MATHEMATICS

1. NUMBER SYSTEM

1. Non-negative counting numbers excluding zero are called Natural Numbers.
   N = {1, 2, 3, 4, 5, ...}
2. All natural numbers including zero are called Whole Numbers.
   W = {0, 1, 2, 3, 4, 5, ...}
3. All natural numbers, negative numbers and 0, together are called Integers.
   Z = {.......– 3, – 2, – 1, 0, 1, 2, 3, 4, ...}
4. The number 'a' is called Rational if it can be written in the form of $\frac{r}{s}$ where 'r' and 's' are integers and $s \neq 0$.
5. Rational number between two given numbers 'a' and 'b' is given by $\frac{a+b}{2}$
6. The number 'a' which cannot be written in the form of p/q is called irrational, where p and q are integers and $q \neq 0$ or you can say that the numbers which are not rational are called Irrational numbers.
7. The decimal expansion of a number is either terminating or non-terminating recurring.
8. A number whose decimal expansion is terminating or non-terminating recurring is rational.
9. A number whose decimal expansion is non-terminating non-recurring is irrational
10. Both rational and irrational numbers together in a set are called Real Numbers.
11. There is a unique real number corresponding to every point on the number line.

    also, corresponding to each real number, there is a unique point on the number line.
12. The sum, difference, product and quotient of two rational numbers will be rational
13. If we add or subtract a rational number with an irrational number then the outcome will be irrational.
14. If we multiply or divide a non-zero rational number with an irrational number then also the outcome will be irrational.
15. For a positive real number a and b

    i) $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ iii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = a-b$

    iv) $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = a^2-b$ v) $\left(\sqrt{a}+\sqrt{b}\right)^2 = a+2\sqrt{ab}+b$

16. To rationalise the denominator of $\frac{1}{\sqrt{a}+b}$, we multiply this by $\frac{\sqrt{a}-b}{\sqrt{a}-b}$ where a and b are integers.

Laws of Exponents for Real Numbers :

17. $a^m \times a^n = a^{m+n}$

18. $\frac{a^m}{a^n} = a^{m-n}, m > n$

19. $\left(a^m\right)^n = a^{mn}$

20. $a^m \times b^m = (ab)^m$

21. $a^0 = 1, a \neq 0$

22. $\frac{1}{a^n} = a^{-n}, a \neq 0$

23. $\frac{a^m}{a^n} = 1, m = n$

24. $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, m < n$

25. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

26. $\sqrt[n]{a} = a^{1/n}$

27. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

2. POLYNOMIALS

1. Polynomial is an algebraic expression which includes constants, variables and exponents. It is the expression in which the variables have only non-negative integral powers.
2. A polynomial p(x) in one variable x is an algebraic expression in x of the form ,
   $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n$ where $a_0, a_1, a_2, ...a_n$ are real numbers and n is a non-negative integer.
3. The degree of the polynomial is the highest power of the variable in a polynomial
4. A polynomial of one term is called a monomial.
5. A polynomial of two terms is called a binomial.
6. A polynomial of three terms is called a trinomial.
7. A polynomial of degree one is called a linear polynomial.
8. A polynomial of degree two is called a quadratic polynomial.
9. A polynomial of degree three is called a cubic polynomial.
10. A real number “a” is a zero of a polynomial p(x), if p(a) = 0. In this case “a” is also called a root of the equation p(x) = 0.

11. The zero of the polynomial is basically the x-intercept of the polynomial.

12. Every linear polynomial in one variable has a unique zero.

13. A non-zero constant polyniomial has no zero.

14. Remainder Theorem : Dividend = (Divisor × Quotient) + Remainder

15. If p(x) and g(x) are two polynomials in which the degree of $p(x) \geq$ degree of g(x) and $g(x) \neq 0$ are given then we can get the q(x) and r(x) so that:

    $p(x) = g(x).q(x) + r(x)$, where r(x) = 0 or degree of r(x) < degree of g(x).

16. If p(x) is any polynomial of degree greater than or equal to one and let 'a' be any real number and p (x) is divided by the linear polynomial x – a, then the remainder is p(a).

17. Factor Theorem :If p(x) is a polynomial with degree ≥ 1 and a is a real number, then

    i. (x - a) is a factor of p(x), if p(a) = 0, and

    ii. p(a) = 0 if (x – a) is a factor of p(x).

18. $(a+b)^2 = a^2 + 2ab + b^2$

19. $(a-b)^2 = a^2 - 2ab + b^2$

20. $(a+b)(a-b) = a^2 - b^2$

21. $(x+a)(x+b) = x^2 + (a+b)x + ab$

22. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

23. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

24. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

25. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

26. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$

27. If $a+b+c=0$ then $a^3 + b^3 + c^3 = 3abc$

3. CO-ORDINATE GEOMETRY

1. If we take two number lines, one horizontal and one vertical, and then combine them in such a way that they intersect each other at their zeroes, and then they form a Cartesian Plane.
2. The horizontal line is known as the X-axis and the vertical line is known as the Y-axis.
3. The point where these two lines intersects each other is called the origin. It is represented as 'O'.
4. OX and OY are the positive directions as the positive numbers lie in the right and upward direction.
5. The Cartesian plane is divided into four quadrants named as Quadrant I, II, III, and IV in an anticlockwise direction from OX.
6. The x - coordinate of a point is marked by drawing perpendicular from the Y-axis measured a length of the x-axis .It is also called as the Abscissa.
7. The y - coordinate of a point is marked by drawing a perpendicular from the x-axis measured a length of the Y-axis .It is also called as the Ordinate.
8. The coordinates of the origin is (0, 0).
9. **The relationship between the signs of the coordinates of a point and the quadrant of a point in which it lies.**

