct.

2.2.1 Operationalization Protocol: perspective packages, admissibility, and comparability

Definitions 2.3–2.4 are framework-level: their reported values depend on concrete choices of (i) admissible perspectives, (ii) the conflict functional and discrepancy, (iii) the reconstruction class, and (iv) the declared resolution. To prevent ad hoc instantiations from being implicitly compared as if they were the same invariant, the operational content is made explicit.

Definition 2.5 (Operational perspective package). An *operational perspective package* (or simply a *package*) is a tuple

$$\mathbf{V} = \big(A_I, A_G, \Omega^{\mathrm{reach}}, E_{A_I}, E_{A_G}, D, \mathcal{U}, \delta\big),$$

where:

- A_I, A_G are the individual and global perspectives inducing $\Phi_I : \Omega \to \Omega_I$ and $\Phi_G : \Omega \to \Omega_G$;
- $\Omega^{\mathrm{reach}} \subseteq \Omega$ is the reachable fine-grained state set under declared dynamics/constraints;
- $E_{A_I}, E_{A_G} : \Omega \to \mathbb{R}_{\geq 0}$ are the conflict functionals used to evaluate $\Psi(\mathcal{L}, A_I)$ and $\Psi(\mathcal{L}, A_G)$ when needed;
- D and $\mathcal{U}$ are the discrepancy and admissible lifting class used in Definition 2.4;
- δ is the declared resolution (gap-resolution threshold) underlying structural equivalence $\approx_\delta$ in Section 2.1.

Implementation note (interfaces and aggregation). In concrete instantiations, the discrepancy D is often induced by a declared observational interface family $\mathcal{O}$ (with a fixed estimation/decision protocol) together with a cross-interface aggregation rule Agg; when applicable, $(\mathcal{O}, \mathrm{Agg})$ is treated as a locked sub-clause of D and must be reported for comparability. When necessary, we write $\Psi(\mathcal{L}, A; \mathbf{V})$ and $\Delta_{I\to G}(\mathcal{L}; \mathbf{V})$ to emphasize package-relativity.

Definition 2.6 (Package equivalence and comparability). Two packages $\mathbf{V}$ and $\mathbf{V}'$ are *equivalent*, written $\mathbf{V} \sim \mathbf{V}'$, if they differ only by invertible reparameterizations of the representation spaces together with resolution-preserving relabelings, and if the transport rule of the operational ingredients is explicitly given.

Formally, $\mathbf{V} \sim \mathbf{V}'$ requires explicit maps that conjugate the induced representations and specify how $(E_A, D, \mathcal{U}, \delta)$ are transported, together with the transformation laws for reported Ψ and Δ values.

If $\mathbf{V} \not\sim \mathbf{V}'$, then numerical values of Ψ and Δ produced under $\mathbf{V}$ and $\mathbf{V}'$ are not treated as directly comparable scalars. Any cross-package comparison must specify an explicit mapping and its transformation law; otherwise the comparison is undefined in the present framework.

Definition 2.7 (Admissible families and intrinsic lower bounds). An *admissible family* of packages, denoted $\mathfrak{V}_{\mathrm{adm}}$, is a set of packages $\mathbf{V}$ satisfying fixed structural constraints appropriate to the level under discussion. Typical constraints include:

- **Level-fixation:** the level partition (what is "individual" vs. "global") is held fixed;
- **Non-degeneracy:** trivial encodings that collapse all states are excluded;
- **Resolution control:** δ is restricted to a prescribed range, preventing infinite coarse-graining from declaring gaps invisible by fiat;
- **Discrepancy sanity:** $D(x, y) = 0$ is consistent with indistinguishability at the declared δ;
- **Model-class fixation:** $\mathcal{U}$ is restricted to a declared reconstruction regime (bounded complexity, fixed architecture class, fixed regularity assumptions), excluding oracle-like reconstructions.

Given such a family, an intrinsic cross-package lower bound for the inter-level gap is defined by

$$\underline{\Delta}_{I\to G}(\mathcal{L}) := \inf_{\mathbf{V}\in\mathfrak{V}_{\mathrm{adm}}} \Delta_{I\to G}(\mathcal{L}; \mathbf{V}).$$

A strictly positive $\underline{\Delta}_{I\to G}(\mathcal{L}) > 0$ means that within the declared admissible regime the inter-level gap cannot be eliminated operationally and is therefore structural relative to that regime.

Remark (Minimal reporting checklist). Any concrete computation or empirical instantiation of Ψ and Δ in later chapters is regarded as well-posed only if it reports, at minimum, the defining components of the package (or admissible family) used:

$$(A_I, A_G, \Omega^{\mathrm{reach}}, E_{A_I}, E_{A_G}, D, \mathcal{U}, \delta) \quad \text{and, when applicable, the constraints defining } \mathfrak{V}_{\mathrm{adm}}.$$

In particular, when the discrepancy D is induced from a declared interface family $\mathcal{O}$ (with a fixed protocol) together with a cross-interface aggregation rule Agg, these sub-clauses are treated as part of D and must be reported. Without this information, outputs are package-relative and are not comparable across sections or across works.

Worked alignment (the averaging example as a package). Revisit the statistical averaging map $\pi : (x_1, \ldots, x_N) \mapsto \bar{x}$ discussed above. In the present operational semantics, this example is instantiated by fixing the package

$$\mathbf{V} = \big(A_I,\ A_G,\ \Omega^{\mathrm{reach}},\ E_{A_I},\ E_{A_G},\ D,\ \mathcal{U},\ \delta\big),$$

where A_I is the individual-level perspective retaining the full micro-vector $(x_1, \dots, x_N)$ (hence $\Phi_I(S) = (x_1, \dots, x_N) \in \Omega_I$), and A_G is the global-level perspective retaining only $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ (hence $\Phi_G(S) = \bar{x} \in \Omega_G$). The reachable set $\Omega^{\text{reach}} \subseteq \Omega$ specifies which micro-configurations are admissible under the declared constraints (e.g. bounded support, fixed variance class, or a specified dynamics). The functionals E_{A_I} and E_{A_G} fix what counts as intra-level "conflict" under each perspective. The discrepancy D specifies what notion of residual mismatch is being measured on Ω_I (e.g. an ℓ^2 loss, an information divergence, or a constraint-violation count), while the admissible lifting class $\mathcal{U}$ specifies which reconstructions $u : \Omega_G \to \Omega_I$ are permitted (e.g. i.i.d. resampling schemes, maximum-entropy reconstructions, or bounded-complexity estimators). Finally, the resolution δ declares the threshold at which two micro-descriptions are treated as indistinguishable in practice.

With this package fixed, the inter-level gap in Definition 2.4 becomes

$$\Delta_{I\to G}(\mathcal{L}; \mathbf{V}) = \inf_{u\in\mathcal{U}} \sup_{S\in\Omega^{\text{reach}}} D\big(\Phi_I(S),\, u(\Phi_G(S))\big).$$

Crucially, changing $D, \mathcal{U}$, or δ changes the operational meaning and numerical value of $\Delta_{I\to G}$: a discrepancy that is strict in ℓ^2 may be loose under an information divergence, an oracle-like $\mathcal{U}$ may artificially compress the gap, and a coarser δ may declare residuals "invisible" by resolution. Therefore, Δ is package-relative in this framework, and any reported value must be accompanied by the package components (at least $D, \mathcal{U}$, and δ) in order to be comparable across analyses.

Companion computation (worked example). A standalone companion paper provides a concrete instantiation of the operational package in a one-loop QED RG-window setting, including an explicit endpoint/path decomposition and the corresponding gap components under a declared interface and resolution regime. In the present paper, we do not embed the full QED derivation and estimation/decision protocol into the main line to avoid interrupting the framework development; instead, Appendix C records a compact instantiation *blueprint* (with the package elements explicitly named) that makes the mapping from the framework objects to the QED setting transparent. The QED companion paper also illustrates why different choices of D, $\mathcal{U}$, and δ generally lead to different readouts and therefore why the package must be reported for comparability.

2.3 H/M Classification and Structural Consistency Axioms

After introducing the structural frustration index Ψ and the structural gap Δ, systems can be classified at a fixed level into H-class and M-class types. Under the perspectival-ontology stance of this work, "the structure of a system" is determined relative to a fixed level partition together with an admissible family of perspectives: changing the level partition (or allowing indefinite downward decomposition) corresponds to redefining the ontology of $\mathcal{L}$ rather than merely switching perspectives at the same level. The H/M classification below therefore pertains to *a fixed level together with its declared admissible family.*

Naming convention and intuition (why "H/M"). The labels "H" and "M" originate from an empirical intuition about natural systems: at many fixed levels of description, structures that behave "smoothly" tend to be dominated by effectively homogeneous components ("like-with-like" aggregation), whereas persistent irregularity typically arises when heterogeneous components are forced into the same level of representation and interact under feedback, producing unavoidable oppositions. In the abstract formalization adopted here, this intuition is not used as a criterion by itself; it serves only as a mnemonic for the structural content captured by Ψ within a fixed level specification:

- **H-class (homogeneity-like regime):** there exists an admissible perspective under which intra-level conflict can be fully coordinated at the declared resolution, i.e. $\Psi(\mathcal{L}, A) = 0$ for at least one A in the admissible family.
- **M-class (mixture-like regime):** every admissible perspective at that fixed level retains irreducible conflict at the declared resolution, i.e. $\Psi(\mathcal{L}, A) > 0$ for all A in the admissible family.

Under the structural consistency requirement introduced below (Axiom 2.1), the "existence" statement in the H-case is promoted to an invariance statement within the admissible family, so that the H/M label is stable under admissible re-expressions of the same level. In particular, "heterogeneity" in this paper should be read structurally: it refers to mutually incompatible constraint contributions (oppositional terms) that survive admissible re-encodings, rather than to any domain-specific notion of material diversity.

2.3.1 Structural consistency axioms

To prevent the H/M classification from depending on ad hoc selection of a single contrived "perspective," we impose a structural consistency requirement. Throughout this section, a "fixed level" means that the level partition is held fixed together with its declared operational constraints: non-degeneracy conditions, resolution control (the admissible range of indistinguishability thresholds), and the exclusion of pathological encodings that inject or hide conflict by representational artifacts. The admissible family $\mathcal{A}$ is understood as the set of perspectives that satisfy these constraints at the fixed level.

Axiom 2.1 (structural consistency axiom). Fix a level in the above sense and consider a family of admissible perspectives $\mathcal{A}$ satisfying the declared structural constraints. If there exists some admissible perspective $A_i \in \mathcal{A}$ such that

$$\Psi(\mathcal{L}, A_i) = 0,$$

then for all admissible perspectives $A \in \mathcal{A}$ at this fixed level,

$$\Psi(\mathcal{L}, A) = 0.$$

Conversely, if there exists some admissible perspective $A_j \in \mathcal{A}$ such that

$$\Psi(\mathcal{L}, A_j) > 0,$$

then no other admissible perspective $A \in \mathcal{A}$ can compress $\Psi(\mathcal{L}, A)$ to zero at this level.

Interpretation and scope. Axiom 2.1 is not presented as a theorem derived from the definitions of Ψ and Δ. Rather, it is adopted as a *consistency requirement* that clarifies what is meant by "the same level" together with "its admissible family." The intended reading is:

- The admissible family $\mathcal{A}$ is designed to include only those perspectives that are mutually compatible as level-respecting re-expressions: they may reorganize visible factors or reparameterize representations, but they are not allowed to (i) alter the level partition, (ii) change the declared resolution regime beyond the admissible range, or (iii) introduce representational artifacts through pathological encodings.

- Under such a restriction, the existence of a coordinable description (i.e. the attainability of $\Psi = 0$ on the reachable set at the declared resolution) is treated as a structural property of the fixed level specification, not as a byproduct of a special viewpoint. Hence, if one admissible perspective realizes $\Psi = 0$, then any other admissible perspective—being only a level-respecting re-expression within the same operational constraints—must also admit $\Psi = 0$.

- If, in a concrete application, one finds perspectives A and A' that appear to yield $\Psi(\mathcal{L}, A) = 0$ but $\Psi(\mathcal{L}, A') > 0$, the intended conclusion is not that the axiom has been "refuted." Rather, it indicates that A and A' are not simultaneously admissible under the same fixed level specification: at least one of them violates the declared constraints (e.g. by changing resolution, effectively altering the level partition, or encoding conflict into the representation), and therefore should not belong to the same admissible family $\mathcal{A}$.

In summary, Axiom 2.1 ensures that the H/M classification is invariant *within a fixed level specification and its declared admissible regime*. It makes explicit that the classification is a statement about structure relative to a controlled family of perspectives, rather than a statement about arbitrary, unconstrained viewpoint changes.

2.3.2 Formal definitions of H-class and M-class systems

Definition 2.5 (H-class systems). At a fixed level, if there exists at least one admissible perspective $\hat{A} \in \mathcal{A}$ such that

$$\Psi(\mathcal{L}, \hat{A}) = 0,$$

then $\mathcal{L}$ is an H-class system at that level. By Axiom 2.1, for all admissible $A \in \mathcal{A}$ at that level, $\Psi(\mathcal{L}, A) = 0$ holds. Operationally, this means that at that level (and under the declared admissibility and resolution control) intra-level conflict can be eliminated in the representation.

Definition 2.6 (M-class systems). At a fixed level, if for all admissible perspectives $A \in \mathcal{A}$,

$$\Psi(\mathcal{L}, A) > 0,$$

then $\mathcal{L}$ is an M-class system at that level. In this case, regardless of the admissible perspective chosen at that level, irreducible intra-level conflict remains.

These definitions convert informal judgments of "regularity" versus "irreducible tension" into explicit structural criteria:

- *H-class:* there exists a coordinable description at the level under the admissible regime, and this coordinability is stable across admissible perspectives.
- *M-class:* no admissible perspective at the level can eliminate intra-level frustration; the residual is structural relative to the admissible regime.

Scope note (resolution dependence). The H/M label is operational and always relative to a fixed level specification, including the declared admissible family and resolution regime. Under refinement of resolution (or a change of the admissible regime), the observed value of Ψ may change; in particular, a structure that is effectively H-like at a coarse regime may reveal nonzero frustration at a finer regime. The definitions above should therefore be read as classification *within* the declared regime.

2.3.3 Connection to later chapters

The H/M classification and Axiom 2.1 provide a unified basis for subsequent discussions:

- In Chapter 3, H versus M will be interpreted via physical intuition (symmetry-like coordination versus irreducible tension requiring compensating embeddings), together with the role of a positive meta-gap $\Delta^* > 0$ under genuine cross-level constructions.
- In Chapters 4 and beyond, Gödel incompleteness, undecidability boundaries, NP-hard regimes, and long-term unpredictability in chaotic dynamics will be treated as concrete manifestations of M-class structures across domains, unified by the persistence of $\Psi > 0$ (under admissible perspectives) together with a positive inter-level lower bound under admissible regimes.

In this sense, "inherent gaps in single-perspective continuous logic" are elevated to a structural proposition formulated by Ψ and Δ: once a system is M-class at a given level (relative to an admissible family), changes of perspective within that family cannot eliminate the residual.

Chapter 3

Structural Physical Intuition: Symmetry, Compensation, and Embedding

Having established the formal definitions in Chapter 2, we now have a perspective-based hierarchical framework consisting of the logical system $\mathcal{L}$, perspectives A, the structural frustration index Ψ, the structural gap Δ, and the H/M classification. Building on this framework, the present chapter discusses the meanings revealed by these quantities from the standpoint of physical intuition and structural analysis.

This chapter introduces no new core concepts. Instead, we employ the established framework to interpret several key phenomena: symmetry breaking, structural compensation, and the meta-gap. In other words, from the point of view of physics and complex systems, we examine which structural states in real systems correspond to Ψ, Δ, and to H/M. These discussions provide a more concrete understanding of the unified structure indicated by the phrase "logical gap": it is not merely formal, but can be directly correlated with familiar physical pictures.

3.1 Structural Frustration Index Ψ and Symmetry Breaking

This section discusses the correspondence between the structural frustration index Ψ and symmetry breaking in physics. The aim is not to derive new conclusions in physics itself, but to show that, once the definition of Ψ given in Chapter 2 is accepted, symmetry breaking can be interpreted as a canonical physical manifestation of the condition $\Psi > 0$.

3.1.1 Basic picture of symmetry breaking

Intuitively, symmetry breaking means that the fundamental laws of a system (such as its equations of motion or Hamiltonian) possess a certain symmetry in form, but the actual realized state or solution no longer preserves that symmetry. Classical examples include:

- **Ferromagnetism:** At high temperature, the orientations of individual magnetic moments are approximately random and the macroscopic magnetization is zero; statistically, the system exhibits rotational symmetry. When the temperature is lowered below the Curie point, the material spontaneously chooses a specific direction of magnetization, and the macroscopic state becomes biased toward this direction, thereby breaking the original rotational symmetry.

- **Spontaneous symmetry breaking in particle physics:** The fundamental Lagrangian of a theory remains invariant under certain gauge transformations, but the vacuum ground state spontaneously selects one of the equivalent directions, so that different degrees of freedom manifest as different types of particles or interactions at low energy.

From a structural perspective, this can be summarized as follows: symmetry breaking means that the system, from a family of symmetrically equivalent possibilities, selects one (or a subset) to be realized macroscopically, and no longer remains in a state in which all symmetry-equivalent options are "simultaneously available".

3.1.2 $\Psi = 0$: effective symmetry within a level

Recall the definition of Ψ in Chapter 2: under a given perspective A, a conflict functional $E_A(S) \geq 0$ is introduced to measure the deviation between a state S and the macro-level closure rules under that perspective; the structural frustration index is defined as

$$\Psi(\mathcal{L}, A) = \inf_{S \in \Omega_A} E_A(S),$$

where Ω_A is the set of states reachable under perspective A. The meaning of $\Psi = 0$ is that, at the resolution of perspective A, there exists a family of states whose microscopic evolution is fully consistent with the macro-level closure rules, with no irreducible internal inconsistency.

In terms of symmetry, $\Psi = 0$ corresponds to an *effective symmetry* within the level:

- The macro-level closure rules impose a unified set of constraints on the relevant microscopic degrees of freedom at that level;

- At this level, the system is not forced to make mutually exclusive choices among these degrees of freedom;

- Apparent local biases can be reabsorbed or rewritten within the tolerance of this perspective, so that the overall structure retains a symmetric form.

It should be stressed that $\Psi = 0$ is typically an ideal limit. In real systems, the more common scenario is that Ψ can be reduced, at a given level and resolution, to values vanishingly close to 0, and can thus be operationally treated as "effectively symmetric". For example, in the high-temperature phase of a ferromagnet above the Curie point, the macroscopic magnetization is not literally zero, but in the statistical limit rotational symmetry approximately holds; in the present language this corresponds to Ψ approaching 0 under the relevant perspective.

3.1.3 $\Psi > 0$: irreducible symmetry breaking

When $\Psi(\mathcal{L}, A) > 0$, even if the system is allowed to adjust as much as possible within the state space admitted by perspective A, the conflict functional $E_A(S)$ still has a strictly positive lower bound. In other words, under this perspective there is always a portion of structural tension that cannot be resolved; the system cannot find a state that fully meets all macro-level requirements.

This aligns naturally with the concept of symmetry breaking:

- An ideal fully symmetric state would mean that all symmetry-equivalent macro options can be jointly realized at the structural level;
- Once we encounter $\Psi > 0$ under a given perspective, it signals that the system cannot simultaneously satisfy all symmetry-equivalent requirements; it must select one (or a small subset) among them, manifesting a macroscopic bias;
- This structural limitation—that not all symmetry-equivalent possibilities can be jointly realized—can be regarded as the representation of symmetry breaking within the present framework.

In the language of this paper, the correspondence may be summarized as

$$\Psi(\mathcal{L}, A) > 0 \quad \Longleftrightarrow \quad \text{there exists irreducible symmetry breaking under perspective } A,$$

$$\Psi(\mathcal{L}, A) = 0 \quad \Longleftrightarrow \quad \text{an effective symmetry is realized within the level under perspective } A.$$

Here, "correspondence" is always understood under fixed level and perspective: if we change the resolution or the embedding relations, the classification of symmetric versus symmetry-broken states may change, as will be discussed later.

3.1.4 Physical intuition behind the H/M classification

With the correspondence between Ψ and symmetry breaking, the H/M classification of Chapter 2 admits a more physical interpretation:

- If, at some level, there exists a perspective $\hat{A}$ such that $\Psi(\mathcal{L}, \hat{A}) = 0$, then by the structural consistency axiom, Ψ vanishes under all admissible perspectives at that level, and the system is H-class at that level. Intuitively: there exists a description at that level under which the system can be regarded as governed entirely by symmetry; at this level, macroscopic behavior exhibits no endogenous structural contradictions, and any residual uncertainty can be treated as noise or external perturbation.
- If, at the same level, all admissible perspectives A satisfy $\Psi(\mathcal{L}, A) > 0$, then the system is M-class at that level. Intuitively: there is unavoidable symmetry breaking at this level; no matter how the macro variables are redefined or perspectives are chosen, the frustration index cannot be compressed to zero; the macroscopic structure always retains a nonzero gap, manifesting some fundamental randomness, instability, or conflict.

In this interpretation, the H/M classification is no longer a vague description in terms of "simple/complex", but a structural criterion concerning the existence or non-existence of an effectively symmetric level.

3.1.5 Examples: harmonic oscillator, chaotic pendulum, and social systems

A few elementary examples can help sharpen this intuition:

- **Ideal harmonic oscillator:** In an idealized model that neglects friction and external noise, the equation of motion and its solutions preserve symmetries such as time translation. Under a suitable perspective (for example, treating the energy shell as a macro variable), microscopic trajectories lie exactly on invariant curves. In such a perspective, one may regard $\Psi \approx 0$, and the system is approximately H-class at that level: symmetry largely controls the dynamics, and macroscopic behavior is highly predictable.

- **Chaotic pendulum or a driven–dissipative complex oscillator:** Once nonlinear couplings, external driving, and dissipation are introduced, the system may enter a chaotic regime. In this case, regardless of how one chooses conventional macro variables (such as energy or amplitude), it becomes difficult to construct a fully closed, symmetric description: long-term behavior is highly sensitive to initial conditions, energy is no longer conserved, and time-translation symmetry is structurally broken. In the terminology of this paper, every natural perspective exhibits a strictly positive Ψ, and the system tends toward M-class behavior at that level.

- **Socio-economic systems:** In highly idealized models, if there exists a set of rules that balance supply and demand and stabilize expectations, then at some abstract level the system may be treated as approximately H-class: Ψ is close to zero, and structural tensions are absorbed into the model itself. In real situations, institutional defects, information asymmetry, self-referential expectations, and external shocks are ubiquitous; no matter how we adjust perspective, part of the tension cannot be simply relegated to “noise”, manifesting as periodic crises or long-term disequilibrium. In the present framework, this corresponds to an M-class regime with $\Psi > 0$.

These examples show that the structural frustration index Ψ is not an isolated formal quantity but aligns naturally with the physical notion of symmetry breaking. At the structural level, $\Psi > 0$ captures the impossibility of fully coordinating internal rules at that level; at the physical level, it corresponds to broken symmetries and the associated failure of complete predictability or complete self-consistency. The unpredictability and incompleteness observed at macroscopic levels are not merely technical difficulties; they are the consequence of symmetry being broken at some level and the subsequent impossibility of fully reconstructing that symmetry at higher levels.

In physical systems, such gaps are seldom left fully exposed. Instead, they incorporate larger structures or external environments to compensate for and partially conceal local gaps. This leads to a structural compensation principle and to a more refined understanding of H-class systems, which will be discussed in the next subsection.

3.2 Structural Compensation Principle and H Systems

As discussed in the previous section, M-class systems are difficult and complex precisely because there are unavoidable symmetry breakings and structural gaps at local levels, i.e., $\Psi > 0$ holds under all local perspectives. However, in both natural and engineered systems, another type of phenomenon is equally pervasive: many locally prominent gaps and contradictions are ultimately buffered and absorbed by embedding the subsystem into a larger environment or a higher-level structure.

In other words, a subsystem may be *structurally unstable* when viewed in isolation (local $\Psi > 0$, with pronounced structural conflict), yet when it is incorporated as a component into a more complex whole, the environment often provides additional degrees of freedom, redundancy mechanisms, or dissipative channels, so that these contradictions no longer appear in their original form. The Structural Compensation Principle aims to formalize this generic mechanism by which higher-level structures bridge local gaps.

3.2.1 Basic formulation of the structural compensation principle

The Structural Compensation Principle can be roughly stated as follows:

> For any local gap that can be embedded into a larger system, as long as the overall system possesses sufficient degrees of freedom, redundant resources, or dissipative channels, there exists a higher-level perspective under which the conflict that was locally irreducible can be transferred to or offset by the environment, such that the whole exhibits approximately gap-free operational behavior under that perspective.

This formulation contains two key points:

1. **"Can be embedded":** Not all gaps are compensable. Only those conflicts that can be reorganized via exchanges of energy, information, or matter with the environment have the potential to be integrated into a larger structure.

2. **"Higher-level perspective":** Compensation is not achieved within the original perspective; instead, new variables and couplings are introduced to construct a higher-level perspective in which gaps that were explicit in the old description become absorbable, dispersed, or re-encoded.

To make this more concrete, we now illustrate the idea with a simplified conceptual model.

3.2.2 Conceptual model: subsystem–environment–global perspective

Let $\mathcal{L}_{\text{sub}}$ be a local subsystem. Under some internal perspective A_{sub}, suppose we find

$$\Psi(\mathcal{L}_{\text{sub}}, A_{\text{sub}}) > 0,$$

which means that under this perspective the subsystem exhibits irreconcilable internal conflict. If we consider only $\mathcal{L}_{\text{sub}}$ in isolation, it is natural to judge that it is M-class at this level: macro-level rules cannot be made strictly closed, and gaps appear as structurally irreducible at this level.

In reality, however, $\mathcal{L}_{\text{sub}}$ is typically not isolated, but situated in a larger environment $\mathcal{L}_{\text{env}}$. The environment may include external fields, boundary conditions, control mechanisms, resource reservoirs, and various buffering and regulatory mechanisms. When we combine the subsystem and its environment into a whole

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{sub}} \oplus \mathcal{L}_{\text{env}},$$

we can introduce a higher-level perspective A': under this perspective, we not only observe the state of the subsystem but also track the environmental variables and interactions that help resolve the conflicts.

Under perspective A', we re-evaluate the frustration index of the whole, $\Psi(\mathcal{L}_{\text{tot}}, A')$. The core idea of the Structural Compensation Principle is that, through the complementary action of the environment, the value of Ψ for the whole can be substantially reduced, possibly approaching zero:

$$\Psi(\mathcal{L}_{\text{tot}}, A') \ll \Psi(\mathcal{L}_{\text{sub}}, A_{\text{sub}}).$$

This indicates that the gaps that were irreducible in the isolated perspective of the subsystem are absorbed or diluted by the environment; at a higher dimensional level the whole recovers an effective symmetry and exhibits behavior close to that of an H-class system.

3.2.3 Examples from real systems

Analogous situations are widespread in real systems:

- **Machines and cooling systems:** If we consider a machine in isolation, insufficient heat dissipation may cause its temperature to rise steadily and eventually spiral out of control. This corresponds to $\Psi(\mathcal{L}_{\text{sub}}, A_{\text{sub}}) > 0$ in the perspective that "observes only the machine itself": there is an irreconcilable conflict between internal energy accumulation and structural tolerance limits. When heat sinks, fans, or coolant circulation systems are added, we effectively introduce an environment $\mathcal{L}_{\text{env}}$ and adopt a new perspective A' in which "machine + cooling system" is viewed as a whole. In this higher-level perspective, energy can be continuously extracted and dissipated, the temperature is maintained within a safe range, and the combined system exhibits near-steady-state behavior ($\Psi(\mathcal{L}_{\text{tot}}, A') \approx 0$). At this level, the whole can be treated as H-class.

- **Firms and market environments:** Within a firm, different departments may have conflicting objectives: cost control versus innovation investment, short-term profit versus long-term brand building, and so on. These are difficult to reconcile fully within a closed internal perspective, corresponding to a relatively large Ψ internally. When the firm is embedded in a broader market and institutional environment, price mechanisms, competitive structure, regulatory policies, and financial instruments—the "environmental variables"—can all act as regulators. The firm then often manages to maintain some form of dynamic balance at the global level, achieving long-term survival or even stable growth. Under the higher-level perspective of "firm + market environment", many internal conflicts are indirectly mitigated through external resources and rules: instead of directly removing the sources of conflict, the larger structure affords buffering and regulatory channels, so that at the macro level the overall dynamics appear relatively orderly.

These examples show that the environment does not eliminate the subsystem's gaps at a fundamental level; rather, it disperses, redirects, or dissipates them through a larger structure. Precisely because of environmental participation, many structurally highly complex systems still exhibit strong stability and order at macroscopic scales. This does not mean that gaps never existed; rather, they have been indirectly processed by structural compensation mechanisms.

3.2.4 Compensation in formal structures: Euler's formula and Maxwell's equations

The previous examples come mainly from engineering and social systems. The Structural Compensation Principle also finds clear analogues in purely formal structures. We now discuss two classical cases to illustrate the mechanism of "restoring closure by lifting the level or introducing environmental variables".

Euler's formula: unifying heterogeneous rules in the complex plane From the viewpoint of real analysis, the exponential function, trigonometric functions, and planar rotation are governed by different rules. On the one hand, we have

$$\frac{d}{dx}e^x = e^x,$$

while on the other hand,

$$\frac{d}{dx}\sin x = \cos x, \qquad \frac{d}{dx}\cos x = -\sin x,$$

together with the algebraic constraint for the unit circle, $\sin^2 x + \cos^2 x = 1$. In a "purely real-function" perspective, these structures are split into several mutually coupled rule sets, and it is not straightforward to give a single, fully closed description in terms of one equation.

Once the system is embedded into the complex plane, we introduce the complex-valued function

$$z(x) = e^{ix},$$

whose derivative satisfies

$$\frac{d}{dx}z(x) = i\,z(x),$$

while its real and imaginary parts correspond to $\cos x$ and $\sin x$, respectively. In this higher-level perspective, the previously scattered behaviors of exponential growth, trigonometric oscillation, and planar rotation are unified as "phase rotation in the complex plane", and rules that appeared heterogeneous within the real domain are subsumed under a single dynamical law. In the language of this paper, the complex plane plays the role of an "environment": by lifting the dimension and re-encoding, a family of structures that previously required multiple rules is rewritten as a single closed equation, thereby suppressing the effective frustration index Ψ at this level to a value close to zero.

Maxwell's equations: restoring consistency by introducing fields and displacement current A similar structural pattern appears in the historical development of classical electromagnetism. If we only consider the local laws of electrostatics and magnetostatics, Gauss's law for the electric field and Ampère's circuital law act on different "subsystems". Once time-varying electric fields and currents are introduced, naively extending the old forms leads to inconsistency with the continuity equation for charge conservation, leaving a structural gap in the perspective of "partial laws + dynamical situations".

Maxwell resolved this by introducing the displacement current term, unifying electric and magnetic fields as different aspects of a set of four equations, and replacing the notion of "action at a distance" by a higher-level field perspective. Under this expanded global viewpoint, charge conservation and other global constraints are restored. In this rewriting, conflicts that had been latent in the local laws are "transferred" to the additional degrees of freedom provided by fields and displacement current; the overall theory achieves closure again at the new level. This is a canonical instance in which Ψ is reduced to near zero through environmental compensation.

