Topology of group actions, Morse theory and symplectic invariants

joint work with Laurent Côté

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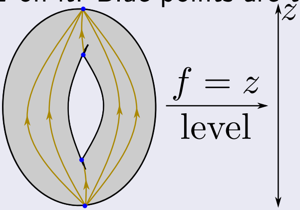
Example

The surface of revolution of $x^2 + (z-1)^2 = 4$ around x-axis (torus), and the Morse function $f = z$ on it. Blue points are the critical points.



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- central focus of modern symplectic topology: shapes the field
- connects symplectic topology with other fields including algebraic geometry, algebraic topology (string topology), non-commutative geometry, low dimensional topology, ...

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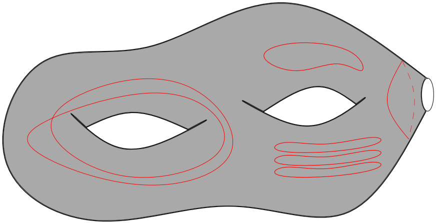

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$SH(M;\mathbb{Z})$ is a generalization of quantum cohomology to non-compact manifolds.

Warning: The "Morse function" on $\mathcal{L}M$ depends on the symplectic geometry of M, and $SH(M;\mathbb{Z})$ is a symplectic invariant. It depends more than the topology of $\mathcal{L}M$.

Floer theory meets homotopy theory

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Large '21: Defined a "space" $SH(M;\mathbb{S})$, which is a symplectic invariant.

Lie group actions?

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Problem: The setup of Morse/Floer theory is not compatible with group actions.

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- We make a major upgrade to a well-known theorem (Viterbo isomorphism).
- We give applications connecting Floer theory to the equivariant homotopy theory- a deep subfield of homotopy theory.

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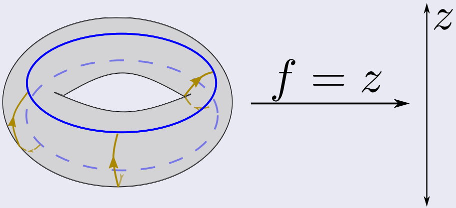
Example

Let N be the torus obtained by revolving $(x-1)^2 + z^2 = 4$ around z-axis. Then, $f = z$ is Morse–Bott and equivariant with respect to rotation. Critical set consists of two blue circles.



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- One can now use Morse–Bott theory to study equivariant cohomology.
- The proof involves construction of a "space" associated to such an abstract object and new tools to induce maps on this "space".
- **Most importantly:** the construction works for "infinite dimensional Morse theory", aka Floer theory.

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- Viterbo isomorphism connects symplectic topology to the topology of loop spaces (to a field called string topology).

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- Connections to mirror symmetry (e.g. conjectures about the complex K-theory of mirror pairs).

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We are working verification of these. This will further bridge symplectic topology to representation theory.

Thank you!