The Ψ–$\Delta_{I\to G}$ Framework: Structural Gaps and Their Dynamics in Multi-Level Systems

Zihe Zhao
Independent Researcher
ybing2093@gmail.com

February 2, 2026

Abstract

This paper introduces a perspective-indexed structural diagnostic for multi-level systems, the Ψ–$\Delta_{I\to G}$ pair, and uses it to organize a common family of persistent limitations appearing across domains (e.g. incompleteness, undecidability, intractability, chaotic predictability loss, and irreducible information loss) as regime-relative consequences of cross-level gluing constraints.

Fix a target system $\mathcal{L}$ together with a declared package V (variables treated as distinguishable, background constraints, admissible aggregations/compressions, discrepancy rules, admissible liftings, and a reporting threshold ε). We define: (i) an intra-level structural frustration index $\Psi(\mathcal{L}, A; V) \geq 0$ as the infimum of a declared nonnegative conflict functional over the declared reachable set, quantifying the minimal residual tension achievable within that perspective; and (ii) a cross-level diagnostic gap $\Delta_{I\to G}(\mathcal{L}; V)$ as the irreducible residual that remains when lower-level structure is optimally compressed into an upper-level description and then reconstructed under the declared admissible lifting class.

A guiding semantic axiom is adopted: for any genuine cross-level relation R, ontological non-identity is encoded by the meta-gap schema $\Delta^*_R > 0$ (Sec. 6.3), so cross-level "=" statements are not read as ontological equalities. Operationally, at a fixed finite reporting resolution ε, one may report resolution-negligibility only as a convention (e.g. $\Delta_{I\to G}(\mathcal{L}; V) \approx_\varepsilon 0$ meaning $\Delta_{I\to G}(\mathcal{L}; V) < \varepsilon$ under the locked clauses of V).

To describe persistence under refinement, Ψ is promoted to a running intra-level readout along a declared evolution parameter γ (time/scale/hierarchy), and an accumulated bookkeeping quantity $\Delta_{\mathrm{acc}}(\gamma)$ is defined by a minimal evolution interface (Sec. 7):

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{run}}(\gamma')\, d\gamma', \qquad \frac{d\Delta_{\mathrm{acc}}}{d\gamma} := \Psi_{\mathrm{run}}(\gamma),$$

Formally, along a declared evolution path, $\Psi_{\mathrm{run}}(\gamma) := \Psi(\mathcal{L}_\gamma, A_\gamma; V_\gamma)$ (Def. 7.1). Interpreting intra-level tension as a local density for an accumulated structural readout under declared evolution.

The paper provides formal definitions, an HTDS dynamical extension, and cross-domain illustrations. It also specifies a reproducible reporting protocol (package cards, admissibility/comparability gating, and explicit undefinedness diagnoses (optionally tagged; Appendix A)) and introduces a resolution-aware notion of fracture location (first detection along an evolution coordinate under a declared package), enabling reuse and comparison of Ψ–$\Delta_{I\to G}$ reports across domains beyond domain-specific content.

Contents

1 Introduction: A Package-Relative Diagnostic for Cross-Level Obstructions

Across logic, computation, dynamical systems, and information processing, a recurring pattern appears: a system admits locally coherent descriptions, yet attempts to connect "parts" to "whole" across genuinely distinct levels by a single continuous inference chain produce persistent residuals. Representative instances include Gödel-type incompleteness [13], undecidability and complexity barriers [4, 32, 33, 5, 17, 12, 26], long-horizon predictability loss in nonlinear dynamics [21, 20, 22, 10, 11], and irreducible loss under coarse-graining and compression [29, 30, 7, 16, 37, 35, 1].

Most classical "no-go" results are established *a posteriori* by domain-specific constructions (diagonalization, reductions, adversarial families, sensitivity arguments, etc.). The present paper addresses a prior structural question:

> Before committing to a specific algorithm, proof technique, or model detail, can one state a reusable, regime-controlled diagnostic indicating when cross-level continuity within a single declared perspective must leave a non-negligible residual (or, under missing declarations, must be reported as undefined)?

Answer sketch. This paper makes the diagnostic explicit by locking an operational package V and defining a pair of package-relative quantities: $\Psi(\mathcal{L}, A; V)$, the minimal intra-level residual tension within a declared perspective, and $\Delta_{I\to G}(\mathcal{L}; V)$, the irreducible residual produced by compression followed by best admissible reconstruction across levels. Cross-level "=" is treated only as a resolution-aware reporting equivalence $\approx_\varepsilon$, and when interface/admissibility/comparability clauses are not declared the correct output state is undefined rather than an implicit surrogate.

The emphasis is not on replacing domain theorems, but on extracting a shared *reporting-level structure* that can be instantiated and compared across domains under explicit declarations.

1.1 Terminology and scope: levels, perspectives, and multi-level systems

Throughout this paper, a *perspective* A means a declared representation scheme: it specifies (i) which variables or structural factors are treated as distinguishable, (ii) which background conditions are held fixed, and (iii) which aggregation/compression or identification rules are used to form the representation. A *level* is an abstraction layer induced by such a declared representation together with a declared resolution regime: different levels correspond to different representation/state spaces (e.g. micro/individual descriptions versus macro/global summaries) connected by generally non-invertible maps. A system is called *multi-level* when its analysis necessarily involves at least two such levels linked by an irreversible passage (coarse-graining, completion, identification, or any other genuinely cross-level gluing).

All statements in this paper are made relative to a declared operational *package* V, which fixes (at minimum) the system boundary and reachable-set convention, the discrepancy conventions, the admissible lifting class, and the reporting threshold ε (and, when needed, explicit comparability/transport clauses). This package-relative stance is not stylistic: without explicit declarations, cross-domain comparisons of structural claims are not well-posed.

1.2 Structural fracture law as a working meta-constraint

The framework is organized around the following structural law, used as a working meta-constraint rather than a theorem:

> **Structural fracture law.** When a logical system simultaneously contains opposing elements, and these elements produce feedback on the system, if one attempts to connect, across levels, the system's whole (macroscopic determinacy) and the system's internal factors (microscopic behavioral logic) by a single continuous-logic principle within a single perspective, then a judgment discontinuity is structurally unavoidable. In short: *opposing elements + system self-feedback + cross-level continuous logic $\Rightarrow$ judgment discontinuity.*

In operational terms, the law functions as a design constraint: once (i) oppositional contributions coexist within a system, (ii) these contributions feed back into the system, and (iii) one insists on a single cross-level continuity principle within a single perspective, the attempt to extend judgments across levels generates an irreducible residual (a cross-level mismatch under the declared regime). The main task of this paper is to turn this pattern into a reusable diagnostic framework with explicit objects, explicit admissibility constraints, and resolution-aware semantics.

In this manuscript, the fracture law is recorded as a motivating meta-constraint; no definition, lemma, or reporting rule depends on it as an axiom. Its operational counterparts are the package-relative objects (Ψ,$\Delta_{I\to G}$) and, when the claim is explicitly about decidability/extendability, the audit criterion of fiber non-constancy (Sec. 6.2, Appendix G).

Operational translation (informal, object-level). In the present framework, "opposing elements" correspond to competing contributions encoded by a declared nonnegative conflict functional E_A under a perspective A; "self-feedback" enters through the declared reachable-set convention Ω_{reach} and the declared interface constraints; and "cross-level continuous logic within a single perspective" corresponds to attempting a cross-level identification/transport without the extra clauses required to license it. The resulting judgment discontinuity is reported either as a non-negligible inter-level residual (e.g. $\Delta_{I\to G}(\mathcal{L};V) \not\approx_\varepsilon 0$ under the locked clauses of V) or as a non-licensing state (undefined) when interface/admissibility/comparability clauses are missing. When the claim is explicitly about decidability/extendability under an audited query set, fiber non-constancy provides a concrete witness localizing the obstruction (Sec. 6.2; Appendix G).

1.3 From a static constraint to an operational diagnostic object

The paper proceeds by separating two distinct sources of obstruction:

- **Intra-level tension:** residual conflict that persists even when one stays within a fixed level and a fixed perspective (no cross-level gluing).
- **Inter-level fracture:** residual mismatch generated by passing between levels (e.g. compressing to a global description and attempting to reconstruct individual structure), even when one allows an admissible class of reconstructions.

This separation is implemented by a perspective-indexed structural object, the Ψ–$\Delta_{I \to G}$ pair. The object is designed to be regime-relative: all statements are evaluated under a declared level specification, an admissible family of perspectives, and a declared resolution regime. This is the minimal requirement for controlled comparability across instantiations.

1.4 Declared regime and comparability layer (package discipline)

To make instantiations comparable rather than viewpoint-dependent, the framework requires that every report be relative to a declared package V, which records the comparability layer. Two consequences are central.

Resolution-aware semantics. Cross-level "=" and the convention "$\Delta_{I \to G} \approx_\varepsilon 0$" are interpreted in reporting by the single authoritative semantics fixed in Normative Rule 2.1.

Non-licensing outputs. When the declared package is insufficient to justify a report (missing admissibility clauses, missing domain restrictions, or missing transport for cross-package comparison), the correct output state is undefined (optionally tagged; Appendix A), rather than an unscoped numerical surrogate; see Normative Rule 2.1.

1.5 Positioning relative to existing frameworks

The Ψ–$\Delta_{I \to G}$ object is not introduced as a competing replacement for existing domain frameworks; it is a package-relative *reporting layer* that can be instantiated within them.

- **Renormalization/coarse-graining viewpoints.** The use of multi-level passages and admissible reconstructions is compatible with RG/EFT practice [16, 37, 38, 36], but the present manuscript treats endpoint matching and cross-level identification as *licensed only under a declared package* (interface, discrepancy, liftings, and resolution), with explicit undefined outputs when those clauses are absent.
- **Information-geometric and divergence-based diagnostics.** Many diagnostics are naturally expressed using relative entropy and its induced local geometry [7, 2, 40]. The framework keeps these tools admissible as package-level choices (e.g. as a discrepancy D), while separating them from the *comparability layer* (transport, domain gates, resolution conventions).
- **Probabilistic instantiation as a reporting interface.** Appendix B records a distribution-valued interface reading in which the already-declared diagnostic $\Delta_{I \to G}(\mathcal{L}; V)$ is translated into declared operational reporting readouts (uniform event-probability gap control under TV; uniform audit advantage/error control under an IPM) once the observation space, admissible lifting class, and discrepancy rule are locked as part of the package. The instantiation introduces no new core object and is invoked only under the declared clauses. Interpretation boundary: the distributions are protocol-/interface-induced objects declared as part of the package; this branch provides deterministic readouts and does not introduce sampling-based statistical inference.

- **Transport and cross-report comparability.** When comparisons across packages are invoked, explicit transport clauses are required; when an optimal-transport style reconciliation is intended, the framework treats it as an explicit declared component rather than an implicit identification [34].

The distinguishing feature is therefore not the choice of discrepancy or the underlying domain model, but the insistence that: (i) reporting semantics is locked by a declared package, (ii) cross-package comparisons require declared transport, and (iii) under missing clauses the correct output is undefined rather than an unscoped surrogate.

1.6 Main objects, contributions, and what is proved

Fix a system $\mathcal{L}$ and a declared package V with reporting threshold ε. Two structural quantities are introduced:

- **Intra-level tension Ψ.** Within a fixed perspective A at a fixed level specification, $\Psi(\mathcal{L}, A; V)$ is the infimum of a declared nonnegative conflict functional over the declared reachable set.

- **Cross-level fracture $\Delta_{I\to G}$.** $\Delta_{I\to G}(\mathcal{L}; V)$ is the minimax residual remaining after compressing to a coarse description and reconstructing via an admissible lifting class under the declared discrepancy rule.

Beyond definitions and reporting discipline, the paper establishes strictly operational non-vanishing mechanisms at the package-relative level: under metric-like discrepancy rules, whenever a macro representation non-injectively identifies interface-distinguishable states within a single macro fiber (i.e., induces fiber non-constancy), the minimax residual $\Delta_{I\to G}(\mathcal{L}; V)$ admits an explicit positive lower bound (Sec. 5.2). This supports witness-based "not resolution-negligible" reports at finite ε without invoking any ontological equality claim.

Separation of roles (definitions vs. proved statements vs. protocol). To support long-term study and controlled reuse, the manuscript keeps four roles explicit:

- **Defined objects:** $\Psi(\mathcal{L}, A; V)$, $\Delta_{I\to G}(\mathcal{L}; V)$, the accumulated bookkeeping quantity $\Delta_{\mathrm{acc}}(\gamma)$ (Sec. 7), and the comparability/undefinedness semantics (Sec. 8).

- **Adopted semantic axioms/postulates:** resolution-aware cross-level "=" is read as $\approx_\varepsilon$ in reporting (Normative Rule 2.1); and genuine cross-level relations carry a non-vanishing meta-gap schema $\Delta^*_R > 0$ (Sec. 6.3).

- **Proved operational claims:** when the required package clauses are declared, certain non-negligibility reports (e.g. positive lower bounds for $\Delta_{I\to G}$ under fiber non-constancy of the coarse representation (relative to the interface content)) are supported by explicit witnesses (Sec. 5.2 and related propositions).

- **Protocol clauses:** package cards, admissibility/comparability gating, and explicit undefined outputs are treated as part of the reporting semantics, not as mathematical conclusions (Sec. 8).

The framework also provides: (i) a reproducibility-oriented reporting protocol (package cards, admissibility/comparability gating, and explicit undefinedness diagnoses (optionally tagged; Appendix A)); (ii) an optional dynamical extension in which accumulated fractures are tracked along a declared evolution coordinate γ; and (iii) cross-domain instantiations structured as package cards to support controlled comparison across domains.

1.7 Roadmap

Section 2 fixes the type discipline and the resolution-aware semantics used throughout. Sections 3–5 introduce the package-relative objects and the core diagnostics. Later sections formalize well-posedness/comparability discipline and comparability/transport conditions, and present worked instantiations under explicitly declared packages.

2 Conventions and Reader Guide

2.1 Symbols and type discipline (system vs. declared package)

This paper distinguishes *target-system objects* from *analysis/reporting objects*. The distinction is semantic (what is being studied vs. how it is reported) and is enforced to avoid silent shifts of meaning across sections.

Target system (object-level). A target system is denoted by $\mathcal{L}$ (static or dynamical, depending on the chapter). Its state space is denoted by Ω, with a declared reachable set $\Omega_{\text{reach}} \subseteq \Omega$ (once the boundary/reachability convention in the declared package is fixed). When a task restricts attention to a regime relevant for a report, we use $\Omega^{\text{rel}}_{\text{reach}} \subseteq \Omega_{\text{reach}}$ (task-relevant reachable subset).

Declared package V (reporting-level). A declared package V is an *audit bundle* that fixes the operational conventions under which $\mathcal{L}$ is analyzed and reported. At minimum, a package declaration pins down:

- the system boundary and the reachable set convention (what is counted as a state in Ω_{reach});
- the level specification and admissible perspective family (what representations are allowed);
- the discrepancy structure used for reporting (micro discrepancy and, when needed, induced macro discrepancy);
- the admissible lifting / reconstruction class (what "best reconstruction" means);
- the reporting threshold ε (the operational meaning of "resolution-negligible");
- the comparability clauses: transport rules (when defined), domain gating, and optional diagnostic tags.

Accordingly, numerical statements about Ψ or $\Delta_{I\to G}$ are always package-relative, even when V is suppressed typographically.

Type discipline (what may depend on what).

- $\mathcal{L}, \Omega, \Omega_{\text{reach}}, \Omega^{\text{rel}}_{\text{reach}}$ are target-system objects (what is being studied).
- A (perspective), E (conflict functional), D (declared discrepancy), $\mathcal{U}$ (admissible liftings), and ε (threshold) are package-level objects (how the study is operationalized).
- Ψ is an intra-level structural quantity *for* $(\mathcal{L}, A)$ but is *reported under* V.
- $\Delta_{I\to G}$ is an inter-level residual defined only after V declares the relevant compression/reconstruction interface and admissible liftings; cross-package comparison requires an explicit transport declaration (otherwise the correct output is "undefined" with a diagnostic).

Partial semantics (well-posedness vs. undefinedness). All reporting-level quantities in this paper are treated as *partial* objects whose meanings are locked only on the domain of declared packages satisfying the relevant clauses. If required clauses are missing (under-specification), if an invoked comparison lacks declared transport/comparability, or if a referenced domain condition is not satisfied, then the corresponding numerical or logical statement is undefined relative to the declared regime and must be reported as such. For compactness, a report may attach a short diagnostic status tag (Appendix A) together with a minimal witness list, but the core semantic point is the same: "undefined" is an output state, not an error to be silently repaired by implicit conventions.

Normative Rule 2.1 (Resolution-aware equality and non-licensing outputs).

- **Cross-level equality.** Whenever an identification *glues distinct levels* (limits, completions, quotients/identifications, coarse-graining followed by reconstruction), any "=" is read in reporting as resolution-aware structural equivalence: $X = Y$ in such a context is reported as $X \approx_\varepsilon Y$ under the declared package V.
- **Ontological non-identity (meta-gap).** For genuine cross-level relations, ontological non-identity is recorded by the meta-gap axiom $\Delta^*_R > 0$ (Def. 6.1) for the invoked genuine cross-level relation R; accordingly, one may not silently replace a cross-level mismatch by a zero report $\Delta_{I\to G} \approx_\varepsilon 0$ without satisfying the licensing conditions under the declared package V.
- **Reporting convention.** The statement $\Delta_{I\to G} \approx_\varepsilon 0$ is *only* a reporting convention, read operationally as "$\Delta_{I\to G} < \varepsilon$ under the declared clauses of V".
- **Refinement behavior.** If the declared resolution is finer than the intrinsic floor for the intended gluing, the $\approx_\varepsilon 0$ convention is unavailable: one must report $\Delta_{I\to G} \not\approx_\varepsilon 0$ (or, when the needed clauses are missing, undefined) rather than forcing an equality claim.
- **Undefinedness / non-licensing.** Any numerical statement or cross-report comparison is licensed only on the domain where the required package clauses (interface, admissibility, and domain gating) are declared; cross-package comparison additionally requires an explicit transport law (Normative Rule 2.2). Otherwise, the correct output state is undefined (optionally with a short diagnostic status tag and a minimal witness list; Appendix A).

Normative Rule 2.2 (Cross-package comparability and explicit transport).

- **no implicit transport (Normative Rule 2.2).** Numerical reports produced under distinct declared packages V and V' are *not* treated as directly comparable scalars by default (even if they share a symbol name).
- **Explicit transport law.** A cross-package comparison (equality/inequality, limits across packages, or reusing a numerical threshold) is licensed only if a transport/identification rule $\tau_{V \to V'}$ is explicitly declared together with its domain of validity and compatibility with the relevant representation maps and interface clauses.
- **Non-comparability output.** If $\tau_{V \to V'}$ is not declared (or does not cover the reported objects), cross-package comparison statements are undefined; an optional diagnostic tag is non-comparable.
- **Resolution alignment.** When a transport is declared, comparison is made only after aligning reporting thresholds (or declaring an alignment rule). Otherwise the appropriate output is undefined (optional tag RES / resolution-mismatch).

Notation collisions (avoidance rules).

- The resolution threshold is ε. Classical constants that use the letter δ (e.g. Feigenbaum) retain their standard notation and are always written with explicit subscripts (e.g. δ_{Feig}) to avoid confusion with the reporting threshold.
- The symbol Δ is reserved for cross-level residuals, but the bare symbol Δ is not used as an alias for any specific gap object; we always keep subscripts/markers (e.g. $\Delta_{I \to G}$, $\Delta_{\text{acc}}(\gamma)$, Δ^*) to prevent cross-module collisions.
- Generic scalar placeholders (dummy variables, coordinates, function arguments) are written as $x, y, \ldots$, not Δ.
- The symbol A is reserved for *perspectives* (Def. 4.4). Matrix operators used in local linearizations are written as J (and, when needed, J_{shift}, J_{meta} in Appendix E).
- The symbol V is reserved for declared packages (audit bundles). The package-locked reporting structure is denoted by $\mathcal{V}$ (Def. 4.11).
- The symbol R is reserved for genuine cross-level relations (Sec. 6.3). Substrate constraint regimes are written as $\mathfrak{R}$ (Def. 4.1) to avoid collision.
- Level-respecting re-expression maps are written as $T_{A_2 \leftarrow A_1}$ (Def. 4.6).
- In Appendix G.3, intra-level evolution semigroups are written as $\{U_\gamma\}_{\gamma \geq 0}$ (not T_γ) to avoid collision with the re-expression symbol $T_{A_2 \leftarrow A_1}$.
- In Appendix F (calculus-only instantiations), the discrete residual is denoted by e_k (Eq. 20); the symbol r_k is reserved for the meta deviation relative to a declared propagation rule (Lemma C.6).
- In Appendix E, normalization-equivalence in fracture space is written as $\equiv_{\text{N}}$; the symbol "$\approx$" is reserved for local analytic/discretization approximations and is not used to denote normalization-equivalence.

2.2 Equality and resolution-aware approximation

This paper uses "=" as an academic writing convention, but its *reporting semantics* depends on whether a statement stays within a single declared calculus/module or crosses a genuine level boundary.

(i) Within a declared calculus/module (single-level inference). When a derivation is explicitly confined to a declared calculus/module (an H-calculus segment in the terminology used later), "=" is read as *equality under the rules of that declared module*: an inference-level identification licensed by the adopted rules and admissibility constraints of the module. This reading is local: it does not by itself assert that a cross-level identification has been made gap-free.

Bare approximation vs. resolution-aware reporting equivalence. Within a declared module, the symbol "≈" (without ε) denotes a local analytic/discretization approximation (e.g. a local linearization $v_{k+1} \approx Jv_k$). It does not carry the reporting semantics of $\approx_\varepsilon$. Only $\approx_\varepsilon$ denotes resolution-aware reportable equivalence under a declared package.

(ii) Cross-level identities (limits, completions, quotients, coarse-graining). Whenever "=" arises from a construction that *glues distinct levels* (e.g. limiting, completion, quotienting/identification, coarse-graining followed by reconstruction), the default reading is *resolution-aware structural equivalence under the declared package*:

$$X = Y \quad \text{(in cross-level context)} \qquad \rightsquigarrow \qquad X \approx_\varepsilon Y,$$

where $\approx_\varepsilon$ denotes "indistinguishable at the declared reporting threshold ε under V".

Normative semantics. All later uses of cross-level equality, resolution-aware approximation, and non-licensing/undefinedness inherit the single authoritative semantics fixed by Normative Rule 2.1.

Two uses of Δ (fixed-slice vs. accumulated). To prevent a common confusion:

- $\Delta_{I \to G}$ denotes a fixed-slice inter-level residual (compression → best admissible reconstruction).
- $\Delta_{\mathrm{acc}}(\gamma)$ (when used) denotes an accumulated scalar along a declared evolution coordinate γ. Its meaning is package-relative and requires that the package declare the carrier/averaging conventions used to define the accumulation. It is not interchangeable with the fixed-slice diagnostic $\Delta_{I \to G}$.

Throughout the manuscript we do not use the bare symbol Δ as an alias for any specific gap object ($\Delta_{I \to G}$, $\Delta_{\mathrm{acc}}(\gamma)$, or Δ^*); we always keep subscripts/markers. Generic scalar placeholders (dummy variables, coordinates, function arguments) are written as $x, y, \dots$, not Δ.

2.3 Reader guide and dependency map

This manuscript is written as a *slim core* plus optional branches. The core fixes the package-relative semantics and reporting discipline; the branches are invoked only when the corresponding questions are asked and the corresponding declarations are supplied.

Main line (slim core; required). Read Sections 3–5 for the core objects and their basic lower-bound mechanisms, together with the regime-locked outputs (including the level-locked H/M classification, §5.4). Read Section 8 for the licensing discipline: when a numerical statement or a cross-report comparison is not justified under the declared clauses, the correct output is *undefined / not licensed* (optionally tagged; Appendix A), rather than a forced surrogate number.

Branch I: audit semantics and $H \to H^+$ transition (Appendix G; conditional). *Trigger.* Invoke this branch only when the statement requires *single-valued decidability* of an audited query set under a locked viewpoint $V^{\text{aud}} = (\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$ (Def. G.1). The key predicate is $\mathrm{Val}(V^{\text{aud}})$ (Def. G.3): when $\mathrm{Val}(V^{\text{aud}}) = 1$ one is in H-closure; when $\mathrm{Val}(V^{\text{aud}}) = 0$ (fiber non-constancy) the non-closure regime holds and an $H \to H^+$ extension is structurally necessary (Lemma G.2). *Reader payoff.* This module explains why a surface-level $\Delta_{I\to G} \approx_\varepsilon 0$ report can become unavailable once an audit viewpoint demands single-valuedness over $\mathcal{Q}$, and it localizes the missing information on endpoint fibers (worked entry point: §G.4).

Branch II: probabilistic instantiation (Example 5.1; Appendix B). *Trigger.* Invoke this branch when the interface maps (Φ_I, Φ_G) are instantiated as distribution-valued readouts and the discrepancy D is chosen as a divergence/metric on distributions (e.g. total variation or an IPM induced by declared tests [8, 24]). *Reader payoff.* This branch turns $\Delta_{I\to G}$ into an operational bound on event-probability gaps and audit advantage/error, clarifying why the package clauses (domain, tests, and threshold) must be locked for reproducible reporting. Interpretation boundary: throughout this branch, P_S and Q_S^u are protocol-/interface-induced distributions declared as part of V; the TV/IPM outputs are deterministic reporting readouts about these objects, not sampling-based statistical inference.

Branch III: stability of discrete macro labels (Section 9; Appendix H). *Trigger.* Invoke this branch when the natural follow-up question is: *why do certain macroscopic (discrete) labels appear stable even when cross-level gluing is not gap-free? Reader payoff.* Chapter 9 provides a package-relative, resolution-aware stability template (Edges A–C) for discrete labels, together with explicit undefinedness/comparability behavior. Appendix H records an aligned checklist and diagnostic stubs for reuse.

Branch IV: geometric/dynamical intuition (Appendix D; conditional). *Trigger.* Invoke this branch only when a two-dimensional embedding is used as an *interpretive aid* (e.g. to visualize self-similar/closure segments or local rotation–scaling dynamics that compress to a logarithmic spiral picture). *Reader payoff.* Appendix D records a package-compatible geometric vocabulary (including a per-mode spiral lemma under a declared linearized generator) and a "when to use / when not to use" boundary. It does not introduce any new slim-core primitive and does not upgrade any visualization into an ontological claim.

Module invocation discipline. Each optional branch is *non-invoked by default.* If its dependency declarations are not supplied, the correct behavior is to fall back to the slim-core outputs and report "undefined / not licensed" for any branch-specific numerical claim, according to Protocol 8.2.