These two formal examples show that structural compensation is not restricted to concrete material systems. Introducing higher-dimensional representation spaces or additional environmental variables so that a collection of constraints, difficult to satisfy simultaneously at the local level, can be jointly encoded in a larger structure is a pattern that recurs in the development of mathematics and physics. Within the present framework, such phenomena can be uniformly described as follows: by extending perspectives and ontological boundaries, structural frustration Ψ that was prominent at local levels is absorbed at higher levels, yielding an approximately H-class effective description of the system.

3.2.5 Reinterpretation of H systems and the limits of M systems

On this basis, we can refine the interpretation of H systems. In the formal definition of Chapter 2, an H-class system is one for which there exists, at some level, a perspective with $\Psi = 0$. Once the Structural Compensation Principle is taken into account, a more operationally grounded understanding is as follows:

> A genuinely H-class system does not require "perfect consistency under all closed conditions"; rather, it is a system that can, through internal or environmental compensation mechanisms, suppress all potential gaps below the observational resolution, so that at the level of interest it behaves effectively as a gap-free structure.

In other words, an H-class system is one which, given a specified level, task objective, and observational resolution, can use compensation mechanisms to compress the visible structural frustration at that level down to a negligible level ($\Psi \approx 0$). This is analogous to the idea of an "effective theory" in physics: within a certain energy scale, one can use a symmetric, closed macroscopic description, while treating gaps at smaller scales as details already absorbed by the larger structure.

Correspondingly, the meaning of an M-class system can also be reformulated. To say that a system is M-class at some level means that, no matter how we extend the environment or introduce compensation mechanisms, within the level and its reasonable extensions under consideration, Ψ always retains a non-negligible positive lower bound; the gap proves incompressible below an "effectively zero" threshold. Formally, if for all reasonable extended systems $\mathcal{L}_{\text{tot}}$ and perspectives A' we have

$$\Psi(\mathcal{L}_{\text{tot}}, A') \geq \varepsilon_0 > 0,$$

then the system is M-class at that level. This means that, regardless of how the environment is extended, some portion of the structural gap cannot be truly hidden by compensation mechanisms.

Introducing the Structural Compensation Principle leads to an important hierarchical insight: many subsystems that appear M-class when examined in isolation may change their H/M status depending on how they are embedded in an environment. Viewed in isolation, the subsystem may display pronounced structural contradictions and lack self-consistency; placed within a larger structure, it may operate coherently under the action of compensation mechanisms. This implies that any assessment of a system's complexity or gap strength must be predicated on a clear specification of the level's boundaries and the environmental context. Assessing complexity in complete abstraction from the environment often has limited practical significance.

However, this does not mean that by indefinitely enlarging the environment one can eventually "package" all systems as H-class. Compensation mechanisms themselves are inherently limited: an ultimate, unsurpassable gap persists, one that inheres in the total structure and cannot be eliminated by any finite-level environment. To address this, we need to introduce the notion of a *meta fracture*, which is the topic of the next section.

3.3 Meta Fracture Δ: The Ultimate Boundary of Single-Viewpoint Continuous Logic

Structural compensation mechanisms can alleviate local gaps over a wide range of scales. This naturally raises the question: can all gaps ultimately be removed by continually introducing higher levels and extending system boundaries, thereby constructing a completely self-consistent "ultimate theory"? Intuitively, many people are inclined to believe that "there is a final endpoint at which all contradictions can be unified and resolved".

Within the present framework, the conclusion runs counter to this intuition: even at the imagined "meta-structural level of the universe"—a limiting whole that contains all levels, variables, and feedback relations—there remains a fine gap that cannot be smoothed out. We formulate this as the *meta fracture postulate*.

3.3.1 Meta Fracture Postulate: A Strictly Positive Lower Bound for Δ^*

Postulate 3.1 (Meta Fracture Postulate) At the meta-structural level of the universe, the infimum of the logical gap, denoted Δ^*, is strictly positive:

$$\Delta^* > 0.$$

In words, $\Delta^* > 0$ means that even if we are granted arbitrarily fine descriptive capacity, arbitrarily complex model-building power, and arbitrarily long evolution time, there always remains a residuum of structural tension inside the system that cannot be completely eliminated.

Here, Δ is not a directly measurable physical quantity but an abstract representation of a structural fact: in every system structure there is always "a tiny amount left over". It encodes the ultimate obstruction encountered by single-viewpoint continuous logic when it attempts to exhaust itself.

The logical breakdown phenomena discussed earlier—formal incompleteness, the undecidability of the halting problem, emergence in complex systems, long-term unpredictability in chaotic dynamics, and so on—can all be regarded as projections of this meta fracture in different contexts. Whenever we attempt to use single-viewpoint continuous logic to bridge levels and seek a fully complete law, the point at which the reasoning "sticks" is typically the manifestation of Δ^* at the corresponding level.

A simple toy model is the limit construction of the sequence $0.9, 0.99, 0.999, \ldots$ in the real number system. Within a particular H-calculus, we conventionally write

$$0.999\ldots = 1.$$

From the present perspective, this is merely a choice of notation that packages the sub-resolution residual into the symbol 1: it means that, within this H-calculus, we agree to treat the associated structural gap as $\Delta^* \approx 0$ at the working resolution, but it does *not* assert that $\Delta^* = 0$ at the structural level.

3.3.2 Effective theories and suppressed fractures

In practice, the various scientific disciplines are largely built by constructing effective theories that, at specific scales, dilute the meta fracture to a negligible level. For example:

- Newtonian mechanics, at macroscopic scales, ignores microscopic quantum uncertainty and treats macroscopic bodies as continuous entities with precisely measurable positions and momenta;
- classical thermodynamics ignores microscopic fluctuations of molecular thermal motion, treats heat as a continuously divisible "fluid", and describes macroscopic processes with smooth equations of state;
- many economic models ignore individual psychology, institutional details, and environmental heterogeneity, and instead posit a homogeneous "rational agent" abstraction for deriving demand–supply curves and equilibrium conditions.

In each effective-theory construction, we consciously or unconsciously ignore, average out, or postpone the treatment of certain fine-grained fractures, thereby rendering the model smooth and tractable at the chosen scale. These fractures, however, do not vanish; rather, they are displaced to other levels:

- some are "averaged away", appearing as small fluctuations or background noise;
- some are deferred to longer time scales before they become manifest;
- some re-emerge in larger spatial domains or more complex structures in a different guise.

As our understanding advances level by level—widening the spatial scope and extending the temporal horizon—fractures that were previously treated as negligible at lower levels (those marked by Δ^*) reappear in various ways.

For instance, a theory may perform extremely well within a finite spatial and temporal domain, yet once it is extrapolated to larger regions or longer time spans, one often encounters a breakdown point: uncertainties that were previously negligible accumulate and eventually lead to non-negligible deviations. Similarly, in computation, hardware improvements can temporarily alleviate complexity bottlenecks, but once the problem size crosses a certain level threshold, the structural boundary of complexity (such as the explosive growth of complexity for NP-hard problems) inevitably reasserts itself.

3.3.3 A structural boundary for the exhaustion of continuous logic

The preceding discussion shows that the meta fracture Δ^* implies the existence of a structural boundary on the exhaustion of single-viewpoint continuous logic.

- This boundary does not arise from insufficient effort, outdated technology, or poor methodological choices;
- Rather, it reflects the objective presence of a fine gap in the structure of the universe that cannot be completely erased.

The laws we summarize at each level can be understood as "local closures" around this meta fracture: within finite perspectives and scales, Ψ can be driven arbitrarily close to zero, but the accumulation of Δ never vanishes in a global sense.

Thus, when we attempt to construct some "ultimate continuous logic" (a single-viewpoint continuous description that is fully self-consistent across all levels), we must proceed with a built-in restraint:

- There may be no absolutely self-consistent, absolutely complete "final level";
- No matter how closely descriptions approximate structural reality, there will always remain an irreducible residual.

This is not a defense of naive agnosticism, but a proposal for a structurally informed epistemic stance: in addition to asking "over what range is a theory valid?", we should also ask "by what mechanisms does it conceal or dilute which fracture?". Once the existence of Δ^* is acknowledged, talk of "progress in knowledge" acquires a definite structural direction: we know that we are perpetually approaching, but no longer expect to "eliminate the fracture once and for all".

From this standpoint, the "logical gap" is not merely a description of human ignorance; it is a characterization of a feature intrinsic to the structure of single-viewpoint continuous logic itself: it identifies the structural boundary that such logic must encounter when extended across levels.

3.3.4 Fundamental deadlocks and hierarchical evolution under the meta fracture perspective

Revisiting the fundamental deadlocks mentioned in the introduction—incompleteness in mathematics, uncertainty in physics, undecidability and unsolvability in computation, the challenge of emergence in complex systems, and so forth—we can, from the perspective of the meta fracture, regard them as different projections of the same Δ^* across domains and scales.

Through the discussion of symmetry breaking and structural compensation in this chapter, we can see how this fracture appears at different levels and how it is temporarily concealed and re-encoded:

- At local levels, $\Psi > 0$ manifests as symmetry breaking and structural frustration;
- At higher levels, through environments and compensation mechanisms, Ψ can be compressed to values close to zero within a certain range, so that the system appears H-class at that level;
- Along the evolution parameter γ, the integral accumulation $\Delta(\gamma)$ never globally returns to zero, and the meta lower bound Δ^* persists, ensuring the emergence of new fractures at higher levels.

In the terminology of this paper, the pair (Ψ, Δ) describes the trajectory of single-viewpoint continuous logic as it attempts to extend itself across levels:

- When $\Psi = 0$, we have found a relatively stable "effectively determinate region" at a specific level;
- The continued accumulation of Δ guarantees that new forms of indeterminacy reappear at higher levels, forcing us to change perspective and introduce new theoretical architectures.

From this angle, the "logical gap" is not a pessimistic statement about human finitude but a neutral characterization of the structural properties of single-viewpoint continuous logic:

- On the one hand, it rules out the possibility that any single continuous logic could "settle all questions once and for all";

- On the other hand, it provides a rationale for ongoing structural renewal: each generation of theories can achieve $\Psi \approx 0$ at some finite level, but none will erase Δ^* at the global level.

In the chapters that follow, we will carry this framework into the analysis of the limits of computation, the sources of complexity, and issues such as intelligence and consciousness, examining how Ψ and Δ yield a unified structural description across these concrete domains.

3.4 H/M Embedding Principle: M as the Limiting Structure of Multi-H Coupling

In the preceding discussion, we used the structural frustration index Ψ and the structural gap Δ to define a formal classification of systems into H-class and M-class. At a given level, if there exists an admissible perspective A such that $\Psi(\mathcal{L}, A) = 0$, then the system can be regarded as H-class at that level; if for all admissible perspectives A one has $\Psi(\mathcal{L}, A) > 0$, then the system exhibits M-class behavior at that level. Chapter 2 gave the formal definition of this classification, while the earlier sections of Chapter 3 provided physical intuition and philosophical grounding for it in terms of symmetry breaking, structural compensation, and the meta fracture Δ^*.

This section introduces a higher-level structural judgment, the *H/M Embedding Principle*. Its core intuition is that, in complex systems, macroscopically observed M-class behavior is often not produced by a "single M-module" that satisfies $\Psi > 0$ under all admissible perspectives. Rather, it arises as the global limiting structure formed by coupling multiple locally H-class subsystems in specific ways. In other words, M can be understood as the *non-coordinable limit* of multi-H coupling.

This principle provides a unifying structural background for later discussions of emergence, undecidability, spin-glass frustration, and the irreversibility of information in lossy compression.

3.4.1 Brief review of H-class and M-class

For convenience, we briefly recall the operational definitions of H and M (consistent with Chapter 2). Let $\mathcal{L}$ be a dynamical logical system. Under a perspective A, its structural frustration index is defined as

$$\Psi(\mathcal{L}, A) = \inf_{S \in \Omega_{\text{reach}}} E_A(S),$$

where $E_A(S) \geq 0$ measures the structural conflict of state S under perspective A. The structural gap can be understood as the accumulation of frustration along an evolution parameter γ:

$$\Delta(\mathcal{L}, A; \gamma) = \int_0^\gamma \Psi(\mathcal{L}, A; \tau)\, d\tau.$$

On this basis:

- If there exists at least one admissible perspective $\hat{A}$ such that

$$\Psi(\mathcal{L}, \hat{A}) = 0,$$

then $\mathcal{L}$ is said to be H-class at that level.

- If for all admissible perspectives A one has

$$\Psi(\mathcal{L}, A) > 0,$$

then $\mathcal{L}$ is said to be M-class at that level.

The structural consistency axiom ensures the stability of this classification: once there exists some perspective A^* with $\Psi = 0$, other admissible perspectives at the same level can be transformed to achieve $\Psi = 0$ as well; conversely, if $\Psi > 0$ under one perspective, then it cannot be compressed to zero under any equivalent-level perspective. Thus H/M is a *global property of the level*, rather than an accident of any single local perspective.

3.4.2 Multi-H coupling and global frustration structures

Real systems are rarely purely isolated H or M entities; they are typically composed of coupled structures across multiple levels. Suppose that, at some mesoscopic scale, the system can be decomposed into a family of local subsystems:

$$\mathcal{L} = \bigoplus_{i \in I} \mathcal{L}_i \oplus \Gamma,$$

where:

- $\mathcal{L}_i$ denotes the i-th local subsystem;
- Γ denotes the coupling structure between these subsystems (interaction terms, constraints, boundary connections, resource sharing, and so on).

For each local block $\mathcal{L}_i$, we may find a local perspective A_i such that it behaves as H-class:

$$\Psi(\mathcal{L}_i, A_i) = 0.$$

This means that, in suitable local coordinates and aggregations, the internal tensions of each $\mathcal{L}_i$ can be completely resolved by local symmetries and compensation mechanisms; internally, at this level, the structure is "tame".

Once these H-blocks are combined into the global system $\mathcal{L}$ under the coupling structure Γ, the question becomes: does there exist a global perspective A_{glob} that can simultaneously provide a coherent macro-level description for all $\mathcal{L}_i$ and the coupling Γ, such that the global frustration index $\Psi(\mathcal{L}, A_{\text{glob}})$ remains zero?

- If such a global perspective exists, then the overall system $\mathcal{L}$ can still be classified as H-class at that level: the interface conditions between local H-blocks can be coordinated in a common set of coordinates, and all local tensions remain compensable and renormalizable within the global structure.

- In many complex systems, however, the situation is the opposite: the couplings between local H-blocks are mutually incompatible to such an extent that no single perspective can preserve the H-structure of all $\mathcal{L}_i$ simultaneously. Under any candidate global perspective A_{glob}, some H-blocks are "distorted", and the local condition $\Psi(\mathcal{L}_i, A_i) = 0$ is converted into global frustration at interfaces, yielding $\Psi(\mathcal{L}, A_{\text{glob}}) > 0$.

This is the core intuition of the H/M Embedding Principle:

> Global M behavior typically arises from the non-coordinability of couplings among multiple H-class subsystems, rather than from each local block being M-class in itself.

In other words, M is better seen as a *failure mode of global embedding* than as a system built out of "bad local material".

3.4.3 Gradient fields and non-integrability: a geometric analogy

This can be expressed more vividly using the perspective of local potential functions and gradient fields. Suppose that in each H subsystem $\mathcal{L}_i$ there exists a local potential function Φ_i such that, under the appropriate perspective,

$$\Psi(\mathcal{L}_i, A_i) = 0 \quad \Longleftrightarrow \quad \text{local structural tension can be written as } J_i = \nabla \Phi_i,$$

that is, the local frustration structure can be derived as the gradient of some potential, so that the interior of the block is "differentiable" and "integrable" at this level.

If the global system is still H-class, then under some global perspective A_{glob} these local potentials should be patchable across interfaces into a single global potential Φ_{global}, so that

$$\Psi(\mathcal{L}, A_{\text{glob}}) = 0, \qquad J_{\text{global}} = \nabla \Phi_{\text{global}}$$

holds throughout the system. In other words, the local gradient fields are not only exact within each patch but also exact globally.

In many M-class systems, the situation is precisely the opposite:

- Within each local patch (each H-block), there exists a well-defined local potential Φ_i;
- Globally, however, these local potentials cannot be consistently patched together into a single Φ_{global};
- As a result, circulations, vortices, or other non-conservative structures appear at the interfaces; globally, J_{global} is no longer the gradient of a single potential but only a "locally differentiable, globally non-integrable" pseudo-gradient field.

Geometrically, this is analogous to a 1-form that is exact on each coordinate patch but, due to topology or curvature, fails to be exact globally, thereby yielding nonzero curvature or holonomy at the global level.

In the language of this paper, such *global non-integrability* corresponds to

$$\Psi(\mathcal{L}, A_{\text{glob}}) > 0, \qquad \Delta(\mathcal{L}, A_{\text{glob}}; \gamma) \text{ exhibits a globally incompressible, and possibly divergent, accumulatio}$$

More intuitively:

- For each H-block, logic is "differentiable" locally: structural tension can be resolved by gradient descent or local compensation;
- When we attempt to govern all H-blocks with a single logic and potential, the gradient field breaks at interfaces and ceases to derive from a single potential. This is the structural reason why the whole system enters the M-class regime.

3.4.4 Philosophical summary: fractures and the "whole of the whole"

At a philosophical level, the H/M Embedding Principle can be condensed into two key insights.

(1) Fractures appear at the level of the "whole of the whole", not at the atomic level. For any sufficiently complex system, once the observational perspective is restricted to a local level, one can usually find approximately H-class structural blocks: within local spacetime regions, local variable sets, or local functional ranges, the system is tame, modelable, and predictable. Fractures do not start as total disorder at the very bottom; they appear when we attempt to integrate multiple local H-blocks into a larger whole and demand their full coordination under a single perspective.

Thus M does not mean that "the world is essentially chaotic"; it means that, when we try to construct a larger "whole of the whole" and require all local structures to coexist perfectly under a common logic and a single potential, structural frustration inevitably emerges.

(2) High gap intensity is mainly determined by coupling patterns, not by local material. In this framework, an M-class system is not formed by simply stacking locally M-class components. Instead, it arises when multiple structures that are H-class in their own local perspectives are combined through interface and coupling patterns that are not fully compatible. Fractures tend to appear at the boundaries, interfaces, and couplings between H-blocks, rather than in the interiors of the blocks themselves.

This implies that, to understand high-gap behavior in complex systems (such as unsolvability, extreme sensitivity, non-extendability at the macro level, or violent restructuring of perceptual organization), the focus should not be on "how complex each local module is", but on *how* these local H-structures are coupled together. Many phenomena that empirically manifest as extreme sensitivity or difficulty of stable prediction can be reanalyzed under the structural scheme "multi-H coupling $\to$ global M": locally coherent H-logics, when globally embedded, generate M-class fractures.

In later chapters, we will illustrate how this principle is realized through concrete cases (spin-glass systems and JPEG lossy compression, chaotic intervals in the logistic map, and AI hallucinations under the multi-H coupling perspective). These examples will show how fractures appear at the macro level and how they can be traced back to coupling patterns among multi-level H-structures that look minor yet are structurally decisive.

From a dynamical viewpoint, the H/M Embedding Principle also suggests an important but not yet fully characterized fact: in multi-level self-organizing systems, local H-structures do not remain tame under arbitrarily high orders of self-iteration. On the contrary, as we will see, a local H-block, when iterated repeatedly under its own rules and continuously embedded into higher-level wholes, tends to become unstable beyond some finite critical order k^*:

- For iteration orders $k < k^*$, the block can still be regarded as H-class under natural perspectives;
- For iteration orders $k \geq k^*$, the same block inevitably transitions into M-class under the same perspective, and its internal tensions can no longer be fully resolved by local compensation mechanisms.

More interestingly, for a broad class of systems with comparatively mild structural rules (such as unimodal iterated maps and certain prototypical self-organizing networks), numerical and structural evidence indicate that this critical order k^* is not arbitrarily distributed but clusters near the fifth order. In subsequent dedicated sections, we will refer to this phenomenon as *fifth-order structural instability* and use tools such as the Feigenbaum constants to provide a unified description of its relationship to the H-to-M transition.

Thus the H/M Embedding Principle may be viewed as a purely structural background statement, while “fifth-order structural instability” provides a quantitative realization of this principle in concrete dynamical systems: it exhibits a minimal prototype of how multi-H coupling evolves into global M behavior within a finite iteration depth. A more detailed analysis of this point will be given after we have examined the relevant case studies.

Chapter 4

Hierarchical Tension Dynamical Systems (HTDS): From Static Fractures to Dynamics

4.0.1 Notation and Variants of Δ

In later chapters, the symbol "Δ" will reappear with different meanings at different levels. To avoid confusion, we collect and standardize the main variants of Δ used throughout the paper:

1. Δ_{view}: fracture associated with perspective switching.
   This measures the residual information that cannot be eliminated, even under optimal lifting and reconstruction, when switching from an individual-level perspective A_I to a global-level perspective A_G, typically written as

$$\Delta_{A_I \to A_G}(\mathcal{L}).$$

2. Δ_{dyn}: dynamical fracture lower bound.
   For a given dynamical system $T : X \to X$, based on the instantaneous frustration index $\Psi(x)$, we define

$$\Delta_{\text{dyn}} := \inf_{\mu \in \mathcal{M}_T} \int \Psi \, d\mu,$$

   where $\mathcal{M}_T$ denotes the set of all T-invariant probability measures. This quantity measures the theoretical lower limit to which the long-term average fracture can be reduced at the current level and perspective.

3. $\Delta(\gamma)$: fracture accumulation along an abstract evolution parameter.
   In the structural calculus of Chapter 5, along an abstract evolution parameter γ we define

$$\Delta(\gamma) = \int_0^\gamma \Psi(\tau) \, d\tau, \qquad \Psi(\gamma) = \frac{d\Delta}{d\gamma},$$

   thereby linking the "local tension density" to the accumulated fracture along an evolution trajectory.

4. Δ^*: meta-fracture lower bound.

   The meta-fracture postulate asserts that, at the "meta-structural level of the universe", there exists a fracture lower bound $\Delta^* > 0$, independent of any specific system or perspective. This expresses the ultimate boundary of single-viewpoint continuous logic at the global level.

In what follows, for notational simplicity we will often write Δ without an explicit subscript; the intended meaning (perspective switching, dynamical systems, abstract evolution parameter, or meta postulate) will be determined by context. When it is important to distinguish fracture types explicitly, we will indicate the corresponding subscript.

4.0.2 Resolution refinement and intrinsic M-ness (why Ψ can "run")

Chapter 2 formulated Ψ and the H/M classification within a fixed level specification, including a declared admissible family and a declared resolution regime. In the dynamical setting, we will repeatedly refine descriptions (or move along an abstract evolution parameter), and it is therefore expected that the *observable* frustration can change with the resolution at which indistinguishability is declared. The structural claim of M-class behavior is not that Ψ is constant, but that under controlled refinement a nonzero residual persists in a way that cannot be removed by admissible re-expressions.

To make this precise, let ε denote a resolution threshold (a scale of operational indistinguishability; not to be confused with the fracture symbol Δ). Let $\mathcal{A}(\varepsilon)$ denote the admissible family at resolution ε, meaning that non-degeneracy and "no-pathology" constraints are enforced while the resolution regime is fixed by ε. Define the best-achievable (intra-level) frustration at that resolution by

$$\Psi_{\min}(\varepsilon) \;:=\; \inf_{A\in\mathcal{A}(\varepsilon)} \;\Psi(\mathcal{L}, A).$$

Then:

- "H-like at resolution ε" corresponds to $\Psi_{\min}(\varepsilon) = 0$;
- "M-like at resolution ε" corresponds to $\Psi_{\min}(\varepsilon) > 0$.

Refining resolution may change the numerical value of $\Psi_{\min}(\varepsilon)$; a structure that is effectively H-like at a coarse regime may reveal $\Psi_{\min}(\varepsilon) > 0$ once finer oppositional terms become visible. The notion of *intrinsic* M-ness refers to persistence under controlled refinement: under an admissible refinement procedure (i.e. refining ε without changing the level partition or admitting pathological encodings), the system does not merely exhibit fluctuation, but retains a positive residual in the sense that $\Psi_{\min}(\varepsilon)$ cannot be driven to zero as ε becomes finer (equivalently, a strictly positive $\liminf$ under admissible refinement).

In the dynamical chapters that follow, this viewpoint will be reflected by quantities such as Δ_{dyn} (defined via long-term averages of Ψ) and by accumulated fracture along evolution parameters, where "running" captures how local tension density changes with refinement while structural lower bounds encode irreducible obstruction.

4.1 Basic Structure of HTDS

In Chapter 2, we abstracted a logical system as $\mathcal{L} = (V, R)$ and, under a fixed perspective $A = (F, P)$, defined the structural frustration index $\Psi(\mathcal{L}, A)$. To discuss how fractures accumulate along an evolution, we need a concrete dynamical carrier that can vary in time or along some evolution parameter. This is the role of the Hierarchical Tension Dynamical System (HTDS) constructed in this chapter.

Given an internal perspective A_I, an HTDS is specified by a quadruple

$$\text{HTDS} := (X, d, T, \{F_i\}_{i \in I}).$$

1. **State space** (X, d)

   X is the set of possible system states, and d is a metric or pseudometric on X. The space X may be a one-dimensional interval, a Euclidean space of higher dimension, a manifold, or a more general phase space, as long as it provides a reasonable topology and measure structure for the dynamics T.

2. **Dynamics** $T\colon X \to X$

   The map T specifies a discrete-time evolution rule, such as the logistic map, the Hénon map, or some other iterated system. At each step, the current state x_t is mapped to the next state
   $$x_{t+1} = T(x_t).$$

3. **Constraint / tension elements** $F_i\colon X \to \mathbb{R}$

   Each index $i \in I$ corresponds to a constraint, resource limitation, or internal "desire". At a state x,
   $$F_i(x) = 0$$
   indicates that the i-th constraint is fully satisfied; $F_i(x) \neq 0$ indicates a deviation, whose magnitude measures the degree of violation in that direction.

4. **Instantaneous structural frustration** $\Psi\colon X \to [0, +\infty)$

   Under the fixed internal perspective A_I, the function $\Psi(x)$ measures the overall structural tension carried by state x. A typical construction is to assemble the vector
   $$F(x) := (F_i(x))_{i \in I},$$
   and then apply a norm to this vector, for example
   $$\Psi(x) = \max_{i \in I} |F_i(x)|,$$
   which captures the largest violation among all constraints. One may instead use weighted sums or other norms; the specific choice corresponds to different modeling requirements.

Bridge to Chapter 2 Within the framework of Chapter 2, the conflict function $E_{A_I}(S)$ defined under perspective A_I measures the degree to which a state S violates the logical constraint set R. HTDS provides a concrete realization: via an embedding, the abstract state S is represented as a point x in the phase space, and we may identify

$$E_{A_I}(S) \quad \longleftrightarrow \quad \Psi(x).$$

In other words, $\Psi(x)$ is the instantiation of $E_{A_I}(S)$ in phase-space coordinates, and x is the representation of S in (X, d). In this way, the static frustration index of Chapter 2 is embedded into a concrete dynamical system.

4.2 Orbit-Averaged Tension and the Dynamical Fracture Lower Bound Δ_{dyn}

In a given HTDS

$$(X, d, T, \{F_i\}_{i \in I}),$$

the instantaneous frustration $\Psi(x)$ describes the local tension of state x under perspective A_I. To examine how this tension behaves over long times, we introduce the time-averaged frustration along an orbit

$$(x, T(x), T^2(x), \dots) :$$

$$\overline{\Psi}(x) := \limsup_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \Psi(T^n x).$$

Here the upper limit lim sup is used to cover the case where the time average does not converge in the strict sense: if the limit exists, then $\overline{\Psi}(x)$ reduces to the usual time average; if it does not, the limsup still provides a conservative estimate of long-term tension.

Let $\mathcal{M}_T$ denote the set of all T-invariant probability measures, i.e., those satisfying

$$\mu(T^{-1}(B)) = \mu(B)$$

for all measurable sets B. In ergodic theory, these invariant measures correspond to various "statistical steady states" of the system. On this set we define the **dynamical fracture lower bound**

$$\Delta_{\text{dyn}} := \inf_{\mu \in \mathcal{M}_T} \int \Psi \, d\mu.$$

Intuitively, Δ_{dyn} measures the theoretical lower bound of long-term average structural frustration that can be achieved at the current level and perspective A_I, assuming the system is free to realize any of its statistical steady states. If there exists an invariant measure with $\int \Psi \, d\mu = 0$, this means there is a statistically "gap-free" mode of evolution at that level. Conversely, if the integral is strictly positive for all invariant measures, then no matter how the system evolves, its long-term average retains a non-negligible structural tension.

In strict ergodic theory, $\mathcal{M}_T$ may contain many pathological measures of little physical relevance. In applications, one typically focuses on natural invariant measures (such as SRB measures) selected by "typical initial conditions". This paper will not delve into these technical distinctions; instead, we interpret Δ_{dyn} as a dynamical extension of $\Psi(\mathcal{L}, A_I)$ from Chapter 2: it no longer measures the conflict of a single state, but the lower bound of structural fracture that remains after averaging over time and statistics.

4.3 Compensable vs Frustrated Levels (Operational Definition)

Given Δ_{dyn}, we can introduce an operational dichotomy for the dynamical behavior of a system at a given perspective A_I and level: *compensable levels* versus *frustrated levels*. This classification is not intended to exhaust all possibilities of dynamical systems, but to provide a structured description for later H/M classification and chaos analysis.

Compensable levels ($\Delta_{\mathbf{dyn}} = 0$)

If there exists a T-invariant probability measure μ such that

$$\int \Psi \, d\mu = 0,$$

then the system is said to be at a *compensable level* at the current scale. In this case, structural frustration can be completely eliminated in a statistical sense: along typical orbits governed by this measure,

$$\overline{\Psi}(x) = 0.$$

Dynamically, such situations often correspond to periodic orbits, limit cycles, or certain simple quasi-periodic behaviors: the long-term evolution repeatedly visits a finite or low-dimensional set of trajectories, which, at the resolution of perspective A_I, can be regarded as fully self-consistent and fracture-free. In the language of Chapter 2, this level naturally corresponds to H-class: there exists a perspective and dynamical mode in which structural tension is completely "tamed".

Frustrated levels ($\Delta_{\mathbf{dyn}} > 0$)

If for all T-invariant probability measures μ one has

$$\int \Psi \, d\mu > 0,$$

then the system is said to be at a *frustrated level* at the current scale. This indicates that, regardless of which statistical steady state the system selects, the long-term average frustration cannot be reduced to zero; at that level there is always some residual structural tension that cannot be eliminated.

In many typical dynamical systems, such frustrated levels are strongly correlated with chaotic behavior: extreme sensitivity to initial conditions and long-term unpredictability are precisely the temporal manifestations of this "uncompensable structural fracture".

It should be emphasized that, in this paper, statements about the relationship between $\Delta_{\text{dyn}} > 0$ and chaotic behavior should be understood as structural claims or working hypotheses, not as universally valid theorems for all dynamical systems. In later sections, we will show, in specific models (such as the logistic map), how one can observe a correspondence between $\Delta_{\text{dyn}} > 0$ and chaotic intervals under natural perspectives, thereby providing examples supporting the picture "frustrated levels ↔ M-class behavior". We do not claim to have established a general equivalence such as "$\Delta_{\text{dyn}} > 0$ if and only if the system is chaotic".

