NARAYANA IIT ACADEMY INDIA

40+ YEARS OF EXCELLENCE

Sec: SR.IIT_*CO-SC(MODEL-B)	PTM-1	Date: 18-09-22
Time: 3HRS	JEE-MAIN	Max. Marks: 300

KEY SHEET

PHYSICS

1	**A**	2	**B**	3	**A**	4	**B**	5	**D**
6	**B**	7	**C**	8	**C**	9	**D**	10	**D**
11	**A**	12	**C**	13	**A**	14	**A**	15	**C**
16	**B**	17	**A**	18	**A**	19	**B**	20	**C**
21	**0**	22	**1**	23	**4**	24	**1**	25	**45**
26	**150**	27	**6**	28	**8**	29	**3**	30	**3**

CHEMISTRY

31	**A**	32	**C**	33	**B**	34	**C**	35	**C**
36	**C**	37	**D**	38	**D**	39	**D**	40	**D**
41	**D**	42	**D**	43	**A**	44	**D**	45	**B**
46	**C**	47	**C**	48	**D**	49	**D**	50	**D**
51	**5**	52	**4**	53	**9**	54	**6**	55	**6**
56	**4**	57	**11**	58	**4**	59	**4**	60	**5**

MATHEMATICS

61	**B**	62	**C**	63	**D**	64	**C**	65	**D**
66	**C**	67	**C**	68	**C**	69	**A**	70	**B**
71	**D**	72	**C**	73	**A**	74	**C**	75	**B**
76	**B**	77	**A**	78	**C**	79	**B**	80	**A**
81	**73**	82	**4**	83	**4**	84	**2**	85	**2**
86	**3**	87	**10**	88	**11**	89	**2013**	90	**5**

SOLUTIONS
PHYSICS

1. Relative velocity x time taken = relative displacement
2. The maximum angle of deviation of a projectile is 2‾ which is $\leq 90^0$ in this case.
3. Decrease of horizontal separation is equal to the horizontal component of relative velocity of 1 wrt 2.
4. $t = \frac{width}{v_b \cos‾}$ and $v_b \sin‾ = v_r$
5. The velocity of the projectile wrt the ground is v in the vertically upward direction. T=2v/g
6. Dot product of velocity and acceleration is zero.
7. Component of acceleration along the velocity is tangential acceleration while the component perpendicular to the velocity is normal acceleration.
8. Velocity increases linearly with time t and then in the opposite direction it decreases.
9. $t = \frac{u_y}{g} + \frac{\sqrt{u_y^2 + 2gH}}{g}$

   $h = H + \frac{u_y^2}{2g}$
10. $a = \frac{v^2 - u^2}{2s}$, $F = ma + mg$
11. a is upwards. $N = mg + ma$
12. Pseudo force doesn't appear as part of action reaction pair.
13. $kx = Ma$, $F - kx = Ma_b$
14. $a = \frac{v^2 - u^2}{2s}$, $S = 1cm$, $F = mg + ma$
15. Horizontal component = $g \sin‾ . \cos‾$
16. Just after string is cut tension unit in it becomes zero, because of maselessness
    Elongation of spring remains same just after because of inertia of A.
17. $a = \frac{m_1 N m_2}{m_1 + m_2} g$
18. Write equilibrium equations
19. $\vec{F}.\hat{i} =$ Component
20. $\sum \vec{F}.\vec{a} = 0$
21. $y = u_y t - \frac{1}{2} g t^2$
22. $t_1 = -\frac{u}{g} + \frac{\sqrt{u^2 + 2gh}}{g}$, $t_2 = \sqrt{\frac{2H}{g}}$
23. $\frac{u^2 \sin 2‾}{g} = \frac{u^2 \sin^2 ‾}{2g}$
24. During collision the vertical velocity component isn't effected.
    $\therefore$ Time of flight is same as the time in free flight.
25. Tension in string = mg
26. $\vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0}$ because of equilibrium
27. $m_2 g - T = M_2 2a$ ……..(1)

$2T - M_1 g \sin^{-} = M_1 a$ ....(2)

28. Tension in the string connecting m_1 is

$$T = \frac{4m_2 m_3}{m_2 + m_3} g = (m_1 g)$$

29. The bob may be accelerating up or down

30 The monkey may be accelerating up or down.

CHEISTRY

MATHS

61. Since $f(2+t) = f(2-t)$

$\Rightarrow$ function is symmetric about the line $x = 2$.