Core statement (slim core commitment). Once a package V is declared, Ψ and $\Delta_{I\to G}$ separate two non-interchangeable sources of obstruction: Ψ records the best achievable intra-level tension floor within a fixed level/perspective, while $\Delta_{I\to G}$ records the residual generated specifically by crossing levels via compression and admissible reconstruction. Under refinement, either an intra-level floor persists at the reporting resolution or an inter-level floor persists (not resolution-negligible), so cross-level identifications are available only as $\approx_\varepsilon$ conventions under declared clauses. When those clauses are not supplied, the framework does not fill gaps by default: the output state is undefined.

3 Framework Overview

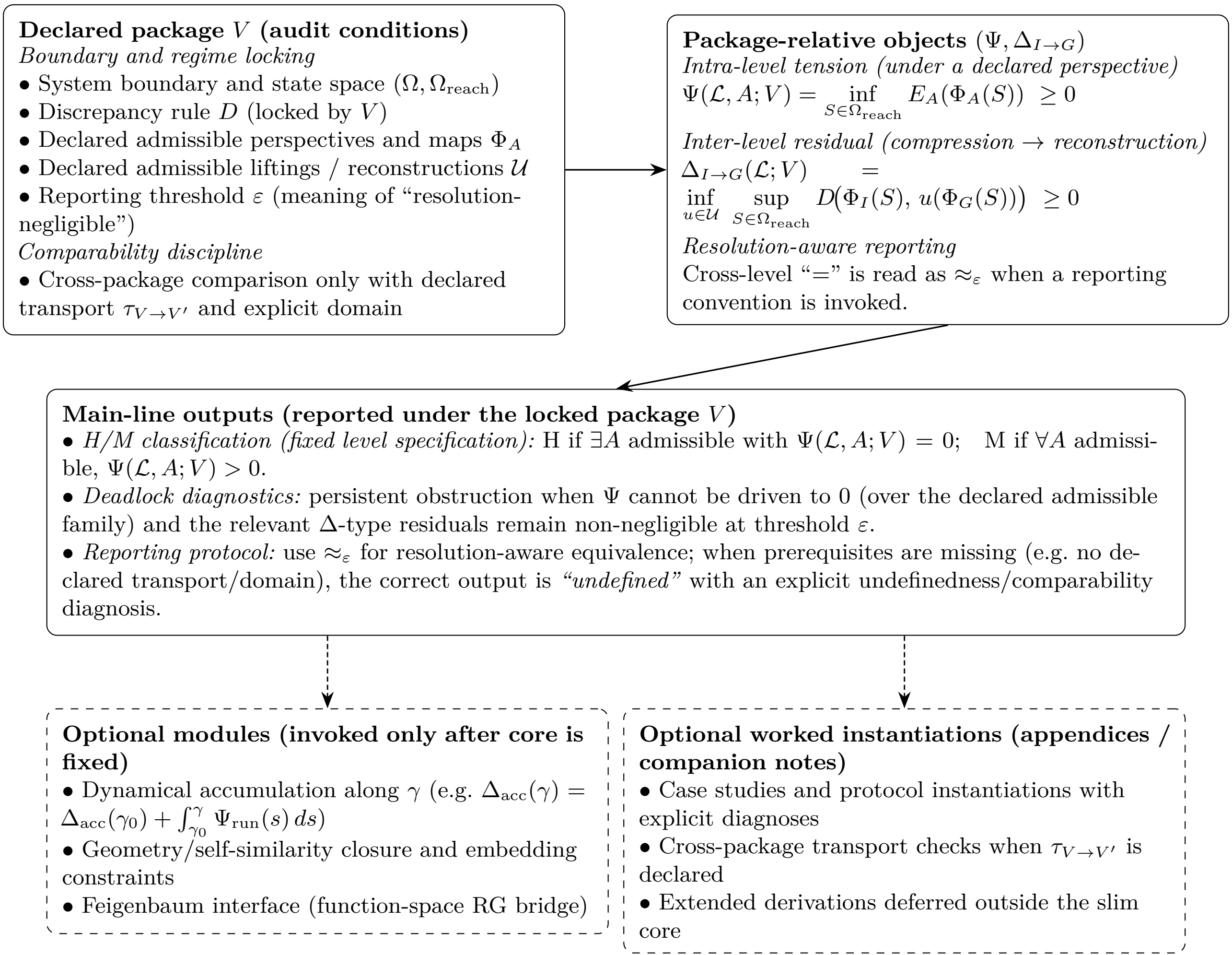

Figure 1: Overview: declared package V (audit conditions) → package-relative objects $(\Psi, \Delta_{I\to G})$ → main-line outputs; optional dynamical extension and worked instantiations are deferred to appendices.

Dependency tree (reading guide). Figure 1 is read as a dependency-locked pipeline: (i) first, one declares a package V (system boundary, reachable set, discrepancies, admissible perspectives/liftings, and reporting threshold ε); (ii) only under that locked package are the structural quantities well-defined and reportable: Ψ is evaluated as an intra-level tension proxy under declared perspectives, while $\Delta_{I\to G}$ is evaluated as an inter-level residual under declared compression/reconstruction admissibility; (iii) main-line outputs (H/M classification, deadlock diagnostics, and stability/instability claims) are then reported using the resolution-aware convention $\approx_\varepsilon$ and the well-posedness/domain/comparability rules; (iv) any cross-package comparison is licensed only under an explicit transport law; otherwise it is undefined (Normative Rule 2.2); (v) optional modules (dynamical accumulation, geometry closure, Feigenbaum interface, extended instantiations) may be invoked only after the core package and object definitions are fixed, and must state any extra dependencies explicitly.

3.1 Minimal notation quick reference (macro-consistent)

For quick reference, we list only the symbols that are most likely to be misread on first contact. All meanings are *package-relative* (i.e. evaluated and reported under a declared package V and its threshold ε), unless explicitly stated otherwise.

Table 1: Minimal notation quick reference (restricted to the high-confusion set; consistent with the current macro layer).

Symbol	**Meaning (as used in this paper)**
$\mathcal{L}$	Target system object (static or dynamical, depending on context).
Ω	Full state space of $\mathcal{L}$ (target-system object).
Ω_{reach}	Reachable set (base) declared under the package; subset of Ω on which reporting is defined.
$\Omega^{\text{rel}}_{\text{reach}}$	Task-relevant subset of Ω_{reach} (regime restriction used by a specific report/diagnostic).
A	Perspective / representation specification (declares distinguishable variables, fixed background, and aggregation/compression rules). Reported statements are always relative to an admissible family of such A.
Ψ	Intra-level structural frustration index (package-reported): minimal irreducible tension achievable within a fixed perspective (via the declared conflict functional and reachable convention).
$\Delta_{I\to G}$	Fixed-slice inter-level structural gap (package-reported): irreducible residual under declared compression $\to$ best admissible reconstruction/lifting. Throughout, we write $\Delta_{I\to G}$ explicitly; the bare symbol Δ is reserved as a generic gap placeholder and is not used as an alias for any specific object. Ontological non-identity for genuine cross-level relations is recorded by the meta-gap axiom $\Delta^*_R > 0$ (for any genuine cross-level relation R); operationally one may report $\Delta_{I\to G} \approx_\varepsilon 0$ only as a resolution convention (i.e. $\Delta_{I\to G} < \varepsilon$ under V).
$\Delta_{\text{acc}}(\gamma)$	Accumulated fracture scalar along a declared evolution coordinate γ (when used). This is distinct from the fixed-slice $\Delta_{I\to G}$ and is meaningful only after the package declares the accumulation carrier and averaging conventions.
ε	Reporting threshold (resolution cutoff) declared as part of V. Statements like "$\Delta_{I\to G} \approx_\varepsilon 0$" are read operationally as "$\Delta_{I\to G} < \varepsilon$ under the declared package."
$\approx_\varepsilon$	Resolution-aware structural equivalence: "indistinguishable at threshold ε under V" (default reading of cross-level "=" in reporting).
Δ^*	Meta-gap schema. For any genuine cross-level relation R, $\Delta^*_R > 0$. Not a tunable tolerance and not package-relative; it prevents cross-level "=" statements from being read as ontological identities (Sec. 6.3).
$\delta_{\text{Feig}},\ \alpha_{\text{Feig}}$	Classical Feigenbaum constants (subscripted to avoid collision with the reporting threshold ε).

4 Formal Objects: Regime, Perspectives, Packages, and Package-Locked Reporting Structures

This section fixes the primitives used throughout the manuscript. The central convention is:

A reportable analysis context is specified by a target system equipped with a declared package; the package locks the reporting structure used for evaluation and comparison.

Accordingly, we distinguish a *substrate regime* (which supplies a fine-grained domain on which representations act) from a *declared package* (which locks what is reportable/comparable and at which resolution). The resulting package-locked reporting structure is denoted by $\mathcal{V}$ and is determined by the pair $(\mathcal{L}, V)$ (equivalently, the substrate regime together with the locked clauses of V) (Def. 4.11).

4.1 Substrate regime and fine-grained domain

Definition 4.1 (Constraint regime)**.** A *constraint regime* is a collection $\mathfrak{R}$ of local constraint rules. Each rule $r \in \mathfrak{R}$ involves only finitely many degrees of freedom and is viewed as a relation/predicate on those degrees of freedom.

Definition 4.2 (Induced support and fine-grained state space)**.** Let $\mathrm{Var}(\mathfrak{R})$ denote the set of degrees of freedom that appear in at least one rule in $\mathfrak{R}$ (the support induced by the regime). A *fine-grained state* is an assignment of values to all degrees of freedom in $\mathrm{Var}(\mathfrak{R})$. The set of all such assignments forms the fine-grained state space Ω.

Remark 4.1 (No global satisfiability assumption). No assumption is made that $\mathfrak{R}$ is jointly satisfiable, nor that the regime occupies only states satisfying all constraints. Partial violations are permitted and treated as structurally relevant.

Definition 4.3 (Reachability)**.** When admissibility/dynamics are declared, the actually attainable fine-grained states form a reachable set

$$\Omega_{\mathrm{reach}} \subseteq \Omega.$$

Unless stated otherwise, worst-case (supremum) statements are taken over Ω_{reach}.

4.2 Perspectives and cross-level interfaces

Definition 4.4 (Perspective and representation map)**.** Given a regime $\mathfrak{R}$ with fine-grained state space Ω, a *perspective* is specified as

$$A = (F, \mathrm{Enc}),$$

where F is the family of structural factors made visible and Enc is the encoding rule implementing the chosen description. Each perspective A induces a representation space Ω_A and a map

$$\Phi_A : \Omega \to \Omega_A.$$

Definition 4.5 (Level specification)**.** A *level* is specified operationally by a *level specification*

$$\Lambda := \big(\Pi_\Lambda,\ \mathcal{A}_\Lambda,\ \varepsilon\big),$$

where:

- Π_Λ is a fixed level partition determining what is treated as "individual" versus "global" at that stage (i.e. which degrees of freedom are retained or suppressed by level-respecting encodings);
- $\mathcal{A}_\Lambda$ is a declared admissible family of level-respecting perspectives (re-expressions that do not change Π_Λ);
- $\varepsilon > 0$ is a declared resolution threshold used for reporting indistinguishability (Def. 4.10).

Remark 4.2 ("Whole" versus "individual" is boundary-induced). Within this formalization, "whole" and "individual" are not intrinsic labels of an object; they are induced by the declared level specification (partition and admissible re-expressions) and by the boundary choices encoded in the package.

Definition 4.6 (Level-respecting re-expression)**.** For two perspectives A_1, A_2 within the same admissible family $\mathcal{A}_\Lambda$, a *level-respecting re-expression* is an invertible map

$$T_{A_2 \leftarrow A_1} : \Omega_{A_1} \to \Omega_{A_2}$$

such that for all $S \in \Omega$,

$$\Phi_{A_2}(S) = T_{A_2 \leftarrow A_1}\big(\Phi_{A_1}(S)\big).$$

When such a $T_{A_2 \leftarrow A_1}$ exists, A_1 and A_2 are treated as equivalent encodings at the same declared level (up to reparameterization).

Definition 4.7 (Designated cross-level pair)**.** We fix two designated perspectives, interpreted as an individual-level and a global-level description:

$$A_I,\ A_G, \qquad \Phi_I : \Omega \to \Omega_I,\ \Phi_G : \Omega \to \Omega_G.$$

Here Ω_I and Ω_G are the respective representation spaces induced by A_I and A_G.

Definition 4.8 (Interface mechanism)**.** A *cross-level interface mechanism* consists of:

- a discrepancy rule $D : \Omega_I \times \Omega_I \to \mathbb{R}_{\geq 0}$ (the notion of mismatch);
- an admissible family of liftings (reconstructions) $\mathcal{U} \subseteq \{u : \Omega_G \to \Omega_I\}$.

Remark 4.3 (Operational degrees of freedom must be locked). Any scalar cross-level diagnostic (e.g. a residual) is not well-posed until the operational degrees of freedom are locked. Minimally, one must lock

$$(\Phi_I, \Phi_G, D, \mathcal{U}, \Omega_{\text{reach}}, \varepsilon),$$

where ε is the declared reporting resolution (Def. 4.10).

4.3 Declared package V as an audit bundle

Definition 4.9 (Declared package)**.** A *declared package* is an audit bundle that locks the operational degrees of freedom under which cross-level statements are made. Concretely, we write

$$V := \big(\mathcal{I},\ \varepsilon,\ \mathcal{Q},\ \mathcal{C}\big),$$

where:

- $\varepsilon > 0$ is the declared reporting threshold (resolution convention);
- $\mathcal{Q}$ is a declared set of audited predicates/queries (optional in purely structural sections, mandatory in audit sections);
- $\mathcal{C}$ is a declared admissible class of constrained histories or admissible trajectories (optional unless dynamics/audit questions are invoked);
- $\mathcal{I}$ is the *interface module* responsible for uniquely locking cross-level diagnostics, and must minimally include the core degrees of freedom

  $$\mathcal{I}\ \supseteq\ (\Phi_I,\ \Phi_G,\ D,\ \mathcal{U},\ \Omega_{\text{reach}}),$$

  where (Φ_I, Φ_G) are the chosen representation maps, D is the discrepancy rule on Ω_I, $\mathcal{U}$ is the admissible lifting family $\Omega_G \to \Omega_I$, and Ω_{reach} is the reachable set over which worst-case statements are evaluated.

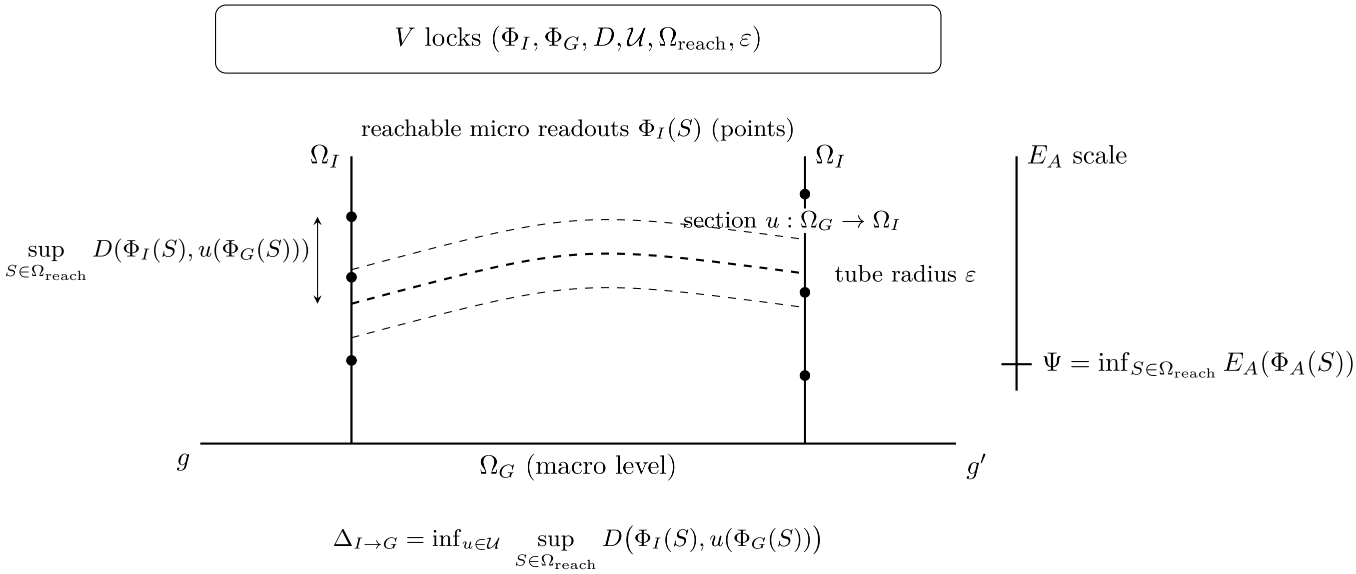

Figure 2: Fiber view of cross-level gluing under a declared interface. The macro representation Φ_G induces fibers above Ω_G, while reachable micro readouts $\Phi_I(S)$ appear as points. A declared lifting $u \in \mathcal{U}$ selects a section; $\Delta_{I \to G}$ is the minimax worst-case residual measured by the declared discrepancy rule D over Ω_{reach}. The dashed band of radius ε visualizes the reporting convention $\Delta_{I \to G} \approx_\varepsilon 0 \Leftrightarrow \Delta_{I \to G} < \varepsilon$ under the locked $(D, \mathcal{U})$. The intra-level quantity Ψ (right) is shown only as a floor marker $\inf_{S \in \Omega_{\text{reach}}} E_A(\Phi_A(S))$ for a declared perspective A with representation map Φ_A and conflict functional E_A, and is distinct from $\Delta_{I \to G}$.

Remark 4.4 (Threshold binding within a declared package). When a declared package V provides a reporting threshold ε, all level specifications Λ invoked inside V are understood to inherit the same ε unless an explicit multi-threshold conversion clause is declared. If thresholds differ across reports without such a conversion clause, comparisons are treated as undefined (optional diagnostic tag RES / resolution-mismatch) in the sense fixed by Normative Rule 2.1.

Definition 4.10 (Resolution-aware indistinguishability)**.** Under a declared threshold $\varepsilon > 0$, the symbol "$\approx_\varepsilon$" denotes the reporting convention for resolution-aware indistinguishability. For a nonnegative reportable scalar $x \geq 0$, the report

$$x \approx_\varepsilon 0$$

is licensed precisely when $x < \varepsilon$ under the conventions locked by V. When a discrepancy rule D is used, the same convention applies to the declared comparisons induced by D.

Protocol 4.1 (Minimal reporting card for V)**.** Any reported value of a package-relative quantity (including later Ψ and $\Delta_{I \to G}$) is regarded as well-posed and comparable only if it reports at least:

1. **Substrate and boundary:** the substrate regime $\mathfrak{R}$ (explicitly or by reference) and the induced fine-grained domain $(\Omega, \Omega_{\text{reach}})$;
2. **Designated perspectives:** the designated perspectives (A_I, A_G) and representation maps (Φ_I, Φ_G);
3. **Reachability:** the reachable set Ω_{reach} (and any declared dynamics or admissibility constraints used to define it);
4. **Interface mechanism:** discrepancy rule D on Ω_I and the admissible lifting family $\mathcal{U}$;

5. **Reporting convention:** threshold ε and the meaning of "$\approx_\varepsilon$" used in the report;
6. **Audit clause (when applicable):** the audited query set $\mathcal{Q}$ and admissible history class $\mathcal{C}$.

Definition 4.11 (Package-locked reporting structure (logical structure))**.** Fix a target system $\mathcal{L}$ (equivalently, a substrate regime together with its induced domain $(\Omega, \Omega_{\text{reach}})$). Given a declared package $V = (\mathcal{I}, \varepsilon, \mathcal{Q}, \mathcal{C})$, the *reporting structure locked by* $(\mathcal{L}, V)$ is the logical structure

$$\mathcal{V} := \big(\Omega_{\text{reach}}, \approx_\varepsilon, \Phi_I, \Phi_G, D, \mathcal{U}, \mathcal{Q}, \models_V\big),$$

where $\models_V$ denotes evaluation/satisfaction carried out under the conventions, encodings, and admissibility constraints locked by V. All quantities reported in this manuscript (including Ψ and Δ-type diagnostics) are to be understood as invariants/obstructions of $\mathcal{V}$ (hence *package-relative*). In most of the manuscript we suppress $\mathcal{V}$ and simply say "under V".

Example 4.1 (Package Card: V_{toy} (XOR 2-bit), report ID: `V_toy_xor`)**.**

- **ID / scope.** Target quantities: $\Delta_{I\to G}(L; V_{\text{toy}})$ and $\Psi(L, A; V_{\text{toy}})$. Boundary: system state is 2-bit; no environment variables.
- **System object.** L: toy 2-bit system. $\Omega = \{00, 01, 10, 11\}$.
- **Audited domain.** $\Omega_{\text{reach}} = \Omega$.
- **Level specifications / perspectives.** For Ψ: Λ = bit-level, A = identity perspective, $\Phi_A = \text{id} : \Omega \to \Omega$. For $\Delta_{I\to G}$: $\Phi_I = \text{id} : \Omega \to \Omega_I (= \Omega)$, $\Phi_G(x_1x_2) = x_1 \oplus x_2 : \Omega \to \Omega_G (= \{0, 1\})$.
- **Conflict functional (choice declared).** Version A: $E_A(x_1x_2) = \mathbf{1}\{(x_1 = x_2 = 1)\}$ (thus $\Psi = 0$ in the infimum reading; the infimum is realized here). Version B: $E_A \equiv 1$ (thus $\Psi = 1$).
- **Interface mechanism.** Discrepancy D: Hamming distance on 2-bit strings. Admissible liftings $\mathcal{U}$: all functions $u : \{0, 1\} \to \Omega$.
- **Reporting resolution / threshold.** Choose ε (e.g. $\varepsilon = 0.5$ or $\varepsilon = 2$). Convention: $\Delta_{I\to G} \approx_\varepsilon 0$ iff $\Delta_{I\to G} < \varepsilon$ under the declared $(D, \mathcal{U})$.
- **Comparability.** No transport/comparability law $\tau_{V\to V'}$ is declared; cross-package comparison is undefined (Normative Rule 2.2).
- **Admissibility constraints.** (toy) none beyond u is a function $\{0, 1\} \to \Omega$ and D is Hamming.

Example 4.2 (Minimal end-to-end pipeline report under V_{toy})**.** Consider the declared package card V_{toy} in Example 4.1. Both $\Psi(\mathcal{L}, A; V_{\text{toy}})$ and $\Delta_{I\to G}(\mathcal{L}; V_{\text{toy}})$ are well-defined because the required clauses $(\Phi_I, \Phi_G, D, \mathcal{U}, \Omega_{\text{reach}}, \varepsilon)$ are explicitly locked.

Computing $\Delta_{I\to G}$ (fixed-slice). For the XOR macro, the fibers are $\Phi_G^{-1}(0) = \{00, 11\}$ and $\Phi_G^{-1}(1) = \{01, 10\}$. Fix any lifting $u : \{0, 1\} \to \Omega$. On the fiber $\Phi_G^{-1}(0)$, the best choice is $u(0) \in \{00, 11\}$, but even then the worst-case Hamming residual is 2 (because $D(00, 11) = 2$). Likewise, on $\Phi_G^{-1}(1)$, the worst-case residual is 2. Therefore

$$\sup_{S\in\Omega_{\text{reach}}} D\big(\Phi_I(S), u(\Phi_G(S))\big) = 2 \quad \text{for every admissible } u,$$

and hence

$$\Delta_{I\to G}(\mathcal{L}; V_{\text{toy}}) = \inf_{u\in\mathcal{U}} 2 = 2.$$

Consequently, under the resolution-aware convention, a report of "$\Delta_{I\to G} \approx_\varepsilon 0$" is licensed exactly when the declared threshold satisfies $\varepsilon > 2$; if $\varepsilon \le 2$ the correct output is "not resolution-negligible" under V_{toy}.

Computing Ψ and the induced H/M regime label. In Example 4.1, Version A of the declared conflict functional yields $\Psi = 0$ in the infimum reading (and in this toy domain the infimum is realized), while Version B yields $\Psi = 1$. Under the level-locked H/M classification (Definition 5.6, with the admissible family declared as part of the level specification), Version A yields an H-type regime, whereas Version B yields an M-type regime. This illustrates that H/M is regime-relative: it is a property of $(\Lambda, \mathcal{A}_\Lambda, V)$, not of $\mathcal{L}$ alone.

Undefinedness check. If, in the package card, either the discrepancy rule D or the admissible lifting family $\mathcal{U}$ is not declared, then $\Delta_{I\to G}(\mathcal{L}; V_{\text{toy}})$ is undefined under the declared regime (under-specification; optional tag U-spec).

Cross-package non-comparability check. Define a second package V'_{toy} that differs only in the discrepancy rule and threshold, e.g. $D' := \frac{1}{2}D$ (half-scaled Hamming) and $\varepsilon' := \frac{1}{2}\varepsilon$, while keeping $(\Phi_I, \Phi_G, \Omega_{\text{reach}}, \mathcal{U})$ unchanged. Then the numerical value of the fixed-slice gap rescales accordingly: $\Delta_{I\to G}(\mathcal{L}; V'_{\text{toy}}) = 1$ (instead of 2). However, in the absence of an explicitly declared transport law $\tau_{V_{\text{toy}}\to V'_{\text{toy}}}$ relating the reporting conventions and units, the statement $\Delta_{I\to G}(\mathcal{L}; V_{\text{toy}}) \le \Delta_{I\to G}(\mathcal{L}; V'_{\text{toy}})$ is undefined (optional tag non-comparable); see Normative Rule 2.2.

4.4 Comparability across packages

Definition 4.12 (Comparability and transport law)**.** Let $V = (\mathcal{I}, \varepsilon, \mathcal{Q}, \mathcal{C})$ and $V' = (\mathcal{I}', \varepsilon', \mathcal{Q}', \mathcal{C}')$ be two declared packages. Cross-package comparison of reported scalars is defined only if a *transport law* is explicitly declared, consisting of:

- reparameterization/identification maps between the relevant representation spaces (e.g. between Ω_I and Ω'_I; between Ω_G and Ω'_G), compatible with the chosen (Φ_I, Φ_G);
- a transformation rule specifying how D and $\mathcal{U}$ are transported (pushforward/pullback) under these identifications;
- a resolution rule specifying how ε relates to ε' for reporting.

If no such transport law is declared, cross-package comparison statements are undefined; see Normative Rule 2.2.

5 Core Quantities: Ψ and the Fixed-Slice Gap $\Delta_{I\to G}$

This section defines the two core quantities of the framework. Both are *package-relative*: their meanings and reported values are uniquely locked only after a declared package V has fixed the relevant interface mechanism and reporting convention (Sec. 4).