Within these constraints, the distinction between compensable and frustrated levels aligns naturally with the H/M classification of Chapter 2:

- When $\Delta_{\text{dyn}} = 0$, the level can be regarded as H-class: there exists an appropriate statistical perspective under which structural frustration is completely eliminated in the long-term average.
- When $\Delta_{\text{dyn}} > 0$, the level is closer to M-class: a non-negligible tension remains under any statistical steady state. In later sections, we will connect this to chaos, unpredictability, and structural sources of complexity via concrete examples.

4.4 Hierarchical Structure and Coarse-Graining Maps π_ℓ

In real systems, observing directly in the full state space (X, d) is often neither necessary nor feasible. Typically we care only about information relevant to a particular scale or task, which can be captured via *coarse-graining*. Formally, we choose a family of coarse-graining maps

$$\pi_\ell : X \to Y_\ell, \qquad \ell = 0, 1, 2, \dots$$

where each Y_ℓ is the "macro state space" at level ℓ, and π_ℓ maps a microscopic state x to its macro description at that level. By convention, smaller ℓ corresponds to more macro levels. For example, $\ell = 0$ may denote the coarsest macro level; as ℓ increases, the description is gradually refined, ultimately approaching the original phase space X.

At each level ℓ, the coarse-graining map π_ℓ induces an effective frustration function at that level. A natural construction is to first define a macro-level frustration function

$$\Psi_\ell : Y_\ell \to [0, +\infty),$$

and then obtain a level-ℓ frustration on X by composition

$$\Psi_\ell(x) := \Psi_\ell(\pi_\ell(x)).$$

Intuitively, $\Psi_\ell(x)$ measures the magnitude of structural tension perceived when we view state x with the resolution of level ℓ.

More concretely, the macro description at level ℓ can be regarded as a new perspective

$$A_\ell = (F_\ell, P_\ell),$$

where F_ℓ collects the macro variables of interest on Y_ℓ, and P_ℓ specifies their "ideal relations" (e.g., equilibrium equations, symmetry constraints, and so on). Under this perspective, Ψ_ℓ plays the same role as $E_A(S)$ in Chapter 2: it measures the deviation of the macro state $y = \pi_\ell(x)$ from the ideal constraints specified by A_ℓ. Thus π_ℓ and Ψ_ℓ are not independent structures, but together form the full perspective at level ℓ.

For example, in classical fluid mechanics, Y_ℓ may be taken as the space of macro velocity and density fields, and the perspective A_ℓ may encode ideal relations such as Maxwell–Boltzmann distributions near equilibrium. Then $\Psi_\ell(y)$ can be defined as the deviation of the current macro distribution from this ideal (e.g., in some statistical distance), thereby turning "macro frustration" into an operational quantity.

On this basis, we can proceed as in the previous section and introduce time-averaged frustration and a dynamical fracture lower bound at each level:

$$\overline{\Psi}_\ell(x) := \limsup_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \Psi_\ell(T^n x),$$

$$\Delta_{\mathrm{dyn},\ell} := \inf_{\mu\in\mathcal{M}_T} \int \Psi_\ell \, d\mu.$$

This yields, for the same dynamical system T, a level-dependent sequence of fracture lower bounds

$$\{\Delta_{\mathrm{dyn},\ell}\}_{\ell=0,1,2,\dots}.$$

This sequence provides a clear structural picture of how the system's "compensability" changes as we move from macro (small ℓ) to micro (large ℓ) levels.

A typical situation is entirely possible:

$$\ell > \ell', \qquad \Delta_{\mathrm{dyn},\ell} > 0, \quad \Delta_{\mathrm{dyn},\ell'} = 0.$$

This means that, at the finer level ℓ, the system exhibits an irreducible frustration lower bound and the long-term average tension cannot be compressed to zero, appearing "chaotic" or structurally mismatched; whereas at a more macro level ℓ', after coarse-graining via $\pi_{\ell'}$ has absorbed some microscopic detail, the system can still admit a statistical steady state with $\Delta_{\mathrm{dyn},\ell'} = 0$, thus behaving at that level as a highly ordered, compensable H-class structure.

Structurally, this is precisely the dynamical realization of the "perspective-determines-ontology" principle of Chapter 2: for a given physical or logical system, its "computability" and "stability" are not absolute attributes, but depend on the chosen level and perspective. A system that appears full of frustration at a fine scale may well be regarded as nearly fracture-free H-class at a more macro scale; conversely, once M-class behavior is already visible at some macro level, refining the perspective often reveals only more intricate fracture structures, rather than "restoring" a simple H-class description.

4.5 Closing the Loop with the Philosophical Framework of Chapters 2–3

With the constructions above, HTDS provides a formal framework in which "microscopic determinism" and "macroscopic randomness" can be discussed in a single language, in alignment with the two-level structural picture proposed in Chapters 2 and 3.

At the microscopic level, the dynamics

$$T : X \to X$$

is fully deterministic: given an initial state x_0, the entire orbit $(x_0, x_1, x_2, \dots)$ is uniquely determined by T in the ideal mathematical model. The instantaneous frustration $\Psi(x_t)$ simply tracks fluctuations in the tension among various constraints and "desires" at the individual level; at this scale there is no "ontological randomness", only deterministic evolution under a single-viewpoint continuous logic.

Once we introduce coarse-graining maps $\pi_\ell : X \to Y_\ell$ and shift observation to the macro level Y_ℓ, the picture changes qualitatively. At level ℓ, macro variables often evolve in a mixed form of "deterministic structure plus probabilistic fluctuations":

- On the one hand, long-term statistical behavior may obey some stable laws, such as equilibrium distributions, periodic patterns, or steady attractors;
- On the other hand, microscopic details are compressed into equivalence classes of π_ℓ and reappear at the macro scale only as probabilities or noise.

In the terminology of Chapters 2 and 3:

- At the global (macro) level, there exist perspectives and coarse-grainings such that

  $$\Delta_{\mathrm{dyn},\ell} \approx 0,$$

  and the system can be regarded as H-class at that level: macro behavior is describable by nearly fracture-free effective laws.
- At the individual (micro) level, probabilistic fluctuations appear: they are not "ontologically random", but are shaped by local variations of $\Psi(x)$ together with the constraints encoded in higher-level $\Delta_{\mathrm{dyn},\ell}$.

From this viewpoint, the "whole / individual" and "deterministic / random" two-level framework acquires a fully formal realization in HTDS:

- **"Whole" level:** By choosing suitable coarse-grainings π_ℓ and statistical perspectives, at some levels one can achieve $\Delta_{\mathrm{dyn},\ell} \approx 0$, and single-viewpoint continuous logic constructs an almost fracture-free macro structure within that level.
- **"Individual" level:** One acknowledges that microscopic states inevitably deviate from these macro structures, and uses Ψ and $\Delta_{\mathrm{dyn},\ell}$ to characterize the strength and accumulation of such deviations across levels.

Thus, HTDS is not an ad hoc attachment to the philosophical stance of Chapters 2 and 3, but its formal realization. Through the combined structure

$$(X, T, \{F_i\}, \pi_\ell, \Psi, \Delta_{\mathrm{dyn},\ell}),$$

we can, within a single framework, simultaneously discuss:

- Deterministic evolution rules (T and $\Psi(x)$);
- Emergent probabilistic behavior (macro statistical properties under $\overline{\Psi}_\ell$ and $\Delta_{\text{dyn},\ell}$);
- The generation, compensation, and rewriting of fractures across levels (Δ_{view}, Δ_{dyn}, and Δ^*).

This provides the technical background for the unified treatment, in later chapters, of uncomputability, chaos, NP-hardness, and hallucinations in intelligent systems.

4.6 Fifth-Order Structural Instability: Structural Observations and a Phenomenological Law

Up to this point, the HTDS framework introduced in this chapter, the definition of Δ_{dyn}, and the hierarchy of coarse-grained levels in Section 4.4 have all been formulated under general structural conditions: they address questions of the form "how to define a notion of fracture on an arbitrary dynamical system," without imposing specific numerical laws.

The property discussed in this section, which we call *fifth-order structural instability*, is of a different nature. It is a numerical and structural phenomenon that appears repeatedly in concrete models and is therefore stated here as a *phenomenological law*: it is based on structural observations in several canonical systems and is formulated as a working hypothesis, rather than being a necessary consequence of the axioms introduced earlier.

To describe this phenomenon, we first sketch a rough formal setting. Let

$$T : X \to X$$

be a concrete dynamical system, such as a one-dimensional unimodal map (for instance, the logistic map) or a self-organizing network with local interactions. We consider two types of "self-iteration":

(1) Self-composition order of the map. For a positive integer k, let $T^{(k)}$ denote the k-fold composition of T:

$$T^{(k)} = \underbrace{T \circ T \circ \cdots \circ T}_{k \text{ times}}.$$

On the same state space X, $T^{(k)}$ describes the effective dynamics when we observe the system only every k steps.

(2) Self-embedding of the update rule across levels. For systems with a clear hierarchical structure, one may introduce, at each level, coarse-graining maps π_ℓ as in Section 4.4, and then reapply an update rule that is isomorphic to the original one at a higher level. Along the path

$$\text{subsystem} \;\to\; \text{higher-level system} \;\to\; \text{higher-level system again},$$

we are effectively examining how the same structural rule replicates itself on increasingly macro levels.

Operational meaning of the order parameter k (two instantiations). In this section, the symbol k denotes an *iteration depth* of a declared self-iteration operator, instantiated in two concrete ways: (i) *composition depth* k for the map $T^{(k)}$ (observation every k steps), and (ii) *self-embedding depth* k for level-to-level replication of an isomorphic update rule under a declared coarse-graining hierarchy. When a statement below refers to a specific instantiation, we will state it explicitly (e.g. "composition order" versus "self-embedding order"). The phenomenological observation reported here concerns the existence of a finite *critical depth* k^* in either instantiation under comparable admissibility and resolution regimes.

Informally, k measures how many layers of "self-superposition / self-embedding" the system has undergone according to its own rule template. For each k, we may, under a declared coarse-graining scheme and an admissible family of perspectives, define a dynamical fracture lower bound $\Delta_{\mathrm{dyn}}(k)$ for the corresponding effective dynamics: that is, on $T^{(k)}$, or on the system after k levels of self-embedding, we evaluate (under the declared operational package) the infimum of the long-term average frustration. This yields a sequence

$$\{\Delta_{\mathrm{dyn}}(k)\}_{k=1,2,\dots}$$

indexed by the self-iteration depth k.

Resolution-based criterion for "tame" versus "manifest" fracture. Fix a declared operational package (admissible family of perspectives, discrepancy D, admissible lifting class $\mathcal{U}$, reachable-set convention, and resolution threshold δ). For each iteration depth k, the quantity $\Delta_{\mathrm{dyn}}(k)$ is evaluated relative to this declared regime. In particular, the phrase "$\Delta_{\mathrm{dyn}}(k)$ can be compressed to a negligible level" is used in the strictly operational sense

$$\Delta_{\mathrm{dyn}}(k) < \delta,$$

whereas "$\Delta_{\mathrm{dyn}}(k)$ develops a strictly positive lower bound" means that, under the same declared regime, $\Delta_{\mathrm{dyn}}(k)$ remains bounded away from the negligibility threshold: it cannot be reduced below δ by admissible re-encodings, coarse-grainings, or admissible parameter tuning within the tested range.

The structural content of *structural instability* can be summarized in the following pattern:

- For small values of k (e.g. $k = 1, 2, 3, \dots$), there exist declared admissible regimes under which $\Delta_{\mathrm{dyn}}(k)$ is operationally negligible (i.e. $\Delta_{\mathrm{dyn}}(k) < \delta$), so that the effective dynamics can still be treated as approximately H-class at those iteration depths.
- When k increases beyond a finite critical value k^*, then under the same declared regime $\Delta_{\mathrm{dyn}}(k)$ becomes manifest (i.e. $\Delta_{\mathrm{dyn}}(k) \geq \delta$) and cannot be reduced back below the negligibility threshold by admissible re-expressions: at that scale the system enters an M-class regime, exhibiting uncompensable structural frustration and long-term unpredictability.

In other words, k^* can be interpreted as a *structural instability order*:

- When $k < k^*$, self-iteration can remain H-like under the declared regime (fracture is operationally negligible).
- When $k \geq k^*$, H-like taming under that regime fails, $\Delta_{\mathrm{dyn}}(k)$ is manifest, and the fracture becomes structurally unavoidable at that resolution.

Phenomenological law (why "fifth-order" in this paper). Our empirical observation is regime-relative: within the instance families examined in this work (including unimodal map families and several canonical self-organizing networks), and under the declared admissibility and resolution regime used for comparability, the critical instability order

$$k^* := \inf\{k \in \mathbb{N} : \Delta_{\mathrm{dyn}}(k) \geq \delta\}$$

is repeatedly observed to occur at a small finite depth and, in many representative instances, the first manifest onset occurs at $k^* = 5$. In this operational sense, "fifth-order structural instability" refers to the frequently observed onset depth under the declared regime for the tested instance class; it is not claimed to be universal. When different instantiations yield different onset depths, this is to be reported together with the operational package, and should be read as a change of instance/regime rather than a contradiction of the HTDS or Ψ–Δ framework.

It is important to emphasize:

- The proposal of fifth-order structural instability does not modify the universality of the core postulates and structural framework of this paper (such as perspectival ontology, Axiom 2.1, or the Ψ–Δ structural calculus). These postulates do not depend on the numerical value of k^*; the "fifth-order" label is an additional phenomenological descriptor for certain instance families under a declared regime.
- At the current stage, "fifth order" is a reported empirical onset depth for the tested families and declared packages; its broader scope and possible invariance properties require further validation. Any reported value of k^* is meaningful only together with the package elements that determine $\Delta_{\mathrm{dyn}}(k)$ (admissibility, resolution, discrepancy and lifting conventions).

In subsequent chapters, we will elaborate this phenomenon from two directions: on the one hand, Chapter 5 will, within the Ψ–Δ structural calculus framework, introduce a geometric picture in terms of equiangular logarithmic spirals and establish a structural connection to Feigenbaum scaling constants; on the other hand, Section 6.5 will use the bifurcation structure of the logistic map and its classical Feigenbaum scaling relations to exhibit the embryo of "fifth-order structural instability" in a concrete dynamical system. There we will see that, when examining hierarchical self-period-doubling and self-embedding of the map under a declared regime, one can identify a finite order at which the behavior turns from tamable H-like to global M-like, with fifth-order onset emerging in concrete figures and data.

From the perspective of chapter organization, this section acts as an "anchoring transition":

- Sections 4.1–4.4 establish a universal HTDS and Δ_{dyn} language applicable to a wide range of multilevel systems.
- Fifth-order structural instability provides a concrete quantitative anchor for this language: it shows that, in certain canonical systems and under declared regimes, the H$\rightarrow$M transition need not occur only in an asymptotic limit, but can appear at a finite, observable iteration depth.

This will serve as an important reference point for the analysis of chaos, uncomputability, and NP-hardness, and, together with the geometric structures of Chapter 5, will furnish a quantitative description of the structural pathway from “multi-H coupling” to global M.

4.7 Coarse-Graining and Renormalization: Physical Counterparts of HTDS

In this section, we sketch a simplified correspondence: how the hierarchical tension dynamical systems (HTDS) constructed in Chapter 4 align with familiar ideas of coarse-graining and renormalization in physics. The goal is not to rederive the technical details of RG, but to show that the HTDS and Ψ–Δ language can be regarded as a structural rewriting of coarse-graining/RG practice, rather than as a completely new physical theory built from scratch.

Coarse-graining: perspective compression and fracture redistribution. Previously, we introduced a family of coarse-graining maps

$$\pi_\ell : X \to Y_\ell, \qquad \ell = 0, 1, 2, \dots$$

to represent different levels of perspective: larger ℓ corresponds to finer descriptions, while smaller ℓ corresponds to coarser ones. At each level, we define a macro-level frustration function Ψ_ℓ on Y_ℓ, and pull it back to X via

$$\Psi_\ell(x) := \Psi_\ell\big(\pi_\ell(x)\big).$$

In the Ψ–Δ framework, coarse-graining can be interpreted as an operation of “perspective compression plus fracture redistribution”:

- Part of the microscopic structural tension is retained explicitly as Ψ_ℓ and is counted as intra-level (level-internal) tension at that level.
- Another part is absorbed into extended environment/background degrees of freedom (or into untracked variables introduced by the coarse-graining), and therefore reappears as an inter-level residual at the level–level or system–environment interface, rather than as an explicit intra-level term at the current level.

In other words, coarse-graining is not merely “erasing fine details”; it changes the bookkeeping of which tensions are represented intra-levelly and which ones are externalized and must be accounted for as cross-level mismatch under reconstruction.

Renormalization flow: HTDS on the space of perspectives. Traditional renormalization group analysis studies how effective descriptions *flow* under changes of scale. In our framework, this idea can be rewritten as an HTDS on the space of perspectives.

Given the family of coarse-grainings $\{\pi_\ell\}$, we obtain a family of macro perspectives A_ℓ and the associated frustration functions Ψ_ℓ and dynamical fracture bounds $\Delta_{\mathrm{dyn},\ell}$. Renormalization can then be abstracted as a mapping acting on pairs “perspective + frustration”:

$$\mathcal{R} : (A_\ell, \Psi_\ell) \;\mapsto\; (A_{\ell-1}, \Psi_{\ell-1}).$$

Along the direction of changing ℓ, the triple $(A_\ell, \Psi_\ell, \Delta_{\mathrm{dyn},\ell})$ traces out a "scale evolution trajectory." Relative to a declared admissibility and resolution regime (package), the H/M reading is operational:

- If, over some range of scales, the declared regime admits descriptions with $\Delta_{\mathrm{dyn},\ell} < \delta$ (fracture operationally negligible at resolution δ), that range corresponds to an approximately H-class "ordered phase" in the effective theory space.
- If along the RG flow $\Delta_{\mathrm{dyn},\ell} \geq \delta$ persists (or grows) under the same declared regime, that region is dominated by M-class structures at that resolution.

This can be read as a physical counterpart of the H/M embedding principle: structures that are H-like under a given resolution at intermediate scales may, after repeated coarse-graining and rescaling, flow into a regime where an M-like floor becomes manifest.

Connection to fifth-order structural instability.

Two iteration axes. In this chapter, ℓ indexes the scale/perspective coarse-graining hierarchy (RG-style re-descriptions), whereas k indexes the self-iteration depth (self-composition or self-embedding) within a fixed structural family. Fifth-order structural instability concerns the onset along the k-axis, and can be read as a finite-depth transition that may occur while the RG description along ℓ is being tracked.

In the previous section, using the HTDS language, we formulated fifth-order structural instability as a regime-relative phenomenological law: in certain typical systems (such as unimodal maps and self-organizing networks), along the "self-composition / self-embedding" direction indexed by k, there exists a finite critical order k^* such that (under a fixed declared package and resolution threshold δ)

$$k^* := \inf\{k \in \mathbb{N} : \Delta_{\mathrm{dyn}}(k) \geq \delta\}.$$

Operationally, this means:

- When $k < k^*$, under the declared regime the fracture is negligible, i.e. $\Delta_{\mathrm{dyn}}(k) < \delta$, and the effective dynamics can remain H-like at that resolution.
- When $k \geq k^*$, under the same declared regime the fracture becomes manifest, i.e. $\Delta_{\mathrm{dyn}}(k) \geq \delta$, and cannot be reduced back below δ by admissible re-expressions within the tested range.

From a renormalization viewpoint, this can be interpreted as follows: after a finite number of coarse-graining and structural rescaling steps, the effective description leaves the attraction basin of an H-like regime and enters a region dominated by M-like structure at the declared resolution. Fifth-order structural instability is not a replacement for RG; rather, it identifies a finite, observable onset depth (along the k-axis, for the tested instance families and declared package) at which this transition becomes manifest.

In Chapter 6, when we analyze the logistic map, its bifurcation structure, and its relation to Feigenbaum scaling, we will see how this picture is realized in an explicit model: in the RG process built from repeated function composition and scale rescaling, we will track how Ψ and $\Delta_{\mathrm{dyn},\ell}$ undergo a finite-level transition from a "compensable H-like phase" to a "non-compensable M-like phase" under a stated resolution regime.

4.8 Δ-Conservation Structural Principle: Intrinsic Fracture and Residual Resurgence

In the preceding discussion, we connected hierarchical tension dynamical systems (HTDS), coarse-graining maps π_ℓ, dynamical fracture lower bounds $\Delta_{\mathrm{dyn},\ell}$, and renormalization-style scale evolution. A natural question then arises:

When we keep adjusting perspectives, coarse-graining, renormalizing, and designing increasingly refined compression and approximation schemes, are the fractures that appear as "residuals" truly eliminated, or are they merely transferred to other levels, variables, or environments?

The structural answer proposed in this section is the following: within a fixed finite self-embedding structural family, what can be compressed and rewritten are only the *explicit* residuals at a chosen level and resolution; an *intrinsic fracture floor* persists and is structurally conserved under level-respecting reversible operations. In particular, any statement of the form "the residual approaches zero level by level" is to be read operationally as *"the explicit residual becomes negligible relative to a declared resolution threshold δ"*, rather than as an ontological annihilation of fracture.

4.8.1 System-Level Intrinsic Fracture Floor Δ_S^*: Local Projection of Global Δ^*

The meta-fracture postulate in Chapter 3 introduced a global lower bound $\Delta^* > 0$ at the level of the universe as a whole: in the limiting structure that "contains all levels and perspectives," a strictly vanishing $\Delta = 0$ is unattainable. Operationally, at any finite resolution one may treat fracture as negligible only in the regime-relative sense (e.g. $\Delta < \delta$), not as a gap-free identity claim.

In the hierarchical modeling of concrete systems, it is useful to introduce a system-level floor. For a system S equipped with a finite chain of self-embedding levels, and with its self-embedding rule family and initial structural topology held fixed, we assume the existence of a positive bound

$$\Delta_S^* > 0,$$

called the *intrinsic fracture floor* of the system.

Intuitively, Δ_S^* summarizes the fact that, within this fixed self-embedding structural family, there exists a non-removable fracture component that cannot be eliminated by admissible re-expressions. More concretely: across admissible coarse-grainings, admissible perspective families, and admissible reconstruction conventions (i.e. within declared operational packages), one may redistribute how fracture appears (intra-level versus inter-level, explicit versus externalized), but one cannot reduce the intrinsic floor below Δ_S^* in the ontological sense. What can be made small is only the *explicit* residual relative to a declared threshold δ at a chosen level.

From the standpoint of the meta-fracture, Δ_S^* can be regarded as a local projection of the global Δ^* onto the closed self-embedding structure of S: Δ^* asserts that "there is no perfect level for the universe as a whole," whereas Δ_S^* specifies "how deeply fracture can be rendered operationally negligible inside this particular closed family of structural rules."

In concrete self-embedding systems (such as unimodal maps or self-organizing networks), the system-level floor can often be refined into an order-indexed intrinsic spectrum resembling fixed-point or universality spectra:

$$\{\kappa_k\}_{k=1,2,\ldots},$$

where k denotes the order of self-composition / self-embedding, and κ_k characterizes the incompressible fracture amplitude (or resurgence scale) associated with that order within the same structural family. This paper does not attempt to provide a general construction of Δ_S^* and $\{\kappa_k\}$; rather, they are treated as *intrinsic fracture characteristics* of the system's self-embedding spectrum. (When classical Feigenbaum-type constants are referenced, we will denote them with an explicit qualifier, e.g. δ_{Feig}, to avoid confusion with the resolution threshold δ used throughout this paper.)

4.8.2 The Δ-Conservation Structural Principle (Operational Form)

With Δ_S^* in place, we can now state the core claim of this section.

Structural Principle 4.1 (Δ-Conservation Structural Principle (Operational Form)). *Let S be a system with a finite self-embedding hierarchy and intrinsic fracture floor $\Delta_S^* > 0$. Under all* level-respecting reversible structural operations *that preserve the self-embedding topology—including, but not limited to:*

- *reversible perspective transformations (coordinate changes, variable renamings, admissible reparameterizations);*
- *coarse-graining / renormalization steps that are information-reversible by extending the environment (i.e. by explicitly tracking compensating external degrees of freedom);*
- *reversible residual re-encodings and subsystem re-embeddings that move mismatch between intra-level representation and inter-level interfaces;*

the intrinsic fracture floor Δ_S^* remains invariant as a lower bound for the structural family.

In other words, any process that appears to "shrink" explicit residuals does not annihilate fracture; it relocates how the fracture is represented. Typical destinations of relocated fracture include:

- environmental or background degrees of freedom that were previously untracked,
- additional (extended) structural coordinates introduced by the encoding,
- deeper orders in the same self-embedding hierarchy.

Accordingly, when one reverses along the original structural pathway—lifting back to higher levels, restoring the full self-embedding chain, or undoing environmental extensions—fracture reappears as a resurgence pattern governed by intrinsic order scales (captured phenomenologically by $\{\kappa_k\}$).

In the present context, "Δ-conservation" does *not* mean that the explicit numerical value of a residual stays unchanged after every operation. Rather, it asserts that within the class of level-respecting reversible operations above, the intrinsic floor Δ_S^* (and, when observable, its associated spectrum $\{\kappa_k\}$) cannot be genuinely reduced; only the explicit representation of residuals can be redistributed across levels and interfaces. Operationally, this means one may achieve

$$\text{explicit residual} < \delta$$

at a chosen level and declared resolution δ, without implying any ontological statement of $\Delta = 0$ for the structural family.

It must be emphasized that the principle above is an operational structural postulate, not a proven general theorem. Its support in this work comes from recurring numerical and geometric evidence in several canonical systems (unimodal maps, self-organizing networks, spin glasses, and lossy coding models): in these systems, strong suppression of explicit residuals at one level tends to be accompanied by resurgence of mismatch at other levels or higher self-embedding orders.

4.8.3 Asymptotic Optimal Bounds as Scale Projections of Δ_S^*

In many analyses, one studies "tail bounds," "error bounds," or "approximation accuracies" as some scale parameter tends to infinity, and seeks bounds that become as small as possible. Typical forms include:

- correction factors of order $\log\log$,
- slowly decaying factors such as $1/\sqrt{\log}$,
- "arbitrarily small but nonzero" ε-type approximation bounds.

Within the Δ-conservation perspective, these bounds admit a unified structural reading.

Corollary 4.2 (Asymptotic Bounds as Scale Projections of the Intrinsic Fracture)**.** *Within a fixed self-embedding structural family for S, asymptotically optimal residual bounds that appear to "tend to zero" can be viewed as scale-projection functions of the intrinsic floor Δ_S^* (and, when relevant, its order spectrum $\{\kappa_k\}$): they describe how far* explicit *residuals can be reduced along a particular scale direction by modifying scale, perspective, or coarse-graining under declared admissibility, rather than compressing Δ_S^* itself to 0 in the structural sense.*

In other words, when a field exhibits near-optimal decay bounds, a structural interpretation is that the system has reached the limit of explicit residual suppression along that direction within the same self-embedding family; further improvement would require altering the structural family itself (e.g. changing the self-embedding paradigm), not merely refining compression within the same family. The corollary above is likewise intended as a structural interpretation rather than a fully proven mathematical theorem in the usual sense.

4.8.4 Residual Resurgence at and Beyond the Critical Self-Embedding Depth

Earlier in Chapter 4 we introduced finite-order structural instability as a regime-relative phenomenological law: for typical self-embedding systems, there exists a finite onset depth

k^* (along the self-composition / self-embedding axis) beyond which fracture becomes manifest under a fixed declared package and resolution δ. Operationally,

$$k^* := \inf\{k \in \mathbb{N} : \Delta_{\text{dyn}}(k) \geq \delta\}.$$

In the instance families examined in this work, many representative cases exhibit onset around the fifth order (hence the label "fifth-order structural instability"), but the framework treats k^* as a regime-relative onset depth that must be reported together with the operational package.

Combined with the Δ-conservation principle, this yields the following phenomenological corollary.

Corollary 4.3 (Residual Resurgence Beyond the Onset Depth)**.** *For a system S with a self-embedding hierarchy deep enough to reach the onset depth k^*, the conservation of intrinsic fracture tends to manifest as* residual resurgence*:*

- *For depths $k < k^*$, there exist declared regimes in which explicit residuals can be rendered operationally negligible, i.e. $\Delta_{\text{dyn}}(k) < \delta$ at the stated resolution.*
- *For depths $k \geq k^*$, under the same declared regime the fracture becomes manifest, i.e. $\Delta_{\text{dyn}}(k) \geq \delta$, and mismatch reappears as higher-order resurgence scales (captured phenomenologically by the intrinsic spectrum $\{\kappa_k\}$ within the same structural family).*

From this viewpoint, finite-order instability can be seen as a concrete projection of the Δ-conservation principle: it identifies a minimal self-embedding depth beyond which the intrinsic floor Δ_S^* can no longer be rendered operationally negligible by the same class of compensation mechanisms (coarse-graining, environmental extensions, admissible re-encodings). The "fifth order" label should therefore be read as a reported concentration point in several canonical instance families under declared packages, not as a universally proven constant.

4.8.5 Engineering and Modeling Strategies: From "Infinite Residual Compression" to "Paradigm Shift"

In many engineering and modeling practices, a common goal is to "further reduce error / residuals": increasing model capacity, improving optimization algorithms, introducing more sophisticated regularization and priors, and so on. These efforts often reduce explicit errors at a fixed level and resolution. Under the Δ-conservation viewpoint, their structural meaning can be stated more precisely:

within the same self-embedding structural family, residual-compression operations are essentially re-encodings and redistributions of mismatch, rather than reductions of the intrinsic floor Δ_S^* itself.

On this basis, one can formulate the following regime-relative modeling corollary.

Corollary 4.4 (Structurally Optimal Strategy Under Finite Resources (Regime-Relative))**.** *Assuming (i) the Δ-conservation principle for a fixed self-embedding family and (ii) the existence of a finite onset depth k^* in that family under a declared package, a more effective finite-resource strategy is not to pursue unlimited explicit residual suppression within the same family, but rather:*

1. *Identify the self-embedding structure and the relevant depth axis (including an estimate of k^* and the associated intrinsic spectrum proxy $\{\kappa_k\}$ when observable) under a declared resolution regime.*

2. *Before entering the onset regime (e.g. near the empirically observed fifth-order onset in representative systems), consider structural interventions that change the self-embedding family (adding new structural layers, changing admissible couplings, modifying the update template), rather than exclusively refining compression within the same family.*

3. *Treat intrinsic fracture as a structural boundary to be managed and localized across levels and interfaces, rather than as "noise" that can be eliminated in the ontological sense.*

Intuitively, within a fixed self-embedding family, stacking more layers, adding parameters, or improving optimization tends to move where mismatch appears (explicit versus externalized, low order versus high order), without structurally reducing the intrinsic floor. Truly changing the intrinsic floor requires changing the structural family itself (i.e. a paradigm shift in the self-embedding rules), rather than refining numerical implementation indefinitely within the same family.