Quadrant	Coordinate	Sign in the quadrant
I	(+, +)	1st quadrant is enclosed by the positive x-axis and the positive y-axis.
II	(-, +)	2nd quadrant is enclosed by the negative x-axis and the positive y-axis.
III	(-, -)	3rd quadrant is enclosed by the negative x-axis and the negative y-axis.
IV	(+, -)	4th quadrant is enclosed by the positive x-axis and the negative y-axis

10. If $x \neq y,\ then\ (x, y) \neq (y, x)$, and (x, y) = (y, x), if x = y

4 INTRODUCTION TO EUCLID'S GEOMETRY

1. Though Euclid defined a point, a line, and a plane, the definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined.
2. Axioms or postulates are the assumptions which are obvious universal truths. They are not proved.

3. Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.
4. Some of Euclid's axioms were :
   1. Two things which are equal to the same thing are equal to one another.
   2. If equals are added to equals, the wholes are equal.
   3. If equals are subtracted from equals, the remainders are equal.
   4. Things which coincide with one another are equal to one another.
   5. The whole is greater than the part.
   6. Things which double of the same things are equal to one another.
   7. Things which are halves of the same things are equal to one another.
5. The assumpitions which are very specific in geometry are called postulates.
6. Euclid's postulate were :

   Postulate 1 : A straight line may be drawn from any one point to any other point.

   Postulate 2 : A terminated line can be produced indefinitely.

   Postulate 3 : A circle can be drawn with any centre and any radius.

   Postulate 4 : All right angles are equal to one another.

   Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.
7. Two equivalent versions of Euclid's fifth postulate are :

   i)'For every line *l* and for every point P not lying on *l*, there exists a unique line *m* passing through P and parallel to *l*'.

   ii) Two distinct intersecting lines cannot be parallel to the same line.

5. LINES AND ANGLES

1. A part (or portion) of a line with two end points is called a line-segment.
2. The ray is a part of line that has one end point and goes on infinitely in only one direction.
3. If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points.
4. An acute angle measures between 0^0 and 90^0.
5. A right angle is exactly equal to 90^0.
6. An angle greater than 90^0 but less than 180^0 is called an obtuse angle.
7. A straight angle is equal to 180^0.

8. An angle which is greater than 180^0 but less than 360^0 is called a reflex angle.
9. Two angles whose sum is 90^0 are called complementary angles.
10. Two angles whose sum is 180^0 are called supplementary angles.
11. Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm.
12. If a ray stands on a line, then the sum of two adjacent angles so formed is 180^0, then such a type of angles are called linear pair of angles.
13. If the sum of two adjacent angles is 180^0, then the non-common arms of the angles form a line.
14. If two lines intersect each other, then the vertically opposite angles are equal.
15. A line which intersects two or more lines at distinct points is called a transversal.
16. Interior angles on the same side of the transversal are also referred to as consecutive interior angles or allied angles or co-interior angles.
17. If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
18. If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.
19. If a transversal intersects two parallel lines, then each pair of alternative interior angles is equal.
20. If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.
21. Lines which are parallel to the same line are parallel to each other.
22. The sum of the angles of a triangle is 180^0.
23. If the side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.
24. An exterior angle of a triangle is greater than either of its interior opposite angles.

6. TRIANGLES

1. Congruent figures ('congruent' means equal in all respects or figures whose shapes and sizes are both the same.
2. Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.
3. (SAS congruence rule): Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.
4. Angles opposite to equal sides of an isosceles triangle are equal.

5. The sides opposite to equal angles of a triangle are equal.
6. (SSS congruence rule): If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
7. (ASA Congruence rule): If two angles and included side of one triangle are equal to two angles and included side of other triangle, then two triangles are congruent.
8. (RHS congruence rule): If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruence.
9. If two sides of a triangle are equal, the angle opposite to the longer side is larger (or greater).
10. The sum of any two sides of a triangle is greater than the third side.
11. In a triangle the angle opposite to largest side is bigger angle, the angle opposite to smaller side is smallest.
12. The difference of two sides of a triangle is less then third side.
13. Each angle of an equilateral triangle is 60^0.

7. LINEAR EQUATION IN TWO VARIABLES

1. The equation of a straight line is the linear equation. It could be in one variable or two variables.
2. The equation with one variable in it is known as a Linear Equation in One Variable. its general form is $ax + b = 0$, where a, b and c are real numbers and $a \neq 0$.
3. An equation with two variables is known as a Linear Equation in Two Variables. The general form of the linear equation in two variables is ax + by + c = 0 where a and b are coefficients and c is the constant and $a \neq 0$, $b \neq 0$.
4. A linear equation in two variables has infinitely many solutions.
5. Equation of y-axis is $x = 0$ and the equation of x-axis is $y = 0$
6. The graph of x = a is a line parallel to y axis and at a distance of 'a' units from origin.
7. The graph at y = a is a line parallel to x - axis and at a distance of 'a' units from origin.
8. An equation of the type $y = mx$ represents a line passing through the origin.
9. The graph of every linear equation in two variables is a straight line.