Main-line convention. In the slim core, the gap symbol refers to the fixed-slice diagnostic $\Delta_{I\to G}(\mathcal{L}; V)$. Accumulated quantities and ontological guardrails are always written explicitly as $\Delta_{\text{acc}}(\gamma)$ and Δ^*_R (with R suppressed when unambiguous), respectively.

5.1 Intra-level structural parameter Ψ

Definition 5.1 (Conflict functional (declared))**.** Fix a system $\mathcal{L}$ with fine-grained state space Ω and reachable set Ω_{reach}. Let A be a declared perspective at a fixed level specification (Def. 4.5) with representation map $\Phi_A : \Omega \to \Omega_A$.

A *conflict functional* under A is a declared map

$$E_A : \Omega_A \to \mathbb{R}_{\geq 0},$$

whose interpretation is fixed as part of the declared regime (either explicitly listed in V or deterministically induced by locked clauses of V).

Definition 5.2 (Ψ (intra-level structural parameter))**.** Fix a declared package V and a perspective A admissible at a fixed level. Assume V declares (or induces) a conflict functional E_A in the sense of Def. 5.1, and fixes the reachable set Ω_{reach}.

The intra-level structural parameter is defined by

$$\Psi(\mathcal{L}, A; V) := \inf_{S \in \Omega_{\text{reach}}} E_A\big(\Phi_A(S)\big).$$

(Equivalently, $\Psi(\mathcal{L}, A; V) = \inf_{y \in \Phi_A(\Omega_{\text{reach}})} E_A(y)$.) **Operability gate.** If the declared reachable set Ω_{reach} is empty, numerical reporting of $\Psi(\mathcal{L}, A; V)$ is undefined under Protocol 8.2 (optional tag: ADM).

Remark 5.1 (Allowed values and interpretation)*.* By construction, $\Psi(\mathcal{L}, A; V) \in [0, \infty]$ and in particular Ψ may be equal to 0.

Neutral reading. $\Psi(\mathcal{L}, A; V) = 0$ means that, within the declared reachability and within the chosen conflict functional, there exist reachable states whose residual conflict can be made arbitrarily small under A. $\Psi(\mathcal{L}, A; V) > 0$ means that a strictly positive lower bound persists under the same declared regime. No cross-level claims are implied by Ψ alone.

Regime dependence. Changing any of the locked clauses (e.g. reachable set, admissibility, or the choice of E_A) generally changes the operational meaning and value of Ψ. Therefore, any reported Ψ must be accompanied by the relevant package elements (Sec. 4.3, Protocol 4.1).

5.2 Inter-level structural gap $\Delta_{I\to G}$

Definition 5.3 ($\Delta_{I\to G}$ (inter-level structural gap))**.** Fix a system $\mathcal{L}$ with state space Ω and reachable set Ω_{reach}. Let V be a declared package whose interface module $\mathcal{I}$ (Def. 4.9) includes a designated cross-level pair

$$(\Phi_I : \Omega \to \Omega_I,\ \Phi_G : \Omega \to \Omega_G),$$

a discrepancy rule $D : \Omega_I \times \Omega_I \to \mathbb{R}_{\geq 0}$, and an admissible lifting family $\mathcal{U} \subseteq \{u : \Omega_G \to \Omega_I\}$.

The inter-level structural gap is defined as

$$\Delta_{I\to G}(\mathcal{L}; V) := \inf_{u \in \mathcal{U}} \sup_{S \in \Omega_{\text{reach}}} D\big(\Phi_I(S),\, u(\Phi_G(S))\big).$$

Operability gate. If $\Omega_{\text{reach}} = \varnothing$ or $\mathcal{U} = \varnothing$ under the declared regime, numerical reporting of $\Delta_{I\to G}(\mathcal{L}; V)$ is undefined under Protocol 8.2 (optional tag: ADM).

Definition 5.4 (Macro fibers and interface diameter)**.** Under the interface pair (Φ_I, Φ_G) declared in V, for any macro value $g \in \Phi_G(\Omega_{\text{reach}})$ define the *macro fiber*

$$F_g \ := \ \{S \in \Omega_{\text{reach}} :\ \Phi_G(S) = g\}.$$

Its interface image is $\Phi_I(F_g) \subseteq \Omega_I$. If the declared discrepancy rule D satisfies a triangle inequality on Ω_I (e.g. if D is a pseudometric), define the *fiber diameter*

$$\operatorname{diam}_D(g) \;:=\; \sup_{S,S' \in F_g} D\big(\Phi_I(S), \Phi_I(S')\big) \;\in [0,\infty].$$

See Section 5.3 for a geometric reading of fibers, diameter, and lifting radius.

Assumption 5.1 (Triangle-type inequality for the declared discrepancy)**.** The declared discrepancy rule D satisfies a triangle inequality on Ω_I: for all $x, y, z \in \Omega_I$,

$$D(x,z) \;\le\; D(x,y) + D(y,z).$$

This assumption is satisfied by common metric discrepancies used in auditing (e.g. total variation or IPM distances). When D does not satisfy such an inequality, the lower-bound mechanism below is not available in this form and must be replaced by a discrepancy-specific bound.

5.3 Geometric intuition: projection and fibers

It is helpful to read the cross-level interface as a many-to-one projection. The map $\Phi_G : \Omega_{\mathrm{reach}} \to \Phi_G(\Omega_{\mathrm{reach}})$ collapses multiple micro states into a single macro report. For any $g \in \Phi_G(\Omega_{\mathrm{reach}})$, the fiber

$$F_g := \{S \in \Omega_{\mathrm{reach}} : \Phi_G(S) = g\}$$

collects the micro states that are indistinguishable at the reporting level.

A lifting $u \in \mathcal{U}$ selects a representative $u(g)$ for each macro value g. Given the interface metric D on the internal-side images, the worst-case representative radius at g is

$$r_u(g) := \sup_{T \in F_g} D\big(\Phi_I(T), u(g)\big).$$

By Assumption 5.1, any single representative controls pairwise variation on the same fiber: for all $S, S' \in F_g$,

$$D\big(\Phi_I(S), \Phi_I(S')\big) \le D\big(\Phi_I(S), u(g)\big) + D\big(u(g), \Phi_I(S')\big) \le 2\, r_u(g).$$

Taking the supremum over $S, S' \in F_g$ yields the structural bound

$$\operatorname{diam}_D(g) \le 2\, r_u(g).$$

Consequently, whenever a fiber has nonzero diameter, no lifting can be lossless in worst case: the residual is enforced by the fiber thickness under D, rather than by estimation noise.

Finally, the audit module fixes a finite reporting resolution ε. Within a locked audit view $V^{\mathrm{aud}} = (\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$, statements such as "reportable as 0" refer only to the thresholded convention at resolution ε (i.e., the relevant residual is below ε under V^{aud}), and do not express any ontological equality across levels.

Theorem 5.1 (Fiber-diameter lower bound for $\Delta_{I\to G}$)**.** *Assume Assumption 5.1. Then the package-relative inter-level gap satisfies*

$$\Delta_{I\to G}(\mathcal{L}; V) \;\ge\; \tfrac{1}{2} \sup_{g \in \Phi_G(\Omega_{\mathrm{reach}})} \operatorname{diam}_D(g).$$

In particular, if there exists g with $\operatorname{diam}_D(g) > 0$*, then* $\Delta_{I\to G}(\mathcal{L}; V) > 0$.

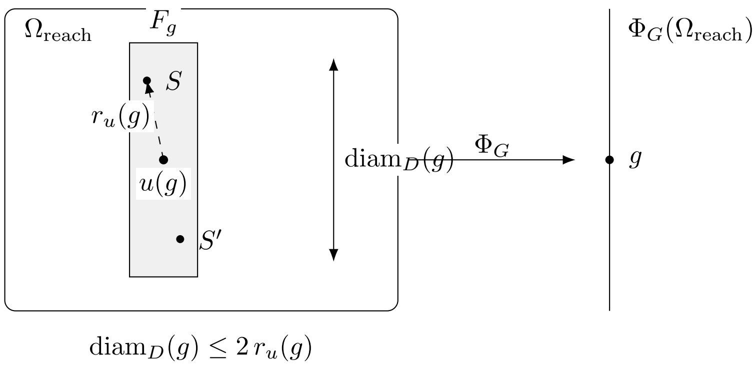

Figure 3: Projection geometry of a cross-level interface. The reporting map Φ_G collapses micro states into a macro value g, producing a fiber F_g. A lifting u selects a representative $u(g)$; the fiber thickness $\text{diam}_D(g)$ controls the unavoidable worst-case representative radius $r_u(g)$ via $\text{diam}_D(g) \le 2r_u(g)$ (Assumption 5.1).

Proof. Fix an admissible lifting $u \in \mathcal{U}$ and any $g \in \Phi_G(\Omega_{\text{reach}})$. For any $S, S' \in F_g$, Assumption 5.1 gives

$$\begin{aligned} D\big(\Phi_I(S), \Phi_I(S')\big) &\le D\big(\Phi_I(S), u(g)\big) + D\big(u(g), \Phi_I(S')\big) \\ &\le 2 \sup_{T \in F_g} D\big(\Phi_I(T), u(g)\big). \end{aligned}$$

Taking the supremum over $S, S' \in F_g$ yields

$$\begin{aligned} \text{diam}_D(g) &\le 2 \sup_{T \in F_g} D\big(\Phi_I(T), u(g)\big) \\ &\le 2 \sup_{T \in \Omega_{\text{reach}}} D\big(\Phi_I(T), u(\Phi_G(T))\big). \end{aligned}$$

Now take the infimum over $u \in \mathcal{U}$ and then the supremum over g. $\square$

Corollary 5.1 (Resolution witness for "not $\approx_\varepsilon 0$" reports)**.** *Assume Assumption 5.1. If there exist $S, S' \in \Omega_{\text{reach}}$ such that $\Phi_G(S) = \Phi_G(S')$ and $D(\Phi_I(S), \Phi_I(S')) \ge 2\varepsilon$, then $\Delta_{I \to G}(\mathcal{L}; V) \ge \varepsilon$ and hence $\Delta_{I \to G}(\mathcal{L}; V) \not\approx_\varepsilon 0$ under the reporting convention of Normative Rule 2.1.*

Remark 5.2 (Fiber witnesses as audit artifacts)*.* A pair (S, S') with $\Phi_G(S) = \Phi_G(S')$ and large interface discrepancy constitutes a *fiber witness.* When a report asserts that $\Delta_{I \to G}(\mathcal{L}; V)$ is not resolution-negligible at threshold ε, including such a witness (or a reproducible procedure for generating it under V) provides an explicit, package-locked explanation of why the "$\approx_\varepsilon 0$" convention is unavailable.

Example 5.1 (Running example: probabilistic instantiation (distribution-valued interface))**.** *No new primitive is introduced.* This example records a distribution-valued reading of the already-declared diagnostic $\Delta_{I \to G}(\mathcal{L}; V)$. It is a *reporting interface*: structural non-vanishing mechanisms (e.g. fiber witnesses and the fiber-diameter bound of Theorem 5.1) remain primary, while the probabilistic reading supplies declared operational reporting vocabulary once the relevant clauses are declared.

Boundary note. The symbols P_S and Q_S^u denote declared distribution-valued readouts under the package V (including any declared noise/channel semantics). They are not empirical estimators, and no sampling interpretation is used unless an additional estimation protocol is explicitly declared as part of V.

Declared setup. Fix an observation space $(\mathcal{Y}, \Sigma)$. For each audited micro-state $S \in \Omega_{\text{reach}}$, the interface map yields a distribution $P_S := \Phi_I(S) \in \mathcal{P}(\mathcal{Y})$. For an admissible lifting $u \in \mathcal{U}$, set $Q_S^u := u(\Phi_G(S)) \in \mathcal{P}(\mathcal{Y})$. With a declared distribution discrepancy D, the diagnostic retains its minimax form

$$\Delta_{I\to G}(\mathcal{L}; V) = \inf_{u\in\mathcal{U}} \sup_{S\in\Omega_{\text{reach}}} D(P_S, Q_S^u).$$

Operational reporting (TV/IPM). If $D = D_{\text{TV}}(P, Q) := \sup_{B\in\Sigma} |P(B) - Q(B)|$ [8], then a report $\Delta_{I\to G}(\mathcal{L}; V) < \varepsilon$ licenses the statement: there exists an admissible $u_\varepsilon \in \mathcal{U}$ such that

$$\sup_{S\in\Omega_{\text{reach}}} \sup_{B\in\Sigma} |P_S(B) - Q_S^{u_\varepsilon}(B)| < \varepsilon,$$

so all event-probability gaps are ε-bounded at the declared resolution. If instead $D = d_{\mathcal{T}}$ is an IPM induced by a declared test family $\mathcal{T} \subseteq \{f : \mathcal{Y} \to [0, 1]\}$ [24, 8], then $\Delta_{I\to G}(\mathcal{L}; V) < \varepsilon$ implies the existence of u_ε for which all admissible audit predicate gaps within $\mathcal{T}$ are $< \varepsilon$; standard bounds then translate this into a uniform constraint on audit advantage/error within $\mathcal{T}$. [7]

For the aligned structural reading (certificate lower bounds via fiber diameter and via audit single-valuedness), and for extended probabilistic details, see Appendix B.

5.4 H/M classification (level-locked, Ψ-based)

Terminology note (two H/M domains used in this manuscript).

- **H-type / M-type (level-locked, Ψ-based; this subsection).** This is the *classification output* of the slim core at a fixed level specification Λ: it answers whether $\Psi(\mathcal{L}, A; V) = 0$ in the infimum reading within the declared admissible family $\mathcal{A}_\Lambda$ (Def. 5.6), i.e., whether the declared conflict functional can be made arbitrarily small under the declared package. If admissible-family invariance does not hold, the correct output is *undefined* with an admissibility diagnostic (Remark 5.4).
- **H-closure / $H \to H^+$ transition (audit-level; Appendix G).** This is an *audit module* that answers a different question: under a locked audit viewpoint $V^{\text{aud}} = (\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$, are audited queries in $\mathcal{Q}$ *single-valued* on endpoint indistinguishability classes (Def. G.3)? If not, the non-closure regime forces a structurally necessary extension (Lemma G.2). This is not a second definition of the level-locked H/M label; it is a conditional branch invoked only when audit single-valuedness is part of the statement.

Name	**Question answered**	**Where defined**
H-type / M-type	At a *fixed* level specification Λ and declared admissible family $\mathcal{A}_\Lambda$, can $\Psi(\mathcal{L}, A; V)$ be made arbitrarily small (i.e. $\Psi = 0$ in the infimum reading)?	Def. 5.6 (this subsection).
H-closure / $H \to H^+$	Under a locked audit viewpoint $V^{\text{aud}} = (\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$, are audited queries single-valued on endpoint indistinguishability classes? If not, what extension carries the missing information?	Appendix G.

Table 2: Two H/M domains used in this manuscript (summary).

Definition 5.5 (Admissible family at a fixed level)**.** Fix a level specification Λ (Def. 4.5). The associated *admissible family* $\mathcal{A}_\Lambda$ is the declared set of perspectives that are treated as level-respecting re-expressions at Λ. Equivalently, for $A_1, A_2 \in \mathcal{A}_\Lambda$ there exists an invertible re-expression map $T_{A_2 \leftarrow A_1}$ as in Def. 4.6.

Assumption 5.2 (Level-consistency of Ψ within an admissible family)**.** For a fixed level specification Λ and a fixed declared package V, the dichotomy "$\Psi(\mathcal{L}, A; V) = 0$ versus $\Psi(\mathcal{L}, A; V) > 0$" is invariant under admissible re-expressions $A \in \mathcal{A}_\Lambda$.

Concretely, if $A_1, A_2 \in \mathcal{A}_\Lambda$ are related by an admissible invertible re-expression, then

$$\Psi(\mathcal{L}, A_1; V) = 0 \quad \Longleftrightarrow \quad \Psi(\mathcal{L}, A_2; V) = 0.$$

Remark 5.3. Assumption 5.2 is a structural requirement on what counts as "the same level" together with its admissible family: the family is declared so as to exclude pathological encodings that would artificially create or hide conflict by changing the effective level partition or by degeneracy.

Remark 5.4 (Admissible-family inconsistency and undefined gating). Assumption 5.2 is not an invariance convenience claim; it is part of what it means for $\mathcal{A}_\Lambda$ to function as a *level-locked admissible family* at the fixed level specification Λ under the declared package V. Accordingly, whether the infimum defining $\Psi(\mathcal{L}, A; V)$ vanishes ($\Psi(\mathcal{L}, A; V) = 0$) within $\mathcal{A}_\Lambda$ is intended to be a *structural property* of the regime $(\mathcal{L}, \Lambda, \mathcal{A}_\Lambda, V)$, rather than an artifact of a particular re-expression $A \in \mathcal{A}_\Lambda$.

If, under the declared regime, the dichotomy $\Psi(\mathcal{L}, A; V) = 0$ versus $\Psi(\mathcal{L}, A; V) > 0$ depends on the choice of $A \in \mathcal{A}_\Lambda$ (i.e., Assumption 5.2 does not hold), then $\mathcal{A}_\Lambda$ cannot be treated as simultaneously admissible at Λ under V. In this case, the H/M label at Λ is *undefined* for the quadruple $(\mathcal{L}, \Lambda, \mathcal{A}_\Lambda, V)$, and Definition 5.6 is not applied.

At a declared reporting resolution ε (Protocol 4.1), such a non-admissibility event can be witnessed by a pair $A_1, A_2 \in \mathcal{A}_\Lambda$ together with an admissible invertible re-expression map $T_{A_2 \leftarrow A_1}$ (Def. 4.6) for which

$$\Psi(\mathcal{L}, A_1; V) < \varepsilon \quad \text{and} \quad \Psi(\mathcal{L}, A_2; V) \geq \varepsilon,$$

(or vice versa). In reproducible reporting, this non-admissibility mode is recorded as an **ADM** diagnostic tag (Def. A.1), with a minimal obstruction witness consisting of $(A_1, A_2, T_{A_2 \leftarrow A_1})$ and the minimal package clauses required to reproduce the computation (Protocol 4.1), rather than reporting an H/M label.

Definition 5.6 (H/M classification at a fixed level (package-relative))**.** Fix a level specification Λ with admissible family $\mathcal{A}_\Lambda$ and fix a declared package V.

- **H-type at Λ (under V):** there exists an admissible perspective $A \in \mathcal{A}_\Lambda$ such that

  $$\Psi(\mathcal{L}, A; V) = 0.$$

  (Read as an infimum statement; see Remark 5.5.) Under Def. 5.5 and Assumption 5.2, this is equivalent to $\Psi(\mathcal{L}, A; V) = 0$ for all $A \in \mathcal{A}_\Lambda$.

- **M-type at Λ (under V):** for all admissible $A \in \mathcal{A}_\Lambda$,

  $$\Psi(\mathcal{L}, A; V) > 0.$$

Remark 5.5 (Reading of $\Psi = 0$ as an infimum statement). Throughout, the statement $\Psi(\mathcal{L}, A; V) = 0$ is an *infimum statement*: it means the declared conflict functional can be made arbitrarily small over the declared reachable set under the declared package, and does not assert the existence of a state with zero conflict. At finite reporting resolution ε, an operational witness is a report of the form $\Psi(\mathcal{L}, A; V) < \varepsilon$ under the same declared package.

Remark 5.6 (Scope and regime dependence). The H/M label is always relative to (i) the fixed level specification Λ, (ii) its declared admissible family $\mathcal{A}_\Lambda$, and (iii) the locked clauses of the declared package V (notably the reachable set and the conflict functional). Under refinement of the declared regime, the reported value of Ψ may change, and the H/M label may change accordingly. This is a regime-level statement, not an absolute label of $\mathcal{L}$.

Remark 5.7 (Notational caution: H/M labels vs. audit $H \to H^+$ extensions). The H/M label in Def. 5.6 is a Ψ-based classification at a fixed level specification. It is unrelated to the notation "$H \to H^+$ extension" used in Appendix G to denote an explicit state-space augmentation required for audit single-valuedness.

6 Structural Fracture Law and Consequences

6.1 Structural fracture law (canonical statement)

> **Structural fracture law.** When a logical system simultaneously contains opposing elements, and these elements produce feedback on the system, if one attempts to connect, across levels, the system's whole (macroscopic determinacy) and the system's internal factors (microscopic behavioral logic) by a single continuous-logic principle within a single perspective, then a judgment discontinuity is structurally unavoidable. In short:
>
> opposing elements + system self-feedback + cross-level continuous logic $\Rightarrow$ judgment discontinuity.

6.2 Immediate consequences stated in Ψ–$\Delta_{I\to G}$ language

The structural fracture law is a statement about cross-level inference carried out under a single, continuity-biased logic within a locked viewpoint. In the present framework, such an attempt corresponds to enforcing a *single-valued cross-level gluing* from a global representation back to an individual representation, without introducing additional structural carriers beyond what is already declared at the chosen level.

Consequence 1: limitation of single-valued cross-level gluing. Fix a declared package V with a designated cross-level pair (Φ_I, Φ_G), discrepancy rule D, admissible lifting family $\mathcal{U}$, reachable set Ω_{reach}, and reporting threshold ε (cf. Sec. 4, Def. 4.9 and Def. 5.3). A "continuous-logic gluing" claim can be formalized as the existence of an admissible lifting $u \in \mathcal{U}$ such that the induced residual discrepancy is uniformly negligible on Ω_{reach},

$$\sup_{S\in\Omega_{\text{reach}}} D\big(\Phi_I(S), u(\Phi_G(S))\big) < \varepsilon.$$

Under the fracture-law meta-constraint, when the system contains opposing elements with self-feedback and one attempts to connect the macroscopic description and microscopic factors by a single continuity-biased inference principle, such single-valued gluing is *not stable as a cross-level identification rule*: either the residual cannot be kept uniformly negligible on Ω_{reach} under the declared admissible regime, or the attempt implicitly relies on additional structure that is not present in the locked description.

In operational terms, the diagnostic manifestation is that the inter-level gap

$$\Delta_{I\to G}(\mathcal{L}; V) = \inf_{u\in\mathcal{U}} \sup_{S\in\Omega_{\text{reach}}} D\big(\Phi_I(S), u(\Phi_G(S))\big)$$

does not admit a resolution-negligible report within the locked regime once the cross-level inference is required to be single-valued on the audited domain.

Consequence 2: intra-level oppositionality and the role of Ψ. The "opposing elements" clause in the fracture law corresponds, at a fixed level specification, to the persistence of incompatible constraint contributions under admissible re-expressions. Under the regime locked by V (including the reachable set and the declared conflict functional), such persistence is captured by $\Psi(\mathcal{L}, A; V) > 0$ across the admissible family at that level (Def. 5.2 together with the level-locked admissibility clause). In particular, the condition $\Psi(\mathcal{L}, A; V) > 0$ for all admissible $A \in \mathcal{A}_\Lambda$ is exactly the M-type label of Def. 5.6 (package-relative).

This is the Ψ-side signature of oppositionality: it indicates that no admissible re-expression at the fixed level eliminates the residual conflict under the declared regime.

Consequence 3: audit formulation as non-single-valuedness (module link). When the claim being made is explicitly about decidability/extendability of an audited predicate set (rather than being limited to reconstruction error), the same structure can be stated as *fiber non-constancy* under a locked audit viewpoint. Concretely, Appendix G formalizes this as the existence of $q \in \mathcal{Q}$ and $h_1, h_2 \in \mathcal{C}$ such that $h_1 \sim_\varepsilon h_2$ but $q(h_1) \neq q(h_2)$ (Def. G.5), which is the audit-level expression of a cross-level judgment that cannot be extended as a single-valued rule under the locked viewpoint.

Remark 6.1 (What the fracture law does and does not assert). The fracture law does not assert that a particular numerical value of $\Delta_{I\to G}(\mathcal{L}; V)$ must exceed ε for every conceivable package. Rather, it asserts a structural obstruction to *single-valued, continuity-biased* cross-level gluing when oppositionality and self-feedback are present, unless one introduces additional structure (explicit carriers, memory, or an augmented state space) beyond the original locked description.

6.3 Meta-gap Δ^* (ontological axiom)

Definition 6.1 (Δ^*_R (meta-gap schema))**.** For each genuine cross-level relation R, the *meta-gap* Δ^*_R is an ontological structural constant attached to R. It encodes the principle that, whenever two levels are genuinely distinct (i.e. not related by an admissible invertible re-expression within a single level specification), cross-level identification under R is not an ontological equality.

Formally, the framework adopts the axiom schema:

$$\text{(genuine cross-level relation } R) \quad \Longrightarrow \quad \Delta^*_R > 0.$$

When R is unambiguous from context, we may write Δ^* for Δ^*_R.

Δ^*_R is not defined as an inf–sup diagnostic and is not a package-relative quantity. Its role is semantic: it prevents cross-level "=" statements from being read as ontological identities.

Remark 6.2 (Visibility is resolution-dependent; ontology is not). Although Δ^*_R is fixed and strictly positive (for the invoked genuine cross-level relation R), it may lie below a declared reporting threshold and therefore be unresolvable in practice. That is, one may have a regime in which cross-level reports treat certain residues as negligible at resolution ε, while $\Delta^*_R > 0$ remains unchanged as an ontological axiom. Accordingly, *diagnostic* threshold comparisons are carried out using the package-relative gap $\Delta_{I\to G}(\mathcal{L}; V)$ (Sec. 5.2), not Δ^*_R.

7 Minimal Dynamical Interface (Optional, Slim)

Module summary.
Scope. Provide a minimal, optional interface for tracking an accumulated structural quantity $\Delta_{\text{acc}}(\gamma)$ along a declared evolution parameter γ, without adding new core primitives.
Dependencies. A declared evolution parameter γ, a declared evolution path $\mathfrak{P}_\gamma$, and an explicit transport/comparability clause across γ.
Outputs. A reproducible accumulated readout $\Delta_{\text{acc}}(\gamma)$ and its running driver $\Psi_{\text{run}}(\gamma)$, together with resolution-aware reporting of claims such as $\Delta_{\text{acc}}(\gamma_1) \approx_\varepsilon \Delta_{\text{acc}}(\gamma_0)$.
Non-invocation rule. If the required evolution/transport clauses are not declared, this module is not invoked and any accumulated-gap claim must be reported as undefined under the reporting protocol.

7.1 Declared evolution parameter and accumulated quantity

Definition 7.1 (Declared evolution path and accumulated structural quantity)**.** Let $\gamma \in [\gamma_0, \gamma_1]$ be a declared evolution parameter. A *declared evolution path* is a γ-indexed family

$$\mathfrak{P}_\gamma := (\mathcal{L}_\gamma,\ A_\gamma,\ V_\gamma),$$

such that for each γ:

- $\mathcal{L}_\gamma$ is the system object at stage γ (e.g. an effective description under refinement/coarse-graining/RG [16, 37, 38, 27, 36], if such a reading is intended);
- A_γ is a declared perspective at the corresponding fixed level specification;
- V_γ is a declared package whose locked clauses make $\Psi(\mathcal{L}_\gamma, A_\gamma; V_\gamma)$ well-posed (Sec. 5).