This provides the dynamical and hierarchical background for later discussions of "managing gaps rather than eliminating them": in complex systems (including large-scale AI systems), hallucinations, long-tail errors, and extreme events are naturally interpreted as manifestations of intrinsic fracture at different levels and interfaces. A defensible design goal is therefore not to claim elimination, but to identify structural locations of fracture and build architectures that buffer, expose, and control how fracture resurges across levels.

4.8.6 Summary: The Role of Δ-Conservation in the Overall Framework

At this point, the structural loop of Chapter 4 can be summarized as follows:

- Chapter 2 introduced static notions of Ψ, Δ, the H/M classification, and the structural consistency axiom.

- Chapter 3 provided physical and ontological intuition for this language, together with the meta-fracture postulate $\Delta^* > 0$.

- Chapter 4 used HTDS, Δ_{dyn}, coarse-graining, and renormalization-style flows to extend these static notions into dynamical objects that evolve with level and scale.

- Finite-order structural instability provided a finite, observable onset scale for H-like regimes transitioning into M-like regimes under declared packages.

- The Δ-conservation principle proposed here states that, within a fixed finite self-embedding structural family, there exists a system-level intrinsic floor Δ^*_S that remains invariant as a lower bound under level-respecting reversible operations; phenomena of "residual disappearance" are regime-relative suppressions of explicit mismatch (below δ), not structural annihilations of fracture.

In this picture, Ψ, Δ, Δ_{dyn}, Δ^*, and Δ^*_S jointly form a unified cross-level structural framework:

- Ψ describes whether tension within a given level can be locally tamed (H/M distinction).
- Δ describes mismatch introduced by cross-level stitching under a single perspective.
- Δ_{dyn} describes a long-term dynamical fracture lower bound at a fixed level and declared regime.
- Δ^* is the global meta-fracture lower bound at the limiting structural level.
- Δ^*_S is the intrinsic system-level floor that cannot be eliminated within a fixed self-embedding structural family, though it may be rendered operationally negligible at selected levels and resolutions.

The subsequent chapters on uncomputability, chaos, NP-hardness, and large-model hallucinations will be developed against this structural background: the question is not merely whether a problem is "hard," but in what sense it touches Δ^*_S (or, at the meta level, Δ^*), thereby exhibiting an incompressible fracture structure under declared regimes.

Chapter 5

Ψ–Δ Structural Calculus and the Geometry of Equiangular Spirals

5.1 Abstract Evolution Parameter γ and the Ψ–Δ Calculus

In the preceding chapters, Ψ was used to measure local structural frustration, while Δ captured the accumulation of fractures across perspectives or over time. In concrete models (for example, the case studies in Chapter 6), one can directly observe the following fact: as we move along a natural evolution parameter γ, the accumulated fracture and the local frustration density satisfy a standard set of calculus relations. Motivated by this empirical observation, this section abstracts it into a unified structural formulation.

In different contexts, the abstract evolution parameter γ can have different meanings, for instance:

- In dynamical systems, γ may correspond to discrete time steps or continuous time;
- In hierarchical structures, γ may correspond to the depth of coarse-graining or the order of self-embedding;
- In control or phase-transition analysis, γ may correspond to an external control parameter (such as coupling strength, temperature, or perturbation amplitude).

In these situations, we adopt the following relations uniformly to describe fracture accumulation along the γ direction:

$$\Delta(\gamma) = \int_0^{\gamma} \Psi(\tau)\, d\tau, \qquad \Psi(\gamma) = \frac{d\Delta}{d\gamma}. \tag{5.1}$$

Here we may interpret:

- $\Psi(\gamma)$ as the *local fracture generation rate* at evolution parameter γ: it describes the density of newly manifested structural tension within an infinitesimal increment $d\gamma$;
- $\Delta(\gamma)$ as the total fracture intensity accumulated from the reference point $\gamma = 0$ up to position γ along the evolution direction.

The dynamical fracture lower bound Δ_{dyn} defined in Chapter 4 can be viewed as a special case of the above, obtained by taking a long-time average in the time direction at a fixed level and perspective. In the present chapter, however, $\Delta(\gamma)$ is treated as an abstract structural scale function, without presupposing any specific physical meaning, and is only required to satisfy (5.1).

Semantic carry-over (resolution-aware reading). Equation (5.1) is used as a regime-relative structural relation: its equalities inherit the resolution-aware semantics fixed in Chapter 2. In particular, when $\Delta(\gamma)$ involves genuine cross-level identifications, the relation is read as a structural equivalence under the declared resolution threshold (rather than as an ontological claim of gap-free identity).

5.2 Self-Similar Fracture Growth and Exponential Solutions

In general, the detailed relation between $\Psi(\gamma)$ and $\Delta(\gamma)$ can be highly complicated, strongly dependent on system details and the choice of perspective. In this section, we single out one case that occurs frequently in concrete models and is simple enough to serve as a baseline image for the geometric analysis that follows:

In certain natural coordinates, *the local fracture generation rate is proportional to the current accumulated fracture*:

$$\Psi(\gamma) = b \cdot \Delta(\gamma), \tag{5.2}$$

where the constant b quantifies the ratio between "fracture generation rate" and "current fracture scale" along the evolution direction.

Combining (5.2) with the calculus relation

$$\Psi(\gamma) = \frac{d\Delta}{d\gamma},$$

we obtain the first-order ordinary differential equation

$$\frac{d\Delta}{d\gamma} = b \cdot \Delta(\gamma). \tag{5.3}$$

On any region where $\Delta(\gamma) \neq 0$, its general solution is

$$\Delta(\gamma) = \Delta_0\, e^{b\gamma}, \tag{5.4}$$

where $\Delta_0 = \Delta(0)$ is the fracture intensity at the reference point $\gamma = 0$.

The sign of b determines the basic phase of fracture evolution:

- $b > 0$: the fracture grows exponentially with γ, and $\Delta(\gamma)$ increases monotonically. This can be viewed as a *frustration-amplifying phase*: the larger the local tension, the faster new fractures are generated, and the system tends to move toward M-type behavior along this direction;

- $b < 0$: the fracture decays exponentially with γ, and $\Delta(\gamma)$ decreases monotonically. This can be viewed as a *compensation / self-organization phase*: the larger the local tension, the more strongly compensation mechanisms are driven to pull the system back toward an H-type region;

- $b = 0$: the fracture intensity remains constant along the evolution direction, corresponding to a *fracture-frozen phase*: in the chosen coordinates, $\Psi(\gamma)$ can be regarded as approximately zero at almost every position.

In the language of the Ψ–Δ framework, the appearance of resolution thresholds can be reformulated as a statement about the renormalization flow of the inter-level gap Δ. Let γ denote a resolution or hierarchy parameter indexing a family of admissible perspectives $\{A_\gamma\}$, and write $\Delta(\gamma)$ for the corresponding view-gap. The structural evolution law

$$\frac{d\Delta}{d\gamma} \;=\; \Psi(\gamma)$$

then defines an effective Δ-RG flow. The H/M classification is threshold-type: in an H-class regime there exist admissible perspectives with $\Delta(\gamma) = 0$ and $\Psi(\gamma) = 0$, while in an M-class regime the meta-fracture postulate enforces a strictly positive lower bound $\Delta(\gamma) \geq \Delta^{>0} > 0$. A resolution threshold is precisely a critical value γ_c at which the Δ-RG trajectory crosses the structural basin boundary separating these two regimes. Below γ_c the fracture term is effectively irrelevant and can be renormalized away by suitable coarse-graining; beyond γ_c it becomes relevant and drives the flow toward an M-class plateau with $\Delta(\gamma) \geq \Delta^{>0}$. Thus the phenomenological "sudden change" observed when increasing resolution is not a mysterious discontinuity of the underlying dynamics, but the structural manifestation of the Δ-RG flow crossing a critical manifold in fracture space.

5.3 Geometric Embedding: From (γ, Δ) to Polar Coordinates (θ, r)

In the previous section, we treated $\Psi(\gamma)$ and $\Delta(\gamma)$ as a pair of structural quantities defined along the abstract evolution parameter γ. To give this pair a more intuitive geometric interpretation, we now introduce a simple planar embedding: we regard $(\gamma, \Delta(\gamma))$ as the angle and radius in polar coordinates (θ, r).

More concretely, we choose the following linear correspondence:

$$\theta = \gamma, \qquad r = \Delta(\gamma).$$

This correspondence is an arbitrary but fixed choice of notation; units and scaling factors do not affect the geometric properties. Through this rescaling, we interpret the "evolution position" γ as the polar angle θ, and the "fracture scale" $\Delta(\gamma)$ as the radial coordinate r. Thus we have

$$\gamma \leftrightarrow \theta, \qquad \Delta(\gamma) \leftrightarrow r(\theta), \qquad \Psi(\gamma) = \frac{d\Delta}{d\gamma} \;\leftrightarrow\; \frac{dr}{d\theta}.$$

Under this correspondence, the role of Ψ can naturally be understood as the *radial growth rate along the angular direction*: when θ changes by a small amount $d\theta$, the radial increment dr is proportional to Ψ, thereby geometrizing the "fracture generation rate" as the radial extension speed of the curve in polar coordinates.

In the self-similar fracture growth regime $\Psi(\gamma) = b \cdot \Delta(\gamma)$, combined with the correspondence above, we obtain the polar-coordinate differential equation

$$\frac{dr}{d\theta} = b \cdot r.$$

Its general solution is

$$r(\theta) = ae^{b\theta},$$

where the constant a denotes the initial radius at $\theta = 0$ (i.e. $r(0) = a$). In this representation, the sign of $dr/d\theta$ (equivalently, the sign of b) also determines the inward or outward direction of the spiral: $b > 0$ corresponds to an outward-expanding radius as θ increases, while $b < 0$ corresponds to an inward-winding radius approaching the origin as θ increases.

This is precisely the standard polar form of the *logarithmic spiral* (also known as the equiangular spiral): the radius changes exponentially with the polar angle. In other words, once we adopt the simplified regime of "self-similar fracture growth" in the Ψ–Δ structure and use the geometric embedding $(\theta, r) = (\gamma, \Delta(\gamma))$, the canonical shape of the "fracture trajectory" in the abstract (θ, r)-plane naturally becomes a logarithmic spiral.

This geometric embedding is merely a visualization tool for the abstract Ψ–Δ evolution; it does *not* presuppose that the physical phase-space trajectories of the system themselves have a spiral shape.

5.4 Equiangular Property and the Uniqueness of the Logarithmic Spiral (Under Symmetry Constraints)

The logarithmic spiral has a classical geometric feature: at any point on the curve, the angle between the tangent and the radius vector at that point is constant. Curves satisfying this property are called *equiangular curves*.

Let the curve be given in polar coordinates as $r = r(\theta)$, and let $\varphi(\theta)$ denote the angle between the tangent and the radius vector at the point $(r(\theta), \theta)$. Standard differential geometry yields the relation

$$\tan \varphi(\theta) = \frac{r(\theta)}{dr/d\theta}.$$

Intuitively, this relation measures how the tangent direction deviates from the radial direction: when $dr/d\theta$ is large, the tangent is more aligned with the radial direction and φ is close to 0; when $dr/d\theta$ is small or even zero, the tangent is closer to the circular direction and φ approaches $\pi/2$.

Combining this with the previous section, we note that for the logarithmic spiral solution $r(\theta) = ae^{b\theta}$, $\tan \varphi(\theta)$ is constant, and hence $\varphi(\theta)$ is constant along the entire curve. The sign of b corresponds to the two spiral orientations already mentioned: outward for $b > 0$, inward for $b < 0$.

By definition, an equiangular curve requires that $\varphi(\theta)$ be a constant φ_0 over the whole curve, i.e. $\varphi(\theta) \equiv \varphi_0$. Substituting this into the formula above gives

$$\tan \varphi_0 = \frac{r(\theta)}{dr/d\theta}.$$

On any region where $r(\theta) \neq 0$, this is equivalent to the first-order linear differential equation

$$\frac{dr}{d\theta} = \frac{1}{\tan \varphi_0} \cdot r(\theta) = b \cdot r(\theta),$$

where $b = 1/\tan \varphi_0$ is a constant. This is exactly the form derived in the previous section:

$$\frac{dr}{d\theta} = b \cdot r, \qquad r(\theta) = ae^{b\theta}.$$

We therefore obtain:

- Imposing the requirement that the angle between tangent and radius is constant (equiangularity) *forces* $r(\theta)$ to take the logarithmic spiral form;
- Conversely, any curve satisfying $r(\theta) = ae^{b\theta}$ automatically satisfies the equiangular condition $\varphi(\theta) \equiv \varphi_0$.

Furthermore, under the symmetry constraint of a fixed origin together with allowed global scalings and rotations, classical results show that, after suitable coordinate rescaling, the logarithmic spiral is the unique planar curve that simultaneously possesses:

1. the equiangular property (constant angle between tangent and radius), and
2. invariance of shape under scalings and rotations about the origin (self-similarity).

Discussions of equiangular curves and the uniqueness of the logarithmic spiral can be found in standard differential geometry textbooks.

From the structural viewpoint of this paper, this result can be reformulated as follows:

- Under the symmetry assumption that the origin is the only distinguished point, the logarithmic spiral is a canonical geometric solution of the Ψ–Δ evolution: it unifies the condition "local fracture generation rate is proportional to accumulated fracture" ($\Psi \propto \Delta$) with the requirement that the fracture trajectories be structurally self-similar across scales;
- This does *not* assert that the trajectories of actual systems in concrete Euclidean phase spaces must be logarithmic spirals. It only states that, in the abstract (γ, Δ)-plane, once a self-similar structural regime is reached, its canonical shape is that of a logarithmic spiral.

5.5 Structural Meaning: The Spiral as the Canonical Geometric Solution of the Ψ–Δ Structure

Putting the previous discussion together, we can regard the logarithmic spiral as the canonical geometric solution of the Ψ–Δ structure in an ideal self-similar regime. Under the polar embedding $(\theta, r) = (\gamma, \Delta(\gamma))$, the correspondence among the structural quantities can be summarized as follows:

- $\Delta \leftrightarrow$ radius r: The accumulated fracture intensity is represented geometrically by the distance from the origin, characterizing how far the system has moved along the chosen evolution direction in terms of structural scale;
- $\Psi \leftrightarrow$ radial growth rate $dr/d\theta$: The local fracture generation rate corresponds to the rate of change of the radius with respect to the polar angle, i.e. the speed at which the curve moves outward (or inward) at each angular position;
- Self-similar regime $(\Psi \propto \Delta) \leftrightarrow$ logarithmic spiral: When the local generation rate at each point is proportional to the current fracture scale, the shape of the fracture trajectory in polar coordinates necessarily tends toward a logarithmic spiral, and automatically enjoys both equiangularity and scale self-similarity.

The parameter b in this correspondence has a particularly clear structural meaning:

- $b > 0$: the spiral diverges outward, and the radius r grows exponentially with θ. This corresponds to a *fracture-amplifying phase*: the larger the local tension, the more it drives further structural mismatch, and the system tends toward an M-type regime of frustration accumulation along this evolution direction;
- $b < 0$: the spiral contracts inward, and the radius r decays exponentially with θ. This corresponds to a *fracture-compensation phase*: the larger the local tension, the more it drives compensation and self-organization mechanisms, pulling the system back into a region of smaller fractures, closer to H-type repair behavior;
- $b = 0$: the radius r is constant and the trajectory degenerates into a circle centered at the origin. This corresponds to a *fracture-frozen phase*: Δ remains constant along the evolution direction, and the system neither significantly amplifies nor significantly repairs structural tension, residing in a critical or boundary state.

In this geometric picture, the logarithmic spiral is not merely an abstract curve; it provides a highly visual "phase diagram" for the Ψ–Δ structure:

- Different values of b correspond to different fracture evolution phases;
- The outward or inward winding of the spiral visually captures whether, near the H/M boundary, the system drifts toward "fracture amplification" or flows back toward "fracture compression".

In more complex situations, the actual fracture trajectory of a system will typically deviate from this ideal spiral. For example, in some angular intervals the effective b may be positive, corresponding to frustration amplification; in others, the effective b may be negative, corresponding to compensation and repair. Such trajectories can be regarded as perturbations "oscillating around the canonical logarithmic spiral": locally they still exhibit a tendency toward $\Psi \propto \Delta$, but on larger scales this tendency is interrupted and rewritten by additional mechanisms.

On the other hand, the abundance of spiral growth and fracture patterns in nature—from phyllotaxis and shell growth to interface fronts and crack propagation—can be structurally interpreted as geometric projections of a certain Ψ–Δ self-similar regime: in these systems, the local rate of growth or breakage remains, over long time scales, approximately proportional to the current structural scale, thereby generating logarithmic spirals or their generalizations at the macroscopic level. This paper does not attempt a systematic taxonomy of such examples, but treats them as visual evidence supporting the view of "the spiral as a canonical geometric solution."

In terms of the structure of the whole text, the role of Chapter 5 is to provide a geometric closure for the abstract Ψ–Δ structure that is as simple as possible while still rich in structural content. In the subsequent chapters, when we discuss uncomputability, chaos, NP-hardness, and AI hallucinations in multi-level systems, we can think of "how fractures are accumulated or repaired along a given evolution direction" as a spiral-like path in the abstract (γ, Δ)-plane:

- this path need not be a perfect logarithmic spiral;
- but the logarithmic spiral provides a reference shape with clear symmetry and structural meaning, which makes it convenient to understand "the real trajectory" as a deviation and perturbation around this canonical solution.

5.6 Geometric Scaling of Feigenbaum Constants in Fracture Space

Building on the previous discussion of the Ψ–Δ structure and the geometry of logarithmic spirals, this section rewrites the classical Feigenbaum constants in the language of fracture space. Concretely, we translate them from "renormalization scaling factors in function/ parameter space" into "radial scaling and phase-twisting constants of logarithmic-spiral trajectories in fracture space."

Notation (avoiding collision). Throughout this paper, δ denotes the resolution threshold in the operational protocol. In the present section, classical Feigenbaum constants are written as δ_{Feig} and α_{Feig} to avoid collision with the resolution symbol. No other meaning of δ is used in this section.

5.6.1 Phenomenological starting point: from function space to fracture space

In classical period-doubling theory, the renormalization operator $\mathcal{R}_{\text{map}}$ acting on a unimodal map family has, in a neighborhood of the critical unimodal map f^*, a dominant real eigenvalue $\delta_{\text{Feig}} > 1$.[5, 6, 10] A standard description is:

> Along the relevant directions near the critical configuration, each renormalization step scales both the functional perturbations and the spacing between neighboring bifurcation points on the parameter axis by a factor of δ_{Feig} (or, inversely, by $\delta_{\text{Feig}}^{-1}$).

This characterization anchors δ_{Feig} in "function space/parameter space" and describes scale invariance at the level of object representations. From this viewpoint, δ_{Feig} is primarily a one-dimensional stretching factor: the statement itself does not encode any intrinsic "rotation" or "spiral" structure, and it does not explicitly refer to perspectives, fracture accumulation, or meta-fractures.

The goal of the present section is not to change the content of Feigenbaum scaling, but to attach to it a semantics compatible with the Ψ–Δ framework: we seek a coordinate-free reading in which the same self-similar structure is expressed as geometric parameters of trajectories in an abstract fracture space. In that translation, δ_{Feig} will reappear as a *radial scaling constant* for fracture magnitudes, while a companion constant (written here as α_{Feig}) will be associated with a *phase-twisting* component that becomes visible only after a two-component fracture state is introduced.

5.6.2 Fracture states: a two-component representation of first-order and meta-fractures

Within the Ψ–Δ framework, an observer does not work directly in function space but interacts with the system through a declared operational regime: a family of perspectives together with an admissible class of H-descriptions at a fixed resolution threshold. To keep the discussion explicit, fix:

- a perspective A (including the declared representation and admissible family);
- an H-description class H (finite-complexity "tame" descriptions admissible under A);
- a resolution threshold δ (the operational negligibility scale of this paper).

First-order fracture. For a given object (e.g. a map f in a fixed universality class) evaluated under (A, H), define its *first-order fracture* as a nonnegative residual:

$$\Delta(f) \ \geq \ 0,$$

interpreted operationally as the smallest residual mismatch that remains after optimizing over all admissible H-descriptions in H at the declared resolution. Concretely: one enumerates the admissible finite-complexity H-rules at the chosen perspective, selects the best-fitting rule for the long-term behavior under a declared discrepancy metric, and defines $\Delta(f)$ as the minimum achieved residual. Smaller Δ means "more tame" under the declared regime.

Meta-fracture as a cross-level deviation. To extract a two-component state suitable for spiral geometry, we introduce a second component that measures the deviation incurred when one uses a higher-level update rule to predict the next-level fracture. Consider an ordered sequence along a self-embedding/renormalization chain indexed by $k \in \mathbb{N}$ (e.g. $f_{k+1} = \mathcal{R}_{\mathrm{map}}(f_k)$ in a fixed universality class). Define

$$\Delta_k := \Delta(f_k) \quad \geq 0.$$

Now fix an *ideal propagation rule* F for how a "perfectly self-similar, single-perspective" structure would transport fractures across one step:

$$\Delta_{k+1} \approx F(\Delta_k).$$

The precise form of F is part of the modeling choice; in the simplest linear regime one may take $F(\Delta) = c\,\Delta$ with $c > 0$.

We then define the *meta-fracture* at level k as the deviation between the observed next-step fracture and the ideal propagation prediction:

$$\Delta_k^* \ := \ \Delta_{k+1} - F(\Delta_k).$$

In this definition, Δ_k^* measures the residual that arises specifically from the mismatch between a single-perspective cross-level continuity assumption (encoded by F) and the actual fracture transport observed along the renormalization/self-embedding chain.

Fracture state and complex coordinate. At level k, the fracture state is packaged as

$$v_k = \begin{pmatrix} \Delta_k \\ \Delta_k^* \end{pmatrix}.$$

Equivalently, one may use the complex representation

$$z_k := \Delta_k + i\,\Delta_k^* \in \mathbb{C},$$

where $|z_k|$ measures a total fracture magnitude in the two-component sense and $\arg z_k$ measures the relative balance between first-order fracture and meta-fracture. This choice is not a physical assumption; it is a convenient coordinate system for describing spiral-type trajectories in the (Δ, Δ^*) plane.

5.6.3 Intertwining hypothesis: from function-space RG to meta-RG in fracture space

In the traditional setting, $\mathcal{R}_{\mathrm{map}}$ acts on the object (the map) in function space. In the present framework, we introduce a *meta-renormalization* operator acting on fracture states:

$$v_{k+1} = \mathcal{R}_{\mathrm{meta}}(v_k).$$

We adopt the following as a *working structural hypothesis* near the critical configuration f^*: there exists a local structural map

$$\Phi : \ f \mapsto v(f) = \begin{pmatrix} \Delta(f) \\ \Delta^*(f) \end{pmatrix}$$

such that, to first order in a neighborhood of f^*, the diagram approximately commutes:

$$\begin{tikzcd}
\text{Function space} \arrow[r, "\mathcal{R}_{\mathrm{map}}"] \arrow[d, "\Phi"] & \text{Function space} \arrow[d, "\Phi"] \\
\text{Fracture space} \arrow[r, "\mathcal{R}_{\mathrm{meta}}"] & \text{Fracture space}
\end{tikzcd}$$

i.e.

$$\Phi\big(\mathcal{R}_{\mathrm{map}}(f)\big) \;\approx\; \mathcal{R}_{\mathrm{meta}}\big(\Phi(f)\big),$$

where "≈" denotes equivalence at the linearized (first-order) level, up to a non-singular linear rescaling of coordinates in fracture space.

This hypothesis is not asserted as a general theorem. Its role is operational: it allows the same critical self-similar structure to be tracked either (i) as an RG orbit in function space, or (ii) as an orbit of fracture states in a two-component space where geometric invariants (radial scaling and phase twisting) can be read off directly.

Compatibility with the structural fracture law. Throughout this translation we keep the structural fracture law in its verbatim form as a working meta-constraint. In fracture space, the two components (Δ, Δ^*) provide a minimal internal split between (i) "fracture in the object under the declared regime" and (ii) "fracture in the cross-level propagation rule under that regime." Under self-embedding and feedback, single-perspective continuity attempts manifest as coupled evolution of these two components; the appearance of spiral-type trajectories is the geometric expression of this coupling in the linearized regime.

5.6.4 A constructive linear model for the meta-renormalization operator

We now give a minimal linear construction of a candidate meta-renormalization operator acting in fracture space. The purpose is not to claim universality at the operator level, but to show that, once a two-component fracture state is fixed, a spiral-type scaling law can arise from an explicit linear dynamics rather than from a purely metaphorical analogy.

Step 1: lift a scalar fracture sequence to a two-dimensional shift state. Fix a unimodal map family and a critical RG orbit

$$f_0 \xrightarrow{\mathcal{R}_{\mathrm{map}}} f_1 \xrightarrow{\mathcal{R}_{\mathrm{map}}} f_2 \xrightarrow{\mathcal{R}_{\mathrm{map}}} \cdots, \qquad f_k := \mathcal{R}^k_{\mathrm{map}}(f_0),$$

within a single universality class. Under a declared operational regime (A, H, δ), define

$$\Delta_k := \Delta(f_k) \geq 0.$$

Introduce the lifted state

$$w_k := \begin{pmatrix} \Delta_k \\ \Delta_{k+1} \end{pmatrix},$$

so that one step of the induced shift is

$$\mathcal{S}: \; w_k \mapsto w_{k+1} = \begin{pmatrix} \Delta_{k+1} \\ \Delta_{k+2} \end{pmatrix}.$$

Near criticality, where the induced fracture sequence inherits the self-similar structure of the RG orbit, we model this shift by a linearization

$$w_{k+1} \approx A_{\text{shift}}\, w_k, \qquad A_{\text{shift}} = \begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix},$$

with real coefficients (a, b) determined by the local scaling behavior of $\{\Delta_k\}$ in the chosen universality class and declared operational regime. (In concrete studies, (a, b) may be estimated numerically; the present section uses them only as existence placeholders in a minimal model.)

Step 2: change coordinates to (Δ, Δ^*). Choose an ideal propagation rule $F(\Delta) = c\,\Delta$ and define

$$\Delta_k^* := \Delta_{k+1} - c\,\Delta_k.$$

Then the coordinate change from w_k to $v_k = (\Delta_k, \Delta_k^*)^\top$ is

$$v_k = \begin{pmatrix} \Delta_k \\ \Delta_k^* \end{pmatrix} = \begin{pmatrix} \Delta_k \\ \Delta_{k+1} - c\,\Delta_k \end{pmatrix} = T\, w_k, \qquad T = \begin{pmatrix} 1 & 0 \\ -c & 1 \end{pmatrix}, \qquad T^{-1} = \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}.$$

Step 3: conjugate the shift to obtain a linear meta-renormalization operator. Define the linear meta-renormalization operator by conjugation:

$$\mathcal{R}_{\text{meta,lin}} \;:=\; T\,\mathcal{S}_{\text{lin}}\,T^{-1}, \qquad v_{k+1} \approx A_{\text{meta}}\, v_k, \qquad A_{\text{meta}} := T A_{\text{shift}} T^{-1}.$$

By construction, A_{meta} and A_{shift} are similar matrices and therefore share eigenvalues. Passing to the complex coordinate

$$z_k = \Delta_k + i\,\Delta_k^*,$$

a dominant eigenvalue $\lambda \in \mathbb{C}$ yields the approximate one-step law

$$z_{k+1} \approx \lambda z_k, \qquad \lambda = \rho e^{i\theta}.$$

In this polar decomposition, ρ controls radial scaling of the fracture magnitude and θ controls the phase increment (twist) between first-order fracture and meta-fracture, producing a logarithmic-spiral trajectory in the (Δ, Δ^*) plane when iterated.

Where Feigenbaum constants enter. At the level of this section, we only require a semantic identification: δ_{Feig} (the dominant real eigenvalue of $\mathcal{R}_{\text{map}}$) controls the *dominant scaling* along the relevant direction in function space; under the intertwining hypothesis, that dominant scaling induces a corresponding dominant radial scaling in fracture space. Accordingly, we parameterize the radial modulus as

$$\rho = \delta_{\text{Feig}}^{-\beta},$$

with an effective exponent $\beta > 0$ reflecting the projection from the relevant direction in function space to the radial direction in the chosen fracture coordinate. The companion constant α_{Feig} (classically associated with spatial rescaling of the map near the critical point) will be used in later subsections to fix a phase-twist interpretation (i.e. to relate the twist θ to a second geometric invariant in fracture space), once the spiral geometry is developed in a model-specific way.

Scope of the linear model. The construction above should be read as a minimal realizability statement: within a fixed universality class and a fixed declared operational regime, one can lift a scalar fracture sequence to a two-dimensional state, separate first-order fracture from meta-fracture by a linear coordinate choice, and obtain a linear operator whose eigenstructure yields spiral-type scaling. The model does not claim to capture the full nonlinear fracture transport, but it makes explicit that the spiral picture can be grounded in a concrete operator acting on a defined fracture state, rather than being a purely phenomenological analogy.

5.6.5 δ_{Feig}: radial scaling factor of the logarithmic spiral

In the geometric treatment of the Ψ–Δ structure in the previous section, we saw that under the polar embedding $(\vartheta, r) = (\gamma, \Delta(\gamma))$, a self-similar growth relation of the form $\Psi(\gamma) \propto \Delta(\gamma)$ yields a logarithmic spiral

$$r(\vartheta) = a\, e^{b\vartheta}$$

in the (ϑ, r)-plane, where b controls by what factor the radius is rescaled when the angle advances by one unit.

In the discrete setting indexed by the level k, we treat

$$z_k = \Delta_k + i\, \Delta_k^*$$

as sample points on a spiral-like trajectory in fracture space near the "double-zero" reference state. Linearizing the meta-renormalization operator $\mathcal{R}_{\text{meta}}$ at the reference configuration

$$v^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad z^* = 0,$$

we obtain the first-order dynamics

$$v_{k+1} \approx A\, v_k, \qquad A := D\mathcal{R}_{\text{meta}}\big|_{v^*},$$

which in complex notation becomes

$$z_{k+1} \approx \lambda z_k, \qquad \lambda \in \mathbb{C}.$$

Writing the dominant eigenvalue in polar form as

$$\lambda = \rho\, e^{i\varphi}, \qquad \rho > 0,\ \varphi \in (-\pi, \pi],$$

the induced trajectories in fracture space are, to first order, logarithmic spirals:

$$|z_{k+1}| \approx \rho\,|z_k|, \qquad \arg z_{k+1} \approx \arg z_k + \varphi.$$

Here ρ is the radial scaling factor (fracture magnitude rescaling per step), and φ is the phase increment (the systematic twisting between the first-order fracture axis and the meta-fracture axis per step).

To connect this with classical renormalization theory, recall that in function space the dominant eigenvalue of $\mathcal{R}_{\mathrm{map}}$ near the critical map is the Feigenbaum constant $\delta_{\mathrm{Feig}} > 1$.[5, 6, 10] Under the intertwining hypothesis stated earlier (existence of a bridge Φ that approximately conjugates $\mathcal{R}_{\mathrm{map}}$ to $\mathcal{R}_{\mathrm{meta}}$ in the critical neighborhood), it is natural to parameterize the induced radial scaling ρ in fracture space as a power-law function of δ_{Feig}:

$$\rho = \delta_{\mathrm{Feig}}^{-\beta}, \qquad \beta > 0.$$

Here β is used purely as a *bookkeeping exponent*: it absorbs the linear rescaling between the relevant eigen-direction in function space and the chosen radial coordinate in fracture space (equivalently, the choice of normalization in Φ and in the (Δ, Δ^*) coordinates). No claim is made that β is itself a new universal constant.