In addition, the path must include an explicit transport/comparability law across γ in the sense of Def. 4.12, so that quantities reported at different γ are interpreted within a single declared convention.

Define the *running intra-level readout* along the path by

$$\Psi_{\text{run}}(\gamma) \ := \ \Psi(\mathcal{L}_\gamma,\ A_\gamma;\ V_\gamma).$$

The *accumulated structural quantity* along the evolution parameter is then defined by

$$\Delta_{\text{acc}}(\gamma) \ := \ \Delta_{\text{acc}}(\gamma_0) \ + \ \int_{\gamma_0}^{\gamma} \Psi_{\text{run}}(s)\, ds,$$

whenever the integral is well-defined. Equivalently, in differential form,

$$\frac{d}{d\gamma}\Delta_{\text{acc}}(\gamma) \ := \ \Psi_{\text{run}}(\gamma) \quad \text{with an initial condition } \Delta_{\text{acc}}(\gamma_0) \text{ declared.}$$

Separation from the fixed-slice inter-level diagnostic. $\Delta_{\text{acc}}(\gamma)$ is an accumulated bookkeeping quantity attached to the declared evolution path $\mathfrak{P}_\gamma$. It is distinct from the fixed-slice inter-level gap $\Delta_{I\to G}(\mathcal{L}_\gamma; V_\gamma)$ of Def. 5.3, which is defined at each γ by an inf–sup reconstruction residual under the locked interface module of V_γ.

Symbol	Role in this manuscript	Gating / notes
$\Delta_{I\to G}(\mathcal{L};V)$	Fixed-slice inter-level diagnostic (inf–sup reconstruction residual) under a locked interface module.	Requires $(\Phi_I,\Phi_G,D,\mathcal{U},\Omega_{\mathrm{reach}},\varepsilon)$ to be declared; $\approx_\varepsilon$ semantics fixed by Normative Rule 2.1.
$\Delta_{\mathrm{acc}}(\gamma)$	Accumulated bookkeeping quantity along a declared evolution parameter γ (integral / discrete accumulation interface).	Requires a declared evolution path $\mathfrak{P}_\gamma$ and an explicit transport/comparability clause across γ. Not identified with $\Delta_{I\to G}$ unless an explicit bridge is declared.
Δ_{dyn}	Optional invariant-measure lower bound used only in the dynamical module (Appendix C).	Not part of the slim core; requires additional dynamical declarations (invariant measure class, admissibility gates).
Δ^*_R	Meta-gap schema (ontological axiom) forbidding ontological equality across genuine cross-level relations.	Semantic guardrail, not an operational inf–sup diagnostic; never used as a numerical threshold criterion.

Table 3: Disambiguation of Δ-roles used in this paper. Numerical reporting is always package-relative; ontological statements use Δ^*_R (with R suppressed when unambiguous) only as a semantic axiom (Sec. 6.3).

Remark 7.1 (Domain of validity and reporting convention)*.* The interface in Def. 7.1 is operational and path-dependent: it requires a declared evolution parameter, a declared path $\mathfrak{P}_\gamma$, and an explicit transport law across γ.

Discrete variant. If γ indexes a discrete sequence (e.g. $\gamma = n \in \mathbb{N}$), one may use the declared discrete accumulation rule

$$\Delta_{\mathrm{acc}}(n) = \Delta_{\mathrm{acc}}(n_0) + \sum_{k=n_0}^{n-1} \Psi_{\mathrm{run}}(k),$$

under the same comparability/transport requirements.

Resolution. Any statement of the form "$\Delta_{\mathrm{acc}}(\gamma) \approx_\varepsilon 0$" (if used) is a report under the declared threshold convention, and does not assert an ontological identity. Geometric closures, self-similar solutions, and any scaling-constant connections are deferred to appendices (or later chapters) to keep the main line slim.

Remark 7.2 (Structural isomorphism with dynamical formalisms (role-level alignment))*.* This paper permits cross-formalism alignment only at the level of *logical roles* under an explicit declaration package V (including its declared reporting threshold ε). Accordingly, any correspondence with familiar dynamical lexicons (e.g. "force," "motion," "energy," "action") is to be read as a *structural isomorphism of interfaces*, not as a literal identification of physical quantities across theories.

Fix a declared evolution parameter γ and a declared path $\mathfrak{P}_\gamma = (\mathcal{L}_\gamma, A_\gamma, V_\gamma)$ as in Def. 7.1, together with the transport/comparability clauses required to interpret reports along γ when such interpretation is invoked (Def. 4.12). The accumulation interface

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{run}}(s)\, ds \qquad \text{(or its discrete analogue)} \tag{1}$$

exhibits a generic "local–to–global" skeleton: a *local running readout* $\Psi_{\mathrm{run}}(\gamma)$ generates a *global accumulated effect* $\Delta_{\mathrm{acc}}(\gamma)$ along a declared evolution parameter. In this role-level sense, Ψ_{run} may be regarded as a *driver* (a local increment generator), while Δ_{acc} records the corresponding *accumulated response.*

Separately, the cross-level diagnostic $\Delta_{I\to G}(\mathcal{L}_\gamma; V_\gamma)$ (Def. 5.3) plays a distinct role: it reports whether a declared cross-level gluing/alignment remains resolution-negligible under the package V_γ (with its declared reporting threshold, written ε_γ when needed). When $\Delta_{I\to G}$ is not resolution-negligible, the audited description cannot be treated as gap-free at that declared resolution; the correct output is a licensed report of non-negligibility (status: not resolution-negligible) under V_γ. If the locking clauses required to form $\Delta_{I\to G}$ are missing in V_γ, the correct output is undefined under the declared regime (package under-specified; optional tag U-spec, Appendix A). This diagnostic role is distinct from the accumulation role in (1); collapsing it into (1) requires an explicitly declared bridge (and, when comparisons are invoked, the corresponding transport/comparability clauses and diagnostic).

In particular, statements of the form "the driver is Ψ" or "the motion is $\Delta_{I\to G}$" are admissible only as shorthand for the above role-level alignment under an explicit V, and must not be read as an ontological equation between heterogeneous quantities. Any cross-package or cross-regime comparison of these roles requires an explicitly declared transport/comparability law (Def. 4.12); otherwise, the comparison is undefined under the declared regime (missing transport; optional tag NC, Appendix A). If reporting thresholds or resolutions are compared without a declared conversion rule, the comparison is undefined under the declared regime (resolution mismatch; optional tag RES, Appendix A).

8 Well-posed Reporting and Licensed Comparisons

This section states the minimal declaration conditions needed for reproducible reporting, and specifies when cross-report comparisons are licensed. In this framework, Ψ and $\Delta_{I\to G}$ (and any other Δ-variant when explicitly invoked) are not treated as context-free invariants: their meanings are uniquely locked only after the relevant clauses of a declared package have been stated. When the required clauses are not declared (or when cross-report comparisons invoke undeclared transport/domain/resolution conversions), the corresponding numerical statement is *undefined relative to the declared regime.* For compactness, we optionally attach short diagnostic status tags (Appendix A), but the core semantics is simply partiality (well-posed vs. undefined) and licensed vs. unlicensed comparisons.

8.1 Package card template (fillable)

The minimal reporting card (Protocol 4.1) specifies what must be *declared.* For long-term reuse, it is often convenient to record these clauses in a single "package card" block with a short identifier. The following template is a fillable form; it introduces no new primitives.

Package Card Template (minimal). Fill the right column; omit rows that are not invoked.

Field	**Declaration (fill)**
ID / scope	____________________
Target system $\mathcal{L}$ and boundary	____________________
Audited domain / reachability Ω_{reach} (and how fixed)	____________________
Level specification(s) and perspectives (A_I, A_G) / (Φ_I, Φ_G)	____________________
Conflict functional for Ψ: E_A (and hidden clauses, if any)	____________________
Discrepancy rule for $\Delta_{I\to G}$: D (and induced interface sub-clauses)	____________________
Admissible liftings $\mathcal{U}$ (and admissibility constraints)	____________________
Reporting threshold ε (and meaning of $\approx_\varepsilon$; see Normative Rule 2.1)	____________________
Audit clause (only if invoked): $\mathcal{Q}, \mathcal{C}$	____________________
Comparability / transport (only if cross-package comparisons are invoked)	____________________
Invoked optional modules (e.g. audit, dynamics, stability)	____________________

8.2 What must be declared to make Ψ and $\Delta_{I\to G}$ well-defined

Protocol 8.1 (Minimal declaration requirements)**.** Any report of $\Psi(\mathcal{L}, A; V)$ or $\Delta_{I\to G}(\mathcal{L}; V)$ is considered well-defined only if the following elements are explicitly declared (or are deterministically induced by locked clauses that are themselves declared):

1. **Target system and boundary.** State the target system object $\mathcal{L}$ and the boundary of the discussion: what is treated as inside the system versus environment/background.

2. **Reachability / audited domain.** Declare the reachable set $\Omega_{\mathrm{reach}} \subseteq \Omega$ (Def. 4.3) or declare the rule that determines it (dynamics, admissibility constraints, initial set, and any conditioning).

3. **Level specification and perspectives.** Declare the fixed level specification(s) used (Def. 4.5) and the perspective(s) used to compute the quantity:

   - for Ψ: the perspective A (or admissible family at that level) and the associated representation map Φ_A;

- for $\Delta_{I\to G}$: the designated cross-level pair (A_I, A_G) and the induced maps (Φ_I, Φ_G) (Def. 4.7).

4. **Conflict functional for Ψ.** Declare the conflict functional E_A (Def. 5.1), including any weighting, normalization, or hidden clauses (e.g. interface aggregation rules) that affect its value.

5. **Interface mechanism for $\Delta_{I\to G}$.** Declare the discrepancy rule D on the individual-level representation space and the admissible lifting family $\mathcal{U}$ (Def. 4.8 and Def. 5.3). If D is induced by a measurement/observation interface family together with an aggregation protocol, these sub-clauses must be declared as part of D.

6. **Reporting threshold and convention.** Declare the reporting threshold ε and the semantics of resolution-aware reporting, including the meaning of "$\approx_\varepsilon$" (Normative Rule 2.1).

7. **Admissibility constraints and non-degeneracy gates.** Declare any constraints defining admissible families (e.g. exclusion of degenerate encodings, bounded complexity of liftings, fixed architecture class, fixed regularity assumptions, or restricted parameter ranges). If an intrinsic bound is claimed by taking an infimum over a family, the family itself must be declared (cf. Def. 4.12).

Operability (non-emptiness). The declared audited domain Ω_{reach} and any declared admissible family invoked by the relevant inf/sup definition (in particular the lifting family $\mathcal{U}$ when reporting $\Delta_{I\to G}$) must be nonempty. If $\Omega_{\text{reach}} = \varnothing$ or an invoked admissible family is empty, numerical reporting under the declared regime is undefined (optional tag: ADM) rather than a surrogate value induced by empty-set conventions.

If any of the above is absent, the symbols "Ψ" and "$\Delta_{I\to G}$" denote under-specified *families* of quantities rather than a single well-posed value. In that case, numerical reporting under the declared regime is undefined; the appropriate report is an explicit statement of under-specification (optionally tagged as U-spec, Appendix A) rather than an unscoped numerical surrogate.

8.3 Partiality and licensed comparisons (when numerical reporting is undefined)

The framework distinguishes two outcomes: (i) a numerical report (when the quantity is uniquely locked and well-defined under a declared package and domain), and (ii) an explicit *undefinedness* or *non-licensing* statement (when the attempted report or comparison is not meaningful under the declared clauses). The latter is a semantic classification, not a value judgment.

Definition 8.1 (Undefinedness / non-licensing modes (minimal set))**.** The following modes cover the minimal reasons used in this paper for treating a numerical report or comparison as undefined under the declared regime:

1. **Under-specification:** one or more required clauses of Protocol 8.1 are missing, so the quantity is not uniquely locked.

2. **Admissibility violation:** the attempted computation invokes an element (perspective, discrepancy, lifting, encoding, or interface sub-clause) that lies outside the declared admissible family; the report is undefined relative to the declared regime.

3. **Domain mismatch:** the audited domain (reachable set, history class, or query set) differs across reports without an explicit reconciliation rule; comparisons or aggregations across domains are undefined.

4. **Non-comparability (missing transport):** two values are produced under packages V, V' without a declared transport/comparability law in the sense of Def. 4.12; equality/inequality comparisons are not licensed.

5. **Resolution mismatch:** reporting thresholds or audit resolutions differ without a declared conversion rule; cross-report comparisons are not licensed under the declared resolution-aware semantics.

Status	**Meaning under the declared regime**	**Minimal witness (optional)**
ok	Quantity is well-posed and the numerical report is licensed (package clauses locked, admissible instantiation, domain matches; and transport/resolution clauses declared if a cross-report comparison is performed).	Package card (Protocol 4.1) + reproduction notes.
U-spec	Under-specification: at least one clause needed to lock the quantity or the comparison is missing.	List missing declaration items (Protocol 8.1).
ADM	Admissibility violation: an invoked component lies outside the declared admissible family.	Identify the offending component and the declared admissible family.
DOM	Domain mismatch: reachable/audited domains differ across reports with no declared reconciliation rule.	Specify both domains and the missing reconciliation clause.
NC	Non-comparability: cross-package comparison is requested without a declared transport law.	Specify (V, V') and the missing transport/comparability clause.
RES	Resolution mismatch: cross-report comparison is requested with different ε (or audit resolutions) and no declared conversion rule.	Specify the resolutions and the missing conversion rule.

Table 4: Status-style reporting (optional). These labels are short names for the undefinedness / non-licensing modes (Def. 8.1) and inherit the domain-gating protocol (Protocol 8.2).

Protocol 8.2 (Licensing rule for reporting and cross-report comparison)**.** When a numerical report (or a cross-report comparison) is requested, the following checks determine whether the request is licensed under the declared regime:

1. **Locking check:** verify that the package clauses required by Protocol 8.1 have been declared. If not, the request is undefined by under-specification (optional tag: U-spec, Appendix A).

2. **Admissibility check:** verify that instantiated components lie in the declared admissible family. If not, the request is undefined by admissibility violation (optional tag: ADM, Appendix A).

3. **Domain check:** verify that the audited domain matches the declared one, or that an explicit reconciliation rule is given. If not, the request is undefined by domain mismatch (optional tag: DOM, Appendix A).

4. **Operability check:** verify that the declared audited domain is nonempty and that any admissible family invoked by the requested definition is nonempty on that domain (in particular, $\mathcal{U} \neq \varnothing$ when reporting $\Delta_{I\to G}$). If not, the request is $\mathsf{undefined}$ by admissibility/degeneracy (optional tag: ADM, Appendix A).

5. **Comparability check (cross-report only):** when comparing across packages, verify that a transport law is declared (Def. 4.12). If not, the comparison is not licensed (optional tag: NC, Appendix A).

6. **Resolution check (cross-report only):** verify that thresholds are identical or related by a declared conversion rule. If not, the comparison is not licensed (optional tag: RES, Appendix A).

7. **Otherwise:** report the numerical value together with the minimal package card (Protocol 4.1) and any additional clauses needed for reproducibility.

Remark 8.1 (Neutrality of undefinedness diagnoses). An undefinedness/non-licensing statement indicates that a numerical report or comparison is not meaningful under the *declared* package clauses, domain, and resolution semantics. It does not evaluate the system under study; it specifies which declaration or reconciliation clause would be required to make the requested numerical statement well-posed.

8.4 Canonical audited example: Markov coarse-graining and fiber non-constancy

This subsection records a short, non-toy instantiation showing how the protocol-level outputs (ok vs. $\mathsf{undefined}$) arise from an explicit witness. Full definitions and a worked extension are in Appendix G.4.

Setup (finite Markov chain under a locked endpoint). Let X be a finite state space with Markov kernel $K(x,\cdot)$ [25]. Fix a declared coarse map $\phi : X \to Y$ and take the endpoint operator for one-step histories $h = (x_0, x_1)$ to be $\mathsf{End}(h) := \phi(x_0) \in Y$ as in Appendix G.4. Lock a discrete endpoint discrepancy and choose $\varepsilon < 1$, so endpoint indistinguishability reduces to equality of macro labels.

Audited predictive queries. For each event $A \subseteq Y$, consider the one-step macro predictive query

$$q_A(h) := \mathbb{P}(\phi(X_1) \in A \mid X_0 = x_0). \tag{2}$$

This is a natural audited family when the intended statement treats macro-level predictions as the reportable content.

Witness for non-single-valuedness. If there exist micro states $x, x' \in X$ with $\phi(x) = \phi(x')$ such that the induced macro predictive laws differ, i.e. $\bar{K}_x(\cdot) \neq \bar{K}_{x'}(\cdot)$ (§G.4.2), then there exists an event A with $q_A(h_1) \neq q_A(h_2)$ for histories h_1, h_2 sharing the same endpoint label. Concretely, the validity predicate $\mathrm{Val}(V^{\mathrm{aud}})$ of Def. G.3 does not hold and the non-closure regime holds (Prop. G.1).

Protocol consequence. Under this witness, any claim that relies on *single-valued endpoint-based decidability* of the audited predictive queries at the locked resolution is not licensed: the appropriate output is an explicit non-licensing diagnosis ($\mathsf{undefined}$ under Def. 8.1, optionally tagged) rather than an endpoint-matching surrogate. If the intended claim requires restoring single-valued decidability, the protocol prescribes an $H \to H^+$ extension carrying within-fiber information (§G.4.3).

Reading. The example isolates a typical mechanism behind "endpoint matching is insufficient": a coarse endpoint non-injectively identifies multiple micro- states into a single macro label (i.e., a non-injective coarse map), while the audited predictive content varies within the fiber. In this regime, the residual is not removed by reparameterization or by redefining a number; it is localized on the fiber and must be either reported as non-negligible (under a declared discrepancy) or reported as undefined if the relevant clauses are not declared.

9 Structural Stability of Discrete Macroscopic Labels

9.1 Scope and intent

The slim core develops the Ψ–$\Delta_{I\to G}$ framework as a regime-relative diagnostic of cross-level gluing limitations and perspective-induced residuals. A natural follow-up question is why certain *macroscopic properties* nevertheless appear stable in practice. This chapter addresses one clean case: the macroscopic property is a *discrete label* (topological type / mode), and the reported label is produced by a declared operational package.

The goal here is not to add new primitives to the core. The goal is to provide a *package-relative, resolution-aware* stability template with explicit reporting behavior:

- when label-stability claims are well-defined under the declared package(s);
- what structural evidence suffices to justify stability at the declared reporting threshold;
- what must be reported when cross-package comparability is undefined.

All "equality" statements in this chapter inherit the resolution-aware semantics and the diagnostic / domain-gating rules of the reporting protocol (cf. Sec. 8).

9.2 Dependency declarations and objects

Fix a declared operational package V and its reporting threshold ε (as part of V, cf. Sec. 8). The present chapter uses only objects that are either declared by V or induced by declared components:

- the full state space Ω and the reachable set $\Omega_{\mathrm{reach}} \subseteq \Omega$ (as declared in V);
- a task-relevant subset $\Omega^{\mathrm{rel}}_{\mathrm{reach}} \subseteq \Omega_{\mathrm{reach}}$ (declared for this chapter; it is a reporting subset, not a new primitive);
- a micro-level discrepancy D_V on Ω_{reach} (as declared in V);
- an optional update / coarse-graining operator $\mathcal{C}_V : \Omega_{\mathrm{reach}} \to \Omega_{\mathrm{reach}}$ (when V declares one);
- a representation map $R_V : \Omega_{\mathrm{reach}} \to \mathcal{Y}_V$ used by the labeling pipeline (declared in V);
- a macroscopic discrepancy D^{macro}_V on $\mathcal{Y}_V$ (declared or induced by V);
- a decision rule $\mathfrak{M}_V : \mathcal{Y}_V \to \mathcal{S}$ for a discrete label set $\mathcal{S}$ (declared as part of the labeling pipeline under V).

Checklist (minimal declared inputs for this chapter). At minimum, the following must be explicitly declared to license any stability claim at resolution ε:

- the audited domain Ω_{reach} and the task-relevant subset $\Omega_{\text{reach}}^{\text{rel}}$ (or an explicit rule that $\Omega_{\text{reach}}^{\text{rel}} = \Omega_{\text{reach}}$);
- the micro discrepancy D_V and the reporting threshold ε;
- the label pipeline $R_V : \Omega_{\text{reach}} \to \mathcal{Y}_V$ and $\mathfrak{M}_V : \mathcal{Y}_V \to \mathcal{S}$.

Optional-but-common additions are: a preprocessing/coarse-graining operator $\mathcal{C}_V$, a feasible representation region $K_V \subseteq \mathcal{Y}_V$, and an induced/declared macro discrepancy D_V^{macro}. For an aligned checklist together with diagnostic stubs (undefinedness/comparability behavior), see Appendix H and §H.5.

No additional cross-level axioms are introduced. In particular, this chapter does not require ontological claims of gap-free identity; it uses only package-relative comparability and resolution-aware reporting.

9.3 Macro-label assignment and resolution-aware equivalence

Let $\mathcal{S}$ be a discrete label set ("modes" / "types").

Definition 9.1 (Macro-label assignment (package-relative)). A *macro-label assignment* is a map

$$\mathfrak{m}_V : \Omega_{\text{reach}} \to \mathcal{S},$$

interpreted as the label reported for a reachable micro-state under the declared package V.

Remark 9.1 (Pipeline induction and package relativity). The definition does not prescribe how $\mathfrak{m}_V$ is computed. In applications, $\mathfrak{m}_V$ is induced by a declared pipeline

$$S \in \Omega_{\text{reach}} \;\mapsto\; R_V(S) \in \mathcal{Y}_V \;\mapsto\; \mathfrak{M}_V(R_V(S)) \in \mathcal{S},$$

and therefore depends on declared representation choices, admissible preprocessing, and the reporting threshold. Label equality is always read under the declared package and threshold, in the sense fixed by the reporting protocol (Sec. 8).

9.4 Two robustness notions: micro-robustness and package-robustness

Definition 9.2 (Micro-robust stability at resolution ε). Fix V and its reporting threshold ε. The label assignment $\mathfrak{m}_V$ is *micro-robust at resolution* ε if there exists a perturbation radius $\rho > 0$ (specified using the discrepancy structure of V) such that for all $S, S' \in \Omega_{\text{reach}}$,

$$D_V(S, S') < \rho \quad \Longrightarrow \quad \mathfrak{m}_V(S) = \mathfrak{m}_V(S').$$

Admissible package neighborhood (for cross-package robustness). Let $\mathfrak{N}(V)$ be an admissible neighborhood of packages (e.g. small variations of permissible liftings, representation choices, or coarse-graining steps). A cross-package robustness statement is meaningful only when the neighborhood explicitly declares how packages are compared.

Declared transport and its domain. For each $V' \in \mathfrak{N}(V)$, assume the neighborhood declares:

- a transport map $\tau_{V\to V'}$ and a declared domain of definition $\mathrm{Dom}(\tau_{V\to V'}) \subseteq \Omega_{\text{reach}}$;

- a comparability clause specifying when the cross-package label comparison is defined (i.e. which parts of the pipeline are kept fixed, and what counts as an admissible variation).

Definition 9.3 (Package-robust stability (defined-case))**.** The label assignment is *package-robust on* $\mathfrak{N}(V)$ if for every $V' \in \mathfrak{N}(V)$ and every $S \in \mathrm{Dom}(\tau_{V\to V'})$,

$$\mathfrak{m}_V(S) = \mathfrak{m}_{V'}\big(\tau_{V\to V'}(S)\big).$$

Definition 9.4 (Cross-package label statement (three-way reporting output))**.** Fix V, $V' \in \mathfrak{N}(V)$, and $S \in \Omega_{\mathrm{reach}}$. The cross-package statement

$$\mathfrak{m}_V(S) \stackrel{?}{=} \mathfrak{m}_{V'}\big(\tau_{V\to V'}(S)\big)$$

is reported as one of the following, consistent with the diagnostic / domain-gating rules in Sec. 8:

1. **Defined + True**: $S \in \mathrm{Dom}(\tau_{V\to V'})$ and the equality holds.
2. **Defined + False**: $S \in \mathrm{Dom}(\tau_{V\to V'})$ and the equality does not hold (a counterexample is reported).
3. **Undefined**: either $S \notin \mathrm{Dom}(\tau_{V\to V'})$, or no valid transport/comparability clause is declared; in this case, equality is not reported and an "undefined" diagnostic is provided.

Remark 9.2 (no implicit transport (Normative Rule 2.2))*.* If no transport rule is declared, cross-package comparison is *undefined* and must be reported via diagnostics rather than equality claims (cf. Sec. 8).

9.5 Three-edge sufficient template for robust discrete stability

This subsection gives a structural sufficient-condition template for micro-robust stability. Each edge is package-relative and must be verified under the declared package.

Definition 9.5 (Edge A: contraction before label extraction)**.** Edge A holds (on the task-relevant domain) if V declares an operator $\mathcal{C}_V : \Omega_{\mathrm{reach}} \to \Omega_{\mathrm{reach}}$ and there exist constants $\kappa \in [0,1)$ and $\rho_A > 0$ such that for all $S, S' \in \Omega^{\mathrm{rel}}_{\mathrm{reach}}$,

$$D_V\big(\mathcal{C}_V(S), \mathcal{C}_V(S')\big) \;\le\; \kappa\, D_V(S, S').$$

Remark 9.3 (Reading of Edge A)*.* Edge A is a statement about damping of micro-variation in the declared micro discrepancy, restricted to $\Omega^{\mathrm{rel}}_{\mathrm{reach}}$ (the portion of Ω_{reach} relevant for the labeling task).

Definition 9.6 (Edge B: feasible-set narrowing in representation space)**.** Edge B holds if there exist a declared subset $\mathcal{K}_V \subseteq \mathcal{Y}_V$ and a radius $\rho_B > 0$ such that:

1. (*Concentration*) for all $S \in \Omega^{\mathrm{rel}}_{\mathrm{reach}}$, one has $R_V(S) \in \mathcal{K}_V$;
2. (*No boundary reachability*) for all $S \in \Omega^{\mathrm{rel}}_{\mathrm{reach}}$ and all $S' \in \Omega_{\mathrm{reach}}$ with $D_V(S, S') < \rho_B$, the points $R_V(S)$ and $R_V(S')$ lie in the same decision region of $\mathfrak{M}_V$ when restricted to $\mathcal{K}_V$.

Remark 9.4 (Reading of Edge B)*.* Edge B states that, within the portion of representation space actually visited by the task, admissible micro-perturbations do not move the pipeline output across the decision boundaries of the declared label rule.

Definition 9.7 (Edge C: label separation margin at reporting resolution)**.** Write $\mathfrak{m}_V(S) = \mathfrak{M}_V(R_V(S))$. Edge C holds if there exists a margin $\eta > \varepsilon$ such that for any two distinct labels $m \neq m'$, the corresponding representation regions

$$\mathcal{R}_m := \{y \in \mathcal{K}_V : \mathfrak{M}_V(y) = m\} \qquad \text{and} \qquad \mathcal{R}_{m'} := \{y \in \mathcal{K}_V : \mathfrak{M}_V(y) = m'\}$$

satisfy the separation condition

$$\inf\{ D_V^{\mathrm{macro}}(y, y') : y \in \mathcal{R}_m, \ y' \in \mathcal{R}_{m'} \} \ \geq \ \eta.$$

Remark 9.5 (Reading of Edge C)*.* Edge C encodes a reporting-level statement: distinct labels are separated by a margin strictly above the declared threshold, in the macroscopic discrepancy actually used by the package.

Remark 9.6 (Optional alignment with attractor-based language)*.* Readers from dynamical-systems and complex-systems traditions often describe persistent macroscopic patterns in terms of *attractors* and *basins*. The present section does not require such notions; however, Edges A–C can be read as a package-relative translation of a common "basin stability" intuition once the declared operator $\mathcal{C}_V$ is instantiated as a task-relevant update (time-advance / coarse-step) and the declared discrepancies are chosen accordingly.

Under this optional reading: (i) Edge A corresponds to damping of micro-variation visible in the declared micro discrepancy D_V under repeated application of $\mathcal{C}_V$ on $\Omega^{\mathrm{rel}}_{\mathrm{reach}}$; (ii) Edge B corresponds to remaining inside a basin *core* in representation space, in the sense that admissible micro-perturbations do not induce decision-boundary crossings of $\mathfrak{M}_V$ within $\mathcal{K}_V$; and (iii) Edge C corresponds to basin separability at reporting resolution, i.e. a separation margin $\eta > \varepsilon$ in the declared macroscopic discrepancy D_V^{macro}.

This alignment is not an equivalence. An attractor-style description may hold while the declared labeling pipeline fails to satisfy Edge B/C at the declared resolution (e.g. a decision boundary slices through a basin in $\mathcal{Y}_V$), and conversely Edges A–C may hold for stable reporting labels without implying the existence of a classical attractor as an invariant set.

Proposition 9.1 (Triangle closure: Edges A–C imply micro-robust stability)**.** *Assume Edges A–C hold under a declared package V with reporting threshold ε. Then the induced macro-label assignment $\mathfrak{m}_V$ is micro-robust at resolution ε.*

Proof sketch (structural, package-relative). Edge A provides contraction (on $\Omega^{\mathrm{rel}}_{\mathrm{reach}}$) of micro-level perturbations under the declared operator $\mathcal{C}_V$. Edge B ensures that, in the representation space region relevant to the task, admissible perturbations do not cross decision boundaries of $\mathfrak{M}_V$. Edge C supplies a positive separation margin $\eta > \varepsilon$ between distinct label regions in the macroscopic discrepancy. Together, these imply the existence of a $\rho > 0$ such that $D_V(S, S') < \rho$ forces $\mathfrak{m}_V(S) = \mathfrak{m}_V(S')$. □

Remark 9.7 (Template status)*.* Proposition 9.1 is a sufficient-condition template. Each edge must be instantiated and supported relative to the declared package (choice of discrepancies, representation, admissible preprocessing, and threshold).

9.6 Operational reporting: diagnostics and non-admissibility tags

This subsection aligns stability claims with the diagnostic / domain-gating discipline of the reporting protocol (Sec. 8). A stability claim is reported together with the package-relative evidence supporting the relevant edges, or else it is reported as undefined.

Definition 9.8 (Stability diagnostic bundle (package-relative))**.** A *stability diagnostic bundle* for V on $\Omega^{\mathrm{rel}}_{\mathrm{reach}}$ consists of:

1. an (A-diagnostic) specifying $\mathcal{C}_V$ (if used), the domain $\Omega^{\mathrm{rel}}_{\mathrm{reach}}$, and constants (κ, ρ_A) together with a verification method/bound supporting Edge A;

2. a (B-diagnostic) specifying R_V, $\mathcal{K}_V$, ρ_B, and the declared decision rule $\mathfrak{M}_V$ together with evidence for Edge B;

3. a (C-diagnostic) specifying D_V^{macro} and a margin $\eta > \varepsilon$ together with evidence for Edge C.

Remark 9.8 (Undefined cases). If any required object is not declared (e.g. no D_V, no R_V, no decision rule, or no declared transport for a cross-package claim), then the corresponding stability statement is *undefined under the declared package(s)* and must be reported with an undefined-diagnostic listing the missing declarations.

Lemma 9.1 (Label flips imply edge/diagnostic non-admissibility (operational split))**.** *Fix V and threshold ε. Suppose there exist states $S, S' \in \Omega^{\mathrm{rel}}_{\mathrm{reach}}$ such that*

$$D_V(S, S') < \rho \quad \textit{but} \quad \mathfrak{m}_V(S) \neq \mathfrak{m}_V(S').$$

Then, under the reporting protocol of Sec. 8, at least one of the following must be reported:

1. *Edge A is not supported on $\Omega^{\mathrm{rel}}_{\mathrm{reach}}$ at the claimed $\kappa < 1$ (no valid A-diagnostic);*

2. *Edge B is not supported at the claimed radius (no valid B-diagnostic for boundary non-crossing);*

3. *Edge C is not supported at threshold ε (no valid separation margin $\eta > \varepsilon$ consistent with the observed flip).*

Remark 9.9 (Resolution-aware reading). Lemma 9.1 is a reporting-level statement: it does not introduce or require any ontological equality claim across a genuine cross-level relation (guardrailed by the meta-gap axiom schema $\Delta^*_R > 0$ for the genuine cross-level relation R under discussion). It only states that a label flip inside declared micro-balls is incompatible with simultaneously supporting Edges A–C at the declared reporting threshold.

9.7 Cross-package stability: defined comparisons versus undefined comparisons

Let $V' \in \mathfrak{N}(V)$. Cross-package stability statements are meaningful only when the neighborhood declares a transport rule and a comparability clause.

- If $\tau_{V\to V'}$ is declared and $S \in \mathrm{Dom}(\tau_{V\to V'})$, then the statement

  $$\mathfrak{m}_V(S) = \mathfrak{m}_{V'}\big(\tau_{V\to V'}(S)\big)$$

  is *defined* and is reported as True/False, accompanied by the relevant evidence or a counterexample.

- If no valid transport/comparability clause is declared (or $S \notin \mathrm{Dom}(\tau_{V\to V'})$), the statement is *undefined*. In this case, the report must not contain an equality claim; it must contain an undefined diagnostic specifying the missing declarations, consistent with Sec. 8.

9.8 What this chapter does not claim

This section does not claim that macroscopic stability must always arise, nor that Edges A–C are necessary. It provides a clean, package-relative pathway for explaining stability of discrete macroscopic labels when such stability is observed, and it specifies reporting behavior (including undefined cases) consistent with the protocol and diagnostic discipline of the core framework.

The attractor/basin alignment remark is interpretive only: no existence, uniqueness, or convergence-to-invariant-set claim is made or required. All stability statements here are package-relative properties of the declared labeling pipeline at the declared reporting threshold.

10 Discussion

This paper isolates a minimal core that makes cross-level statements auditable and reproducible without presupposing a particular domain (physics, computation, dynamical systems, or cognitive systems). The core establishes: (i) a clean separation between the *target system object* and the *declared package* used to analyze it; (ii) two package-relative quantities, the intra-level structural parameter $\Psi(\mathcal{L}, A; V)$ and the inter-level diagnostic gap $\Delta_{I\to G}(\mathcal{L}; V)$, each defined only after the relevant operational degrees of freedom are locked; and (iii) a reporting semantics in which resolution-aware statements (e.g. $\approx_\varepsilon$) are treated as declarations of observability under a threshold, not as ontological identities.

Within this core, the structural fracture law is recorded verbatim and is rewritten in Ψ–$\Delta_{I\to G}$ language as a limitation on single-valued, continuity-biased cross-level gluing under opposing elements with self-feedback. The law is expressed in a neutral structural form: when cross-level identification is demanded as a single-valued rule under a locked viewpoint, non-negligible residuals are expected (diagnostically: $\Delta_{I\to G} \not\approx_\varepsilon 0$ under the declared regime) unless additional structure is introduced explicitly (e.g. extended carriers, memory, or an augmented state space). To prevent symbol overloading, the core also distinguishes the package-relative diagnostic $\Delta_{I\to G}$ from the ontological meta-gap axiom schema $\Delta^*_R > 0$, where Δ^*_R functions only as a semantic contract against reading cross-level "=" as an ontological identity.

A central practical output of the slim core is the usage protocol (Sec. 8): it specifies the minimal declarations required for Ψ and $\Delta_{I\to G}$ to be well-defined and comparable, and it defines diagnostic-based domain gating for cases where numerical reporting is undefined (e.g. under-specification, non-comparability without a transport law, admissibility violations, domain or resolution mismatch). This converts many common cross-level disputes into explicit declaration tasks: either a report is licensed under a declared package and domain, or a diagnostic is issued stating which clause is missing.

Several components are intentionally deferred to keep the main line slim and to avoid conflating interface statements with model-specific constructions. In particular, the following are deferred:

- **Dynamical refinements:** concrete dynamical carriers (e.g. HTDS constructions), long-time averages, invariant-measure formulations (such as Δ_{dyn}), and any claims that depend on ergodic selection or typicality assumptions [3, 39].
- **Geometric closure and self-similarity:** logarithmic-spiral closures, self-similar solutions, and any geometric realization of the Δ_{acc}–Ψ_{run} interface beyond the minimal accumulation relation of Sec. 7.
- **Feigenbaum interface and scaling constants:** any operator constructions, scaling laws, or constant-matching arguments, together with the required admissibility and comparability clauses for reproducible numerical alignment.

- **Case studies and applications:** domain instantiations (e.g. logistic map [22], RG windows [37, 38], complexity barriers [5, 17, 26], or AI error modes), including the concrete choice of D, $\mathcal{U}$, and resolution regimes and any data/figures used for auditability.

Accordingly, the core is read as a minimal semantic and operational scaffold: it defines what must be declared, what can be computed, what can be compared, and what must be reported as undefined under a given regime. Later chapters and appendices supply model-specific constructions and empirical interfaces while remaining constrained by the same package and reporting protocol.

A Diagnostic status tags and optional witnesses

Module summary.
Scope. Provide optional short tags and optional minimal witnesses for the undefinedness / non-licensing modes used in the main text.
Dependencies. None beyond the core reporting protocol (Sec. 8) and Def. 8.1.
Outputs. A lightweight status vocabulary (ok, U-spec, ADM, DOM, NC, RES) and suggested minimal witness fields.
Non-invocation rule. If omitted, the manuscript may report undefinedness in full prose; the core semantics is unchanged.

This appendix provides an *optional* compact labeling scheme for the undefinedness/non-licensing modes used in the main text (Def. 8.1). These tags are not part of the core semantics; they serve as short names for common regimes of under-specification or unlicensed comparison.

Status-based presentation (optional). A report may be presented in the lightweight form

$$\text{Report} := (\text{status},\ \text{payload},\ \text{witness}),$$

where status $\in \{\mathsf{ok}, \mathsf{U\text{-}spec}, \mathsf{ADM}, \mathsf{DOM}, \mathsf{NC}, \mathsf{RES}\}$, payload is the numerical value when ok (and omitted otherwise), and witness is an optional minimal record that makes the relevant missing clause or mismatch explicit.

Definition A.1 (Diagnostic status tags)**.** The diagnostic tags used in this paper are:

1. **U-spec (under-specification):** one or more clauses required to lock the quantity (Protocol 8.1) are missing.

2. **ADM (admissibility violation):** the attempted computation invokes an element outside the declared admissible family.

3. **DOM (domain mismatch):** the audited domain differs across reports without an explicit reconciliation rule.

4. **NC (non-comparability):** two reports are produced under packages V, V' without a declared transport/comparability law (Def. 4.12); equality/inequality comparisons are not licensed.

5. **RES (resolution mismatch):** thresholds/resolutions differ without a declared conversion rule; cross-report comparisons are not licensed under the declared resolution-aware semantics.

Remark A.1 (Minimal witnesses (optional))*.* When a diagnostic tag is used, the following witness data are sufficient (but not required) to make the mode explicit:

- U-spec: list the missing declaration items relative to Protocol 8.1 (or to the minimal package card, Protocol 4.1).
- ADM: specify the offending component (e.g. A, D, $u \in \mathcal{U}$, interface sub-clause) and the declared admissible family.
- DOM: specify the two audited domains and the missing reconciliation clause.
- NC: specify the pair of packages (V, V') and the missing transport/comparability declaration.
- RES: specify the compared thresholds/resolutions and the missing conversion rule.

Worked example (relative-entropy endpoint–path gaps). An explicit instantiation of a divergence-based diagnostic under the reporting protocol, including a status-style report and an operational witness for a reported gap, is given in [40].

B Extended probabilistic instantiation of $\Delta_{I\to G}$

Module summary.
Scope. Clarify how the package-locked residual $\Delta_{I\to G}(\mathcal{L}; V)$ functions as a certificate-level structural limit, and how a distribution-valued interface reading translates it into declared operational reporting readouts (TV/IPM) *without* replacing the witness mechanisms of the slim core. Interpretation boundary: this module provides distribution-level readouts under declared clauses; it does not introduce sampling-based statistical inference.
Dependencies. A declared package V fixing Ω_{reach}, admissible liftings $\mathcal{U}$, a discrepancy rule D, and a reporting resolution ε. For the probabilistic branch, additionally a declared observation space $(\mathcal{Y}, \Sigma)$ and distribution-valued interface clauses.
Outputs. (i) Certificate lower bounds via fiber witnesses / fiber diameter and via audit single-valuedness; (ii) probabilistic reporting readouts: uniform event-gap control under TV and uniform audit predicate / advantage-error control under an IPM.
Non-invocation rule. If the interface is not distribution-valued (or the required clauses are absent), the probabilistic branch is not invoked; the slim core remains unaffected.

B.1 Minimax residual as a certificate-level structural limit

The slim core treats $\Delta_{I\to G}(\mathcal{L}; V)$ as a package-locked minimax residual:

$$\Delta_{I\to G}(\mathcal{L}; V) = \inf_{u\in\mathcal{U}} \sup_{S\in\Omega_{\text{reach}}} D\big(\Phi_I(S),\ u(\Phi_G(S))\big),$$

where $\Omega_{\text{reach}} = \Omega_{\text{reach}}(V)$, $\mathcal{U} = \mathcal{U}(V)$, and $D = D_V$ are fixed by the declared package. In general high-dimensional settings one should not treat this as a point-estimation target. That is, $\Delta_{I\to G}(\mathcal{L}; V)$ is a certificate-level minimax residual defined by the declared package, not a random estimator with a sampling distribution. The primary role of $\Delta_{I\to G}$ in reporting is certificate-level: explicit witnesses can force resolution-non-negligibility under the authoritative reporting semantics (Normative Rule 2.1).

A canonical certificate is provided by the fiber-diameter mechanism. Under the triangle inequality assumption on D, Theorem 5.1 gives the witness lower bound

$$\Delta_{I\to G}(\mathcal{L}; V) \ \ge\ \tfrac{1}{2} \sup_{g\in\Phi_G(\Omega_{\text{reach}})} \operatorname{diam}_D(g),$$

and Corollary 5.1 upgrades this to a resolution-aware non-licensing statement: if there exist $S, S' \in \Omega_{\text{reach}}$ with $\Phi_G(S) = \Phi_G(S')$ and $D(\Phi_I(S), \Phi_I(S')) \ge 2\varepsilon$, then $\Delta_{I\to G}(\mathcal{L}; V) \ge \varepsilon$ and hence $\Delta_{I\to G}(\mathcal{L};V)\not\approx_\varepsilon 0$ under the declared reporting convention. In this sense, a fiber witness (Remark 5.2) functions as an audit artifact: it explains, in package-locked terms, why the "$\approx_\varepsilon 0$" convention is unavailable at the stated resolution.

B.2 Alignment with audit semantics: single-valued decidability and fiber non-constancy

Appendix G records an audit-level branch that is invoked only when a claim is explicitly about decidability/extendability under a locked audit viewpoint $V^{\text{aud}}=(\mathsf{End},\varepsilon,\mathcal{Q},\mathcal{C})$. The key predicate is the single-valuedness criterion $\mathrm{Val}(V^{\text{aud}})$ (Def. G.3): $\mathrm{Val}(V^{\text{aud}})=1$ corresponds to audit closure, while $\mathrm{Val}(V^{\text{aud}})=0$ is the non-closure regime and is identified as fiber non-constancy at the locked audit resolution. In the non-closure regime, an $H\to H^+$ extension is structurally necessary (Lemma G.2) rather than an implicit repair.

This audit layer is consistent with the slim-core witness logic above: a non-closure witness supplies an explicit obstruction to single-valued audit decidability under the declared viewpoint, and the correct output is either an extension workflow or a non-licensing report. Throughout, "reportable as 0" is used only as a resolution-aware reporting convention at finite ε, not as an ontological identity.

B.3 Probabilistic instantiation as reporting interface (TV/IPM)

Setup (declared in V). Fix an observation space $(\mathcal{Y},\Sigma)$. Under the declared interface, each audited state $S\in\Omega_{\text{reach}}$ induces an outcome distribution $P_S:=\Phi_I(S)\in\mathcal{P}(\mathcal{Y})$. Given the coarse description $\Phi_G(S)$ and an admissible lifting $u\in\mathcal{U}$, the reconstructed outcome distribution is $Q_S^u:=u(\Phi_G(S))\in\mathcal{P}(\mathcal{Y})$. The diagnostic retains its minimax form:

$$\Delta_{I\to G}(\mathcal{L};V)=\inf_{u\in\mathcal{U}}\ \sup_{S\in\Omega_{\text{reach}}} D(P_S,Q_S^u),$$

with D declared as a discrepancy on distributions.

Remark B.1 (interpretation boundary; no sampling semantics). In this branch, P_S and Q_S^u are distribution-valued readouts induced by the interface clauses declared in V (e.g., measurement/channel/noise semantics, or any explicitly declared uncertainty model). The TV/IPM consequences are deterministic reporting readouts about these declared objects and the minimax form; they do not introduce sampling-based statistical inference. In particular, ε is a reporting threshold (resolution convention) rather than a significance/confidence parameter. The $\sup_{S\in\Omega_{\text{reach}}}$ form is a worst-case certificate across reachable states; any average-case semantics requires an additional explicit state-weighting clause to be declared. If distributions are to be approximated from data, the estimation protocol (including its error notion) must be declared as an additional package component; otherwise no estimator-level interpretation is licensed.

(i) Event-probability gaps via total variation. If V declares total variation distance $D_{\text{TV}}(P,Q):=\sup_{B\in\Sigma}|P(B)-Q(B)|$ [8], then

$$\Delta_{I\to G}(\mathcal{L};V)=\inf_{u\in\mathcal{U}}\ \sup_{S\in\Omega_{\text{reach}}}\ \sup_{B\in\Sigma}|P_S(B)-Q_S^u(B)|.$$

Accordingly, a report $\Delta_{I\to G}(\mathcal{L};V)<\varepsilon$ licenses the existence of an admissible $u_\varepsilon\in\mathcal{U}$ such that

$$\sup_{S\in\Omega_{\text{reach}}}\ \sup_{B\in\Sigma}|P_S(B)-Q_S^{u_\varepsilon}(B)|<\varepsilon,$$

i.e. all event-probability gaps are ε-bounded at the declared reporting resolution.

(ii) Audit predicates and advantage/error control via an IPM. Let V declare an admissible audit predicate family $\mathcal{T} \subseteq \{f : \mathcal{Y} \to [0,1]\}$, and define the induced IPM [24, 8]

$$d_{\mathcal{T}}(P,Q) := \sup_{f\in\mathcal{T}} |\mathbb{E}_P[f] - \mathbb{E}_Q[f]|.$$

If V sets $D := d_{\mathcal{T}}$, then

$$\Delta_{I\to G}(\mathcal{L};V) = \inf_{u\in\mathcal{U}} \sup_{S\in\Omega_{\text{reach}}} d_{\mathcal{T}}(P_S, Q_S^u),$$

and a report $\Delta_{I\to G}(\mathcal{L};V) < \varepsilon$ licenses the existence of $u_\varepsilon \in \mathcal{U}$ such that

$$\sup_{S\in\Omega_{\text{reach}}} \sup_{f\in\mathcal{T}} |\mathbb{E}_{P_S}[f] - \mathbb{E}_{Q_S^{u_\varepsilon}}[f]| < \varepsilon.$$

To translate $d_{\mathcal{T}}$ into an audit advantage/error readout, consider the equal-prior one-shot decision task induced by the declared predicate family $\mathcal{T}$ [7] with $Y \sim P$ under $H = 0$ and $Y \sim Q$ under $H = 1$. This is a decision-theoretic interpretation of the IPM under the declared test family, not an additional statistical testing protocol. Any $f \in \mathcal{T}$ induces a randomized decision rule that outputs 1 with probability $f(Y)$. Its equal-prior error probability is

$$P_e(f;P,Q) = \tfrac{1}{2}\Big(\mathbb{E}_P[f] + \mathbb{E}_Q[1-f]\Big) = \tfrac{1}{2}\Big(1 + \mathbb{E}_P[f] - \mathbb{E}_Q[f]\Big).$$

Define the achievable audit advantage within $\mathcal{T}$ as $\mathrm{Adv}_{\mathcal{T}}(P,Q) := \frac{1}{2} - \inf_{f\in\mathcal{T}} P_e(f;P,Q)$. Then

$$\mathrm{Adv}_{\mathcal{T}}(P,Q) \;\le\; \tfrac{1}{2}\, d_{\mathcal{T}}(P,Q),$$

with equality if $\mathcal{T}$ is closed under complement ($f \in \mathcal{T} \Rightarrow 1 - f \in \mathcal{T}$). Under the declared instantiation $D = d_{\mathcal{T}}$, a report $\Delta_{I\to G}(\mathcal{L};V) < \varepsilon$ therefore bounds the worst-case audit advantage (after admissible reconstruction) by $\varepsilon/2$ at the declared resolution, and yields the corresponding equal-prior audit error lower bound.

Interpretation. The probabilistic branch is a reporting interface: it converts package-locked residual statements into declared distribution-level readouts (event-gap bounds; predicate-gap and advantage/error bounds) once the relevant clauses are declared. This is a translation of the same package-locked residual into a more interpretable output vocabulary; it does not introduce sampling-based statistical inference or replace the slim-core witness mechanism. When a fiber witness (or an audit non-closure witness) is available, the certificate lower bounds above already lock the appropriate non-licensing or extension conclusions within the same package.

C Extended Ψ–$\Delta_{I\to G}$ Dynamics (HTDS details)

Module summary.
Scope. Collect optional dynamical material (HTDS-style details) that refines the minimal evolution interface, including variant bookkeeping and optional long-time lower bounds.
Dependencies. A declared evolution axis γ, a declared transport/comparability clause across γ, and (when Δ_{dyn} is used) additional dynamical declarations such as invariant-measure admissibility.
Outputs. Variant-safe statements about $\Delta_{\text{acc}}(\gamma)$; optional definitions such as Δ_{dyn}; and auxiliary lemmas for regime-relative dynamical interpretations.
Non-invocation rule. If dynamical declarations are not supplied, this module is not invoked and no dynamical claims are licensed beyond the minimal interface in Sec. 7.

This appendix collects optional dynamical material that is not required for the slim core. All statements here remain package-relative and resolution-aware. In particular: (i) the fixed-slice diagnostic $\Delta_{I\to G}(\mathcal{L};V)$ (Def. 5.3) is distinct from (ii) the accumulated quantity $\Delta_{\mathrm{acc}}(\gamma)$ along a declared evolution path (Def. 7.1); and both are distinct from (iii) the ontological meta-gap axiom schema $\Delta^*_R > 0$ (Def. 6.1).

C.1 Notation and Δ-variants used in the dynamical module

Fixed-slice cross-level diagnostic. At each stage (or scale) γ, the fixed-slice diagnostic is $\Delta_{I\to G}(\mathcal{L}_\gamma;V_\gamma)$ defined by an inf–sup residual under the declared interface module.

Accumulated quantity along an evolution parameter. Along a declared evolution path $\mathfrak{P}_\gamma = (\mathcal{L}_\gamma, A_\gamma, V_\gamma)$ with an explicit transport law, the accumulated quantity is $\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{run}}(s)\,ds$ (Def. 7.1).

Optional: dynamical lower bound. When a concrete dynamical carrier is declared (HTDS below), one may define a long-time lower bound (denoted here by Δ_{dyn}) as an optional extension. This quantity is not part of the slim core and is used only when invariant-measure or orbit-average semantics are explicitly declared.

C.2 HTDS construction as an instantiation layer (optional)

Definition C.1 (HTDS (optional dynamical carrier))**.** A *hierarchical tension dynamical system* (HTDS) is a tuple

$$\mathrm{HTDS} := (X, d, T, \{F_i\}_{i\in I})$$

where (X,d) is a metric/pseudometric state space, $T : X \to X$ is a discrete-time evolution map, $\{F_i\}_{i\in I}$ is a declared family of constraint/tension elements, and an instantaneous tension readout $\Psi_X : X \to [0,\infty)$ is constructed from $\{F_i\}$ (e.g. by a declared norm rule). All clauses must be declared to make subsequent quantities reproducible.

Remark C.1 (Bridge to the core). HTDS serves only as an instantiation layer: it provides concrete coordinates in which the conflict functional E_A of the core can be realized as $\Psi_X(x)$ under a declared embedding of abstract states into X. This appendix does not modify the core semantics of $\Psi(\mathcal{L}, A; V)$.

C.3 Orbit averages and an optional dynamical lower bound

Definition C.2 (Orbit-averaged tension (optional))**.** Given HTDS $(X, d, T, \{F_i\})$ with $\Psi_X : X \to [0,\infty)$, define the orbit-averaged tension by

$$\overline{\Psi}(x) := \limsup_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \Psi_X(T^n x).$$

Definition C.3 (Δ_{dyn} (optional invariant-measure lower bound))**.** Let $\mathcal{S}_T$ denote the set of T-invariant probability measures on X (under the declared measurable structure). Define

$$\Delta_{\mathrm{dyn}} := \inf_{\mu\in\mathcal{S}_T} \int \Psi_X \, d\mu.$$

This quantity is meaningful only under a declared admissibility gate on $\mathcal{S}_T$ (if any) and with explicit reporting conventions.