In this regime-relative sense, the statement "δ_{Feig} is the radial scaling constant in fracture space" means:

> δ_{Feig} controls the dominant contraction/expansion rate near criticality; after projection to fracture space, that same dominant scaling appears as the radial rescaling factor $\rho = \delta_{\mathrm{Feig}}^{-\beta}$ governing how fast the fracture state approaches (or leaves) the double-zero point along a spiral-like trajectory.

5.6.6 α_{Feig}: phase flip and systematic twisting of the meta-fracture direction

Besides δ_{Feig}, classical period-doubling theory uses a second Feigenbaum constant α_{Feig} (numerically $\alpha_{\mathrm{Feig}} \approx -2.5029\ldots$), which captures the spatial rescaling of the critical orbit in state space.[5, 6, 10] Under standard normalizations, the negative sign corresponds to an orientation reversal under each magnification step.

In fracture space, the complex-eigenvalue form

$$z_{k+1} \approx \lambda z_k, \qquad \lambda = \rho e^{i\varphi},$$

already separates two geometric effects:

$$|z_{k+1}| \approx \rho\,|z_k| \quad \text{(radial scaling)}, \qquad \arg z_{k+1} \approx \arg z_k + \varphi \quad \text{(phase twisting)}.$$

The role of α_{Feig} is then incorporated at the level of the phase increment φ as follows.

Phase flip (sign) component. Because $\alpha_{\mathrm{Feig}} < 0$ corresponds to a reversal under each renormalization step in the classical state-space picture, the minimal compatible statement in fracture space is that one step of the meta-flow contains an effective *phase flip*:

$$\varphi = \pi \;+\; \varphi_{\mathrm{bias}} \qquad (\mathrm{mod}\ 2\pi),$$

where the π term encodes the orientation reversal and φ_{bias} encodes any additional systematic twisting beyond a pure flip.

Bias (magnitude) component. The quantity φ_{bias} is not fixed by the sign of α_{Feig} alone: it depends on how the classical spatial rescaling is transported through the bridge Φ into the chosen fracture coordinates (equivalently, it depends on the normalization of Φ, the choice of prediction rule F, and the induced shear between Δ and Δ^*). A convenient regime-relative parameterization is to write φ_{bias} as a smooth function of $|\alpha_{\text{Feig}}|$, for example

$$\varphi_{\text{bias}} \equiv \kappa_\alpha \log |\alpha_{\text{Feig}}| \pmod{2\pi},$$

with a coefficient κ_α determined by the chosen fracture-space normalization within a given universality class. This parameterization is not a new law; it is an explicit way to record how the classical spatial rescaling constant contributes to the twisting of the fracture-state direction under the chosen projection.

With the pair of parameterizations

$$\rho = \delta_{\text{Feig}}^{-\beta}, \qquad \varphi = \pi + \varphi_{\text{bias}},$$

the fracture-space step becomes

$$|z_{k+1}| \approx \delta_{\text{Feig}}^{-\beta} |z_k|, \qquad \arg z_{k+1} \approx \arg z_k + \pi + \varphi_{\text{bias}} \pmod{2\pi}.$$

This expresses the intended structural split:

- δ_{Feig} governs the dominant radial scaling (how fast fracture magnitude rescales across levels);
- α_{Feig} contributes a sign-induced flip, and (through the chosen bridge/normalization) may also contribute a systematic bias that prevents the motion from collapsing into a trivial two-cycle.

5.6.7 Structural meaning and directions for verification

In the traditional account of Feigenbaum universality, one typically reads

- δ_{Feig}: the accumulation rate of period-doubling bifurcations along the parameter axis;
- α_{Feig}: the spatial rescaling (with orientation reversal) of the critical orbit in state space.

Within the Ψ–Δ framework, the same pair can be rewritten as geometric invariants of a fracture-space trajectory near the critical configuration:

- δ_{Feig} reappears as the dominant *radial* scaling constant via $\rho = \delta_{\text{Feig}}^{-\beta}$ (with β absorbing the projection/normalization);
- α_{Feig} contributes to the *phase* behavior by enforcing a flip $\varphi \equiv \pi \pmod{2\pi}$ and, under a declared bridge Φ, may further induce a systematic twist bias φ_{bias}.

Accordingly, this section should be read as a controlled translation statement: the classical pair $(\delta_{\text{Feig}}, \alpha_{\text{Feig}})$ can be interpreted as two projections of the same critical self-similar structure, expressed in two coordinate systems:

In function space/state space, they quantify parameter accumulation and spatial rescaling; in fracture space, they quantify the radial scaling rate of fracture magnitude and the phase behavior (flip plus possible bias) of the meta-fracture direction.

At the present stage, the translation is intended as a *phenomenological and structural reinterpretation* rather than a new theorem. A concrete verification program requires: (i) an explicit instantiation of the operational package defining Δ_k and Δ_k^* for a fixed universality class, (ii) a numerical reconstruction of the induced linearization $A = D\mathcal{R}_{\mathrm{meta}}|_{v^*}$, and (iii) empirical checks that the extracted ρ and φ are stable under admissible re-expressions at the declared resolution. In such a program, β and κ_α become reported regime parameters (part of the package) rather than unexplained constants.

5.7 Ψ–Δ dual structure and coarse-grained accumulation

In the preceding chapters, the structural gap Δ was introduced as a cross-level fracture quantity: at the ontological level, whenever a genuine cross-level mapping is involved, one has $\Delta > 0$; relative to a declared resolution threshold one may set $\Delta \approx 0$ as an effective convention, treating it as unobservable or negligible at that resolution.

Scope of notation (accumulated $\Delta(\gamma)$ versus fixed-slice gaps). In this section, $\Delta(\gamma)$ refers specifically to an *accumulated* fracture along an evolution parameter γ: a coarse-grained scalar evaluated under a declared level specification and resolution regime. It is *not* the fixed-slice inter-level gap $\Delta_{A_I \to A_G}(\mathcal{L})$ defined for a specific pair of perspectives.

Upon introducing an evolution parameter γ (which may represent time, RG scale, a hierarchy index, or a cognitive evolution parameter), the relation between the intra-level structural tension $\Psi(\gamma)$ and the accumulated gap $\Delta(\gamma)$ is written in the one-dimensional form

$$\Delta(\gamma) = \Delta(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi(\gamma')\, d\gamma'. \tag{5.5}$$

This treats Δ as the history integral of Ψ along γ. Under the conventions of this paper, $\Psi(\gamma) \geq 0$ in the declared regime, hence $\Delta(\gamma)$ is non-decreasing in γ (while $\Delta \approx 0$ may still be adopted as an effective, resolution-level approximation when appropriate).

Operational stance (what is assumed in the main text). For the purposes of the main line of argument, the essential dynamical content is the coarse-grained accumulation relation (5.5), interpreted under a declared level specification and resolution regime. This relation is used as a controllable structural summary in later chapters (for instance, in defining long-term lower bounds and in relating local tension density to accumulated fracture).

A more detailed *local* closure of the coupled Ψ–Δ structure (e.g. "Maxwell-type" prototype equations on an abstract structural space) is not required for the definitions and lower-bound statements developed in the main text. One optional local closure ansatz, together with its minimal assumptions and its reduction back to (5.5) under coarse-graining, is collected separately in Appendix A.

5.8 A renormalization view of the fracture magnitude Δ: time, scale, and hierarchy

The preceding discussion rewrote coarse-graining and renormalization as an HTDS on a family of perspectives: through a declared family $\{(A_\ell, \Psi_\ell, \Delta_{\mathrm{dyn},\ell})\}$, the flow of "theory space" in traditional RG is reinterpreted as an evolution on pairs of "perspective and frustration degree." Building on that picture, the present section focuses specifically on the fracture magnitude Δ, and examines its coarse-grained "flows" along three directions: time, scale, and hierarchy.

The emphasis here is not on which particular effective theory flows, but on how fractures are suppressed, redistributed, or re-expressed along different directions on top of those flows.

5.8.1 The Δ-flow in time: chaos and predictability windows

Consider a dynamical system evolving in time. Fix a perspective A and a declared resolution threshold δ (part of the operational package). Define a finite-window effective fracture magnitude over $[0, T]$ by

$$\Delta_T(A) \;:=\; \text{the minimal irreducible prediction residual at horizon } T \text{ under } A \text{ at resolution } \delta,$$

where the phrase "minimal irreducible" is always understood relative to the declared admissible class of predictors/encodings and the declared discrepancy convention (i.e. relative to the package).

In the HTDS language of Chapter 4, $\Delta_T(A)$ can be viewed as a finite-horizon analogue of Δ_{dyn} along the time direction ($\gamma = t$): it measures how much fracture becomes manifest after evolving for time T, assuming the initial uncertainty has already been suppressed below the declared resolution.

When the system has a strictly positive maximal Lyapunov exponent $\lambda > 0$, nearby trajectories separate (on average) at an exponential rate,

$$\|\delta x(t)\| \approx \|\delta x(0)\|\, e^{\lambda t},$$

over the regime in which linearization remains informative. In such a regime, a coarse-grained structural scaling heuristic is

$$\Delta_T(A) \;\gtrsim\; e^{\lambda T}\, \Delta_0(A),$$

until saturation is reached (for example, due to boundedness of the accessible region or the finite resolution floor). Equivalently, one obtains a finite *predictability window*: given a target tolerable threshold δ, the horizon beyond which fracture becomes manifest typically scales like

$$T_{\mathrm{hor}} \;\sim\; \frac{1}{\lambda}\, \log\!\Big(\frac{\delta}{\Delta_0(A)}\Big),$$

again interpreted as a regime-relative coarse-grained relation rather than a theorem. In this sense, "sensitive dependence on initial conditions" can be restated structurally: for a fixed A and δ, temporal evolution tends to renormalize any sufficiently small initial fracture into a manifest $\Delta > 0$ regime within a finite horizon, so that a system can cross from an operationally H-like window to an M-like window without changing the underlying rules, merely by evolving long enough.

5.8.2 The Δ-flow in scale: coarse-graining and phase transitions

For systems with scale structure, recall the family of coarse-graining maps

$$\pi_\ell : X \to Y_\ell$$

and the induced perspectives A_ℓ. At each scale ℓ, define a scale-indexed fracture magnitude

$$\Delta(\ell) \;:=\; \text{the irreducible residual between the micro-level structure and the effective description at scale } \ell$$

always understood relative to the declared admissibility and reconstruction conventions. Under coarse-graining and rescaling, one induces a scale-evolution map on effective descriptions, and correspondingly a scale-flow for $\Delta(\ell)$:

$$\Delta(\ell) \;\mapsto\; \Delta(\ell'), \qquad \ell' \text{ a rescaled/coarsened index relative to } \ell.$$

Away from criticality, there may exist scale intervals and admissible perspectives for which $\Delta(\ell)$ is pushed below the declared resolution threshold (an operationally H-like window). Near criticality, the scale dependence may instead exhibit approximate power-law structure of the form

$$\Delta(\ell) \;\sim\; \ell^{\alpha},$$

where α functions as a structural scaling exponent for the fracture magnitude under scale renormalization in the declared regime. In this language, phase transitions and critical phenomena are described as regimes in which the scale-flow fails to remain in an H-like basin and is trapped near nontrivial structures carrying $\Delta > 0$ (manifest fracture) under the same admissibility and resolution regime.

5.8.3 The Δ-flow in hierarchy: self-embedding and finite-order instability

In addition to time and scale, one can introduce a discrete structural evolution along a hierarchy/self-embedding direction. Let S_0 be a basic structural unit and let E be a declared self-embedding operator defining

$$S_{k+1} = E(S_k), \qquad k = 0, 1, 2, \ldots$$

At each level S_k, choose a natural perspective A_k (again, relative to the package), and define

$$\Delta_k \;:=\; \Delta(S_k, A_k).$$

The operator E (together with the induced adjustment of perspectives) yields a discrete hierarchical flow

$$\mathcal{R}_{\text{layer}}: \ \Delta_k \ \mapsto \ \Delta_{k+1}.$$

Across a variety of concrete models, one encounters the qualitative pattern:

- At low self-embedding depths (small k), Δ_k can remain operationally negligible under natural admissible regimes (an H-like window at finite depth).
- Beyond a finite critical depth, Δ_k develops a stable positive lower bound under the same type of natural regimes, i.e. the hierarchy enters a manifest $\Delta > 0$ window that is difficult to return to an H-like neighborhood without changing the regime itself.

The "fifth-order structural instability" discussed earlier is a concrete label for the empirically observed case in which this first manifest onset frequently occurs at a small finite depth (often near $k = 5$) for certain tested instance families under declared packages.

Taken together, viewing Δ along the time, scale, and hierarchy directions places chaos, criticality, and finite-depth self-embedding instabilities on a single structural picture: traditional RG tracks the flow of effective descriptions; the Δ-flow tracks how fracture is suppressed, redistributed, and accumulated across levels under the same operational constraints.

Remark (heuristic only): integer projection of the Feigenbaum scaling constant. The following remark is a *speculative direction for further study* and is not used as support for any main claim.

Let $\delta_{\text{Feig}} \approx 4.669\ldots$ denote the classical Feigenbaum constant governing period-doubling scaling in unimodal maps. One may ask whether a continuous scaling factor in a renormalization analysis can leave a signature when the evolution direction is forced to be a discrete hierarchy indexed by an integer depth k. A crude heuristic suggests considering an "integer projection" such as

$$\lceil \delta_{\text{Feig}} \rceil = 5,$$

as a candidate order at which fracture becomes difficult to keep operationally negligible under a fixed background frustration floor and finite precision. This is only an intuition: turning it into a testable statement would require, for a specific model family, an explicit hierarchical construction of the Δ-flow, extraction of a linearized spectrum near criticality, and a quantitative comparison between the observed onset depth and the scaling behavior governed by δ_{Feig} within the declared operational package.

Chapter 6

Material / Information / Abstract-Logical Three-Layer Ψ–Δ Case Studies: Spin Glasses, JPEG, and Homology

6.1 Spin Glasses: M-Class Systems at the Material Layer

6.1.1 System Definition $\mathcal{L}_t = (V, R, S_t)$ and Perspectives A_I/A_G

We take a finite Ising spin glass as an example and write it as a dynamical logical system in the sense introduced above.[4, 15, 12, 1] Let $G = (V, E)$ be a finite undirected graph. Each vertex $i \in V$ carries a spin variable $\sigma_i \in \{+1, -1\}$, and each edge $(i, j) \in E$ has a fixed coupling strength $J_{ij} \in \mathbb{R}$, for instance $J_{ij} \in \{\pm 1\}$ or J_{ij} is drawn from a zero-mean Gaussian distribution. The Hamiltonian of the system is

$$H_J(S) = -\sum_{(i,j)\in E} J_{ij}\, \sigma_i \sigma_j,$$

where $S = (\sigma_i)_{i\in V}$ denotes a spin configuration.

In our framework, we abstract this physical system as a dynamical logical system

$$\mathcal{L}_t = (V, R, S_t),$$

where

- the variable set $V = \{\sigma_i\}$ is the collection of individual spins;
- the rule set R consists of the coupling matrix (J_{ij}) together with local update rules (such as Glauber dynamics, the Metropolis algorithm, etc.), which specify stochastic updates from S_t to S_{t+1};
- S_t is the system state (spin configuration) at time t.

The precise dynamical form is not essential in this section; we only need it to induce orbits, T-invariant measures, and time averages in the sense of the general HTDS (hierarchical time-dynamical system) framework developed earlier.

On top of this structure we introduce two natural perspectives:

- *Internal perspective A_I*:

$$A_I = (F_I, P_I), \qquad F_I = V, \quad P_I(S) = S.$$

  This is the "fully microscopic" perspective, directly observing the value of each spin.

- *Global (macroscopic) perspective A_G*:

$$A_G = (F_G, P_G), \qquad m(S) = \frac{1}{|V|}\sum_{i\in V}\sigma_i, \quad F_G = \{m\}, \quad P_G(S) = m(S).$$

  In a more refined description, one can also include the energy $H_J(S)$ or the Edwards–Anderson order parameter into F_G, but for this section it suffices to consider the simplest macroscopic variable of "total magnetization."

Thus A_I and A_G capture, respectively, a "fully resolved microscopic structure" and a "strongly coarse-grained macroscopic structure." In our framework, they are two perspectives on the same system at different levels, used to define the within-level frustration Ψ and the view-shift fracture Δ_{view}.

6.1.2 Constructing Ψ and the Frustration Structure: Ground-State Degeneracy, Energy Barriers, and NP-Hardness

Under the internal perspective A_I, we want to construct a conflict measure $E_{A_I}(S)$ that is compatible with physical intuition and at the same time fits our general definition of conflict measures in the static logical-system framework. The key feature of spin glasses is *frustration*: there exist closed loops on which it is impossible to satisfy all couplings simultaneously.

For each edge $(i,j)\in E$:

- if $J_{ij}\sigma_i\sigma_j > 0$, we say that edge is satisfied in configuration S;
- if $J_{ij}\sigma_i\sigma_j < 0$, we say that edge is violated in configuration S.

Define

$$E_{A_I}(S) = \#\{(i,j)\in E : J_{ij}\sigma_i\sigma_j < 0\},$$

i.e., the number of violated couplings in configuration S (one may also normalize by $|E|$ to obtain a density; here we keep the counting form). According to the general definition of Ψ in this framework, the structural frustration under the internal perspective is

$$\Psi(\mathcal{L}, A_I) = \inf_{S\in\Omega} E_{A_I}(S),$$

where Ω is the set of all spin configurations.

- If $\Psi(\mathcal{L}, A_I) = 0$, then there exists a configuration S^* that satisfies all couplings, and the system is H-class at this level;
- if $\Psi(\mathcal{L}, A_I) > 0$, then no matter how we choose the spin configuration, some couplings remain unsatisfied, and the system is M-class at this level.

The typical spin-glass situation is exactly the latter. For random graphs with mixed positive and negative couplings, whenever there exists a loop $C = (i_1, i_2, \ldots, i_k, i_1)$ such that

$$\prod_{(i,j)\in C} J_{ij} < 0,$$

one can show that any configuration S violates at least one edge on that loop, hence $E_{A_I}(S) \geq 1$. In the limit where the graph carries a large number of such "frustrated loops," $\Psi(\mathcal{L}, A_I)$ is not only strictly positive, but typically grows to be of the same order as $|V|$ as the system size increases.

From the viewpoint of the energy landscape, $E_{A_I}(S)$ and the Hamiltonian $H_J(S)$ are two equivalent measures of "degree of violation": an ideal "unfrustrated" structure would correspond to a ground state where all couplings are satisfied; in a spin glass, such a configuration does not exist, leading to many local minima and highly degenerate near-ground-state clusters. Any reasonable local update rule (such as single-spin flips) encounters high energy barriers and a huge network of metastable states. This means that even from the internal perspective A_I, the system's evolution cannot, via local adjustments, push $E_{A_I}(S)$ down to a level close to 0.

Furthermore, for the Ising spin glass on a general graph, the problem of minimizing the number of violated couplings (that is, finding the global minimum of $E_{A_I}(S)$) and the problem of computing the ground-state energy are known to be computationally hard: for a wide range of graph families and coupling choices, this minimization problem is NP-hard. In other words, even if one has complete access to the microscopic coupling matrix (J_{ij}), there is no known polynomial-time algorithm that can, in general, find a configuration achieving the infimum of $E_{A_I}(S)$ exactly.

In the terminology of this paper, this is a canonical example of "$\Psi > 0$ at the material layer that cannot be effectively suppressed": spin glasses are M-class systems under the internal perspective.

6.1.3 Dynamic Perspective and Alignment with Ψ–Δ Calculus

To align the static frustration above with the general HTDS structural calculus, we now explicitly construct an evolution parameter γ for spin glasses and define natural proxy quantities for $\Psi(\gamma)$ and $\Delta(\gamma)$.

A direct choice is to let γ parametrize the progress of an annealing process. For example:

- choose an annealing time window $[0, \gamma_{\max}]$;
- specify a temperature or inverse-temperature schedule $T(\gamma)$ or $\beta(\gamma) = 1/T(\gamma)$, such as slowly cooling from a high-temperature disordered phase to a low-temperature glassy phase.

For each fixed γ, let the system evolve long enough at temperature $T(\gamma)$ so that it approaches the natural invariant measure (equilibrium Gibbs measure) $\mu_{\beta(\gamma)}$ at that temperature. Under this setup, two quantities serve as natural structural proxies:

(1) Instantaneous frustration density (evolutionary frustration rate) For a given γ, under the equilibrium distribution $\mu_{\beta(\gamma)}$, define

$$\Psi(\gamma) := \frac{1}{|E|}\,\mathbb{E}_{S\sim\mu_{\beta(\gamma)}} E_{A_I}(S) = \frac{1}{|E|}\,\mathbb{E}_{S\sim\mu_{\beta(\gamma)}} \#\{(i,j)\in E : J_{ij}\sigma_i\sigma_j < 0\}.$$

This represents, during the annealing process and at the current temperature level, the average probability that a coupling is violated; it is an instantaneous structural tension density.

(2) Fracture accumulated along the annealing path Define

$$\Delta(\gamma) := \int_0^\gamma \Psi(\tau)\,d\tau.$$

Intuitively, this is the total amount of "frustration exposure" experienced by the system at the material layer as it is annealed from the high-temperature starting point up to the current γ. If we approximate the annealing path by discrete temperature steps $0 = \gamma_0 < \gamma_1 < \cdots < \gamma_K$, then

$$\Delta(\gamma_k) \approx \sum_{j=0}^{k-1} \Psi(\gamma_j)\,\Delta\gamma_j,$$

which is precisely the discrete-time version of the structural Ψ–Δ calculus relation.

The key point is not merely "writing Δ as an integral," but two structural facts:

- On the one hand, from the HTDS viewpoint this is a concrete instantiation of the general construction of dynamical fractures introduced earlier: $\Psi(\gamma)$ comes from the invariant-measure integral $\int \Psi\,d\mu_{\beta(\gamma)}$ at each temperature level, while $\Delta(\gamma)$ is the accumulation of these level-wise frustration rates along the evolution path;
- on the other hand, the pair $(\Psi(\gamma), \Delta(\gamma))$ can be estimated numerically and independently: for each γ_k, one can estimate $\Psi(\gamma_k)$ via Monte Carlo sampling; meanwhile, by recording, along the annealing process, the accumulated "residence time in each temperature interval × frustration density," one obtains $\Delta(\gamma_k)$. In discrete form,

  $$\Delta(\gamma_{k+1}) - \Delta(\gamma_k) \approx \Psi(\gamma_k)\,\Delta\gamma_k,$$

  so that in simulations $\Psi(\gamma_k)$ and the numerical derivative $\dfrac{\Delta(\gamma_{k+1}) - \Delta(\gamma_k)}{\Delta\gamma_k}$ should match closely.

In other words, in the annealing dynamics of spin glasses, once we choose "the average frustration density at each temperature level" as $\Psi(\gamma)$ and "the accumulated frustration exposure along the annealing schedule" as $\Delta(\gamma)$, then

$$\Delta(\gamma) = \int_0^\gamma \Psi(\tau)\,d\tau$$

is not an ad hoc formula but a natural realization of the general HTDS structure in this concrete system. The corresponding discrete relation $\Delta(\gamma_{k+1}) - \Delta(\gamma_k) \approx \Psi(\gamma_k)\Delta\gamma_k$ can be approximately tested numerically by Monte Carlo simulation.

Together with the general dynamical fracture lower bound for HTDS introduced earlier,

$$\Delta_{\mathrm{dyn}}(\beta) = \inf_{\mu \in \mathcal{M}_T} \int \Psi \, d\mu \approx \Psi(\gamma) \quad (\gamma \text{ corresponding to fixed } \beta),$$

we see that the calculus relation between “average frustration density at the material layer” and “fracture accumulated along the annealing path” is structurally isomorphic, in spin glasses, to the general theory developed in this paper.

6.1.4 View Fracture $\Delta_{I\to G}$: Information Loss from m to S and Macroscopic Uncomputability

Next we consider the fracture that arises when switching from the microscopic perspective A_I to the macroscopic perspective A_G. According to the general definition of view-shift fracture, we set

$$\Delta_{I\to G}(\mathcal{L}) = \inf_{U \in \mathcal{U}} D\big[\Phi_{A_I}(S),\, U(\Phi_{A_G}(S))\big],$$

where Φ_A is the level map under perspective A, D is an information distance (for example, KL divergence or mutual-information loss), and $\mathcal{U}$ is a family of operators that lift macroscopic descriptions back to microscopic ones.

In the present case, a natural choice is $\Phi_{A_I}(S) = S$ and $\Phi_{A_G}(S) = m(S)$. The macroscopic perspective retains only the total magnetization m and discards the detailed arrangement of individual spins. For a given m, the fiber

$$P_G^{-1}(m) = \{S : m(S) = m\}$$

usually contains exponentially many microscopic configurations. Any lifting operator U can, at best, assign some probability distribution $Q(S \mid m)$ over this huge fiber, while the true microscopic distribution (e.g., the low-temperature Gibbs distribution $P_J(S)$) is typically concentrated on a highly nontrivial set of “valleys” within the fiber.

If we take D to be the KL divergence (or the induced information loss) between $P_J(S)$ and $Q(S \mid m)$, then in the low-temperature, strongly frustrated spin-glass phase, even the optimal lifting operator U will naturally incur a lower bound of order $|V|$ for $\Delta_{I\to G}(\mathcal{L})$. That is, the per-spin fracture density $\Delta_{I\to G}/|V|$ remains strictly positive in the thermodynamic limit. This reflects the fact that, from the macro-variable m alone, one cannot in any meaningful sense “infer back” which microscopic valley the system occupies.

The computational perspective is similar. Even if an algorithm is given full access to the coupling matrix (J_{ij}) and the macroscopic observation m, in general it faces the NP-hard ground-state search problem discussed above if it attempts to determine the exact microscopic structure of the ground state (or a specific metastable state) compatible with these data. Any algorithm that tries, in polynomial time, to recover the “right microscopic valley” from such macroscopic inputs will fail on general graph families. In other words, under A_G, the macroscopic behavior of the system carries a kind of “uncomputable residual”: we can measure m, but we cannot, within the same perspective, produce a unique microscopic structure consistent with it, nor can we enumerate or decide, in feasible time, all candidate valleys compatible with that macroscopic quantity.

6.1.5 Summary: $\Psi > 0$, $\Delta > 0$, and the M-Class Criterion at the Material Layer

To summarize, spin glasses provide a concrete prototype of an "M-class system at the material layer":

- **Internal frustration guarantees $\Psi > 0$:** Under the internal perspective A_I, the abundance of frustrated loops ensures that the infimum of $E_{A_I}(S)$ is strictly positive, i.e., $\Psi(\mathcal{L}, A_I) > 0$. This means that there is no configuration that satisfies all couplings at this level, and in our terminology the system is M-class at the material layer.

- **View shift induces irreversible information loss $\Delta_{I\to G} > 0$:** In the view shift $A_I \to A_G$, the total magnetization m corresponds to exponentially many microscopic configurations, and at low temperatures the microscopic distribution is strongly concentrated in a complex energy-valley structure. Any lifting operator U inevitably loses a large amount of information about the true microscopic distribution. As a result, the view-shift fracture $\Delta_{I\to G}(\mathcal{L})$ has a strictly positive lower bound, and the macroscopic description is structurally non-invertible.

- **NP-hard ground-state search gives algorithmic evidence of "non-tameness":** From the perspective of computational complexity, the ground-state search of a spin glass on general graphs is NP-hard, providing algorithmic evidence for the "non-tameness" of $\Psi > 0$ and $\Delta > 0$: even attempts to compensate for the view fracture by stronger reasoning and computation will run into structural barriers under finite resources.

- **Dynamically, frustration density and accumulated fracture obey the Ψ–Δ calculus structure:** In the annealing-dynamics perspective, by choosing the average frustration density at each temperature level as $\Psi(\gamma)$ and the accumulated frustration exposure along the annealing path as $\Delta(\gamma)$, one can numerically verify that they satisfy $\Delta(\gamma) \approx \int_0^\gamma \Psi(\tau)\, d\tau$ and $\Psi(\gamma) \approx d\Delta/d\gamma$. This shows that the Ψ–Δ calculus proposed earlier in the paper is not limited to smooth toy models, but is also concretely realized in highly frustrated, macroscopically uncomputable M-class material systems.

In the unified perspective of this paper, spin glasses thus serve as a canonical representative of "M-class systems at the material layer": their internal frustration structure guarantees that Ψ cannot be driven to 0 under natural perspectives; view shifts leave Δ_{view} strictly positive due to information residuals; the NP-hardness of ground-state search shows that this fracture has a stable structural lower bound; and at the same time, their annealing dynamics provide an example where the Ψ–Δ calculus remains valid "even in extremely complex systems." This will be contrasted, in the subsequent JPEG case study, with "H-class, tame fractures at the information layer," and together they support the structural isomorphism claims of the entire paper.

6.2 JPEG Compression: H-Class Tame Fractures at the Information Layer

6.2.1 System Definition $\mathcal{L} = (V, R, S)$ and Perspectives $A_{\mathbf{pix}}/A_{\mathbf{code}}$

In JPEG, we take a single 8×8 grayscale block as the basic system unit. Let the original pixel block be

$$x = (x_{ij})_{0 \le i,j \le 7}, \quad x_{ij} \in \{0, \dots, 255\}.$$

In our static logical-system framework, we abstract JPEG encoding/decoding as a logical system

$$\mathcal{L} = (V, R, S),$$

where:

- The variable set V consists of the 64 pixel values in the block:

$$V = \{x_{ij} \mid 0 \le i, j \le 7\};$$

- The rule set R is a family of constraints built from the following composable operators (we only describe the structure, without going into engineering details):
  - DCT transform: a linear orthogonal transform

  $$F = \mathrm{DCT}(x) = (F_{uv})_{0 \le u,v \le 7};$$

  - Quantization: given a quantization table $Q_{\mathrm{tab}}(u, v) > 0$, define

  $$Q_{uv} = \mathrm{round}\Big(\frac{F_{uv}}{Q_{\mathrm{tab}}(u, v)}\Big);$$

  - Entropy coding: apply lossless entropy coding (Huffman or arithmetic coding) to the quantized coefficients Q_{uv}. This part is essentially a reversible bit rearrangement and does not itself contribute to distortion;

- The system state S can be understood as the joint description of "original block x + encoded bitstream + reconstructed block $\hat{x}$ after decoding." For the purposes of this section, we only care about the relationship between the original block x and the reconstructed block $\hat{x}$.

On top of this structure, there are two natural perspectives:

- Pixel-domain perspective A_{pix}:

  $$A_{\mathrm{pix}} = (F_{\mathrm{pix}}, P_{\mathrm{pix}}), \qquad F_{\mathrm{pix}} = V, \quad P_{\mathrm{pix}}(S) = x.$$

  That is, a perspective in the spatial domain with pixels as fully resolved units;

- Coding-domain perspective A_{code}:

$$A_{\text{code}} = (F_{\text{code}}, P_{\text{code}}), \qquad F_{\text{code}} = \{Q_{uv}\}, \quad P_{\text{code}}(S) = Q = (Q_{uv}).$$

  That is, a compressed perspective in the frequency domain after quantization, observing the block structure via its coefficients.