Remark C.2 (Typicality and admissibility gates). If the intended semantics uses a restricted class of “natural” invariant measures (e.g. SRB [28, 39]), or if typical initial conditions are assumed, that restriction must be declared as part of the package/regime. Otherwise, Δ_{dyn} is reported with an explicit diagnostic stating which measure class is used.

C.4 Multi-view interpretations of Δ-flow (optional)

This subsection records alternative readings of "Δ-flow" depending on what the evolution parameter γ represents:

- **Scale/refinement flow:** γ indexes resolution refinement or coarse-graining depth [16, 37, 38], with an explicit transport law for comparability across γ.
- **Model-selection / admissible-regime flow:** γ indexes restricted families of packages, so $\Psi_{\mathrm{run}}(\gamma)$ is a controlled best-achievable readout under regime constraints.
- **Time flow (HTDS):** γ is physical/discrete time; then Ψ_{run} is an instantaneous readout and Δ_{acc} is a time-integrated accumulation.

C.5 Additional lemmas (optional instantiation layer)

This appendix collects auxiliary statements that are *not* required to define the core framework objects, but are repeatedly used when one instantiates the framework in dynamical or self-similar settings. Throughout, every statement is read under the *declared package* semantics: numerical quantities are well-defined only relative to a declared package V (or an explicit admissible family), and any "≈ 0" claim is interpreted in the resolution-aware sense fixed in the core.

Lemma C.1 (Variant separation: fixed-slice gap vs. accumulated gap vs. dynamical bound)**.** *Fix a target system $\mathcal{L}$ and a declared package V.*

1. Fixed-slice inter-level gap. *The quantity $\Delta_{I\to G}(\mathcal{L};V)$ denotes the fixed-slice inter-level gap defined for a declared pair of perspectives (Φ_I,Φ_G) together with a declared discrepancy D and admissible lifting class $\mathcal{U}$.*
2. Accumulated gap along an evolution parameter. *The quantity $\Delta_{\mathrm{acc}}(\gamma)$ denotes an accumulated scalar along a declared evolution parameter γ under a declared evolution interface (Section 7); it is* not *identical to any fixed-slice gap unless an additional identification is explicitly declared.*
3. Dynamical fracture bound. *The quantity Δ_{dyn} (when used) denotes a dynamical lower bound obtained by averaging Ψ along invariant measures (or an analogous long-time construction) under a declared dynamical carrier; it is neither a fixed-slice gap nor an accumulated scalar unless an explicit bridge is stated.*

Consequently, statements involving "Δ" are well-posed only after the intended variant is explicitly specified.

Lemma C.2 (Resolution-aware null reporting and non-implication of ontological zero)**.** *Fix a declared resolution threshold ε as part of V, together with a reporting rule "report* 0 *when the readout is below ε". For any reportable gap quantity X among $\{\Delta_{I\to G}(\mathcal{L};V),\ \Delta_{\mathrm{acc}}(\gamma),\ \Delta_{\mathrm{dyn}}\}$, the statement*

$$X \approx_\varepsilon 0 \quad\Longleftrightarrow\quad X < \varepsilon$$

is a resolution-level convention. *It does not entail an ontological identity $X = 0$ in any genuine cross-level setting, and it is never to be read as a claim that the meta-gap Δ^*_R is* 0 *for any genuine cross-level relation R.*

Lemma C.3 (Accumulation interface: absolute continuity and monotonicity under $\Psi_{\mathrm{run}} \geq 0$)**.** *Fix a declared evolution parameter γ and define $\Delta_{\mathrm{acc}}(\gamma)$ via the minimal interface*

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{run}}(s)\, ds, \qquad \frac{d}{d\gamma}\Delta_{\mathrm{acc}}(\gamma) := \Psi_{\mathrm{run}}(\gamma),$$

interpreted as an operational interface under the declared evolution (Section 7). If $\Psi_{\mathrm{run}}(\gamma) \geq 0$ holds pointwise in the declared regime, then $\Delta_{\mathrm{acc}}(\gamma)$ is non-decreasing in γ and is absolutely continuous on any finite interval where Ψ_{run} is integrable.

Remark C.3 (Discrete γ (hierarchy index)). When γ is a discrete index (e.g. a hierarchy depth $k \in \mathbb{N}$), the same interface is understood in difference form:

$$\Delta_{\mathrm{acc},k+1} - \Delta_{\mathrm{acc},k} := \Psi_k,$$

with the same resolution-aware reporting convention as in Lemma C.2.

Lemma C.4 (Optional local closure ansatz: proportional growth implies exponential accumulation)**.** *Assume an* optional *local closure ansatz holds on an interval in γ under the declared regime:*

$$\Psi_{\mathrm{run}}(\gamma) = b \cdot \Delta_{\mathrm{acc}}(\gamma),$$

where b is a constant fixed by the chosen coordinate/normalization. Then on any region where $\Delta_{\mathrm{acc}}(\gamma) \neq 0$,

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0)\, e^{b(\gamma-\gamma_0)}.$$

This statement is not *a core postulate: it is valid only when the proportionality ansatz is explicitly declared as part of the modeling choice.*

Lemma C.5 (Optional geometric embedding: logarithmic spiral as a closure realization)**.** *Assume a declared geometric embedding is introduced as an* auxiliary visualization map*:*

$$\theta = a_\theta\, \gamma + \theta_0, \qquad r = a_r\, \Delta_{\mathrm{acc}}(\gamma), \qquad a_\theta \neq 0,\ a_r > 0,$$

and suppose the proportional closure ansatz of Lemma C.4 holds with constant b. Then the embedded curve $(r(\theta), \theta)$ satisfies

$$\frac{dr}{d\theta} = \tilde{b}\, r, \qquad r(\theta) = r_0\, e^{\tilde{b}(\theta-\theta_0)}, \qquad \tilde{b} := \frac{b}{a_\theta}.$$

Equivalently, the embedded trajectory is a logarithmic spiral (equiangular spiral). The constant equiangular property is given by

$$\tan\varphi = \frac{r}{dr/d\theta} = \frac{1}{\tilde{b}},$$

where φ is the (constant) angle between the tangent direction and the radius vector. Again, this is an optional *closure/embedding result and carries no claim about physical phase-space geometry.*

Lemma C.6 (Optional two-component fracture state and linear spiral scaling)**.** *Fix a declared self-embedding or renormalization index k [37, 38] and a declared operational regime that produces a scalar fracture readout $\Delta_k \geq 0$ at each k (in the RG bridge of Protocol E.1, one instantiates $\Delta_k := \Delta_{I\to G}(f_k; V)$ under a declared package V). Declare an* ideal propagation rule *F (e.g. $F(x) = c\, x$) and define the meta-fracture component by*

$$r_k := \Delta_{k+1} - F(\Delta_k).$$

Define coordinates $x_k := \Delta_k$ and $y_k := r_k$, and package the state as $v_k = (x_k, y_k)^\top \in \mathbb{R}^2$ (equivalently $z_k := x_k + i\, y_k = \Delta_k + i\, r_k$). Assume a local linearization of the induced meta-update holds near a reference configuration:

$$v_{k+1} \approx J\, v_k,$$

with $J \in \mathbb{R}^{2\times 2}$. If J has a dominant complex eigenvalue $\lambda = \rho e^{i\varphi}$ (in its real Jordan form), then in the corresponding complex coordinate $z_k := x_k + i\, y_k$ one has the first-order law

$$z_{k+1} \approx \lambda z_k,$$

so that iterates form a logarithmic-spiral scaling in the (x, y) plane with radial factor ρ and phase increment φ per step. This lemma is optional *and depends on (i) the declared rule F, (ii) the declared coordinate choice for (x, y) (i.e. $x_k := \Delta_k$, $y_k := r_k$), and (iii) the validity of a local linear approximation.*

Corollary C.1 (Feigenbaum interface as a regime-relative identification (optional))**.** *In unimodal-map period-doubling settings, let δ_{Feig} and α_{Feig} denote the classical Feigenbaum constants [10, 11, 6, 19] (notation separated from the resolution threshold ε). Under an explicitly declared intertwining/transport hypothesis from function-space RG to fracture space (including the normalization choices absorbed into the declaration), one may parameterize the dominant spiral invariants in Lemma C.6 as regime-relative quantities:*

$$\rho = \delta_{\mathrm{Feig}}^{-\beta}, \qquad \varphi \equiv \pi + \varphi_{\mathrm{bias}} \pmod{2\pi},$$

where β and φ_{bias} are treated as declared (reportable) regime parameters rather than new universality claims. This corollary is not used to support any core theorem; it only records a compatible semantic translation once the required transport law is declared.

Remark C.4 (Domain gating and reproducibility)*.* Whenever Lemma C.4–Corollary C.1 are invoked, the text must explicitly state the additional declarations that make them applicable: the chosen Δ-variant, the declared evolution index, the declared embedding/closure rules, and the reporting convention at resolution ε.

C.6 A regime-relative conservation template for an accumulated gap

This subsection records a *template* for stating and auditing "conservation" claims for the accumulated quantity $\Delta_{\mathrm{acc}}(\gamma)$ along a declared evolution parameter γ. It introduces no new core quantities and is *variant-safe*: throughout this subsection we write the accumulated quantity explicitly as $\Delta_{\mathrm{acc}}(\gamma)$ (or Δ_{acc} when γ is clear), and we do not reuse the bare symbol Δ as an alias. In particular, it is not identified with the fixed-slice diagnostic $\Delta_{I\to G}(\mathcal{L}_\gamma; V_\gamma)$ (Def. 5.3), nor with any optional dynamical bound Δ_{dyn}, nor with the ontological meta-gap Δ^*_R.

Prerequisites (declaration gates). Fix a declared reporting threshold $\varepsilon > 0$. A *regime* for accumulated-gap reporting consists of:

1. A declared evolution parameter $\gamma \in [\gamma_0, \gamma_1]$ and a declared evolution path $\mathfrak{P}_\gamma = (\mathcal{L}_\gamma, A_\gamma, V_\gamma)$ as in Def. 7.1.
2. An explicit transport/comparability law across γ (Def. 4.12) with a declared domain, so that values reported at different γ are interpreted within a single declared convention.
3. A declared initialization $\Delta_{\mathrm{acc}}(\gamma_0)$ and a declared running readout $\Psi_{\mathrm{run}}(\gamma) := \Psi(\mathcal{L}_\gamma, A_\gamma; V_\gamma)$, for which the accumulation interface

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{run}}(s)\, ds \quad \text{(or its discrete analogue)}$$

is well-defined on the declared domain.

If any prerequisite is missing, the correct output under the reporting protocol is undefined with an explicit diagnostic listing the missing clauses.

What "conservation" means (resolution-aware). A conservation claim for Δ_{acc} is always *resolution-aware*: it asserts that the net change in Δ_{acc} over a declared interval is negligible at the declared threshold ε, i.e.

$$\Delta_{\mathrm{acc}}(\gamma_1) \;\approx_\varepsilon\; \Delta_{\mathrm{acc}}(\gamma_0) \quad\Longleftrightarrow\quad \big|\Delta_{\mathrm{acc}}(\gamma_1)-\Delta_{\mathrm{acc}}(\gamma_0)\big| < \varepsilon. \tag{3}$$

This is a reporting-level statement and must not be read as an ontological identity.

Template I: Integral (continuous-γ) conservation test

Lemma C.7 (Regime-relative conservation test (continuous-γ))**.** *Fix a declared regime satisfying the gates above on $[\gamma_0, \gamma_1]$. Suppose an audit procedure provides:*

1. *an estimate I of the path integral $\int_{\gamma_0}^{\gamma_1} \Psi_{\mathrm{run}}(s)\,ds$, and*

2. *a declared error bound $E \geq 0$ such that $\left|I - \int_{\gamma_0}^{\gamma_1} \Psi_{\mathrm{run}}(s)\,ds\right| \leq E$.*

If $|I| + E < \varepsilon$, then the conservation report (3) *is licensed and may be output as $\Delta_{\mathrm{acc}}(\gamma_1) \approx_\varepsilon \Delta_{\mathrm{acc}}(\gamma_0)$. If $|I| + E \geq \varepsilon$, the regime does not license a conservation claim at threshold ε.*

Remark C.5 (Local form). A sufficient local condition for (3) is that $\Psi_{\mathrm{run}}(\gamma) \approx_\varepsilon 0$ almost everywhere on $[\gamma_0, \gamma_1]$ under the declared integrability convention, together with an explicit bound translating the pointwise resolution rule into a bound on the integral. This translation is regime-dependent and must be declared (e.g. via a sampling density or an L^1 envelope).

Template II: Discrete-γ conservation test (grid or hierarchy index)

Lemma C.8 (Regime-relative conservation test (discrete-γ))**.** *Let γ be a declared discrete index (e.g. γ_k on a grid or $k \in \mathbb{N}$). Let $h_k := \gamma_{k+1} - \gamma_k$ denote the declared step size. Assume the declared accumulation interface is given in difference form on the declared domain:*

$$\Delta_{\mathrm{acc},k+1} - \Delta_{\mathrm{acc},k} := \Psi_k \cdot h_k \quad \textit{(with the convention specified by the regime).}$$

Suppose an audit provides an estimate of the net increment $S := \sum_{k=0}^{K-1} \Psi_k\, h_k$ together with an error bound $E \geq 0$ controlling numerical/discretization error. If $|S| + E < \varepsilon$, then the report $\Delta_{\mathrm{acc}}(\gamma_K) \approx_\varepsilon \Delta_{\mathrm{acc}}(\gamma_0)$ is licensed. Otherwise it is not licensed at threshold ε.

diagnostic fields (minimal) and undefined triggers

Minimal diagnostic fields. Any report of the form (3) (or its negation) includes a *conservation diagnostic* recording:

1. **Regime identifier:** the declared path $\mathfrak{P}_\gamma = (\mathcal{L}_\gamma, A_\gamma, V_\gamma)$ and the declared interval/domain.

2. **Transport/comparability:** the transport law across γ, its declared domain, and any normalization or coordinate conventions needed for comparing values across γ.

3. **Accumulation interface:** the declared rule implementing Δ_{acc} from Ψ_{run} (integral or discrete difference form), including the declared initial condition.

4. **Estimation data:** the computed value I (or S), the declared error bound E, and the threshold ε used for the final decision.

5. **Decision:** whether $|I| + E < \varepsilon$ (licensed conservation), $|I| + E \geq \varepsilon$ (not conserved at ε), or undefined (missing clauses).

Undefined triggers (domain gating). The conservation statement (3) must be reported as undefined (with an explicit undefined-diagnostic) if any of the following holds:

- no explicit transport/comparability law across γ is declared, or the queried interval lies outside its declared domain;
- the integrability/discretization convention required to evaluate the accumulation interface is not declared or is not valid on the queried domain;
- the reporting threshold ε or the initialization $\Delta_{\mathrm{acc}}(\gamma_0)$ is not declared.

Remark C.6 (Non-collision with Δ-variant separation). This subsection licenses only statements about $\Delta_{\mathrm{acc}}(\gamma)$ along a declared evolution path under a declared transport law. It does not imply any statement about the fixed-slice gap $\Delta_{I\to G}(\mathcal{L}_\gamma; V_\gamma)$, nor does it identify Δ_{acc} with any other Δ-variant without an additional, explicitly declared bridge.

D Logarithmic Spiral Closure and Self-Similar Geometry

Module summary.
Scope. Provide an optional geometric embedding/closure for the accumulated interface $(\gamma, \Delta_{\mathrm{acc}}(\gamma))$, isolating a self-similar regime as a logarithmic spiral under declared embedding parameters.
Dependencies. A declared evolution parameter γ, an accumulated quantity $\Delta_{\mathrm{acc}}(\gamma)$ with its interface, and declared embedding/closure parameters (when invoked).
Outputs. A reproducible planar embedding and optional spiral-type conclusions that are explicitly marked as non-core.
Non-invocation rule. If embedding/closure parameters are not declared, this module is not invoked; the core framework makes no geometric closure claim.

This appendix records an *optional* geometric closure for the accumulated gap interface introduced in Section 7. Nothing in the core framework requires this closure. It is provided as a compact, reproducible embedding that (i) turns the scalar pair $(\gamma, \Delta_{\mathrm{acc}}(\gamma))$ into a planar curve, and (ii) isolates a canonical self-similar regime as a logarithmic (equiangular) spiral.

Throughout, statements are interpreted under the declared package semantics: any equality involving reportable quantities inherits the resolution-aware conventions fixed in the core, and any "$\approx_\varepsilon 0$" statement is purely a reporting convention at the declared threshold ε.

D.1 Objects and scope of this appendix

Fix a declared evolution parameter γ and an accumulated quantity $\Delta_{\mathrm{acc}}(\gamma)$ as in Section 7, together with an intra-level rate $\Psi_{\mathrm{run}}(\gamma)$ satisfying the interface

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{run}}(s)\, ds, \qquad \frac{d}{d\gamma}\Delta_{\mathrm{acc}}(\gamma) := \Psi_{\mathrm{run}}(\gamma) \tag{4}$$

on any interval where the integral is well-defined. In this appendix, $\Delta_{\mathrm{acc}}(\gamma)$ is an *accumulated readout* along γ and is not identified with any fixed-slice inter-level gap $\Delta_{I\to G}(\mathcal{L}; V)$ unless such an identification is explicitly declared elsewhere.

D.2 Hilbert-space mode spirals as a dynamical visualization (optional)

A recurring geometric intuition is that, in high (or infinite) dimension, each "degree of freedom" is read as carrying both a *radial* component (amplification/attenuation) and an *angular* component (phase rotation), so that the natural trajectory in the coefficient plane is a logarithmic (equiangular) spiral rather than a straight line.

This subsection records a clean mathematical formalization of that intuition in a standard setting: *it is a statement about coefficient trajectories under a declared evolution and a chosen representation*, not a claim about the intrinsic geometry of one-dimensional subspaces in a Hilbert space.

Remark D.1 (Purpose within this appendix)*.* This subsection is included only to motivate the use of planar embeddings and logarithmic-spiral segments as a *visualization/closure device* later in Appendix D; none of the slim-core objects or definitions depend on Hilbert-space structure.

Lemma D.1 (Mode-coefficient spirals under diagonal complex generators (optional))**.** *Let $\mathcal{H}$ be a complex Hilbert space with an orthonormal basis $\{e_n\}_{n\in\mathbb{N}}$. Consider a (possibly time-homogeneous) linear evolution*

$$\dot{\psi}(t) = L\psi(t), \qquad \psi(0) = \psi_0 \in \mathcal{H}, \tag{5}$$

where L is a linear operator for which the chosen basis diagonalizes the action:

$$Le_n = \lambda_n e_n, \qquad \lambda_n \in \mathbb{C}. \tag{6}$$

Write $\psi(t) = \sum_n c_n(t)e_n$ with coefficients $c_n(t) = \langle e_n, \psi(t)\rangle$. Then each coefficient satisfies the scalar ODE

$$\dot{c}_n(t) = \lambda_n c_n(t), \qquad c_n(0) = c_{n,0}, \tag{7}$$

and hence

$$c_n(t) = c_{n,0}\, e^{\lambda_n t}. \tag{8}$$

If $\lambda_n = a_n + ib_n$ with $a_n \neq 0$ and $b_n \neq 0$, then the trajectory of $c_n(t) \in \mathbb{C}$ is a logarithmic (equiangular) spiral in the complex plane. More explicitly, writing $c_n(t) = r_n(t)e^{i\theta_n(t)}$,

a choice of branch gives $r_n(t) = |c_{n,0}|\, e^{a_n t}$ and $\theta_n(t) = \arg(c_{n,0}) + b_n t$, and eliminating t yields

$$r_n(\theta) = r_{n,0}\, \exp\Big(\tfrac{a_n}{b_n}(\theta - \theta_{n,0})\Big), \tag{9}$$

which is the polar form of a logarithmic spiral.

Proof. Under (6), the coefficient identity $c_n(t) = \langle e_n, \psi(t)\rangle$ and the evolution (5) imply (7) on any interval where the coefficient dynamics is well-defined. Solving the scalar ODE gives (8). Writing $\lambda_n = a_n + ib_n$ and expressing $c_n(t)$ in polar form yields $r_n(t) = |c_{n,0}|e^{a_n t}$ and $\theta_n(t) = \arg(c_{n,0}) + b_n t$. Eliminating t gives (9). □

Remark D.2 (Interpretation and limitations)*.*

1. **Not an intrinsic axis geometry claim.** The one-dimensional subspace $\operatorname{span}\{e_n\} \subset \mathcal{H}$ remains isometric to a line. The spiral arises only after choosing (i) a representation $\psi = \sum c_n e_n$ and (ii) an evolution with complex growth/rotation in the coefficient c_n.

2. **Per-mode statement.** The spiral statement is made for each coefficient c_n on its own copy of $\mathbb{C}$ (or a two-dimensional invariant block in the real case). The ambient dimension affects only the indexing of modes, not the local form (9).

3. **Pure rotation / pure dilation are degenerate cases.** If $a_n = 0$ and $b_n \neq 0$, then $|c_n(t)|$ is constant and $c_n(t)$ is a circle. If $b_n = 0$ and $a_n \neq 0$, then $c_n(t)$ moves on a ray (no rotation). The logarithmic spiral requires both components.

4. **Real Hilbert spaces and two-dimensional blocks.** If $\mathcal{H}$ is real, spiral-like behavior typically appears on invariant two-dimensional subspaces where the generator acts as a rotation–dilation block. In that setting, the same calculation applies after identifying the block with $\mathbb{R}^2 \cong \mathbb{C}$.

5. **Connection to the embedding/closure device below.** The declared planar map $(\gamma, \Delta_{\mathrm{acc}}(\gamma)) \mapsto (r(\gamma), \theta(\gamma))$ in Definition D.1 can be read as an auxiliary coefficient-plane visualization of a rotation–dilation type evolution. In that view, logarithmic-spiral segments correspond to regimes where radial growth and angular advance are approximately proportional.

Remark D.3 (When this visualization is appropriate). This subsection is an *optional* interpretive aid.

- **Use.** When a declared evolution admits a local linearized form with a rotation–dilation generator (or an equivalent 2×2 real block), and a coefficient-plane view clarifies how "growth" and "phase" co-evolve within a chosen representation.
- **Do not use.** When no such rotation/phase component is declared (purely radial dynamics), or when the required representation and transport clauses are not part of the declared package; in such cases, spiral pictures, by themselves, do not justify any reportable claim.

D.3 Declared planar embedding (auxiliary)

Definition D.1 (Planar embedding of $(\gamma, \Delta_{\mathrm{acc}})$)**.** Declare constants $a_\theta \neq 0$, $a_r > 0$, and offsets $\theta_0 \in \mathbb{R}$, $r_0 \in \mathbb{R}$. Define a planar curve in polar coordinates (r, θ) by

$$\theta(\gamma) = a_\theta \, \gamma + \theta_0, \qquad r(\gamma) = a_r \, \Delta_{\mathrm{acc}}(\gamma), \tag{10}$$

and interpret the geometric trajectory as $\gamma \mapsto (r(\gamma), \theta(\gamma))$.

Remark D.4 (Status of the embedding). The map (10) is an auxiliary visualization/closure device. It does not assert any claim about the physical phase-space geometry of $\mathcal{L}$. All geometric statements below are statements about the embedded curve defined by the declared map.

D.4 Logarithmic-spiral closure as a local proportionality ansatz

Definition D.2 (Proportional closure ansatz)**.** On a declared γ-interval I, assume the following *optional* closure relation holds:

$$\Psi_{\mathrm{run}}(\gamma) = b \cdot \Delta_{\mathrm{acc}}(\gamma), \qquad \gamma \in I, \tag{11}$$

where $b \in \mathbb{R}$ is a declared constant (a regime parameter).

Combining (4) and (11) yields

$$\frac{d}{d\gamma} \Delta_{\mathrm{acc}}(\gamma) = b \, \Delta_{\mathrm{acc}}(\gamma), \qquad \gamma \in I, \tag{12}$$

hence on any subregion where $\Delta_{\mathrm{acc}}(\gamma) \neq 0$,

$$\Delta_{\mathrm{acc}}(\gamma) = \Delta_{\mathrm{acc}}(\gamma_0) \, \exp\bigl(b(\gamma - \gamma_0)\bigr). \tag{13}$$

Under the embedding (10), this becomes a spiral law.

Lemma D.2 (Logarithmic spiral under proportional closure)**.** *Assume Definitions D.1 and D.2 on an interval I, and suppose $\Delta_{\mathrm{acc}}(\gamma) \neq 0$ on I. Then the embedded curve satisfies*

$$\frac{dr}{d\theta} = \kappa \, r, \qquad \kappa := \frac{b}{a_\theta}, \tag{14}$$

hence

$$r(\theta) = r(\theta_0) \, \exp\bigl(\kappa(\theta - \theta_0)\bigr), \tag{15}$$

i.e. the trajectory is a logarithmic spiral in the (r, θ) plane.

Remark D.5 (Inward vs. outward winding). If $\kappa > 0$, then $r(\theta)$ is increasing in θ on the interval. If $\kappa < 0$, then $r(\theta)$ is decreasing in θ on the interval and approaches 0 as $\theta \to +\infty$ along that regime (if the interval extends to $+\infty$ in θ). These are geometric properties of the embedded curve under the declared closure.

D.5 Equiangular property and a characterization

A classical geometric characterization of logarithmic spirals is equiangularity: the angle between the tangent direction and the radius vector is constant along the curve.

Lemma D.3 (Equiangular relation in polar form). *Let a C^1 polar curve be given by $r = r(\theta)$ with $r(\theta) > 0$ on an interval. Let $\varphi(\theta) \in (0, \pi/2) \cup (\pi/2, \pi)$ denote the angle between the tangent direction and the radius vector at angle θ. Then*

$$\tan \varphi(\theta) = \frac{r(\theta)}{dr/d\theta}. \tag{16}$$

Corollary D.1 (Log spiral implies equiangularity). *If $r(\theta) = r_0 e^{\kappa(\theta-\theta_0)}$ with $\kappa \neq 0$, then $dr/d\theta = \kappa r$ and* (16) *yields* $\tan \varphi(\theta) = 1/\kappa$*, hence $\varphi(\theta)$ is constant.*

Corollary D.2 (Equiangularity implies log spiral). *If $\varphi(\theta) \equiv \varphi_0$ is constant with* $\tan \varphi_0 \neq 0$*, then* (16) *implies $dr/d\theta = \kappa r$ with $\kappa = 1/\tan \varphi_0$, hence $r(\theta) = r_0 e^{\kappa(\theta-\theta_0)}$.*

D.6 Self-similarity constraints and a functional characterization

The spiral in Lemma D.2 can also be characterized as the unique orbit shape compatible with a minimal "rotation + scaling" self-similarity constraint.

Definition D.3 (Rotation–scaling self-similarity (shape invariance)). A polar curve $r = r(\theta)$ is said to be rotation–scaling self-similar if there exist constants (ϕ, λ) with $\phi \neq 0$ and $\lambda > 0$ such that for all θ in a declared interval,

$$r(\theta + \phi) = \lambda\, r(\theta). \tag{17}$$

Lemma D.4 (Self-similarity functional equation yields exponential in θ). *Assume $r(\theta) > 0$ and r is continuous on an interval, and suppose* (17) *holds for some fixed (ϕ, λ). If r is additionally differentiable and the relation is stable under small variations of θ within the interval (so that one may differentiate the relation), then r satisfies $dr/d\theta = \kappa r$ with $\kappa = (\log \lambda)/\phi$ and hence is a logarithmic spiral.*

Remark D.6 (Role of symmetry declarations). The implication in Lemma D.4 is a *geometry-side* statement: once rotation–scaling invariance is declared as the relevant symmetry constraint and regularity is declared (continuity/differentiability on an interval), the logarithmic spiral is the compatible curve shape. This does not assert that a given system must satisfy the symmetry constraint; it records what follows if the constraint is adopted.

D.7 Discrete evolution index (hierarchy depth) version

When the evolution parameter is a discrete index (e.g. a hierarchy depth $k \in \mathbb{N}$), the same structure appears in difference form.

Definition D.4 (Discrete proportional closure (difference form)). Let $\Delta_{\mathrm{acc},k}$ be a discrete accumulated readout and define $\Psi_k := \Delta_{\mathrm{acc},k+1} - \Delta_{\mathrm{acc},k}$. Assume an optional proportionality

$$\Psi_k = b\, \Delta_{\mathrm{acc},k}, \tag{18}$$

with declared constant b (regime parameter).

Then $\Delta_{\mathrm{acc},k+1} = (1+b)\Delta_{\mathrm{acc},k}$, hence $\Delta_{\mathrm{acc},k} = \Delta_{\mathrm{acc},0}(1+b)^k$ on any regime where the closure is asserted. Under the embedding $\theta_k = a_\theta k + \theta_0$ and $r_k = a_r \Delta_{\mathrm{acc},k}$, this yields a discrete sampling of a logarithmic spiral.

D.8 Reproducible reporting clauses for spiral closure

Any use of the spiral closure in later case studies is regarded as well-posed only if it reports the following declarations explicitly:

1. the Δ-variant used (*accumulated* Δ_{acc}, not a fixed-slice $\Delta_{I\to G}$ unless such an identification is explicitly declared);
2. the evolution parameter domain (continuous γ or discrete k) and the reference point;
3. the closure ansatz (e.g. $\Psi_k = b\,\Delta_{\mathrm{acc},k}$) and the regime interval on which it is asserted;
4. the embedding constants $(a_\theta, a_r, \theta_0)$ used to map the trajectory into polar coordinates.

Remark D.7 (Relation to the core framework). Appendix D introduces no new core definition. It only records one optional closure path: (4) + (11) + (10) $\Rightarrow$ logarithmic spiral geometry for the accumulated readout. If the proportional closure is not declared (or not supported by an instantiation), none of the spiral claims are invoked.

E Feigenbaum Interface (Optional Module)

This appendix records an *optional* interface module that connects the Ψ–$\Delta_{I\to G}$ framework to classical period-doubling renormalization theory and the associated Feigenbaum constants [10, 11, 6, 19]. No definition or claim in the core text depends on this appendix. The role of this module is purely interpretational and organizational: it provides a controlled mapping from a function-space RG orbit to a "fracture-state" orbit in a low-dimensional auxiliary space where radial scaling and phase behavior can be read as geometric invariants.

All statements here are regime-relative: they are meaningful only after the dependency declarations in Subsection C.0 are fixed and reported.

E.1 Dependency declarations and object list

Protocol E.1 (Optional-module dependencies for the Feigenbaum interface)**.** To make the Feigenbaum interface well-defined, the following additional objects must be explicitly declared.

1. **Instance family and RG operator in object space.** Specify a unimodal map family (or other RG-compatible instance family) and a renormalization operator $\mathcal{R}_{\mathrm{map}}$ acting on that object space. Declare the local regime (neighborhood of the critical configuration) in which linearization is invoked.
2. **Operational package for fracture readouts.** Fix a declared package V (or an admissible family) that makes the scalar fracture readout $\Delta_{I\to G}(\cdot;V)$ well-defined for each object in the family. This includes the discrepancy rule, admissible liftings/predictors, reachable-set convention (if used), and the resolution threshold ε for reporting.
3. **Fracture state representation.** Declare a *two-component* fracture state map

$$\Phi_{\mathrm{state}}:\ f \longmapsto v(f) \in \mathbb{R}^2$$

and specify its components. A minimal choice (used below) is

$$v_k := \begin{pmatrix}\Delta_k\\ r_k\end{pmatrix}, \qquad \Delta_k := \Delta_{I\to G}(f_k;V), \qquad f_k := \mathcal{R}^k_{\mathrm{map}}(f_0),$$

with r_k defined as a *meta* deviation relative to a declared propagation rule F.

4. **Prediction / propagation rule F (declared).** Specify a rule F used to predict the next-step fracture from the current one:

$$F : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}, \qquad \Delta_{k+1} \approx F(\Delta_k),$$

and define the meta component by

$$r_k := \Delta_{k+1} - F(\Delta_k).$$

The choice of F is part of the module declaration (it is not inferred). A common linear choice is $F(x) = c\,x$ with declared $c > 0$.

5. **Bridge/intertwining hypothesis (optional but explicit if used).** If one asserts a commuting/approximate-commuting diagram between object-space RG and fracture-state evolution, one must declare the precise approximation notion (e.g. "up to a nonsingular linear change of coordinates" in a specified neighborhood).

6. **Linearization regime and normalization freedoms.** Declare: (i) the reference configuration at which linearization is taken (typically a critical fixed point or its induced fracture-state reference), and (ii) the allowed coordinate normalizations in fracture space (e.g. $v \mapsto Sv$ for nonsingular S) under which reported invariants are considered equivalent. Normalization-dependent parameters must be labeled as such (module parameters, not invariants).

Remark E.1 (Scope and non-dependence). This appendix supplies an optional semantic bridge. The core framework requires only the definitions of Ψ and $\Delta_{I\to G}$ under a declared package and the usage protocol for reproducible reporting. Readers may omit this appendix without affecting any core statement.

E.2 Semantic bridge to function-space RG (optional)

E.2.1 Object-space RG orbit and induced fracture sequence

Fix an object-space RG operator $\mathcal{R}_{\text{map}}$ and an initial object f_0 in a declared instance family. Define the RG orbit

$$f_{k+1} = \mathcal{R}_{\text{map}}(f_k), \qquad k = 0, 1, 2, \ldots$$

and, under the declared package V, define the induced scalar fracture sequence

$$\Delta_k := \Delta_{I\to G}(f_k; V) \ \geq 0.$$

This scalar sequence is package-relative; any change in discrepancy rule, admissible predictor class, or resolution threshold changes the operational meaning of Δ_k.

E.2.2 Two-component fracture state and meta component

Fix a declared propagation rule F (Subsection E.1). Define the meta deviation

$$r_k := \Delta_{k+1} - F(\Delta_k).$$

Define coordinates $x_k := \Delta_k$ and $y_k := r_k$ and package the state as

$$v_k := \begin{pmatrix} x_k \\ y_k \end{pmatrix} \in \mathbb{R}^2, \qquad \text{or equivalently} \ \ z_k := x_k + i\,y_k = \Delta_k + i\,r_k \in \mathbb{C}.$$

This is an auxiliary representation: it is not a new core quantity. Its purpose is to separate (i) the within-object residual (Δ_k) from (ii) the cross-step mismatch relative to a declared single-rule propagation attempt (r_k).

E.2.3 Intertwining hypothesis (optional)

In a declared neighborhood of the critical configuration (where $\mathcal{R}_{\text{map}}$ is linearizable along its relevant directions), one may adopt the following optional working hypothesis:

Hypothesis E.1 (Bridge/intertwining (module-level))**.** There exists a local map Φ from object space to fracture-state space such that, in the declared neighborhood,

$$\Phi\big(\mathcal{R}_{\text{map}}(f)\big) \;\equiv_{\text{N}}\; \mathcal{R}_{\text{meta}}\big(\Phi(f)\big),$$

where $\mathcal{R}_{\text{meta}}$ acts on fracture states, and "$\equiv_{\text{N}}$" denotes equivalence up to a declared normalization in fracture space (e.g. conjugacy by a nonsingular linear map).

Remark E.2 (What this does and does not assert)*.* Hypothesis E.1 is not required elsewhere. It is used only to justify reading the *dominant* scaling behavior near criticality in fracture space as a projection of the dominant scaling behavior of $\mathcal{R}_{\text{map}}$.

E.3 Constructive example: minimal linear realization (moved from main text)

This subsection provides a minimal linear realization showing how a spiral-type scaling law can arise in a two-component fracture state. The construction is offered as a realizability template, not as a universality claim.

E.3.1 Lift to a 2D shift state

Define a lifted shift state

$$w_k := \begin{pmatrix} \Delta_k \\ \Delta_{k+1} \end{pmatrix}.$$

The induced shift map is $w_{k+1} = (\Delta_{k+1}, \Delta_{k+2})^\top$. In a declared linearization regime, model this by

$$w_{k+1} \;\approx\; J_{\text{shift}}\, w_k, \qquad J_{\text{shift}} = \begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix},$$

with real parameters (a, b) estimated/declared for the instance family under V.

E.3.2 Coordinate change to (x, y)

Fix a linear propagation rule $F(x) = c\,x$ and define

$$r_k := \Delta_{k+1} - c\,\Delta_k.$$

Define $x_k := \Delta_k$ and $y_k := r_k$, and write $v_k = (x_k, y_k)^\top$. Then $v_k = Bw_k$ where

$$B = \begin{pmatrix} 1 & 0 \\ -c & 1 \end{pmatrix}, \qquad B^{-1} = \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}.$$

Conjugating yields a linear meta operator

$$v_{k+1} \approx J_{\text{meta}}\, v_k, \qquad J_{\text{meta}} := BJ_{\text{shift}}B^{-1}.$$

E.3.3 Spiral-type behavior from a complex eigenpair

If J_{meta} has a dominant complex eigenvalue $\lambda = \rho e^{i\varphi}$ (with $\rho > 0$, $\varphi \in (-\pi, \pi]$), then in the complex coordinate $z_k = x_k + i\, y_k$ (equivalently $z_k = \Delta_k + i\, r_k$) one obtains the first-order law

$$z_{k+1} \approx \lambda z_k, \qquad |z_{k+1}| \approx \rho\, |z_k|, \qquad \arg z_{k+1} \approx \arg z_k + \varphi.$$

Thus the state trajectory is a logarithmic-spiral sampling in the (x, y) plane.

Remark E.3 (Normalization dependence). Changing the fracture-state normalization $v \mapsto Sv$ (nonsingular S) conjugates J_{meta} to $SJ_{\mathrm{meta}}S^{-1}$ and preserves eigenvalues. Therefore (ρ, φ) are invariant under such conjugacies, while intermediate parameters such as (a, b, c) are model and normalization dependent.

E.4 Reading δ_{Feig} and α_{Feig} as (ρ, φ)-type invariants

E.4.1 Radial scaling readout

Let δ_{Feig} denote the dominant real eigenvalue of the classical [10, 11, 6, 19] $\mathcal{R}_{\mathrm{map}}$ linearization along its relevant direction (in the declared universality class and normalization used in the RG literature). Under the optional bridge hypothesis (Hypothesis E.1), the module reads the dominant fracture-state radial scaling as a projection of the object-space scaling. Concretely, the radial factor ρ in fracture space is parameterized as

$$\rho = \delta_{\mathrm{Feig}}^{-\beta}, \qquad \beta > 0.$$

Remark E.4 (Status of β). β is *not* asserted to be universal. It is a module parameter absorbing projection and normalization choices (including the fracture-state definition, the rule F, and the scaling of $\Delta_{I\to G}(\cdot; V)$). If a numerical value is reported, it must be reported as part of the module declaration.

E.4.2 Phase behavior readout and the role of sign changes

Classical accounts associate α_{Feig} with spatial rescaling of the critical orbit [10, 11, 6, 19] in state space; under standard normalizations $\alpha_{\mathrm{Feig}} < 0$ introduces an orientation reversal per renormalization step. In the fracture-state evolution $z_{k+1} \approx \rho e^{i\varphi} z_k$, this is represented minimally as a phase-flip component:

$$\varphi \equiv \pi + \varphi_{\mathrm{bias}} \pmod{2\pi},$$

where the π term encodes orientation reversal, and φ_{bias} is any additional twist beyond a pure flip.

E.4.3 Normalization-dependent twist parameterization (optional)

The bias term φ_{bias} depends on how state-space rescaling is transported through the bridge Φ and on how (x, y) are normalized. A convenient recording convention is to parameterize it as

$$\varphi_{\mathrm{bias}} \equiv \kappa_\alpha \log |\alpha_{\mathrm{Feig}}| \pmod{2\pi},$$

with κ_α treated as a declared module parameter.

Protocol E.2 (Minimal reporting for a Feigenbaum-interface claim)**.** If this module is used to claim a mapping "$(\delta_{\mathrm{Feig}}, \alpha_{\mathrm{Feig}})$ correspond to (ρ, φ)", then the report must include:

1. the declared package V defining Δ_k;
2. the definition of r_k and the propagation rule F;

3. the bridge/intertwining hypothesis used (or an explicit statement that none is assumed);
4. the normalization convention in fracture space (what transformations are considered equivalent);
5. the parameters (β, κ_α) explicitly labeled as normalization-dependent if used.

Remark E.5 (No collision with the core resolution symbol ε). In this appendix, ε continues to denote the *resolution threshold* from the operational protocol. The Feigenbaum constant is written as δ_{Feig} for clarity and to avoid confusion with the resolution parameter ε.

F Axis-specific calculus templates (Optional, non-comparability module)

F.1 Scope and reporting rule (calculus-only)

This appendix records three *calculus-only* instantiations of an axis-accumulation template:

$$\Xi_{\mathrm{calc}}(\gamma) = \Xi_{\mathrm{calc}}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\mathrm{calc}}(s)\, ds, \qquad \Psi_{\mathrm{calc}}(\gamma) \approx \frac{d}{d\gamma}\Xi_{\mathrm{calc}}(\gamma), \tag{19}$$

under an explicitly declared evolution axis γ (schedule / strength / hierarchy cutoff).

Non-comparability. $(\Psi_{\mathrm{calc}}, \Xi_{\mathrm{calc}})$ here are axis-specific readouts used to validate portability of the γ-calculus across domains. They are *not* asserted to instantiate protocol-level comparability (Sec. 8), hence cross-case numerical comparison is not supported without an additional declared transport package and admissibility diagnostics.

Numerical-approximation reading. In this appendix, "$\approx$" denotes discretization/estimation error on the declared grid (and any declared sampling procedure). It is not the report-level convention "$\approx_\varepsilon 0$" used for package-relative gap reporting in the main text.

Discrete residual to report. For a grid $\gamma_0 < \gamma_1 < \cdots < \gamma_K$, report

$$e_k := \Big| \Xi_{\mathrm{calc}}(\gamma_{k+1}) - \Xi_{\mathrm{calc}}(\gamma_k) - \Psi_{\mathrm{calc}}(\gamma_k)\,(\gamma_{k+1} - \gamma_k) \Big|. \tag{20}$$

F.2 Spin glasses: annealing-axis calculus with two readout channels

Let $G = (W, E)$ be a finite graph with Ising spins and couplings [9, 31]. Define the violated-edge density $\bar{e}(S) := \#\{(i,j) \in E : J_{ij}\sigma_i\sigma_j < 0\}/|E|$.

Axis. Take $\gamma = \beta$ (inverse temperature) on a grid $\beta_0 < \cdots < \beta_K$ [18].

Rate readout (equilibrium channel). At each β_k, sample an (approximate) Gibbs equilibrium μ_{β_k} and set [23, 14]

$$\Psi_{\mathrm{calc}}(\beta_k) := \mathbb{E}_{S \sim \mu_{\beta_k}}\big[\bar{e}(S)\big]. \tag{21}$$

Accumulation readout (trajectory channel). Run an actual annealing trajectory through $\beta_0 \to \cdots \to \beta_K$ with declared holding times. Let $\bar{\bar{e}}_k$ be the time-average of $\bar{e}(S_t)$ over the realized dynamics on interval k. Define

$$\Xi_{\mathrm{calc}}(\beta_{k+1}) := \Xi_{\mathrm{calc}}(\beta_k) + \bar{\bar{e}}_k\,(\beta_{k+1} - \beta_k), \qquad \Xi_{\mathrm{calc}}(\beta_0) := 0. \tag{22}$$

Residual. e_k (Eq. 20) summarizes discretization, finite sampling, and non-equilibrium lag. No protocol-level diagnostic is claimed.

F.3 JPEG: compression-strength axis and marginal distortion

For an 8×8 block x and its reconstruction $\hat{x}_{\theta(\gamma)}(x)$, let $d(x, \hat{x})$ be block MSE [1, 35, 30, 7].

Axis. γ is a monotone quantization-strength parameter on a grid $\gamma_0 < \cdots < \gamma_K$.

Accumulation/scale readout. For a declared block distribution $\mathcal{D}$ and codec family $\theta(\gamma)$,

$$\Xi_{\mathrm{calc}}(\gamma_k) := \mathbb{E}_{x \sim \mathcal{D}}\Big[d\big(x, \hat{x}_{\theta(\gamma_k)}(x)\big)\Big]. \tag{23}$$

Rate readout. Use finite differences (or a declared local perturbation estimator):

$$\Psi_{\mathrm{calc}}(\gamma_k) := \frac{\Xi_{\mathrm{calc}}(\gamma_{k+1}) - \Xi_{\mathrm{calc}}(\gamma_k)}{\gamma_{k+1} - \gamma_k}. \tag{24}$$

Then e_k is directly computable.

F.4 Homology/cohomology: hierarchy cutoff axis (discrete bookkeeping instantiation)

Let b_k be Betti numbers over declared coefficients [15].

Axis. $\gamma = K \in \{0, 1, \ldots, K_{\max}\}$ is a dimension cutoff.

Accumulation readout. Fix declared weights $w_k \geq 0$ and set

$$\Xi_{\mathrm{calc}}(K) := \sum_{k=1}^{K} w_k\, b_k, \qquad \Xi_{\mathrm{calc}}(0) := 0. \tag{25}$$

Rate readout (discrete increment).

$$\Psi_{\mathrm{calc}}(K) := \Xi_{\mathrm{calc}}(K+1) - \Xi_{\mathrm{calc}}(K) = w_{K+1}\, b_{K+1}. \tag{26}$$

With these declarations, the discrete analogue of (19) holds exactly and $r_K \equiv 0$ (up to arithmetic), serving as a minimal hierarchy-axis realization.

F.5 Minimal summary (orientation only)

Across the three cases, the only shared commitment is the declared axis γ and the paired readouts $(\Psi_{\mathrm{calc}}, \Xi_{\mathrm{calc}})$ with a reproducible residual e_k. No protocol-level comparability is asserted in this appendix.

G Audit Semantics and H→ H^+ Transitions (Optional Module)

Module summary.
Scope. Audit-level closure for a locked viewpoint; characterize when non-closure forces an $H \to H^+$ extension (missing audit information carried on fibers).
Dependencies. A locked audit viewpoint package $V^{\text{aud}} = (\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$ and a requirement that audited queries be single-valued at the declared audit resolution.
Outputs. Validity predicate $\mathrm{Val}(V^{\text{aud}})$, H-closure vs. non-closure regimes, and an $H \to H^+$ workflow via an extended state space with (π, ι).
Non-invocation rule. If no single-valued audit requirement is invoked, this module is not called; the slim-core diagnostic remains the relevant output.

Scope and reporting convention. This appendix records an audit-oriented refinement layered over the slim core. The purpose is to formalize when a *locked viewpoint* supports *single-valued* decidability of a declared audited query set, and when that single-valuedness is not satisfied in a way that forces a *structurally necessary* extension of the description. Throughout, the phrase "reportable as 0" is used only as a *resolution-aware reporting convention*: at the locked finite audit resolution ε (as declared by V^{aud}), the relevant residual lies below the threshold and may be *reported as* 0. This is never an ontological identity. The resolution-aware convention "reportable as 0" is consistent with the projection-and-fiber intuition in Section 5.3.

Independence from the Ψ-based H/M label. The audit notion in this appendix concerns *single-valued decidability under a locked audit viewpoint.* It is a separate layer from the Ψ-based H/M classification: a system may be H-type or M-type in the Ψ-sense, while audit closure may hold or not hold depending on $(\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$.

G.1 Executive summary and trigger conditions

This module is invoked only when the statement being made requires *single-valued decidability* of a declared audited query set.

- **Locked viewpoint.** One locks a package $V^{\text{aud}} = (\mathsf{End}, \varepsilon, \mathcal{Q}, \mathcal{C})$, consisting of an endpoint operator End acting on histories, an audit resolution ε, a declared audited query set $\mathcal{Q}$, and an admissible class $\mathcal{C}$ of histories.
- **Trigger.** The trigger is the requirement that every $q \in \mathcal{Q}$ be decidable as a *single-valued* function of the endpoint semantics at the locked resolution. This is encoded by a validity predicate $\mathrm{Val}(V^{\text{aud}})$.
- **H-closure vs. non-closure.** If $\mathrm{Val}(V^{\text{aud}}) = 1$, audited queries are constant on endpoint indistinguishability classes at resolution ε, so closure holds under the locked viewpoint. If $\mathrm{Val}(V^{\text{aud}}) = 0$, at least one audited query varies *within* an endpoint indistinguishability class; this is the non-closure regime.
- **Structural conclusion.** In the non-closure regime, single-valued audit decidability cannot be restored without adding a structural carrier: an $H \to H^+$ extension introduces an extended state space $\mathcal{X}^+$ together with a projection π and an embedding ι, so that the missing audit information is carried on fibers $\pi^{-1}(x_H)$. At the locked resolution, cross-level structure is then operationally *effective* and cannot be reported as resolution-negligible.
- **Canonical example.** A finite Markov coarse-graining provides a concrete instance: a coarse endpoint label supports an H-level evolution, yet one-step predictive queries vary within fibers, forcing an $H \to H^+$ extension if single-valued audit decidability is demanded.

G.2 Audit skeleton and endpoint semantics

Definition G.1 (Audit skeleton (locked viewpoint package))**.** A locked audit viewpoint package is a quadruple

$$V^{\text{aud}} := (\mathsf{End},\ \varepsilon,\ \mathcal{Q},\ \mathcal{C}),$$

where $\mathsf{End} : \mathcal{H} \to \mathcal{E}$ is an endpoint/operator acting on a history space $\mathcal{H}$, $\varepsilon > 0$ is the locked audit resolution, $\mathcal{Q}$ is a declared set of audited queries defined on $\mathcal{H}$, and $\mathcal{C} \subseteq \mathcal{H}$ is the declared admissible class of audited histories.

Remark G.1 (Relation to the declared package). In applications, End is typically induced from the interface/observation layer together with an explicit aggregation rule. The audit skeleton is used only when single-valued audit decidability of $\mathcal{Q}$ is itself part of the requirement.

Definition G.2 (Endpoint indistinguishability at resolution ε)**.** Fix a declared discrepancy $d_{\mathcal{E}}$ on the endpoint space $\mathcal{E}$. For $h_1, h_2 \in \mathcal{C}$, define endpoint indistinguishability at resolution ε by

$$h_1 \sim_\varepsilon h_2 \quad \iff \quad d_{\mathcal{E}}\big(\mathsf{End}(h_1), \mathsf{End}(h_2)\big) \leq \varepsilon.$$

G.3 Validity, H-closure, and the non-closure regime

Definition G.3 (Validity as single-valued audit decidability)**.** Define $\mathrm{Val}(V^{\text{aud}}) \in \{0, 1\}$ by

$$\mathrm{Val}(V^{\text{aud}}) = 1 \quad \iff \quad \forall q \in \mathcal{Q},\ \forall h_1, h_2 \in \mathcal{C} :\ \big(h_1 \sim_\varepsilon h_2\ \Rightarrow\ q(h_1) = q(h_2)\big).$$

Equivalently, $\mathrm{Val}(V^{\text{aud}}) = 1$ iff every audited query is constant on endpoint indistinguishability classes at the locked resolution.

Definition G.4 (H-closure (intra-level audit closure))**.** H-closure under V^{aud} means that there exists an intra-level representation space $\mathcal{X}_H$ and an evolution semigroup $\{U_\gamma\}_{\gamma \geq 0}$ on $\mathcal{X}_H$ such that:

1. **Intra-level evolution.** For every $\gamma \geq 0$, $U_\gamma : \mathcal{X}_H \to \mathcal{X}_H$.
2. **Audit decidability without extra carriers.** Under the locked viewpoint, each audited query in $\mathcal{Q}$ can be decided as a single-valued function of the current $\mathcal{X}_H$-state at resolution ε, without introducing additional hidden variables or explicit memory beyond what is encoded in $\mathcal{X}_H$.
3. **Resolution-aware reporting.** At the locked finite resolution ε, the cross-level residual may be *reported as* 0 whenever it lies below the declared reporting threshold; this is the meaning of being reportable as 0 in this appendix.

Remark G.2 (One-way evolution is not, by itself, lifting). Non-invertibility or one-way character of $\{U_\gamma\}_{\gamma \geq 0}$ is an intra-level property. It does not, by itself, force an $H \to H^+$ transition. The transition is triggered by audit-level violation of single-valuedness.

Definition G.5 (Non-closure regime (fiber non-constancy))**.** The non-closure regime under V^{aud} holds when $\mathrm{Val}(V^{\text{aud}}) = 0$. Equivalently, there exist $q \in \mathcal{Q}$ and $h_1, h_2 \in \mathcal{C}$ such that

$$h_1 \sim_\varepsilon h_2 \quad \text{but} \quad q(h_1) \neq q(h_2).$$

In this regime, at least one audited query varies within an endpoint indistinguishability class at resolution ε.

G.4 Markov chains: coarse-graining as fiber non-constancy (optional)

Purpose. This worked example exhibits a canonical mechanism: a coarse endpoint representation supports an H-level evolution, yet one-step predictive queries need not be single-valued on endpoint fibers under a locked audit resolution.

G.4.1 Locked audit viewpoint for a finite Markov chain

Micro system. Let X be a finite state space and $K(x, \cdot)$ a Markov transition kernel on X [25]. A one-step history is $h = (x_0, x_1) \in \mathcal{H} := X \times X$.