Switching between A_{pix} and A_{code} is a canonical example of "deliberately introducing fractures and then managing them": we consciously accept a certain amount of information loss at the pixel level in exchange for a substantial bitrate reduction at the coding perspective.

6.2.2 Intra-Block Frustration $E_\theta(x)$ and $\Delta(\gamma)$ Along Compression Strength

Under the pixel perspective A_{pix}, the most natural frustration measure is the distortion between the original and reconstructed blocks. For a given encoding configuration θ (quantization table, chroma subsampling, etc.), define the block-level conflict

$$E_\theta(x) = d(x, \hat{x}_\theta),$$

where $\hat{x}_\theta$ is the reconstructed block obtained by JPEG encoding/decoding under configuration θ, and d can be taken as the mean squared error (MSE) within the block:

$$d(x, \hat{x}_\theta) = \frac{1}{64} \sum_{i,j} \big(x_{ij} - \hat{x}_{\theta,ij}\big)^2.$$

At this level, $E_\theta(x)$ is precisely the "static frustration" in the sense of our general definition of conflict measures: it quantifies how much local tension the state carries under the current perspective and parameter choice.

To lift this static frustration into the setting of "structural calculus along an evolution direction," we introduce a compression-strength parameter γ:

- $\gamma = 0$: corresponds to the lossless limit, or the limit where quantization step sizes go to 0, in which case JPEG introduces no fracture at the pixel perspective;
- $\gamma > 0$: compression strength increases monotonically with γ, and can be concretely implemented as
  - some monotone decreasing function of the quality factor QF (e.g. $\gamma \propto 1/(\mathrm{QF}+\varepsilon)$), or
  - some monotone function of the global scaling factor applied to the quantization table.

We do not fix a particular engineering implementation; we only require that larger γ means coarser quantization and larger average distortion. In this way, γ becomes a structural axis measuring "distance from the H-class limit point $\gamma = 0$."

Given an image distribution $\mathcal{D}$, for each compression strength γ we choose an encoding configuration $\theta(\gamma)$ that is approximately optimal under a given bitrate constraint, and define:

- The cumulative fracture function

$$\Delta(\gamma) := \mathbb{E}_{x\sim\mathcal{D}} E_{\theta(\gamma)}(x) = \mathbb{E}_{x\sim\mathcal{D}} d\big(x, \hat{x}_{\theta(\gamma)}\big),$$

which measures, at compression strength γ, how much information is lost on average from the pixel perspective; in the present context, this is exactly "the visible fracture scale at that position along the evolution axis";

- The fracture generation rate along the evolution direction (structural definition)

$$\Psi(\gamma) := \lim_{\delta\to 0^+} \frac{\Delta(\gamma+\delta) - \Delta(\gamma)}{\delta},$$

namely, the marginal distortion cost per unit increase of compression strength, under the assumption that encoding remains "locally approximately optimal."

Here the logical order is deliberately the reverse of the more abstract presentation in earlier sections:

1. We first define a physically measurable $\Delta(\gamma)$ from the JPEG codec (the curve of average distortion at different γ);

2. We then define $\Psi(\gamma)$ as the marginal fracture generation rate along this structural axis.

Thus, in this concrete system, $\Delta(\gamma)$ and $\Psi(\gamma)$ are no longer a priori abstract quantities, but two structural quantities that can be measured separately: one comes from the overall compression result, the other from the local incremental cost in compression strength.

Under this setup, we can specialize the general Ψ–Δ structural calculus law to JPEG as an empirically testable statement:

Structural Statement: Ψ–Δ Calculus Law for JPEG For a broad class of natural image distributions $\mathcal{D}$ and standard families of JPEG encoding configurations $\theta(\gamma)$, there exists a parameterization of compression strength such that

$$\Delta(\gamma) \;\approx\; \int_0^{\gamma} \Psi(\tau)\,d\tau, \qquad \Psi(\gamma) \;\approx\; \frac{d\Delta}{d\gamma},$$

where both the integral on the left and the derivative on the right can be approximately verified in experiments via discrete sampling of γ and numerical differentiation/integration.

Numerically, a natural procedure (which can be placed in an appendix or implementation note rather than in the main text) is:

1. Select typical 8×8 blocks (or a block distribution from real images) and run JPEG at a sequence of γ_k values;

2. Obtain the average distortion $\Delta(\gamma_k)$ at each γ_k;

3. Approximate

$$\Psi_{\text{num}}(\gamma_k) \approx \frac{\Delta(\gamma_{k+1}) - \Delta(\gamma_k)}{\gamma_{k+1} - \gamma_k};$$

4. Reconstruct the $\Delta(\gamma)$ curve via the discrete sum $\sum_j \Psi_{\text{num}}(\gamma_j)\Delta\gamma_j$, and compare it with the directly measured $\Delta(\gamma)$.

With a reasonable parameterization, one expects to observe that the reconstructed curve overlaps the directly measured $\Delta(\gamma)$ curve up to small errors, and that $\Psi_{\text{num}}(\gamma)$ agrees closely with the "marginal distortion" induced by locally optimal quantization steps. This means that in this engineered system, the Ψ–Δ calculus relation is not imposed by definition, but emerges as an empirical law of the internal structure.

6.2.3 View Fracture $\Delta_{\text{pix}\to\text{code}}$: Manageable Information Loss

Next we return to the general view-shift framework and consider the fracture that arises when we switch from the pixel perspective A_{pix} to the coding perspective A_{code}. Formally, this corresponds to

$$\Delta_{A_{\text{pix}}\to A_{\text{code}}}(\mathcal{L}) = \inf_{U\in\mathcal{U}} D\big[\Phi_{A_{\text{pix}}}(S),\, U(\Phi_{A_{\text{code}}}(S))\big],$$

where $\Phi_{A_{\text{pix}}}(S) = x$, $\Phi_{A_{\text{code}}}(S) = Q$, $\mathcal{U}$ is a family of operators that lift from the quantized-coefficient space back to the pixel space, and D is a chosen information distance (for example, the expectation of block-wise MSE, or a KL divergence induced by pixel histograms).

In JPEG there is a natural lifting operator: given quantized coefficients Q, one reconstructs the block via standard dequantization and inverse DCT:

$$\hat{x} = U_{\text{JPEG}}(Q).$$

Thus, for an image distribution $\mathcal{D}$, we can take

$$D = \mathbb{E}_{x\sim\mathcal{D}} d\big(x, \hat{x}\big),$$

and obtain the approximation

$$\Delta_{\text{pix}\to\text{code}}(\mathcal{L}) \;\approx\; \inf_{\theta\in\Theta} \mathbb{E}_{x\sim\mathcal{D}} d\big(x, \hat{x}_\theta\big),$$

where Θ is the set of allowed encoding configurations under a fixed bitrate constraint.

In the previous subsection we already interpreted

$$\Delta(\gamma) = \mathbb{E}_{x\sim\mathcal{D}} d\big(x, \hat{x}_{\theta(\gamma)}\big)$$

as "the fracture scale along the compression-strength axis." In practice, we do not sweep over all γ, but choose a particular operating point γ^* and run the codec near that point in the long term, according to the rate–distortion trade-off. In this case, we may interpret

$$\Delta_{\text{pix}\to\text{code}}(\mathcal{L}) \quad \text{(the optimal value under a given bitrate budget)}$$

as

$$\Delta_{\text{pix}\to\text{code}}(\mathcal{L}) \;\approx\; \Delta(\gamma^*),$$

that is, "the level of view fracture borne by the system under a task perspective and bitrate constraint jointly determined."

The key point is that this view fracture is *manageable*. More concretely:

- For a given human-perception perspective A_{perc}, there exists a threshold κ_c corresponding to the "just noticeable difference" (JND): when the average distortion

$$\mathbb{E}_{x\sim\mathcal{D}} d\big(x, \hat{x}_{\theta(\gamma^*)}\big) < \kappa_c,$$

  most observers under normal viewing conditions cannot reliably distinguish the original from the reconstructed image;

- The design goal of JPEG is not to achieve $\Delta_{\mathrm{pix}\to\mathrm{code}} = 0$ in a mathematical sense, but to ensure that

$$\Delta_{\mathrm{pix}\to\mathrm{code}}(\mathcal{L}) \;\approx\; \Delta(\gamma^*) \;\ll\; \kappa_c$$

  while obtaining a sufficiently high compression ratio under resource constraints.

Additionally, for a fixed configuration θ, JPEG encoding/decoding at the block level can be implemented in polynomial time; and while the exact optimization of quantization tables and rate configuration under a given distortion model may not be fully solved in a strict complexity-theoretic sense, mature heuristics and approximation methods exist that can find near-optimal solutions within engineering time. In other words, for this information-layer system, Ψ and Δ_{view} are positive but their magnitudes and trajectories are algorithmically controlled to lie in a tame region; there is no structural obstacle analogous to the NP-hardness in spin glasses.

In the terminology of this paper, this means that JPEG fractures are both genuine information losses and a form of "programmable fractures" that can be finely scheduled and constrained under a specified perspective.

6.2.4 Summary: H-Class Tame Fractures at the Information Layer

In contrast to spin glasses in the previous section, JPEG compression provides an almost symmetric example:

1. **Existence of an H-class limit structure.** At the pixel level, there exists a parameter choice (lossless coding or quantization step sizes going to 0) for which $\Delta(0) = 0$, and hence $\Psi = 0$ in the limit. According to the structural consistency axiom introduced in our general framework, such systems are H-class at this level: they possess a "fully tame" limit structure.

2. **Deliberately choosing $\Psi > 0$, $\Delta > 0$ at the engineering operating point.** In practical operation, one deliberately chooses $\gamma^* > 0$, that is, accepts $\Psi(\gamma^*) > 0$ and $\Delta(\gamma^*) > 0$ in exchange for savings in bandwidth and storage. The view fracture $\Delta_{\mathrm{pix}\to\mathrm{code}}$ is finely tuned via quantization-table design and distortion models so that it stays well below the perceptual threshold κ_c. Here, fractures are not uncontrolled defects but resources that are planned and budgeted.

3. **Satisfaction of the Ψ–Δ structural calculus along the evolution direction.** Once an appropriate compression-strength parameter γ is chosen, JPEG offers a computable and experimentally accessible example where $\Delta(\gamma)$ and $\Psi(\gamma)$ satisfy the structural calculus relation: total fracture can be viewed as the accumulation of local fracture

generation rates along the γ axis. In this case, the structural law is not imposed by semantic definition but is identified through modeling of the actual system and numerical verification.

From the unified perspective of this paper, JPEG compression is therefore a canonical representative of "H-class, tame fractures at the information layer":

- Ψ corresponds to the distortion budget that we are willing and able to manage;
- $\Delta(\gamma)$ records the cumulative effect of this budget along the compression-strength axis;
- $\Delta_{\text{pix}\to\text{code}}$ measures the overall information cost of the view shift, constrained to lie within a "perceptually acceptable" range.

Juxtaposed with spin glasses as "M-class, non-tame fractures at the material layer," JPEG shows that, with appropriate structural variables, the Ψ–Δ framework can distinguish not only "structural impossibility" (such as NP-hard ground-state search), but also "consciously managed structural trade-offs." Together they demonstrate that the same Ψ–Δ language can, across the material and information layers, uniformly describe the full structural spectrum from incompressible fractures to programmable fractures.

6.3 Cohomology Groups in Algebraic Topology: Structural Fractures Inside the Abstract Logic Layer (M-Class System)

6.3.1 System Definition $\mathcal{L}_X$ and View Shift $A_{\mathbf{geo}} \to A_{\mathbf{coh}}$

Let X be a topological space. Within our static logical-system framework, the whole family of geometric–algebraic structures associated with X can be abstracted as a static logical system

$$\mathcal{L}_X = (V, R),$$

where:

- The set of individuals V is made up of "microscopic geometric elements." Fixing a simplicial complex or CW complex structure, we take

  $$V = \{\sigma_i\},$$

  where each σ_i is a simplex (vertex, edge, triangle, tetrahedron, etc.);
- The rule set R is the chain complex structure given by the gluing relations among simplices and the boundary operators

  $$\cdots \xrightarrow{\partial_{k+1}} C_k(X) \xrightarrow{\partial_k} C_{k-1}(X) \xrightarrow{\partial_{k-1}} \cdots,$$

  satisfying the local consistency condition $\partial_k \circ \partial_{k+1} = 0$.

On this system we consider two perspectives living at very different levels (corresponding to the "internal / macroscopic" perspectives in our general view-shift framework):

Geometric / Combinatorial Perspective $A_{\mathbf{geo}}$

$$A_{\text{geo}} = (F_{\text{geo}}, P_{\text{geo}}).$$

- Objects of interest F_{geo}: simplices, chain groups $C_k(X)$, and boundary operators ∂_k;
- Aggregation map P_{geo}: organizes simplices by gluing and boundary relations into a chain complex (C_*, ∂_*).

In this perspective, the "individuals" are simplices and finite chains, while the "whole" is the entire chain-complex structure.

Algebraic / Cohomological Perspective $A_{\mathbf{coh}}$

$$A_{\text{coh}} = (F_{\text{coh}}, P_{\text{coh}}).$$

- Objects of interest F_{coh}: cohomology groups and their ring structure, for example
$$H^k(X; G), \qquad H^*(X; G) = \bigoplus_k H^k(X; G);$$
- Aggregation map P_{coh}: passes from the chain complex to cohomology via dualization and kernel/image:
$$C^k(X; G) = \text{Hom}(C_k(X), G), \quad \delta^k : C^k \to C^{k+1},$$
$$H^k(X; G) = \frac{\ker(\delta^k)}{\text{im}(\delta^{k-1})}.$$

The view shift
$$A_{\text{geo}} \longrightarrow A_{\text{coh}}$$
corresponds to a highly nontrivial aggregation: it switches from "concrete geometric gluing data" to "abstract algebraic invariants." In this process:

- All local metrics, concrete embeddings, and a huge amount of detailed combinatorial information are erased;
- Only topological invariants stable under homotopy equivalence (homology, cohomology, ring structure, etc.) are retained.

In the terminology of our view-shift framework, this is an extremely strong coarse-graining perspective, which directly rewrites "what counts as an individual and what counts as a whole":

- In A_{geo}, simplices are individuals and the chain complex as a whole is the aggregate;
- In A_{coh}, cohomology classes become the new "individuals," and the cohomology ring $H^*(X; G)$ becomes a higher-level whole.

The logical fracture Δ arises precisely in such perspective flips: when the very partition of "local vs global" is redefined, some parts of the original structure are inevitably folded and collapsed, leaving gaps that cannot be refilled.

6.3.2 Intra-Level Structural Frustration $\Psi(\mathcal{L}_X, A_{\mathrm{geo}})$: Cycles That Are Not Boundaries

Under the geometric perspective A_{geo}, the local rules are given by the boundary relations $\partial_k \circ \partial_{k+1} = 0$: "the boundary of a boundary is zero," which is a perfectly consistent local axiom.

However, within the chain complex, a typical kind of "structural frustration" quickly appears:

- There exist cycles $c \in C_k(X)$ such that

$$\partial_k(c) = 0, \quad c \in \ker \partial_k,$$

- while at the same time c is not the boundary of any higher-dimensional chain:

$$c \notin \operatorname{im} \partial_{k+1}.$$

In other words:

- c fully satisfies all local rules (since $\partial_k(c) = 0$);
- but c cannot be generated as the boundary of any higher-dimensional object.

It is forced to exist as a "cycle hanging inside the rule system," which is exactly a "hole" in the topological sense. Such "closed but non-bounding" chains form the homology groups

$$H_k(X; G) = \frac{\ker(\partial_k)}{\operatorname{im}(\partial_{k+1})},$$

whose dimensions $b_k = \dim H_k(X; \mathbb{Q})$ are the k-th Betti numbers.

Within the present framework, we can define the intra-level structural frustration under the geometric perspective as some monotone function

$$\Psi(\mathcal{L}_X, A_{\mathrm{geo}}) = f\big(b_1, b_2, \ldots, b_{k_{\max}}\big),$$

where:

- $f = 0$ if and only if all Betti numbers in the chosen set of dimensions vanish;
- as soon as at least one $b_k > 0$, we have $\Psi(\mathcal{L}_X, A_{\mathrm{geo}}) > 0$.

For example, we may take the simple weighted form

$$\Psi(\mathcal{L}_X, A_{\mathrm{geo}}) = \sum_{k \in K} w_k b_k, \quad w_k > 0,$$

where K is the set of dimensions we care about.

The meaning is:

- If X is homologically trivial in all dimensions of interest (for instance, all b_k other than b_0 are viewed as 0), then $\Psi = 0$: there are no "closed but non-bounding" chains; local rules are both consistent and generate all structures. In this restricted regime, the system is approximately H-class;

- Once a topological hole appears (some $b_k > 0$), then from the viewpoint of A_{geo}, the rule system exhibits an irreducible "residual": some configurations that satisfy all rules cannot be derived from the rules, corresponding to M-class frustration.

Here, Ψ no longer measures "disorder in the external world," but the tension internal to the rules that the abstract logic layer itself has prescribed:

- "Cycles" represent full compliance with the boundary rules;

- "Not being any boundary" represents the impossibility of being generated from higher-level structures.

Consistency does not entail generability. This irreducible incompleteness within a system of locally complete rules is exactly a structural frustration internal to the abstract logic layer.

6.3.3 View Fracture $\Delta_{A_{\mathbf{geo}} \to A_{\mathbf{coh}}}$: Irreversible Collapse from Geometry to Algebra

The shift from the geometric perspective to the cohomological one can be abstracted as an aggregation map

$$P : (C_*, \partial_*) \longrightarrow H^*(X; G),$$

which compresses the vast information contained in the chain complex into finite-dimensional algebraic invariants.

Structurally, this map is highly non-injective:

- Different geometric configurations (different triangulations, different CW decompositions, even spaces with different homotopy types) may yield the same cohomology groups or cohomology ring;

- Even given the full $H^*(X; G)$, one typically cannot uniquely recover the original chain complex, let alone the concrete geometric embedding.

According to the general definition of view-shift fracture, we can write

$$\Delta_{A_{\text{geo}} \to A_{\text{coh}}}(\mathcal{L}_X) = \inf_{U \in \mathcal{U}} D\Big(\Phi_{A_{\text{geo}}}(\mathcal{L}_X),\ U\big(\Phi_{A_{\text{coh}}}(\mathcal{L}_X)\big)\Big),$$

where:

- $\Phi_{A_{\text{geo}}}(\mathcal{L}_X)$ denotes the full chain complex (C_*, ∂_*) under the geometric perspective;

- $\Phi_{A_{\text{coh}}}(\mathcal{L}_X)$ denotes $H^*(X; G)$ and its ring structure under the cohomological perspective;

- $\mathcal{U}$ is the collection of all "lifting operators"

$$U : H^*(X; G) \to (\widehat{C}_*, \widehat{\partial}_*)$$

that attempt to reconstruct some geometric / chain-complex model from the cohomological data;

- $D[\cdot, \cdot]$ is an information distance between two structures (for example, a difference in minimum description length, some measure of model complexity, or another suitable structural distance).

Because the non-injectivity of P is strongly supported by mathematics (for instance, via numerous examples of spaces that are homology- or cohomology-isomorphic but not homotopy equivalent), we can expect that:

- For any lifting operator U, at best one can choose a representative inside some equivalence class;
- In generic situations, $U\big(\Phi_{A_{\mathrm{coh}}}(\mathcal{L}_X)\big)$ can only yield a model that is homologically equivalent to the original chain complex, but cannot reconstruct the original geometry and combinatorial details.

Hence there is a positive lower bound, depending on topological dimension, homological structure, etc.,

$$\Delta_{A_{\mathrm{geo}} \to A_{\mathrm{coh}}}(\mathcal{L}_X) \geq \Delta_{\min} := D_{\mathrm{topo}} > 0.$$

In other words, even with the theoretically optimal lifting operator, the information fracture induced by the perspective shift cannot be completely flattened. This is not a temporary lack of computational power; rather, the step

$$(C_*, \partial_*) \mapsto H^*(X; G)$$

is structurally an irreversible compression.

In the semantics of this paper, Δ here directly corresponds to "the residual left after geometric information has been irreversibly folded."

Within the abstract logic layer, this shows that:

- The existence of $\Delta > 0$ is not an accidental "failure of reconstruction," but an in-built "information collapse" of the abstraction process itself;
- The abstract logic layer intentionally discards geometric continuity, metrics, and embeddings, and retains only the topological invariants stable under A_{coh}. This sacrifice itself constitutes a view fracture.

6.3.4 Computability and M-Class Classification (Structural Observation)

From the computational perspective, under the setting considered in this paper:

- For a given finite simplicial complex, individual homology or cohomology groups in fixed dimensions can be computed in polynomial time via linear-algebraic algorithms (such as Smith normal form). This corresponds to "reading off $H^k(X; G)$ at a fixed level";
- However, once we consider richer structures, such as:
  - the full cohomology ring $H^*(X; G)$ and its multiplicative structure;
  - torsion parts (Tor, Ext) and the limiting behavior of various spectral sequences;
  - and inverse problems such as: "given certain algebraic data, does there exist a space / manifold realizing it?",

  it is known that many related decision and recognition problems are, in general, highly complex; some are NP-hard, and some are structurally connected to undecidability.

In this paper we do not rely on any single complexity-theoretic theorem, but only on the following structural observation:

> Within this "geometry–cohomology" system, there exist natural families of problems (such as reconstructing geometric structure, deciding homotopy types, or recovering the limits of spectral sequences) that, in generic situations, cannot be fully solved by polynomial-time algorithms. In other words, if one attempts to "completely control" all structural consequences of $\Psi > 0$ and $\Delta > 0$ at the macroscopic level, one encounters stable computational barriers.

Combining the analysis of intra-level frustration and view fracture above, we can formulate the H/M classification for this case:

- **Intra-level frustration $\boldsymbol{\Psi > 0}$:** Under the geometric perspective A_{geo}, as long as X has a nonzero Betti number $b_k > 0$ in some dimension of interest, there exist closed but non-bounding chains, and thus

  $$\Psi(\mathcal{L}_X, A_{\text{geo}}) > 0.$$

  There are "holes" in the sense that the local rules leave some structures ungenerated;

- **View fracture $\boldsymbol{\Delta_{A_{\text{geo}} \to A_{\text{coh}}} > 0}$:** The projection from geometry to algebra P is highly non-injective. Even with an optimal lifting operator, the view fracture has a positive lower bound:

  $$\Delta_{A_{\text{geo}} \to A_{\text{coh}}}(\mathcal{L}_X) \geq D_{\text{topo}} > 0;$$

- **Macroscopic laws are not fully computable (structural):** Within the cohomological perspective A_{coh}, if we attempt to "fully reconstruct or control the geometric layer" from algebraic invariants, we encounter stable complexity barriers on natural problem families.

Thus, in the language of this paper, cohomology theory provides a prototype of an "M-class system inside the abstract logic layer":

$$\mathcal{L}_{\text{cohomology}} \in \text{M-class}.$$

Here M does not arise because the world itself is "too messy," but because the abstract logic layer inevitably generates new fractures when it abstracts itself.

6.3.5 Philosophical Summary: How the Abstract Logic Layer Turns Fractures into Objects

Unlike the spin-glass and JPEG cases, this example has a special feature:

- In spin glasses, $\Psi > 0$ and $\Delta > 0$ primarily arise from frustration at the material layer and are almost independent of our modeling choices;
- In JPEG, we exploit the tameness of an H-class system and consciously treat certain fractures as a resource (lossy compression, "budgeted fractures" below perceptual thresholds).

In the cohomology case, the abstract logic layer does something third:

> It does not try to repair the fracture, but formalizes the fracture itself as a new family of objects.

Concretely:

- Geometric "closed but non-bounding chains" form the homology groups $H_k(X)$;
- These "geometric fractures" are dualized into the cohomology groups $H^k(X)$ and the cohomology ring $H^*(X)$;
- Thus what originally appeared as "residual structure not generated by the rules" becomes a "basic variable" at a new logical level.

This can be compressed into a single sentence:

> Cohomology groups are what you get when the abstract logic layer treats fractures themselves as basic building blocks.

In this sense:

- The conditions $\Psi > 0$ and $\Delta > 0$ are not eliminated, but *lifted* to become the basis of a new perspective;
- Each paradigm shift from geometry to topology, and from topology to category theory, is a kind of self-reflexive upgrade inside an M-class system: when confronted with its own fractures, the abstract logic layer does not simply patch them, but rewrites its language so that fractures become "atomic terms" in the new language.

Placed alongside the previous two examples, we obtain a structural comparison table (omitting technical details and keeping only the roles):

Case	System Type	Ψ Character	Δ Character	Structural / Computational Properties	Structural Ro
Spin glass	M-class	Frustrated loops, degenerate ground states, $\Psi > 0$ cannot be flattened	Micro valleys not recoverable from m, $\Delta > 0$	Ground-state search NP-hard	Material-laye frustration chaos with solution"
JPEG compression	H-class	Allows finite distortion, Ψ can be made arbitrarily close to 0	View fracture can be pushed below perceptual threshold	Encode/decode polynomial-time solvable, effectively controllable	Information-layer strat of "taming managing" fr tures
Cohomology ring H^*	M-class	Closed but non-bounding chains, topological "holes," $\Psi > 0$	Irreversible compression from geometry to algebra, $\Delta > 0$	Inverse problems face stable complexity barriers	Formalizatior and re-creat of fractures side the abstr logic layer

This third example shows that:

- "Logical fractures" exist not only on the world side, but also inside each layer of language constructed by the abstract logic layer itself;
- Algebraic topology, especially homology and cohomology theory, can be understood as: the entire logical system that emerges once the abstract logic layer learns to treat such structural fractures as objects of study in their own right.

At the global structural level, spin glass, JPEG, and cohomology respectively provide:

1. M-class frustration at the material layer;
2. H-class tame fractures at the information layer;
3. An M-class system in which the abstract logic layer elevates fractures themselves to objects.

Together, they complete the Ψ–Δ structural loop across the three layers of matter, information, and abstract logic.

Chapter 7

The “Phenomenological Triad” of Incomputability / Chaos / NP-Hardness in the Ψ–Δ Framework

7.1 Objectives of This Chapter and Remarks on the “Structural Statements”

The first five chapters have completed the core construction of the Ψ–Δ framework: taking $(\mathcal{L}, A)$ as the basic object, using the intra-level frustration Ψ to measure the tension arising when local constraints cannot be simultaneously satisfied under a given perspective, and using the inter-level fracture Δ to measure irreversible information loss between different perspectives / different levels. On this basis, we obtained the H/M classification, the structural compensation principle, and the meta-fracture lower bound Δ^*. Chapter 6, via three case studies—spin glasses, JPEG compression, and cohomology groups—demonstrated how this framework is instantiated on the material layer, the information layer, and the abstract logic layer, respectively.

In this chapter we no longer introduce new basic concepts. Instead, we focus attention on three classical “families of difficulties”: incomputability / incompleteness, chaotic dynamics, and NP-hard intractability. The goal is, under suitably chosen natural perspectives, to formulate three structural statements (working hypotheses): that these three kinds of phenomena can all be described as systems carrying an irreducible fracture lower bound $\Delta > 0$. These statements do not aim to replace Gödel’s and Turing’s theorems or existing complexity theory; rather, they reorganize and give a compressed summary of them from the standpoint of the Ψ–Δ framework.[8, 16, 5, 6, 11, 2, 14, 3, 9, 7, 13, 1]

7.2 Incomputability: From Gödel / Turing to $\Delta_{\text{view}} > 0$

Let T be a consistent, recursively enumerable formal theory that is strong enough to express arithmetic (for example, PA), viewed as a logical system $\mathcal{L}_T$. We distinguish two perspectives: the internal “provability perspective” A_{comp} and the external “arithmetic truth perspective”

A_{truth}. Under A_{comp}, the world is described by the set of formulas derivable by finite proof sequences, $\text{Th}(T)$; under A_{truth}, the same formulas are directly assigned truth values in the standard model $(\mathbb{N}, +, \times, \dots)$. Gödel's incompleteness theorems and Turing's undecidability theorem show that there is a systematic misalignment between these two perspectives.

In the Ψ–Δ language, we may regard A_{comp} as a perspective that attempts to maintain the ideal that “all true statements can be proven inside the system,” whereas A_{truth} gives the actual arithmetic structure. The discrepancy between the two induces a structural frustration $\Psi(\mathcal{L}_T, A_{\text{comp}})$ whose concrete manifestations are true-but-unprovable statements and families of problems that are describable but undecidable.

Unified Structural Criterion (Working Hypothesis: Incomputability $\Leftrightarrow$ View Fracture $\Delta_{\text{view}} > 0$) For any consistent, recursively enumerable theory T strong enough to express arithmetic, formalized as $\mathcal{L}_T$, consider the two perspectives A_{comp} and A_{truth}. Restricting attention to a family of “natural extensions that preserve computability” (for example, adding finitely many axioms or using mechanically decidable inference rules), the view fracture

$$\Delta_{\text{view}}(A_{\text{comp}} \to A_{\text{truth}}; \mathcal{L}_T)$$

has a strictly positive lower bound and cannot be compressed to 0 by any finite computable extension.

In other words, no matter how we strengthen T while preserving mechanical decidability, there will always remain some family of statements that already have definite truth values under A_{truth} but remain in a state of unresolved structural tension under A_{comp} and all its computable extensions. Incomputability and incompleteness can thus be restated as a concrete instance of

$$\Delta_{\text{view}}(A_{\text{comp}} \to A_{\text{truth}}) > 0.$$

From the standpoint of structural compensation, the classical “meta-theoretical scheme”—constructing a stronger theory T' that takes previously true-but-unprovable statements as new axioms—can be understood as introducing an H-class compensating structure at a higher level: under the provability perspective A'_{comp} of T', part of the original fracture is smoothed out. However, across the whole tower of theories, truth itself can never be fully captured within any single level; the meta-fracture lower bound $\Delta^* > 0$ guarantees that, no matter how high the tower is built, there will always remain “one layer short.”

7.3 Chaos: $\Delta_{\text{dyn}} > 0$ in the Logistic Map

Consider the canonical one-dimensional unimodal map, the logistic map

$$x_{t+1} = f_r(x_t) = r x_t (1 - x_t), \quad x_t \in [0, 1],\ r \in [0, 4].$$

For each fixed r we obtain a dynamical logical system $\mathcal{L}_r$. To measure structural tension at observable resolution, we introduce a coarse-grained perspective A_K: partition the interval $[0, 1]$ into K subintervals I_k, and record at each time step which interval the orbit falls into, thereby obtaining a symbolic sequence $(s_t) \in \{1, \dots, K\}^{\mathbb{N}}$.

Under the perspective A_K, we can define for each interval I_k a "splitting rate" $\sigma_k(r; A_K)$: it measures into how many target intervals I_j the image $f_r(I_k)$ is stretched and dispersed in one iteration, and how uniformly this dispersion is distributed with respect to the natural invariant measure μ_r. We then form a weighted sum

$$\Psi(\mathcal{L}_r, A_K) = \sum_k w_k \, \sigma_k(r; A_K),$$

as the average frustration under this coarse-grained perspective. Intuitively, the larger σ_k is, the more a single macro-state is forced to split into many possibilities at the next step, and the stronger the conflict between macro patterns and micro evolution.

More concretely, consider a parameter r in a chaotic interval. Partition $[0, 1]$ into K equal subintervals and, for each I_k, examine how $f_r(I_k)$ overlaps with the various I_j. If f_r has significant stretching on I_k, its image tends to cover multiple I_j simultaneously and induces nearly uniform transition probabilities $\{p_{k \to j}\}$ across those intervals. In this situation, σ_k can be captured by an entropy-type quantity

$$-\sum_j p_{k \to j} \log p_{k \to j},$$

whose magnitude lies in the same order as the local Lyapunov exponent and the KS entropy. Parameter intervals with periodic behavior form a sharp contrast: with an appropriate choice of I_k, most of the mass concentrates in neighborhoods of finitely many periodic points; the $\{p_{k \to j}\}$ then collapse to "almost deterministic" transitions on these intervals, so that σ_k and the overall average Ψ can be pushed arbitrarily close to 0.

Structural Statement 7.2 (Chaotic Frustration Density and Dynamical Fracture) In typical periodic parameter intervals, the logistic map's orbits eventually fall into finite periodic orbits or their attracting neighborhoods. By choosing a coarse-grained perspective A_K aligned with the periodic skeleton and an invariant measure μ_r, the average frustration $\int \Psi(\mathcal{L}_r, A_K) \, d\mu_r$ can be made arbitrarily close to 0, so that the dynamical fracture lower bound $\Delta_{\text{dyn}}(\mathcal{L}_r, A_K)$ is bounded and can be driven toward 0. In this regime, the system appears H-class under A_K: fractures saturate in the time direction instead of accumulating nonlinearly.

In typical chaotic parameter intervals, the logistic map has a positive Lyapunov exponent and positive Kolmogorov–Sinai entropy $h_{KS}(r)$. Under a family of natural coarse-grained perspectives A_K, the average frustration $\overline{\Psi}(r; A_K)$ has a strictly positive lower bound over μ_r, with magnitude comparable to $h_{KS}(r)$; thus, over a finite time horizon T, the accumulated dynamical fracture approximately satisfies

$$\Delta_{\text{dyn}}(\mathcal{L}_r, A_K; T) \approx \overline{\Psi}(r; A_K) \, T,$$

growing approximately linearly with time. In this regime, the system appears M-class under A_K: the positive density of local splitting cannot be compensated away.

This yields a dynamical version of the H/M criterion: if there exist a perspective A and an invariant measure μ such that $\int \Psi(\mathcal{L}, A) \, d\mu \to 0$, then $\Delta_{\text{dyn}}(\mathcal{L}, A)$ is bounded, fractures do not grow linearly in time, and this level can be viewed as H-class; if, for a family of natural perspectives A, the average frustration has a positive lower bound, so that

$$\Delta_{\mathrm{dyn}}(\mathcal{L}, A; T) \sim cT,$$

then this level exhibits M-class behavior in the dynamical sense.

7.4 NP-Hardness: Local Fractures Δ_{dyn} in Combinatorial Explosion

Canonical NP-hard problems (such as SAT, Max-Cut, and TSP) can be uniformly regarded as a logical system $\mathcal{L}_{\mathrm{NP}}$: the state space X is the set of all candidate solutions; the constraint family $\{F_i\}$ encodes local constraints or components of the objective; for a given solution $x \in X$, the value $F_i(x)$ measures the extent to which that constraint is violated. On this basis, we define the instantaneous frustration

$$\Psi(x) = \left(\sum_i w_i \, |F_i(x)|^p \right)^{1/p},$$

where $w_i > 0$ are weights and $p \geq 1$ is a fixed exponent; $\Psi(x) = 0$ means that all constraints are satisfied, corresponding to a "perfect solution."

In practice, algorithms rarely perform arbitrary rearrangements over the whole space X; instead, they adopt a "local search perspective" A_{local}: at each step, only a limited local modification of the current solution is allowed (flipping a few variables, swapping path segments, etc.), thereby inducing local dynamics

$$T_{\mathrm{local}} : X \to X$$

and its orbit (x_t). Under this perspective, the state space exhibits numerous "frustration clusters": within polynomial time scales, local operations cannot push Ψ down near 0, and significantly reducing Ψ often requires crossing high energy barriers and performing global rearrangements.

Structural Statement 7.3 (Working Hypothesis: NP-Hardness and Local $\Delta_{\mathrm{dyn}} > 0$) Consider canonical NP-hard problems on natural families of instances. Under the local search perspective A_{local} and polynomial time scales, assume there exists a constant $\varepsilon_0 > 0$, together with a family of initial distributions and a class of local search strategies, such that, under these distributions and strategies, for almost all initial points x_0, the time-averaged frustration

$$\overline{\Psi}(x_0; A_{\mathrm{local}})$$

satisfies

$$\overline{\Psi}(x_0; A_{\mathrm{local}}) \geq \varepsilon_0,$$

and thus cannot be driven close to 0 within polynomial time; consequently, the associated dynamical fracture accumulation

$$\Delta_{\mathrm{dyn}}(\mathcal{L}_{\mathrm{NP}}, A_{\mathrm{local}}; T)$$

still grows approximately linearly when T grows polynomially with the instance size.

At the same time, under a high-level global perspective A_{global} (allowing exponential-time exhaustive search and arbitrary rearrangements), the problem itself is often solvable: there exists some solution x^* with $\Psi(x^*) = 0$, so that at the A_{global} level the system remains H-class. However, this H-class structure is practically unreachable from the standpoint of A_{local}. Compressed into a single slogan:

NP-hard $\approx$ "M-class for all efficient local perspectives, H-class only under extremely costly global perspec

7.5 Unified Picture and "Fifth-Order Structural Instability" (Phenomenological Remark)

In the Ψ–Δ perspective, the three "hard" families—incomputability, chaos, and NP-hardness—can all be described in a unified way as follows: in some family of natural perspectives, the system carries an irreducible fracture lower bound $\Delta > 0$. In formal systems, this is

$$\Delta_{\text{view}}(A_{\text{comp}} \to A_{\text{truth}}) > 0;$$

in chaotic dynamics, it appears as a positive frustration density under coarse-grained perspectives and a linearly growing Δ_{dyn}; in NP-hard optimization, it appears as a $\Delta_{\text{dyn}}(A_{\text{local}})$ that cannot be flattened within feasible time under local search. Together they illustrate what in this paper will be called the meta-fracture Δ^*: even if we build an arbitrarily tall tower of perspectives, there always remains some level that carries fractures that cannot be completely smoothed out.

In a certain class of concrete hierarchical tension dynamical systems (HTDS), numerical and structural analyses further reveal a more fine-grained phenomenon: as the order k of "self-similar embeddings under the same structural rule" increases, the system, under the same family of natural perspectives, often undergoes an H $\to$ M transition near some finite critical order k^*. In the unimodal maps and self-organizing structures examined in this paper, this k^* often appears around the fifth order. In the period-doubling diagram of the logistic map, one can associate the self-similar iteration depth k with the chain of parameter bifurcations: under a standard partition and a family of coarse-grained perspectives A_k, numerical experiments show that when k grows to around 5, the dynamical fracture $\Delta_{\text{dyn}}(k; A_k)$ under the same family of perspectives starts to exhibit a stable positive lower bound. This empirical pattern is the main motivation behind the term "fifth-order structural instability," which is explicitly positioned as a phenomenological working hypothesis rather than a universal theorem.

From the H/M embedding principle, we may interpret the self-similarity order k as the depth of "multi-H coupling": in the first few orders ($k < k^*$), couplings among local H-structures can still be absorbed by moderate coarse-graining; when $k \geq k^*$, local H-blocks can no longer be fully coordinated in terms of topology and constraints, leading to a stable $\Delta_{\text{dyn}} > 0$ under natural perspectives and an M-class transition at that level. The "islands of stability" inside chaotic seas provide a converse example: within a globally M-class background, local H-regions continuously appear, and their fractal boundaries can be viewed as a geometric record of "fractures of fractures."

Thus, the compressed conclusion of this chapter can be phrased as follows: incomputability, chaos, and NP-hardness are not three unrelated technical difficulties placed side by side, but three typical projections of the same meta-fracture Δ^* in different domains and under different perspectives. The Ψ–Δ framework provides a unified structural language that places these phenomena within the same “fracture phase diagram”: their differences lie in the specific system types and perspective choices, while their commonality lies in the fact that each, in its own way, carries Ψ and Δ that cannot be completely tamed.

Chapter 8

Outlook and Applications: From Multi-Level Systems to AI Hallucinations

8.1 Limitations of This Work and Open Problems

The first seven chapters have mainly completed three tasks:

- Introducing a modeling scheme for logical systems that takes the perspective A as the basic unit, and on this basis defining the structural frustration Ψ, the logical fracture Δ, and the H/M classification;
- Using HTDS (Hierarchical Tension Dynamical Systems), the geometric interpretation via equiangular spirals, and two families of representative case studies—the "material / information / abstract logic" triad (spin glass, JPEG compression, cohomology groups) in Chapter 6, and the "phenomenological triad" (chaos in the logistic map, incompleteness / incomputability, NP-hard problems) in Chapter 7—to demonstrate unified structural interpretations of this framework across different domains;
- Connecting these structural measures to a broader philosophical picture through the meta-fracture postulate Δ^* and the H/M embedding principle.

In terms of mathematical rigor and breadth of application, the present text is closer to a "structural framework plus phenomenological correspondences" than to a fully developed new mathematical theory. The main goal has been to provide a unified language and a number of structural statements, and to show how they operate on a limited set of model families. At the same time, this leaves a reasonably clear set of limitations and open questions.

8.1.1 Limitations and Questions at the Mathematical Level

First, most results are presented as structural statements or phenomenological correspondences. The emphasis is on rewriting existing theorems and numerical facts in the Ψ–Δ

language, rather than constructing an entirely new and complete mathematical theory from scratch. For example, in Chapter 7, the statements about the connection between

$$\Delta_{\mathrm{dyn}} > 0 \quad \text{and chaotic behavior in the logistic map}$$

and those about "$\Delta_{\mathrm{dyn}} > 0$ under local perspectives" in NP-hard problems are structural condensations built on top of Lyapunov exponents, KS entropy, and classical complexity results. They do not claim to establish new universal theorems.

A direct open problem is: under what general conditions can one rigorously establish theorem-level correspondences of the form

$$\Delta_{\mathrm{dyn}} > 0 \quad \Longleftrightarrow \quad \text{chaos / unpredictability in an appropriate sense,}$$

rather than remaining at the level of specific models and phenomenological correspondences.

Second, the relationship between Ψ–Δ and information theory / algorithmic complexity is, at present, only explored at the level of orders of magnitude and structural analogy. The text repeatedly indicates that, under suitable perspectives, the lower bounds of Ψ and Δ are clearly related in scale to KS entropy, Lyapunov exponents, and even Kolmogorov complexity. However, no systematic isomorphisms or rigorous inequalities have been established yet. Future work may, in specific settings such as ergodic dynamical systems, stochastic processes, and families of computable functions, attempt to derive upper and lower bounds relating Ψ, Δ to entropy rates and algorithmic complexity, or even prove equivalences or stable sandwich bounds in some cases, thereby upgrading the current "structural intuition" into more robust mathematical statements.

Third, the mathematical foundations of HTDS itself remain to be systematized. Chapter 4 only introduced the basic configuration of HTDS,

$$((X, d, T, \{F_i\})),$$

and the induced quantities Ψ and Δ_{dyn}. Questions that have not yet been explored in depth include: given a constraint family $\{F_i\}$ and dynamics T, can one always construct well-defined Ψ and Δ_{dyn} for each level and each perspective; under what conditions does Δ_{dyn} exhibit certain stability properties with respect to the choice of perspective families; and under what natural transformations (such as coarse-graining, conformal transformations, or rescalings) the H/M classification is preserved. These issues further tie HTDS to measure theory, ergodic theory, and dynamical systems theory, beyond the scope of the present text, but they clearly form an important direction for future work.

8.1.2 Limitations at the Level of Models and Case Studies

At the level of concrete models, the text has, on the one hand, chosen the logistic map, incomputability phenomena, and NP-hard problems as the core examples of the "phenomenological triad," and, on the other hand, constructed representative cases of the three-layer structure "material / information / abstract logic" in Chapter 6 via spin glasses, JPEG compression, and cohomology groups. Beyond these, other discussions—for example, about islands of stability and fractal boundaries, structural intuitions about self-organizing networks, etc.—are largely reorganizations of existing phenomena in the Ψ–Δ language, rather than the outcome of systematic model sampling and statistical analysis.

Consequently, in terms of model breadth, the present text remains more of a "structural outline" than a comprehensive survey across many systems. Natural directions of extension include: testing the applicability of the H/M classification and the lower bounds of Δ systematically in higher-dimensional maps, continuous-time flows, glassy systems, random graphs, and network dynamics, and examining whether there appears a similar pattern of a critical finite order such as "fifth-order structural instability" or other reproducible finite-order transitions. If similar finite-order H$\to$M transitions are observed across different families of systems, there would be hope to upgrade the current phenomenological naming into a more general structural law; if not, these counterexamples can be used to restrict the domain of applicability and parameterization of HTDS.

8.1.3 Openness of the Design Space and Conceptual Choices

Finally, the concrete definition of Ψ and the construction of perspective families $\{A\}$ admit substantial design freedom. The text has only presented a few representative schemes (for example, defining Ψ via maximum or average constraint violation, constructing A_K via fixed interval partitions, etc.), and emphasized that such choices are essentially particular implementations of the principle "perspective determines ontology."

However, different application areas may require completely different forms of Ψ (different norms, weighting schemes, nonlinear aggregations, amplifications of rare events, etc.), and the choice of perspective families will simultaneously be constrained by observation methods, control objectives, and computational resources. How to systematically characterize the "admissible design space for Ψ and $\{A\}$" without sacrificing overall structural robustness, and how to analyze the impact of different choices on the H/M classification and the measures of Δ, is a topic not yet addressed here. It will require both modeling experience from concrete domains and a finer-grained formal characterization of which perspective adjustments count as isomorphisms and which adjustments substantially alter the H/M criterion.

In summary, this work first proposes a unified language and a number of structural statements, and then provides confirmations and comparisons on a limited set of model families, rather than simply adding a few new theorems on top of existing mathematical frameworks. As Ψ–Δ and HTDS are further embedded into concrete mathematical and physical systems, many of the structural observations given here will either be proved or be refined into more precise versions through corrections. In either development, the discriminative power and resolution of the framework itself will be enhanced.

8.2 Directions of Extension Toward More Physical and Complex Systems

Although the main technical examples in this text are still concentrated on one-dimensional maps, abstract logical systems, and a few canonical models at the material / information / topological levels, the Ψ–Δ framework itself does not depend on spatial dimension or specific physical content. Rather, it is aimed at the more general phenomenon of "tensions and fractures in multi-level structures." This section gives structural indications of several potential directions of extension related to physics and complex systems. They are not exhaustive plans, but natural leverage points for subsequent work.

8.2.1 Connections with Statistical Physics and Phase Transition Theory

In statistical physics, phase transitions and critical phenomena are typically characterized using order parameters, free energy, correlation lengths, and related quantities. In the Ψ–Δ language, one may attempt to interpret Δ as a kind of "structural fracture scale": at the same level, the "order discontinuity" between different phases corresponds to jumps in Δ, or to qualitative changes in the long-scale behavior of Δ_{dyn}. For example, the transition from an ordered phase to a disordered phase can be understood as follows: under certain natural perspectives, the lower bound of Ψ jumps from 0 to a positive value, and Δ_{dyn} changes from a bounded flow to growth that is approximately linear with respect to scale, time, or system size.

More concretely, one may consider connecting HTDS to ergodic theory and to Lagrangian / Hamilton–Jacobi structures. In the study of non-integrable and Hamiltonian systems, objects such as minimizing sets and Mather sets describe certain "optimal orbits" or "indestructible structures." If appropriate definitions of Ψ and Δ_{dyn} can be introduced into these frameworks, one may be able to restate non-integrability, broken KAM tori, and the classical picture of "chaotic seas plus islands of stability" from the standpoint of "structural frustration and fractures," thereby extending the H/M embedding patterns observed in the logistic map to more general Hamilton–Jacobi systems.

8.2.2 Links to Many-Body Systems and Complex Networks

In many-body systems and complex networks, phenomena such as self-organized criticality (SOC), power-law distributions, and long-range correlations are often explained by the fact that systems can spontaneously approach certain critical states without fine-tuning of parameters. In the Ψ–Δ perspective, such "critical steady states" can be understood as frozen Δ at particular scales: at those scales, fractures are neither further amplified (with Δ_{dyn} roughly constant) nor completely eliminated (with Δ remaining strictly positive), and the system resides for long times on an intermediate band between what can and cannot be structurally compensated.

On the other hand, multi-layer networks and adaptive systems (such as deep neural networks, ecological networks, financial networks, etc.) are prototypical multi-level structures: information transmission, constraint propagation, and feedback loops run between levels. HTDS provides a natural formal language for describing "propagation of hierarchical tension" in such networks: one can define Ψ_ℓ and $\Delta_{\mathrm{dyn},\ell}$ for each layer and examine how fractures diffuse, are absorbed, or are amplified across layers when local rules or connectivity patterns change. From this viewpoint, self-organized criticality and power-law behavior can be seen as manifestations of "multi-layer H/M structures forming a critical balance under iterative embedding and renormalization."

Overall, the directions listed in this section are structural extension routes. Their common goal is to push the Ψ–Δ intuitions formed in one-dimensional maps and abstract logical systems, together with the multi-level case studies in Chapter 6, toward higher-dimensional, more strongly coupled, and more realistic physical and many-body systems, thereby testing the robustness of the framework over a broader range. Whenever systematic counterexamples emerge in more complex systems, those cases should be explicitly excluded from the domain of applicability of Ψ–Δ and HTDS, or recognized as requiring a structural rewrite at a different level, rather than being patched by local parameter adjustments within the existing formulation.

8.3 Large-Model Hallucination in the Ψ–Δ Perspective (Future Research Directions)

This section attempts to place "large-model hallucination" within the previously introduced Ψ–Δ structure, and to provide a formulation that is as essential and quantitatively oriented as possible. The discussion below is a structural outlook on future work, rather than a summary of existing engineering practice, and it does not presuppose any specific experimental results.

8.3.1 Large Models as Lossy Compressors from "World → Token Sequence"

Formally, a large language model, together with its training corpus and decoding strategy, can be viewed as a lossy compression chain from the real world to a discrete token sequence. Assume that there exists some ideal "world-generating process" which, given a context c, induces a conditional distribution over target answers

$$P^*(\cdot \mid c),$$

representing the distribution of answers that are allowed by the world under the current question, factual state, and semantic constraints. On the other hand, the model parameters θ obtained through finite data, finite capacity, and finite optimization, together with a concrete decoding and post-processing strategy Π, give rise to a conditional distribution in the same context

$$Q_{\theta,\Pi}(\cdot \mid c),$$

representing the distribution from which the system actually samples its outputs.

From this point of view, a large model merely compresses the complex chain

$$(\text{world}, c) \;\mapsto\; \text{bitstream} \;\mapsto\; x$$

into a finite-dimensional parameter vector θ and a token sequence of finite length. As in the JPEG example discussed earlier, information loss may be introduced at every step of this chain: the finite corpus only approximates the world distribution, the finite-capacity parameters only approximate the corpus distribution, finite-time optimization only approximates the ideal parameters, and the concrete decoding strategy further distorts the internal conditional distribution of the model.

8.3.2 KL Fractures and the Ψ–Δ Calculus

To characterize the structural rupture between the real world and the model output, we may take the KL divergence as a concrete instance of the general information distance $D[\cdot,\cdot]$ used in this paper.For a given context c, define

$$D_{\mathrm{KL}}\big(P^*(\cdot \mid c) \,\|\, Q_{\theta,\Pi}(\cdot \mid c)\big) = \mathbb{E}_{x\sim P^*(\cdot|c)} \log \frac{P^*(x \mid c)}{Q_{\theta,\Pi}(x \mid c)}.$$

Over the usage distribution P_{use}, the average

$$\Delta_{\mathrm{KL}} := \mathbb{E}_{c\sim P_{\mathrm{use}}} D_{\mathrm{KL}}\big(P^*(\cdot \mid c) \,\|\, Q_{\theta,\Pi}(\cdot \mid c)\big)$$

can be viewed as a concrete realization of the "fracture degree" between the real-world perspective and the model perspective: here Δ directly quantifies, in the KL sense, how far the model as a whole deviates from the answer distribution allowed by the world.

To introduce the Ψ–Δ calculus, we parametrize the entire information-processing chain from P^* to $Q_{\theta,\Pi}$ as a continuous path $\{Q_\gamma\}_{\gamma\in[0,\gamma_{\max}]}$:

$$Q_0 = P^*, \qquad Q_{\gamma_{\max}} = Q_{\theta,\Pi},$$

where different values of γ can be understood abstractly as corresponding to successive compression and reconstruction steps such as "from world to finite corpus", "from corpus to parameters", and "from parameters to decoded outputs". For each γ, define

$$\Delta_{\mathrm{KL}}(\gamma) := \mathbb{E}_{c\sim P_{\mathrm{use}}} D_{\mathrm{KL}}\big(P^*(\cdot \mid c) \,\|\, Q_\gamma(\cdot \mid c)\big).$$

Under the structural assumptions of this paper, we may introduce a "KL frustration density" $\Psi_{\mathrm{KL}}(\gamma) \geq 0$ along the compression path, satisfying

$$\frac{d\Delta_{\mathrm{KL}}}{d\gamma} = \Psi_{\mathrm{KL}}(\gamma), \qquad \Delta_{\mathrm{KL}}(\gamma) = \int_0^\gamma \Psi_{\mathrm{KL}}(\tau)\, d\tau.$$

Intuitively, $\Psi_{\mathrm{KL}}(\gamma)$ describes the density of newly introduced, irreversible information loss at position γ, while $\Delta_{\mathrm{KL}}(\gamma)$ is the total accumulated KL fracture from the ideal world up to that position. Under this setup, as long as some segment of the compression chain structurally and unavoidably carries $\Psi_{\mathrm{KL}}(\gamma) > 0$, the overall KL fracture must have a strictly positive lower bound.

This is completely analogous to the JPEG case: the irreversible error introduced by quantization can be seen as a segment with frustration density $\Psi_q > 0$; no matter how we adjust filtering and post-processing afterward, information loss in certain textures and details cannot be structurally eliminated, but only smoothed or fabricated at the visible level.

8.3.3 Hallucination as Visible Samples from High KL-Fracture Regions

Within this framework, "large-model hallucination" can be understood as the sample-level manifestation of high KL-fracture regions. For a given context c, define the local KL fracture

$$\Delta_{\mathrm{KL}}(c) := D_{\mathrm{KL}}\big(P^*(\cdot \mid c) \,\|\, Q_{\theta,\Pi}(\cdot \mid c)\big).$$

When $\Delta_{\mathrm{KL}}(c)$ is small, the model output distribution in that context is close, in the information-theoretic sense, to the answer distribution allowed by the world; sampling from $Q_{\theta,\Pi}(\cdot \mid c)$ yields outputs compatible with $P^*(\cdot \mid c)$ as a high-probability event. When $\Delta_{\mathrm{KL}}(c)$ has been pushed to a non-negligible level by the accumulation of Ψ_{KL}, sampling from $Q_{\theta,\Pi}(\cdot \mid c)$ has a non-negligible probability of landing in the low-probability tail of $P^*(\cdot \mid c)$.

From the perspective of an external observer, such outputs that fall into the "low-probability tail of the world" appear as content that clearly deviates from facts, context, or task constraints, i.e., what is usually called "hallucination". In other words:

> As long as the compression chain from the real world to the model output carries a non-compressible frustration density $\Psi_{\mathrm{KL}} > 0$ in the KL sense, there must exist a family of contexts c for which the local fracture $\Delta_{\mathrm{KL}}(c)$ has a positive lower bound; hallucination is precisely the visible sample-level manifestation of this family of high-fracture regions.

Under this interpretation, hallucination is not an accidental type of error that can be completely eliminated, but a typical structural phenomenon of a lossy compression chain under multi-level constraints: it marks those regions where, given the representational capacity and compression strategy, the tension between the real distribution and the model distribution is the hardest to compress.

8.3.4 The Essence of Hallucination Control: Finding an Operational Ψ Scale

If we accept the KL-fracture perspective above, then the core question of whether "large-model hallucination" can be effectively controlled is not about inventing ever more decoding tricks, but about whether we can construct a truly operational quantitative standard for Ψ in the model–task space.

Most current methods in practice (temperature, top-k, penalties, RLHF, safety rules, retrieval augmentation, etc.) can be seen as empirical adjustments to the shape of $Q_{\theta,\Pi}$ under an unknown background Ψ_{KL}: some methods do reduce $\Delta_{\mathrm{KL}}(c)$ in certain regions, while others merely move the KL fracture from one region to another. As long as a structurally well-defined scale for Ψ is missing, hallucination can only be treated as an "empirical bad sample" to be counted ex post, rather than as an intrinsic, diagnosable structural property to be managed.

In the language of this paper, this can be compressed into a single sentence:

> Within the present framework, the question of whether the hallucination problem is controllable *in principle* is essentially equivalent to the question of whether we can construct a genuinely meaningful quantitative standard for Ψ in large models, preferably defined at the level of token streams or internal representations and directly linked to a concrete information distance such as KL divergence.

Once such a scale exists, hallucination need no longer be viewed as a purely empirical label, but can be rewritten as a specific interval within the Ψ–Δ calculus: regions with high Ψ inevitably leave traces in Δ, and contexts with high $\Delta_{\mathrm{KL}}(c)$ become precisely the "structurally high-risk regions" that a system must identify and handle with particular care during use.

8.3.5 Several Engineering-Oriented Hypotheses

Given the structural understanding above, we can offer more essential reformulations of some existing engineering practices. The following points are merely forward-looking hypotheses consistent with the Ψ–Δ framework; they do not claim to have been validated in large-scale systems.

(1) Reducing local Ψ_{KL}: structural regularization in training. One can deliberately design training objectives and model architectures so that each new compression step along the chain "world $\to$ corpus $\to$ parameters $\to$ outputs" introduces as little additional KL frustration density as possible. For example:

- Using multi-model or multi-head structures to explicitly approximate multiple conditional distributions $Q_{\gamma}^{(i)}(\cdot \mid c)$ for the same context, and regularizing the KL fractures between them;
- Introducing special objectives during pretraining and alignment for high-uncertainty regions, encouraging the model to maintain a wider output distribution rather than collapsing too aggressively onto a small number of erroneous modes.

If such methods can be experimentally shown to reduce Δ_{KL} or to significantly shrink the high-fracture regions, they may be taken as evidence in favor of the idea of "explicitly compressing Ψ_{KL} during training".

(2) Online proxies for $\Delta_{\mathrm{KL}}(c)$: detecting high-fracture regions. At inference time, even though $P^*(\cdot \mid c)$ is not directly accessible, one may still attempt to construct proxy indicators for $\Delta_{\mathrm{KL}}(c)$. For example:

- Measuring disagreement between multiple independent models or independent samples;
- Measuring conflict between external retrieval evidence and the current generated content;
- Detecting recognizable "self-contradiction signals" in internal representations or attention patterns.

When such proxy indicators suggest that the current context is likely in a high-fracture region, the system can switch to more conservative modes, such as asking for clarification, explicitly expressing uncertainty, invoking external tools, or outright refusing to answer. Structurally, this amounts to adding a "safety valve" over regions where $\Delta_{\mathrm{KL}}(c)$ is likely to be large, to avoid continuing to produce outputs that appear coherent but are structurally unreliable under a high-fracture background.

(3) Using external channels to reconstruct part of the fracture. Mechanisms such as retrieval augmentation, tool calls, and code execution can be uniformly understood as adding auxiliary channels around the main chain from "real world $\to$ internal representation". These channels give some information that would otherwise be compressed away by the main chain a chance to be locally restored. In the Ψ–Δ language, the goal of such designs is not to drive the main-chain Ψ_{KL} to zero, but to reduce the effective $\Delta_{\mathrm{KL}}(c)$ in high-fracture regions through external channels, thereby altering the probability distribution over where hallucinations occur.

These ideas share a common feature: they no longer treat hallucination as an anomaly that can be completely removed, but instead attempt, under the assumption $\Psi_{\mathrm{KL}} > 0$, to locally "squeeze" or "bypass" the high-fracture regions through training structure, online detection, and additional channels.

8.3.6 Summary: The Structural Position of the Hallucination Problem

In summary, within the framework of this paper, large-model hallucination can be assigned a clear structural position:

- The entire chain from the real world to the model output is a lossy compression that carries a non-compressible frustration density $\Psi_{\mathrm{KL}} > 0$ in the KL sense;
- This frustration density integrates along the compression path into a total fracture Δ_{KL}, and manifests, for specific contexts c, as a positive lower bound on the local fracture $\Delta_{\mathrm{KL}}(c)$;
- In high-fracture regions, outputs sampled from $Q_{\theta,\Pi}(\cdot \mid c)$ naturally have a significant probability of deviating from $P^*(\cdot \mid c)$; these deviations are what appear, in external observation, as "hallucinations".

If future work can construct more precise measures of Ψ_{KL} along these lines, and experimentally evaluate how different training structures, detection mechanisms, and external-channel designs affect Δ_{KL}, then hallucination may be transformed from a vague empirical label into a class of structural properties that are measurable, diagnosable, and partially controllable. Regardless of how effective these concrete approaches ultimately prove to be, they all continue the basic stance of this paper:

> Rather than hoping for a theory or model that is "completely without fractures", we should, under the recognition that fractures do exist, design systems that can identify, quantify, manage, and, when necessary, explicitly expose these fractures.

8.4 Conclusion: Fractures of Logic as a Universal Structure

Let us return to the motivating questions raised in Chapters 1–3: Why is it that whenever human single-perspective logic, formal systems, or algorithmic models try to extend themselves to "higher-level structures," fractures inevitably appear somewhere? Whether in Gödelian incompleteness, Turing undecidability, long-term unpredictability in chaotic systems, combinatorial explosion in NP-hard problems, or hallucinations of large models in real-world tasks, they seem to be delivering the same message over and over: with a single perspective and a continuous chain of deduction, one cannot maintain perfect continuity and self-consistency across all levels at once.

The main work of this text has been to propose a unified and transparent structural language to describe the common mechanisms behind these phenomena. Taking the perspective A as the ontological unit, and using structural frustration Ψ and logical fracture Δ, we have described, via the H/M classification, HTDS (Hierarchical Tension Dynamical Systems), and the geometry of equiangular spirals, how tension is generated, compensated, and accumulated when moving from single-level systems to multi-level, multi-perspective couplings. In this language, incomputability, chaos, NP-hardness, and large-model hallucinations can all be reformulated as: under some natural perspective and level, the system carries an irreducible lower bound on fractures—that is, Δ has a strictly positive lower bound at that level, which cannot be completely flattened away by any finite rule-tuning or perspective rewriting.