Coarse endpoint map. Fix a declared coarse map $\phi : X \to Y$, where Y is finite, and define

$$\mathsf{End}(h) := \phi(x_0) \in \mathcal{E} := Y.$$

Lock the discrete endpoint discrepancy $d_{\mathcal{E}}(y, y') := \mathbf{1}\{y \neq y'\}$ and take any $\varepsilon < 1$, so that $h_1 \sim_\varepsilon h_2$ reduces to equality of macro labels $\phi(x_0^{(1)}) = \phi(x_0^{(2)})$.

Admissible class. Declare $\mathcal{C} \subseteq \mathcal{H}$ (e.g. all reachable one-step pairs under a declared boundary/admissibility rule).

Audited query set. For any event $A \subseteq Y$, declare the one-step macro predictive query

$$q_A(h) := \mathbb{P}\big(\phi(X_1) \in A \mid X_0 = x_0\big) = \sum_{x \in X:\ \phi(x) \in A} K(x_0, x).$$

Let $\mathcal{Q} := \{q_A :\ A \subseteq Y\}$ (or any declared subfamily).

G.4.2 Non-closure mechanism: within-fiber predictive variation

Define the induced macro one-step predictive distribution from a micro state x:

$$\bar{K}_x(y') := \sum_{x' \in X:\ \phi(x') = y'} K(x, x'), \qquad y' \in Y,$$

so $q_A(h) = \bar{K}_{x_0}(A)$.

Proposition G.1 (Non-closure from within-fiber predictive variation)**.** *Assume $\varepsilon < 1$ with the discrete endpoint discrepancy. If there exist $x, x' \in X$ with $\phi(x) = \phi(x')$ and an event $A \subseteq Y$ such that $\bar{K}_x(A) \neq \bar{K}_{x'}(A)$, and if corresponding one-step histories based at x and x' lie in $\mathcal{C}$, then* $\mathrm{Val}(V^{\mathrm{aud}}) = 0$.

Proof. Choose histories $h_1 = (x, x_1)$, $h_2 = (x', x_1') \in \mathcal{C}$. Since $\phi(x) = \phi(x')$, we have $h_1 \sim_\varepsilon h_2$. But $q_A(h_1) = \bar{K}_x(A) \neq \bar{K}_{x'}(A) = q_A(h_2)$. Hence an audited query is not constant within an endpoint indistinguishability class, so $\mathrm{Val}(V^{\mathrm{aud}}) = 0$. □

Reading. The endpoint representation $\mathsf{End}(h) = \phi(x_0)$ can support a consistent H-level evolution on Y, but it does not, in general, support single-valued audit decidability of one-step predictive queries at the locked resolution: the ambiguity is localized on endpoint fibers.

G.4.3 Structurally necessary extension (H$\to H^+$)

When $\mathrm{Val}(V^{\mathrm{aud}}) = 0$ and single-valued audit decidability is demanded, the missing within-fiber information must be carried explicitly. A concrete admissible extension is:

$$\mathcal{X}_H := Y, \qquad \mathcal{X}^+ := \{(y, \eta) : \ y \in Y, \ \eta \in \mathcal{P}(X), \ \mathrm{supp}(\eta) \subseteq \phi^{-1}(y)\}.$$

Define

$$\pi(y, \eta) := y, \qquad \iota(y) := (y, \eta_y^0),$$

where η_y^0 is a declared representative distribution supported on the fiber $\phi^{-1}(y)$ (e.g. uniform on the fiber, or a declared conditional prior).

In $\mathcal{X}^+$, the audited query becomes single-valued by construction:

$$q_A(y, \eta) := \sum_{x \in X} \eta(x) \sum_{x' \in X: \ \phi(x') \in A} K(x, x') = \sum_{x \in X} \eta(x) \, \bar{K}_x(A).$$

The extension refines points projecting to the same macro label y by carrying a minimal structural complement η that resolves within-fiber predictive variation.

G.4.4 Optional alignment with a slim-core interface

This Markov instantiation may be aligned with a slim-core interface by taking $\Phi_G(x) = \phi(x)$ and $\Phi_I(x) = \bar{K}_x \in \mathcal{P}(Y)$, choosing a declared discrepancy D on $\mathcal{P}(Y)$, and selecting an admissible lifting family $U \subseteq \{u : Y \to \mathcal{P}(Y)\}$. If within-fiber predictive variation is not negligible under the declared discrepancy at the locked resolution, then the cross-level residual is not reportable as resolution-negligible at that resolution once single-valued audit decidability is enforced.

G.5 From non-closure to a structurally necessary extension (H$\to H^+$)

Lemma G.1 (One-way evolution does not force an extension)**.** *If audited queries remain single-valued under the locked viewpoint, then evolution may be realizable within $\mathcal{X}_H$ while maintaining audit closure for $\mathcal{Q}$. Consequently, one-way (semigroup) evolution alone does not entail an $H \to H^+$ transition. At a fixed finite audit resolution, a cross-level residual may remain resolution-negligible and may be reported as* 0 *under the declared convention.*

Lemma G.2 (Non-closure forces a structurally necessary extension)**.** *Assume* $\mathrm{Val}(V^{\mathrm{aud}}) = 0$ *under the locked viewpoint and that single-valued audit decidability of $\mathcal{Q}$ is required. Then there exist:*

- *an extended state space $\mathcal{X}^+$,*
- *a surjective forgetful map $\pi : \mathcal{X}^+ \to \mathcal{X}_H$,*
- *and an embedding $\iota : \mathcal{X}_H \hookrightarrow \mathcal{X}^+$ with $\pi \circ \iota = \mathrm{id}$,*

such that audited queries become single-valued decidable as functions of the $\mathcal{X}^+$-state at the locked resolution. Moreover, no viewpoint-preserving description confined to $\mathcal{X}_H$ can restore single-valued audit decidability under the locked viewpoint. In this extended regime, cross-level structure is operationally effective *at the locked resolution and is not reportable as resolution-negligible.*

Remark G.3 (Structural vs. redundant extensions)*.* Not every enlargement of the representation space constitutes an $H \to H^+$ transition. A redundant enlargement is decoupled from the locked audit requirement and does not affect single-valued audit decidability. A structural extension is triggered precisely by within-fiber variation of audited queries and is introduced for the sole purpose of restoring single-valued audit decidability over $\mathcal{Q}$. No product form $\mathcal{X}^+ \cong \mathcal{X}_H \times \mathcal{Y}$ is assumed unless explicitly justified.

Remark G.4 (Discrete character of lifting). A structurally necessary lifting need not occur continuously in an evolution parameter; it may occur as a discrete transition once the locked viewpoint demands single-valued audit decidability over the audited query set.

G.6 Optional: effective domain of a $\Delta_{\rm acc}$–$\Psi_{\rm run}$ relation

This subsection is optional and is included only if a dynamical interface relating the accumulated readout $\Delta_{\rm acc}$ to a running driver $\Psi_{\rm run}$ is declared in the main text.

Corollary G.1 (Effective domain of a $\Delta_{\rm acc}$–$\Psi_{\rm run}$ dynamical relation)**.** *Once cross-level structure is operationally effective under the locked viewpoint (i.e. in an $H \to H^+$ regime), the declared $\Delta_{\rm acc}$–$\Psi_{\rm run}$ dynamical interface (when present) enters its effective domain and may be reported as*

$$\Delta_{\rm acc}(\gamma) = \Delta_{\rm acc}(\gamma_0) + \int_{\gamma_0}^{\gamma} \Psi_{\rm run}(\gamma')\, d\gamma'.$$

Within intra-level audit closure at the locked resolution, the same formal relationif writtencarries no auditable incremental content at that resolution under the declared reporting convention.

G.7 Audit summary

- **Closure is an audit property.** Audit closure is defined by single-valued decisability of $\mathcal{Q}$ on endpoint indistinguishability classes at the locked resolution.
- **One-way evolution is not the trigger.** Non-invertibility or one-way evolution does not by itself force an $H \to H^+$ extension.
- **The trigger is within-fiber variation.** If audited queries vary within endpoint fibers under the locked viewpoint, single-valued audit decidability does not hold and the non-closure regime holds.
- **$H \to H^+$ is a structural carrier move.** Restoring single-valued audit decidability requires an extension that carries the missing audit information on fibers. In that regime, cross-level structure is operationally effective at the locked resolution and is not reportable as resolution-negligible.

G.8 Higher-order compensation principle (H$\to$ H^+ as structural complement)

Principle. When a locked audit viewpoint demands single-valued audit decidability over $\mathcal{Q}$, any non-single-valuedness that is invisible in the intra-level representation $\mathcal{X}_H$ is not removed. Instead, it is *encoded* in a structurally necessary extension $\mathcal{X}^+$ whose additional degrees of freedom resolve the ambiguity. The extension introduces no new core quantities; it only adds the minimal structural carrier required by the locked audit requirement.

Fibers as the locus of missing audit information. Given an extension $(\mathcal{X}^+, \pi, \iota)$, the projection π identifies the intra-level component of an extended state. For any $x_H \in \mathcal{X}_H$, the fiber

$$\mathcal{F}(x_H) := \pi^{-1}(x_H) \subseteq \mathcal{X}^+$$

collects all extended states that project to the same intra-level state. Operationally, non-closure indicates that at least one audited query cannot be decided as a single-valued function of x_H alone at the locked resolution. The compensating coordinates distinguishing points within $\mathcal{F}(x_H)$ supply exactly the missing audit information.

Proposition G.2 (Structural complement interpretation)**.** *Assume non-closure under a locked audit viewpoint and suppose an $H \to H^+$ extension restores single-valued audit decidability over $\mathcal{Q}$. Then:*

1. ***Non-reducibility to $\mathcal{X}_H$.*** *At least one audited query cannot be represented as a single-valued function of x_H alone on the declared admissible class at the locked resolution under any viewpoint-preserving decision rule.*

2. ***Complement carried by fibers.*** *The missing audit information is encoded in the fiber structure: the extension refines the representation by distinguishing states that project to the same x_H but differ in the extended coordinates.*

3. ***Consistency on the embedded subspace.*** *Where intra-level decisions are already single-valued under the locked viewpoint, extended decision rules can be chosen to agree with the intra-level ones on the embedded subspace $\iota(\mathcal{X}_H)$.*

Remark G.5 (Minimality is operational, not geometric)*.* Minimality here is operational: $\mathcal{X}^+$ introduces no extra structure beyond what is required to restore single-valued audit decidability at the locked resolution. The map π is the declared forgetting map; ι identifies the intra-level component retained when compensation is not invoked.

H Package-Relative Alignment of Edges A–C and Resolution-Aware Stability

Module summary.
Scope. Support Chapter 9 by aligning the A–C edge conditions with package clauses and by recording diagnostic stubs for undefinedness/non-licensing in stability claims.
Dependencies. A declared package V for the labeling pipeline and (for cross-package claims) explicit transport/comparability and resolution conversion clauses.
Outputs. Checklist-style alignment, diagnostic stubs, and resolution-aware reporting conventions for stability claims.
Non-invocation rule. If stability claims are not made, this appendix is not invoked.

H.1 Scope and type discipline

This appendix provides a self-contained alignment of the "three-edge" sufficient template (Edges A–C) with the paper's package-relative semantics and resolution-aware reporting discipline. All statements here are *package-relative*: they are to be interpreted only after a declaration package V is fixed, including its admissible domain restrictions and reporting threshold ε.

We distinguish:

- **System objects:** a state space Ω, a reachable set $\Omega_{\text{reach}} \subseteq \Omega$, and (optionally) a task-relevant subset $\Omega_{\text{rel reach}} \subseteq \Omega_{\text{reach}}$.

- **Package objects (declared by V):** a micro-difference D_V on Ω_{reach} (metric or pseudo-metric as declared), a reporting threshold $\varepsilon > 0$, a representation map $R_V : \Omega_{\text{reach}} \to Y_V$, a decision rule $M_V : Y_V \to \mathcal{S}$ producing discrete macro labels, and an optional pre-processing/coarse-graining operator $C_V : \Omega_{\text{reach}} \to \Omega_{\text{reach}}$. Additional objects (e.g. a macro-difference D_V^{macro} on Y_V, a declared feasible subset $K_V \subseteq Y_V$) are included when used.

The induced (package-relative) discrete macro label assignment is

$$m_V(S) := (M_V \circ R_V)(S), \qquad S \in \Omega_{\text{reach}}. \tag{27}$$

No cross-package equality claims are permitted without explicitly declared comparability (transport) data; see §H.7.

H.2 Resolution-aware micro-robust stability

Definition H.1 (Micro-robust stability at resolution ε)**.** Fix a declaration package V with micro-difference D_V and reporting threshold ε. The induced label map $m_V : \Omega_{\text{reach}} \to M$ is *micro-robust (at resolution ε)* if there exists $\varepsilon > 0$ such that for all $S, S' \in \Omega_{\text{reach}}$,

$$D_V(S, S') < \varepsilon \quad \Longrightarrow \quad m_V(S) = m_V(S'). \tag{28}$$

Remark H.1 (Reporting-level status)*.* Definition H.1 is a reporting-level property relative to (V, ε). It does not assert any gap-free identity across levels; it specifies when the *declared* macro-label pipeline is stable under micro-perturbations measured by the *declared* D_V, at the *declared* reporting threshold ε.

H.3 Edges A–C as a sufficient template

Edges A–C are stated on the task-relevant domain $\Omega_{\text{rel reach}}$ (when declared). If $\Omega_{\text{rel reach}}$ is not declared, all domain-qualified claims below are *undefined* under the reporting protocol (§H.5).

Edge A: contraction prior to label extraction

Definition H.2 (Edge A (pre-label contraction))**.** Assume V declares an operator $C_V : \Omega_{\text{reach}} \to \Omega_{\text{reach}}$. Edge A holds on $\Omega_{\text{rel reach}}$ if there exist constants $\kappa \in [0, 1)$ and $\rho_A > 0$ such that for all $S, S' \in \Omega_{\text{rel reach}}$,

$$D_V\big(C_V(S), C_V(S')\big) \;\le\; \kappa\, D_V(S, S'). \tag{29}$$

Edge B: feasible-set narrowing and boundary non-reachability

Definition H.3 (Edge B (feasible representation region and no boundary crossing))**.** Assume V declares $R_V : \Omega_{\text{reach}} \to Y_V$ and $M_V : Y_V \to \mathcal{S}$. Edge B holds on $\Omega_{\text{rel reach}}$ if there exist a declared subset $K_V \subseteq Y_V$ and $\rho_B > 0$ such that:

1. **Concentration:** for all $S \in \Omega_{\text{rel reach}}$, $R_V(S) \in K_V$.
2. **No boundary crossing (within K_V):** for all $S \in \Omega_{\text{rel reach}}$ and all $S' \in \Omega_{\text{reach}}$, if $D_V(S, S') < \rho_B$, then $M_V(R_V(S)) = M_V(R_V(S'))$ when both representations are considered in the declared region K_V.

Edge C: label separation margin above the reporting threshold

Definition H.4 (Edge C (macro separation margin $\eta > \varepsilon$))**.** Assume V declares a macro-difference D_V^{macro} on Y_V and a feasible representation region $K_V \subseteq Y_V$. For each label $m \in \mathcal{S}$, define the induced region

$$R_m \;:=\; \{\, y \in K_V :\; M_V(y) = m \,\}. \tag{30}$$

Edge C holds if there exists $\eta > \varepsilon$ such that for all distinct labels $m \neq m'$,

$$\inf \big\{\, D_V^{\text{macro}}(y, y') :\; y \in R_m,\; y' \in R_{m'} \,\big\} \;\ge\; \eta. \tag{31}$$

Remark H.2 (Why $\eta > \varepsilon$)*.* Edge C is explicitly resolution-aware: it requires that distinct labels be separated by a margin strictly exceeding the declared reporting threshold ε, ensuring that label distinctions are supported at the declared resolution under the declared macro-difference.

H.4 Closure theorem: A–C implies micro-robust stability

Proposition H.1 (Edges A–C imply micro-robust stability (package-relative))**.** *Fix a declaration package V and reporting threshold ε. Suppose Edges A–C (Definitions H.2–H.4) hold on $\Omega_{\text{rel reach}}$ with the declared objects $(D_V, \varepsilon, R_V, M_V)$ (and, where used, $C_V, K_V, D_V^{\text{macro}}$). Then the induced label map m_V is micro-robust at resolution ε in the sense of Definition H.1; i.e. there exists $\varepsilon > 0$ such that* (28) *holds.*

Remark H.3 (Template status)*.* Proposition H.1 is a *sufficient* template. It does not claim that A–C are necessary for stability, nor does it prescribe a unique choice of package objects. The conclusion is valid only for the declared package V and its declared ε.

H.5 Protocol alignment: diagnostic bundles and undefined outputs

Stability reporting is subject to the paper's general discipline: statements are reportable only when the required objects are declared and the relevant domain conditions are met. Otherwise, the output must be reported as `undefined` with a structured diagnostic indicating the missing declarations or domain violations.

Minimal diagnostic bundle

When a stability claim "m_V is micro-robust at resolution ε" is reported via Proposition H.1, the report includes an *A/B/C diagnostic bundle*:

- **A-diagnostic (if Edge A is invoked):** the declared operator C_V, the domain $\Omega_{\text{rel reach}}$, and verified constants (κ, ρ_A) supporting (29).
- **B-diagnostic:** the declared representation R_V, the declared feasible region $K_V \subseteq Y_V$, the constant ρ_B, and a verifiable argument that boundary crossing does not occur as required in Definition H.3.
- **C-diagnostic:** the declared macro-difference D_V^{macro} on Y_V, a margin $\eta > \varepsilon$, and verifiable support for (31).

Undefined triggers

A stability statement is `undefined` under (V, ε) if any of the following holds:

- Required package objects are not declared (e.g. D_V, ε, R_V, M_V, or, when used, C_V, K_V, D_V^{macro}).
- The task-relevant domain $\Omega_{\text{rel reach}}$ is referenced but not declared (or is not a subset of Ω_{reach} as required).
- The intended claim is cross-package but no explicit transport/comparability data are declared (see §H.7).

In each case, the report outputs `undefined` together with an *undefined-diagnostic* listing missing objects, missing domains, or missing comparability clauses.

H.6 Operational contrapositive: flip events localize diagnostic non-admissibility

Lemma H.1 (Flip events force A/B/C non-admissibility or undefined status (reporting-level))**.** *Fix (V, ε). Suppose there exist $S, S' \in \Omega_{\text{rel reach}}$ and $\varepsilon > 0$ such that*

$$D_V(S, S') < \varepsilon \quad \textit{but} \quad m_V(S) \neq m_V(S'). \tag{32}$$

Then it is not possible to simultaneously provide valid A/B/C diagnostics for the same declared parameters on the same declared domain. Equivalently, under the protocol, at least one of the A/B/C diagnostic components is not satisfied (or the statement is undefined *due to missing declarations or domain mismatch).*

Remark H.4 (Interpretation boundary). Lemma H.1 is a reporting-level incompatibility statement relative to (V, ε). It does not license any gap-free interpretation across levels. It identifies which part(s) of the A/B/C template cannot be supported under the observed flip event.

H.7 Cross-package stability and transport comparability

All claims comparing stability across distinct packages V and V' require explicitly declared comparability data, including (at minimum) a transport map $\tau_{V \to V'}$ (or an equivalent alignment rule), its declared domain $\mathrm{Dom}(\tau_{V \to V'})$, and the declared criteria under which objects (metrics, representations, labels, thresholds) are comparable. Without such declarations, cross-package comparisons (including "m_V is more stable than $m_{V'}$" or "the same stable phenomenon appears under V and V'") are undefined and must be reported with an undefined-diagnostic.

H.8 One-line audit checklist

For a reportable A–C based stability claim, the minimal declared inputs typically include:

$$\big(\Omega_{\mathrm{rel\,reach}},\, D_V,\, \varepsilon,\, R_V,\, M_V\big) \quad + \quad \big(C_V \text{ optional}\big) \quad + \quad \big(K_V,\, D_V^{\mathrm{macro}}\big) \text{ when invoked,} \tag{33}$$

together with the corresponding A/B/C diagnostic bundle or an explicit undefined output with an undefined-diagnostic.

References

[1] Digital compression and coding of continuous-tone still images: Requirements and guidelines. Recommendation T.81, ITU-T, 1992. JPEG standard (baseline reference).

[2] Shun-ichi Amari and Hiroshi Nagaoka. *Methods of Information Geometry*, volume 191 of *Translations of Mathematical Monographs*. American Mathematical Society and Oxford University Press, 2000.

[3] George D. Birkhoff. Proof of the ergodic theorem. *Proceedings of the National Academy of Sciences of the United States of America*, 17(12):656–660, 1931. `doi:10.1073/pnas.17.2.656`.

[4] Alonzo Church. An unsolvable problem of elementary number theory. *American Journal of Mathematics*, 58(2):345–363, 1936. `doi:10.2307/2371045`.

[5] Stephen A. Cook. The complexity of theorem-proving procedures. In *Proceedings of the Third Annual ACM Symposium on Theory of Computing (STOC '71)*, pages 151–158. Association for Computing Machinery, 1971.

[6] Pierre Coullet and Charles Tresser. Itérations d'endomorphismes et groupe de renormalisation. *Comptes Rendus de l'Académie des Sciences, Série A*, 287:577–580, 1978.

[7] Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory*. Wiley-Interscience, 2 edition, 2006.

[8] R. M. Dudley. *Real Analysis and Probability*, volume 74 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 2002.

[9] S. F. Edwards and P. W. Anderson. Theory of spin glasses. *Journal of Physics F: Metal Physics*, 5(5):965–974, 1975. `doi:10.1088/0305-4608/5/5/017`.

[10] Mitchell J. Feigenbaum. Quantitative universality for a class of nonlinear transformations. *Journal of Statistical Physics*, 19(1):25–52, 1978. `doi:10.1007/BF01020332`.

[11] Mitchell J. Feigenbaum. The universal metric properties of nonlinear transformations. *Journal of Statistical Physics*, 21(6):669–706, 1979. `doi:10.1007/BF01107909`.

[12] Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, 1979.

[13] Kurt Gödel. Über formal unentscheidbare sätze der principia mathematica und verwandter systeme I. *Monatshefte für Mathematik und Physik*, 38(1):173–198, 1931. `doi:10.1007/BF01700692`.

[14] W. K. Hastings. Monte carlo sampling methods using markov chains and their applications. *Biometrika*, 57(1):97–109, 1970. `doi:10.1093/biomet/57.1.97`.

[15] Allen Hatcher. *Algebraic Topology*. Cambridge University Press, 2002.

[16] Leo P. Kadanoff. Scaling laws for ising models near T_c. *Physics*, 2:263–272, 1966.

[17] Richard M. Karp. Reducibility among combinatorial problems. In Raymond E. Miller and James W. Thatcher, editors, *Complexity of Computer Computations*, pages 85–103. Plenum Press, 1972.

[18] Scott Kirkpatrick, C. Daniel Gelatt, and Mario P. Vecchi. Optimization by simulated annealing. *Science*, 220(4598):671–680, 1983. `doi:10.1126/science.220.4598.671`.

[19] Oscar E. Lanford. A computer-assisted proof of the feigenbaum conjectures. *Bulletin of the American Mathematical Society*, 6(3):427–434, 1982. doi:10.1090/S0273-0979-1982-15008-X.

[20] Tien-Yien Li and James A. Yorke. Period three implies chaos. *The American Mathematical Monthly*, 82(10):985–992, 1975. doi:10.2307/2318254.

[21] Edward N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20(2):130–141, 1963. doi:10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2.

[22] Robert M. May. Simple mathematical models with very complicated dynamics. *Nature*, 261:459–467, 1976. doi:10.1038/261459a0.

[23] Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller. Equation of state calculations by fast computing machines. *The Journal of Chemical Physics*, 21(6):1087–1092, 1953. doi:10.1063/1.1699114.

[24] Alfred Müller. Integral probability metrics and their generating classes of functions. *Advances in Applied Probability*, 29(2):429–443, 1997. doi:10.2307/1428011.

[25] J. R. Norris. *Markov Chains*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 1997.

[26] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley, 1994.

[27] Joseph Polchinski. Renormalization and effective lagrangians. *Nuclear Physics B*, 231(2):269–295, 1984. doi:10.1016/0550-3213(84)90287-6.

[28] David Ruelle. A measure associated with Axiom-A attractors. *American Journal of Mathematics*, 98(3):619–654, 1976. doi:10.2307/2373810.

[29] Claude E. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27(3):379–423, 1948. doi:10.1002/j.1538-7305.1948.tb01338.x.

[30] Claude E. Shannon. Coding theorems for a discrete source with a fidelity criterion. In *IRE National Convention Record*, volume 4, pages 142–163, 1959.

[31] David Sherrington and Scott Kirkpatrick. Solvable model of a spin-glass. *Physical Review Letters*, 35(26):1792–1796, 1975. doi:10.1103/PhysRevLett.35.1792.

[32] Alan M. Turing. On computable numbers, with an application to the entscheidungsproblem. *Proceedings of the London Mathematical Society*, 42(1):230–265, 1936. doi:10.1112/plms/s2-42.1.230.

[33] Alan M. Turing. On computable numbers, with an application to the entscheidungsproblem. a correction. *Proceedings of the London Mathematical Society*, 43(1):544–546, 1937. doi:10.1112/plms/s2-43.6.544.

[34] Cédric Villani. *Optimal Transport: Old and New*, volume 338 of *Grundlehren der mathematischen Wissenschaften*. Springer, 2009. doi:10.1007/978-3-540-71050-9.

[35] Gregory K. Wallace. The JPEG still picture compression standard. *Communications of the ACM*, 34(4):30–44, 1991. doi:10.1145/103085.103089.

[36] Steven Weinberg. Phenomenological lagrangians. *Physica A*, 96(1-2):327–340, 1979. doi:10.1016/0378-4371(79)90223-1.

[37] Kenneth G. Wilson. Renormalization group and critical phenomena. I. renormalization group and the kadanoff scaling picture. *Physical Review B*, 4(9):3174–3183, 1971. `doi:10.1103/PhysRevB.4.3174`.

[38] Kenneth G. Wilson and John Kogut. The renormalization group and the ϵ-expansion. *Physics Reports*, 12(2):75–199, 1974. `doi:10.1016/0370-1573(74)90023-4`.

[39] Lai-Sang Young. What are SRB measures, and which dynamical systems have them? *Journal of Statistical Physics*, 108(5-6):733–754, 2002. `doi:10.1023/A:1019762724717`.

[40] Zihe Zhao. Relative-entropy endpoint–path gaps as a diagnostic of rg coarse-graining failure. Manuscript, January 2026. `doi:10.5281/zenodo.18414030`.