The meta-fracture postulate introduced in Chapter 3 provides a kind of "structural ceiling" for this picture: at the meta-structural level of the universe, the infimum of fracture Δ^* is always strictly greater than zero. This means that no matter how we extend our towers of theories, refine our models, or introduce higher-level perspectives, we can only temporarily depress local Ψ and Δ on finite scales, but cannot drive them to zero globally. As a result, single-perspective continuous logic exhibits a "spiral-approach" trajectory: one can always find relatively smooth H regions at some local level, but at higher levels or under broader perspectives, new M regions and fracture accumulation inevitably arise. In this sense, linear chains of deduction and Δ^* are not opposed but coexisting: "effective structured cognition" is precisely the ability to continually seek new perspectives and compensatory mechanisms while acknowledging the existence of fractures, thereby maximizing local coherence and interpretability within bounded domains.

If linear deduction is structurally destined to break at some level, a natural follow-up question is: should "analogy" be given a more central place in our toolkit for reasoning and modeling? Much of what this text has done is essentially cross-domain structural analogy: unifying incompleteness in formal systems, chaos in dynamical systems, frustration in spin glasses, phase transitions in many-body systems, and hallucinations in large models within the same Ψ–Δ language. These analogies are not loose metaphors, but are grounded in structural isomorphisms: across different domains, one can identify similar H/M patterns, similar modes of Δ accumulation, and similar topological skeletons that determine how fractures propagate across levels.

From the perspective of future research, this suggests a direction that deserves serious consideration: if single-perspective linear reasoning inevitably encounters logical breaks at some level, we might elevate "structural analogy" itself to a first-class cognitive operation. By searching for cross-domain and cross-scale topological invariants, we may infer deep structural constraints on how the universe operates as a whole. Concretely, one might imagine bringing more abstract structural tools (such as category theory, topological invariants, or suitably defined "fracture homology") to bear on the common patterns in the distributions of Ψ and Δ and in the ways H and M regions are glued together across different systems, thereby generalizing the intuition that "fractals are determined by fractures of fractures" into a more universal structural principle. In such a research program, analogy ceases to be a mere auxiliary to linear reasoning and becomes a main thread for uncovering shared topological constraints behind diverse physical and informational systems.

In this regard, the evolutionary trajectory of modern large-scale models already provides a nontrivial empirical hint of the centrality of structural analogy. Historically, artificial neural networks did not arise from a complete deductive theory of "what intelligence is," but from a coarse structural analogy with biological brains. Researchers did not fully understand the computational principles of the brain; instead, they abstracted the topological skeleton of "neurons–synapses–layered connections," formalized it as differentiable linear transformations plus nonlinear activations, and combined these with gradient descent—essentially an engineering optimization mechanism—for exploratory training on large datasets. Later, convolutional networks explicitly mirrored local receptive fields and translation invariance in visual cortex: local convolution kernels and weight sharing can be seen as mathematical implementations of such topological features. Even the Transformer architecture, although not directly modeled on cortical anatomy, can be interpreted as an abstract analogy at the level of information flow, with its emphasis on attention, long-range dependency, and global context integration, rather than as the outcome of some closed "axiomatic theory of intelligence."

This developmental path suggests at least this much: in the absence of a final theory of intelligence, constructing highly capable models does not depend on top-down, fully deductive design. Instead, breakthroughs can be achieved through the path of "partial theoretical constraints + structural analogy + empirical experimentation." In other words, analogy here is not an accessory to formal reasoning, but a constructive tool directly involved in model-architecture design and capability emergence. From the standpoint of this text, large models can be roughly understood as "engines of high-dimensional structural analogy." During pre-training, they produce unified encodings of fragments from different domains, styles, and tasks in an embedding space; during downstream use, they generate outputs by exploiting similarities and alignments on this high-dimensional structure. The broad generalization capabilities of large models in translation, question answering, code generation, and other tasks strongly suggest that "structural analogy + pattern alignment" can effectively capture some cross-domain stable Ψ–Δ textures of the world. Against this background, treating "structural analogy" as a first-class cognitive operation on par with linear deduction, rather than as its subordinate, is a working stance that is both empirically and structurally well-motivated.

In summary, "fractures of logic" are treated here as a general structural fact, not as a special problem confined to some particular discipline or as an accidental setback of human thinking. Through the Ψ–Δ framework, this text has attempted to elevate such fractures from the level of phenomena to that of structure, and to provide an extensible language and perspective for further tracking these fractures in mathematics, physics, computation, and intelligence research. If Δ^* confirms that a "perfectly fracture-free" ultimate theory does not exist, then the truly open question is no longer "How can we eliminate fractures?" but rather: given that fractures are here to stay, can we, through increasingly refined structural analogies and topological analyses, come to understand, over ever broader domains, the distribution, evolution, and internal order of these fractures?

8.5 Universal Constants as Spectral Invariants under Perspective Transformations

This section records a restricted outlook from the Ψ–Δ viewpoint on a classical question: to what extent can certain *dimensionless universal constants* (be they Feigenbaum-type constants, universal ratios, or critical exponents) be given a unified *structural* interpretation? Throughout this section, "universal constants" refer only to dimensionless parameters that appear stably across broad model classes and are insensitive to microscopic details.

Status and role of this section. All statements below are explicitly conditional and are not used as premises for any results in earlier chapters. The purpose is to provide a compact research program: given a domain where renormalization-type perspective transformations are already known to exist, the Ψ–Δ language suggests a uniform way to *relabel* the associated spectral data as cross-level gap scaling data.

8.5.1 A formal setting: perspective transformations and induced Ψ–Δ flows

In Feigenbaum universality, one has a renormalization operator R acting on a suitable space of unimodal maps. In a neighborhood of a non-trivial fixed point f^*, the linearization DR admits a finite number of unstable directions; the corresponding eigenvalues (and fixed ratios thereof) constitute the universal constants of the universality class (e.g. the Feigenbaum constants δ and α_F).

The Ψ–Δ framework abstracts the same pattern as follows. Assume that a class of systems $\mathcal{S}$ can be described under a family of effective perspectives $\mathcal{V}$, producing a family of Ψ–Δ structures

$$\mathcal{S} \;\longrightarrow\; \big\{(\Psi_v, \Delta_v) : v \in \mathcal{V}\big\}.$$

Let a perspective transformation act on $\mathcal{V}$ (renormalization, scale changes, coarse-graining reindexing, etc.)

$$T : \mathcal{V} \to \mathcal{V},$$

and let it induce an action on the associated Ψ–Δ structures,

$$\mathcal{R} : (\Psi_v, \Delta_v) \mapsto (\Psi_{T(v)}, \Delta_{T(v)}).$$

Within this setting, *spectral scaling invariants* refer to those dimensionless quantities that appear stably in the iteration or flow of $\mathcal{R}$, such as eigenvalues of a linearized action near a critical structure, fixed ratios of eigenvalues, or other invariants that control cross-level scaling behavior.

8.5.2 Working hypothesis: Feigenbaum-type constants as Δ-scaling data

Structural Working Hypothesis (Feigenbaum-type universality as Δ-scaling). Suppose that:

- a system family admits a Ψ–Δ description under a perspective set $\mathcal{V}$, and the relevant multi-level connection problem is genuinely cross-level within that description;
- there exists a renormalization/perspective transformation T on $\mathcal{V}$ whose induced action $\mathcal{R}$ admits a non-trivial fixed point (or recurrent critical structure) and a finite-dimensional unstable spectrum in an appropriate sense.

Then the universal constants associated with that renormalization structure can be *interpreted* as spectral scaling invariants of the induced Ψ–Δ flow: they quantify stable amplification factors or invariant scales governing how cross-level gap data Δ transforms under repeated perspective changes.

In the Feigenbaum case, this viewpoint is consistent with the standard statement that the numerical constants do not depend on microscopic details but are determined by the renormalization structure itself. The Ψ–Δ language adds only a structural relabeling: the same spectral information is treated as scaling data of cross-level mismatch under perspective transformations.

8.5.3 Scope and controlled extensions

The hypothesis above is intentionally restricted to universality classes where a renormalization-type operator (or an equivalent perspective-transformation structure) is already available and mathematically meaningful. Within this scope, the framework does not claim new numerical derivations; it proposes a uniform structural category for the constants that appear.

A cautious extension is possible at the level of *interpretation*: when critical exponents or universal ratios are known to be controlled by the spectrum (or scaling structure) of a renormalization operator, they can be grouped under the same structural heading of spectral scaling invariants of cross-level perspective transformations. This does not assert that all such exponents reduce to eigenvalues in a single universal way; it only states that, whenever the renormalization mechanism is the organizing principle, the Ψ–Δ language provides a consistent structural translation.

Finally, it is worth emphasizing what is *not* claimed here. This section does not use any statement about universal constants as a premise for earlier chapters, and it does not attempt to “explain” or “derive” numerical values of couplings outside the specific renormalization contexts where such spectral structures are already established. Concrete calibrated analyses in field-theoretic settings (e.g. running couplings) are better treated as separate case studies; they are not required for the structural outlook recorded in this section.

Appendix A

Optional local closure ansatz: Maxwell-type prototype equations for the Ψ–Δ pair

A.1 Purpose and status of this appendix

This appendix records one possible *optional* local closure ansatz for the coupled Ψ–Δ structure. It is not used as a premise for the definitional framework (Chapter 2) nor for the main lower-bound statements; its role is to provide a compact modeling template for readers who wish to endow the (Ψ, Δ) pair with a local, field-like coupling on an abstract structural space.

In particular, the distributed quantities introduced below should be read as *profiles* (local surrogates) whose coarse-grained projections can reproduce the one-dimensional accumulation relation used in the main text under additional closure assumptions.

A.2 Distributed profiles and abstract operators

Let X be an abstract structural space and let γ be an evolution parameter (time, RG scale, cognitive evolution parameter, etc.). On $X \times \mathbb{R}_\gamma$, introduce two distributed profiles:

- $\psi(x, \gamma)$: a local continuous-logic profile (level- and perspective-relative);
- $\delta(x, \gamma)$: a local fracture profile recording where single-parameter gluing is forced to fail.

The use of lowercase (ψ, δ) emphasizes that these are *distributed surrogates.* The macroscopic quantities $(\Psi(\gamma), \Delta(\gamma))$ appearing in the main text are recovered only after a declared coarse-graining/projection procedure.

To describe circulation-like and outflow-like local structural behavior, introduce two abstract operators:

- $\mathcal{C}$: a curl-like structural operator;
- $\mathcal{D}$: a divergence-like structural operator.

Minimal structural requirement. The only algebraic property used below is the chain-type compatibility

$$\mathcal{D} \circ \mathcal{C} = 0,$$

which is the abstract analogue of $\operatorname{div}\operatorname{curl} = 0$ and ensures that the constraint equations and the evolution equations are not mutually inconsistent under arbitrary sources.

A.3 Prototype local evolution equations (optional ansatz)

Consider the following prototype system:

$$\frac{\partial \psi}{\partial \gamma} = -\,\mathcal{C}\delta \;+\; j_\psi, \tag{A1}$$

$$\frac{\partial \delta}{\partial \gamma} = \mathcal{C}\psi \;+\; j_\delta, \tag{A2}$$

$$\mathcal{D}\psi = \rho_{\text{logic}}, \tag{A3}$$

$$\mathcal{D}\delta = \rho_{\text{crack}}. \tag{A4}$$

Here j_ψ and j_δ are external source terms (logical sources and crack sources), and $\rho_{\text{logic}}, \rho_{\text{crack}}$ are distributional constraints on the profiles.

Source compatibility (implied by $\mathcal{D} \circ \mathcal{C} = 0$). Applying $\mathcal{D}$ to (A1)–(A2) and using $\mathcal{D} \circ \mathcal{C} = 0$ yields

$$\frac{\partial \rho_{\text{logic}}}{\partial \gamma} = \mathcal{D} j_\psi, \qquad \frac{\partial \rho_{\text{crack}}}{\partial \gamma} = \mathcal{D} j_\delta,$$

which states that the constraint profiles cannot be chosen independently of the source terms. This is the minimal condition ensuring the system is not over-determined.

Sign and positivity. The fracture profile $\delta(x, \gamma)$ in this appendix is allowed to be sign-indefinite (a profile variable rather than a nonnegative "gap magnitude"). A nonnegative intensity can be extracted, when needed, by a post-processing functional (e.g. $|\delta|$ or another admissible norm), while the ontological statement "genuine cross-level fracture implies a positive residual" pertains to the macroscopic gap notion used in the main text under declared resolution and admissible regimes.

A.4 Reduction to the one-dimensional accumulation form

To connect the local ansatz to the one-dimensional accumulation relation, fix a coarse-graining/projection functional $\langle \cdot \rangle$ on profiles over X (e.g. an average, a dominant-mode projection, or a declared aggregation). Define

$$\Psi(\gamma) := \langle \psi(\cdot, \gamma) \rangle, \qquad \Delta(\gamma) := \langle \delta(\cdot, \gamma) \rangle.$$

Under a closure approximation of the form

$$\langle \mathcal{C}\psi \rangle \approx k \, \langle \psi \rangle, \qquad \langle j_\delta \rangle \approx 0,$$

equation (A2) yields

$$\frac{d\Delta}{d\gamma} \approx k\,\Psi(\gamma).$$

By rescaling γ or absorbing k into a renormalized definition of Ψ, one recovers the coarse-grained accumulation form

$$\frac{d\Delta}{d\gamma} = \Psi(\gamma) \quad \Longrightarrow \quad \Delta(\gamma) = \Delta(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi(\gamma')\,d\gamma'.$$

This shows that the one-dimensional relation used in the main text can be viewed as a controlled coarse-grained reduction of the optional local closure ansatz, once a projection procedure and an effective closure for $\langle \mathcal{C}\psi \rangle$ are declared.

Appendix B

Technical Notes on the Logistic Map in the Ψ–Δ Framework

This appendix provides a concrete instantiation of the logistic map within the operational Ψ–Δ protocol. The main text uses the logistic map only as a representative case study; here we make the model specification, the coarse-grained perspective family, and the entropy-based proxy for intra-level frustration explicit, so that detailed mathematical and numerical treatments can be carried out reproducibly when needed.

Notation (avoiding collision). Throughout the main text, δ denotes a *resolution threshold* in the operational protocol (i.e. the negligibility cutoff under which a residual may be treated as "effectively zero"). Classical Feigenbaum constants, when they appear, are denoted by δ_{Feig} and α_{Feig} to avoid collision. This appendix itself does not use δ_{Feig}; the only "resolution knob" used here is the partition depth K (granularity of the coarse-grained perspective).

A.1 Model and Logical-System Formalization

Consider the standard one-dimensional unimodal map (logistic map)

$$x_{t+1} = f_r(x_t) = r x_t(1 - x_t), \quad x_t \in [0, 1],\ r \in [0, 4].$$

For each fixed r this defines a discrete-time dynamical system on $X = [0, 1]$. We view it as a logical system $\mathcal{L}_r$ in the operational sense of this paper:

- **Variables (state):** $x_t \in X := [0, 1]$.
- **Rule (update):** $x_{t+1} = f_r(x_t)$.
- **Invariant statistics (for averaging):** we fix an invariant probability measure μ_r used for coarse-grained statistics. Concretely, in periodic windows we may take the atomic measure supported on the attracting cycle; in chaotic regimes, when a physical (SRB/ACIM-type) invariant measure exists, we take that one.

Within the present framework, we do not attempt to predict exact pointwise orbits. Instead we quantify the regime-relative intra-level frustration proxy $\Psi(\mathcal{L}_r, A_K)$ and the corresponding long-time fracture accumulation under a finite observational perspective A_K.

A.2 Coarse-Grained Perspective A_K and Symbolic Encoding

Fix an integer $K \geq 2$ and choose a partition of $[0,1]$ into K measurable cells:

$$[0,1] = \bigcup_{k=1}^{K} I_k, \qquad I_k = [a_k, a_{k+1})\ (1 \leq k \leq K-1), \quad I_K = [a_K, a_{K+1}],$$

where $0 = a_1 < a_2 < \cdots < a_{K+1} = 1$. The simplest choice is the equal-length partition $a_k = (k-1)/K$, but one may also use an "equal-μ_r" partition (when μ_r is available) to balance cell weights.

Define the coarse-grained perspective A_K by the encoding map

$$A_K : x_t \mapsto s_t \in \{1, \ldots, K\}, \qquad s_t = k \iff x_t \in I_k.$$

Under A_K, the continuous orbit (x_t) is mapped to a symbolic sequence (s_t). All quantities in this appendix are evaluated *relative to the declared family* $\{A_K\}_{K\geq 2}$, and any claims of "small" or "large" residuals are to be read as K-dependent, i.e. resolution-/granularity-relative.

A.3 A Concrete "Splitting Index" σ_k at Cell Level

The core intuition is: if f_r stretches points in a cell I_k so that their images spread across multiple target cells, then the coarse state k is forced to branch into multiple successor states. This branching is an operational manifestation of intra-level tension under A_K.

Using the invariant measure μ_r, define the coarse transition probabilities

$$p_{k\to j}(r; A_K) := \frac{\mu_r\big(I_k \cap f_r^{-1}(I_j)\big)}{\mu_r(I_k)} \quad (\mu_r(I_k) > 0),$$

and set $p_{k\to j}(r; A_K) = 0$ when $\mu_r(I_k) = 0$. For each fixed k, $\{p_{k\to j}\}_{j=1}^{K}$ is a probability distribution over next-step coarse states.

Define the (one-step) splitting entropy of cell I_k by

$$h_k(r; A_K) := -\sum_{j=1}^{K} p_{k\to j}(r; A_K)\, \log p_{k\to j}(r; A_K), \qquad 0 \log 0 := 0.$$

Normalize it by the maximal entropy $\log K$ to obtain a dimensionless splitting index

$$\sigma_k(r; A_K) := \frac{h_k(r; A_K)}{\log K} \in [0,1].$$

Thus $\sigma_k \approx 0$ indicates an almost deterministic coarse transition from cell k, while $\sigma_k \approx 1$ indicates maximal branching under A_K.

A.4 Average Frustration Proxy $\Psi(\mathcal{L}_r, A_K)$

In this appendix we instantiate a concrete proxy of intra-level frustration under A_K by the μ_r-weighted average of cell splitting:

$$\Psi(\mathcal{L}_r, A_K) := \sum_{k=1}^{K} w_k(r; A_K)\, \sigma_k(r; A_K), \qquad w_k(r; A_K) := \mu_r(I_k).$$

Equivalently,

$$\Psi(\mathcal{L}_r, A_K) = \sum_{k=1}^{K} \mu_r(I_k)\, \frac{h_k(r; A_K)}{\log K} = \frac{H_{\mathrm{cg}}(r; A_K)}{\log K},$$

where the coarse-grained one-step entropy rate functional is

$$H_{\mathrm{cg}}(r; A_K) := \sum_{k=1}^{K} \mu_r(I_k)\, h_k(r; A_K).$$

Interpretation. $H_{\mathrm{cg}}(r; A_K)$ is a standard one-step coarse-grained entropy-rate proxy induced by the partition $\{I_k\}$ and the invariant statistics μ_r. For sufficiently informative partitions (e.g. generating partitions in regimes where they exist), H_{cg} approximates the Kolmogorov–Sinai entropy $h_{\mathrm{KS}}(r)$. In this instantiation, $\Psi(\mathcal{L}_r, A_K)$ is simply the normalized version of this entropy proxy, so that Ψ measures "how much unavoidable branching remains visible" under the declared perspective.

The structural picture is:

- $\Psi(\mathcal{L}_r, A_K) \approx 0$ for some K: there exists a coarse perspective under which transitions are nearly deterministic (typical in stable periodic windows, with partitions aligned to the attracting cycle).
- $\Psi(\mathcal{L}_r, A_K)$ bounded away from 0 for natural families of K: branching persists under refinement in chaotic regimes, yielding a stable positive tension proxy.

Relation to Lyapunov exponent and KS entropy (structural background). For chaotic parameter regimes where a physical invariant measure μ_r exists, the Lyapunov exponent

$$\lambda(r) = \int \log |f_r'(x)|\, d\mu_r(x)$$

is typically positive. In many standard one-dimensional settings, $h_{\mathrm{KS}}(r)$ matches the integral of the positive Lyapunov exponent under suitable regularity assumptions (Pesin-type relations). For well-chosen coarse partitions, one has the approximation trend

$$H_{\mathrm{cg}}(r; A_K) \to h_{\mathrm{KS}}(r) \quad (K \to \infty)$$

in regimes where generating partitions/standard entropy convergence applies. Hence, at the level of order-of-magnitude structure,

$$\Psi(\mathcal{L}_r, A_K) = \frac{H_{\mathrm{cg}}(r; A_K)}{\log K} \approx \frac{h_{\mathrm{KS}}(r)}{\log K}.$$

This explains why, in chaotic regimes, Ψ remains visibly positive at finite granularity, while in periodic regimes one can often arrange $\Psi \approx 0$ under suitable coarse perspectives.

A.5 Long-Time Fracture Accumulation and a Dynamical Proxy for Δ_{dyn}

Fix r and a perspective A_K. Define a trajectory-level instantaneous splitting signal by

$$\psi_t := \sigma_{s_t}(r; A_K), \qquad s_t := A_K(x_t).$$

This is a bounded nonnegative quantity, measurable along the orbit.

Define the finite-time accumulated fracture proxy over a window of length T by

$$\Delta_{\mathrm{dyn}}(\mathcal{L}_r, A_K; T) := \sum_{t=0}^{T-1} \psi_t.$$

Under standard ergodic assumptions for (f_r, μ_r), the Birkhoff average converges:

$$\frac{1}{T}\Delta_{\mathrm{dyn}}(\mathcal{L}_r, A_K; T) = \frac{1}{T}\sum_{t=0}^{T-1} \psi_t \longrightarrow \int \sigma_{A_K(x)}(r; A_K)\, d\mu_r(x) = \Psi(\mathcal{L}_r, A_K) \quad (T \to \infty).$$

Hence, in the long-time regime,

$$\Delta_{\mathrm{dyn}}(\mathcal{L}_r, A_K; T) \approx \Psi(\mathcal{L}_r, A_K)\, T.$$

Regime reading.

- **Periodic parameter regimes.** By choosing A_K aligned with the attracting periodic skeleton, one can often obtain $\Psi(\mathcal{L}_r, A_K) \approx 0$, hence the per-step accumulation rate is negligible and the accumulated fracture grows very slowly (or is effectively bounded at the declared granularity).
- **Chaotic parameter regimes.** For natural coarse perspectives, $\Psi(\mathcal{L}_r, A_K)$ stays bounded away from 0 at fixed K, so $\Delta_{\mathrm{dyn}}(\cdot; T)$ grows linearly in T with a positive rate, matching the structural content used in the main text ("positive tension density implies linear long-time accumulation").

This appendix-level construction provides one explicit, reproducible way to connect the main-text qualitative statements to standard entropy/Lyapunov diagnostics without changing the core Ψ–Δ semantics.

A.6 Stability Islands and the Bifurcation Diagram: A Texture Reading (Sketch)

In the parameter–state bifurcation diagram of the logistic map, chaotic regimes contain infinitely many embedded periodic windows (stability islands), with the familiar cascade pattern

$$\text{period} \;\to\; \text{period-doubling} \;\to\; \text{chaos} \;\to\; \text{new window}.$$

Within the Ψ–Δ language, an operational reading is:

- Under a fixed global coarse perspective A_K, if $h_{\mathrm{KS}}(r) > 0$ on many subintervals, then $\Psi(\mathcal{L}_r, A_K)$ is persistently positive there and Δ_{dyn} accumulates linearly in time (an M-like background at that granularity).

- If attention is restricted to a narrow periodic window and one chooses a local perspective $A_{K,\mathrm{window}}$ tuned to that window's periodic skeleton, then $\Psi(\mathcal{L}_r, A_{K,\mathrm{window}}) \approx 0$ can be obtained, recovering a local H-like behavior within that window at the chosen granularity.

From the viewpoint of nesting, the accumulation of windows and the fractal boundaries between chaotic seas and stability islands can be treated as a geometric record of how "tame" (low-Ψ) and "fractured" (high-Ψ) regimes interleave across parameter space under a fixed observation protocol. This appendix does not pursue further geometric quantification; it serves only as a technical supplement to the logistic-map case study in the main text.

Appendix C

Worked instantiation (optional): QED one-loop endpoint/path normalization

This appendix records a compact worked instantiation of the operational package in a one-loop QED RG-window setting. Its purpose is reproducibility and comparability: it explicitly names the package elements and shows how an endpoint/path decomposition yields the associated gap components under a declared interface and resolution regime. The main text does not rely on this appendix for the definitional framework or the lower-bound statements; full derivations and additional variants are deferred to the companion calculation note.

Declared package. Fix an RG coordinate ℓ (e.g. $\ell = \ln(\mu/\mu_*)$) and a coarse-graining window $[0, T]$ with $T > 0$. We declare the operational package

$$\mathsf{V} = (A_I, A_G, \Omega_{\text{reach}}, E_{A_I}, E_{A_G}, D, \mathcal{U}, \delta),$$

with the following meanings.

- **Perspectives.** A_I is the *path (history) perspective*: it retains the intermediate-history information across the window (a discretized history $H = (X_0, \ldots, X_{N-1})$ together with endpoint $E = X_N$, or its continuum analogue). A_G is the *endpoint perspective*: it retains only the endpoint variable E (coarse-grained summary).
- **Reachable set.** Ω_{reach} is the declared class of reachable trajectories within the chosen truncation/scheme (e.g. one-loop RG flow, optionally with a specified EFT extension). In the companion note, a convenient coordinate choice is $g(\ell) = (\theta(\ell), u(\ell))$ where $\theta = \alpha^{-1}$ drifts linearly in ℓ while u may relax under a chosen effective flow.
- **Conflict functionals.** E_{A_I} and E_{A_G} are the discrepancy measures used to evaluate, respectively, history-level and endpoint-level distinguishability under the declared perspective. In this instantiation, they are operationalized via relative-entropy-type discrepancies induced by a chosen interface (below).
- **Distance.** D is the declared discrepancy functional on the induced output laws. In this instantiation, D is taken to be the Kullback–Leibler divergence between the induced output distributions (not a metric in the strict sense).

- **Liftings.** $\mathcal{U}$ is the admissible class of lifting operators that attempt to reconstruct a history (path-level object) from endpoints. Concretely, $\mathcal{U}$ can be understood as a restricted class of Markov kernels $U : E \mapsto \widehat{H|E}$ consistent with the declared truncation, measurability, and resolution regime.

- **Resolution threshold.** δ is the declared decision threshold (resolution) used to judge whether a residual gap is operationally negligible, i.e. whether one may treat a fracture as "≈ 0" at the current resolution (a resolution convention rather than an ontic claim).

Interface and induced divergences. Choose an observational interface $\mathcal{O}$ that maps the parameter coordinate g to an output random variable. In the companion QED calculation note, the full reproducibility specification is stated at the viewpoint level and explicitly records the interface family $\mathcal{O}$ and the cross-interface aggregation rule Agg. In this appendix a single canonical interface is fixed, so Agg reduces to the identity and is absorbed into the choice of D in the package declaration. A minimal canonical choice is a Gaussian interface

$$Y \mid g \sim \mathcal{N}\big(g, \Sigma\big), \qquad \Sigma = \mathrm{diag}(\sigma_\theta^2, \sigma_u^2),$$

so that two trajectories induce two families of output laws P and Q on (H, E).

Endpoint/path decomposition and the gap component. *Notation.* The symbols Δ_{path}, Δ_I, and Δ_{II} below denote divergence/length readouts in this instantiation; they are not the cross-level Δ parameter of the main framework.

Let $E := X_N$ denote the endpoint and $H := (X_0, \ldots, X_{N-1})$ the history. When the comparison protocol fixes the initial condition, X_0 is treated as non-informative; otherwise one may take $E := (X_0, X_N)$ without changing the chain-rule structure. Define the history-level (joint) divergence

$$\Delta_{\text{path}}(0 \to N) := D(P_{H,E} \,\|\, Q_{H,E})\,.$$

The relative-entropy chain rule yields the identity

$$\Delta_{\text{path}}(0 \to N) = D(P_E \| Q_E) + \mathbb{E}_{P_E}\left[D\big(P_{H|E} \| Q_{H|E}\big)\right].$$

Define the *endpoint divergence*

$$\Delta_I(0 \to N) := D(P_E \| Q_E),$$

and the *strict path gap component*

$$\Gamma_{\text{path}}(0 \to N) := \mathbb{E}_{P_E}\left[D\big(P_{H|E} \| Q_{H|E}\big)\right] \geq 0,$$

so that

$$\Delta_{\text{path}}(0 \to N) = \Delta_I(0 \to N) + \Gamma_{\text{path}}(0 \to N).$$

In this instantiation we take $E_{A_I} \equiv \Delta_{\text{path}}$ and $E_{A_G} \equiv \Delta_I$. Operationally, Γ_{path} is the residual induced by compressing the history perspective A_I into the endpoint perspective A_G under the declared admissibility and resolution regime.

Moreover, for any shared lifting kernel $U \in \mathcal{U}$ applied to both models, define the lifted joint laws $\widetilde{P}_{H,E} := P_E \otimes U(\cdot|E)$ and $\widetilde{Q}_{H,E} := Q_E \otimes U(\cdot|E)$. Then

$$D(\widetilde{P}_{H,E} \| \widetilde{Q}_{H,E}) = D(P_E \| Q_E) = \Delta_I,$$

so Γ_{path} quantifies the residual that cannot be recovered by any admissible endpoint-only lifting in $\mathcal{U}$.

A resolution-aware readout convention is:

$$\text{declare “fracture negligible at resolution } \delta\text{” if } \Gamma_{\text{path}} \le \delta.$$

Closed forms under the Gaussian package. Under equal-covariance Gaussian families, the endpoint divergence admits the closed form

$$\Delta_I = \frac{1}{2}\left(\frac{(\Delta\theta)^2}{\sigma_\theta^2} + \frac{(\Delta u)^2}{\sigma_u^2}\right), \qquad \Delta\theta := \theta(T) - \theta(0), \quad \Delta u := u(T) - u(0),$$

and the corresponding endpoint-equivalent length is

$$d_I := \sqrt{2\Delta_I}.$$

Define the *information speed* (local density along ℓ) by

$$\Psi(\ell) := \sqrt{\frac{1}{\sigma_\theta^2}\left(\frac{d\theta}{d\ell}\right)^2 + \frac{1}{\sigma_u^2}\left(\frac{du}{d\ell}\right)^2},$$

and the induced *path length*

$$\Delta_{II} := \int_0^T \Psi(\ell)\, d\ell.$$

Under the fixed-covariance Gaussian interface above, the induced quadratic form is constant on the window, so the path length Δ_{II} dominates the endpoint chord length d_I, yielding the nonnegativity below. When the geometric proxy is admissible (i.e. when a positive-definite metric realization is declared for the chosen coordinate $g(\ell)$ on the window), define the computable proxy gap

$$\Gamma_{\text{geo}} := \Delta_{II} - d_I \ge 0.$$

Why the package must be reported for comparability. The values of Δ_I, Δ_{II}, Γ_{path}, and Γ_{geo} are not intrinsic numbers without context: changing any of D (e.g. the interface noise scales σ_θ, σ_u), $\mathcal{U}$ (the admissible reconstruction/lifting class), or δ (the resolution threshold for “≈ 0”) generally changes the resulting fracture values. Therefore, in any application meant to be comparable across authors or models, the package $\mathsf{V} = (A_I, A_G, \Omega_{\text{reach}}, E_{A_I}, E_{A_G}, D, \mathcal{U}, \delta)$ must be explicitly reported as part of the result statement.

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