Also, $x^2 + bx + c = 0$ is symmetric about $x = \frac{-b}{2}$.

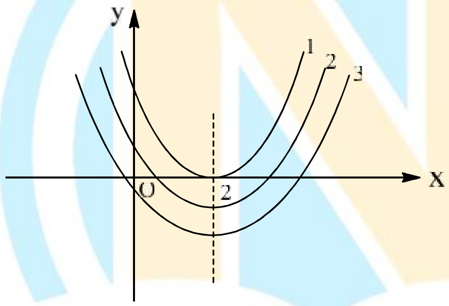
$\therefore \quad \frac{-b}{2} = 2 \Rightarrow b = -4$

$\therefore \quad f(x) = x^2 - 4x + c$

Now, 3 graphs are possible.

In (1) and (2) ‘c’ is positive and in (3) ‘c’ is negative.

$f(0) = c$



Let c is positive.

Now, $f(1) = c - 3$

$f(2) = c - 4$

$f(4) = c$

Say $c = 3$

Then $f(1) = 0;\ f(2) = -1;\ f(3) = 3 \Rightarrow f(2) < f(1) < f(3)$

Again c is negative. Let $c = -3$

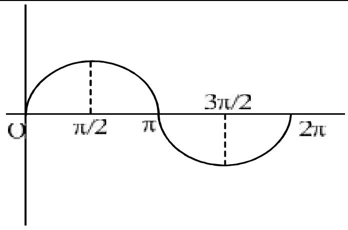
$f(1) = -6;\ f(2) = -7;\ f(4) = -3$

$\therefore \quad f(2) < f(1) < f(4) \Rightarrow (2)$

Also, if $c = 0$ the statement ‘2’ is true.

62. $f(x) = \sin x; 0 \le x \le \frac{®}{2}$

$= 1; \frac{®}{2} \le x \le 2®$



$$g(x) = 0; 0 \le x \le \circledR$$
$$= \sin x;\ \circledR < x < \frac{3\circledR}{2}$$
$$= -1; \frac{3\circledR}{2} \le x \le 2\circledR$$
$$h(x) = 0; 0 \le x < \frac{\circledR}{2}$$
$$= 1; \frac{\circledR}{2} \le x < \frac{3\circledR}{2}$$
$$= 2; x \ge \frac{3\circledR}{2}.$$

Hence, the range of $h(x)$ is $\{0,1,2\}$.

63. $$f(x) = \frac{\sqrt{2}.\sin\left(x + \frac{\circledR}{4}\right) + 2\sqrt{2}}{\text{From -1 to 1}}$$

$\therefore Y =$ Range of function $= \left[\sqrt{2}, 3\sqrt{2}\right]$. Clearly, f will be one–one also, if

$$X = \left[\frac{\circledR}{4}, \frac{5\circledR}{4}\right].$$

64. Let $y = \dfrac{x^2 + x + c}{x^2 + 2x + c}$

$\Rightarrow \quad (y-1)x^2 + (2y-1)x + c(y-1) = 0$

As x is real, so $D \ge 0$.

$\Rightarrow \quad (2y-1)^2 \ge 4c(y-1)^2$

$\Rightarrow \quad 4(c-1)y^2 + 4(1-2c)y + (4c-1) \le 0 \qquad$ ....(i)

But we are given

$$(6y-5)(2y-3) \le 0 \Rightarrow 12y^2 - 28y + 15 \le 0 \qquad \text{.......(ii)}$$

$\therefore$ On comparing Eqs. (i) and (ii), we get $\dfrac{c-1}{3} = \dfrac{1-2c}{-7} = \dfrac{4c-1}{15}$

$\Rightarrow \qquad c = 4$.

65. $$\left||x+2|-3\right| = \operatorname{sgn}\left(1 - \left|\frac{(x-2)(x+6)(x+4)}{(x^2+1)(x+4)(x+6)(x-2)}\right|\right)$$

$$\left||x+2|-3\right| = \operatorname{sgn}\left(1 - \left|\frac{1}{x^2+1}\right|\right) \qquad x \ne 2, -4, -6$$

$$\left||x+2|-3\right| = 1 \Rightarrow |x+2| - 3 = \pm 1$$

$\Rightarrow \quad |x+2| = 4, 2$

$\Rightarrow \quad x + 2 = \pm 4, \pm 2$

$\Rightarrow \quad x = 2, -4, 0, -6$

66. The desired number is ${}^8C_5 = 56$.

Here, for non–decreasing functions from A to B, is

${}^8C_1.1 + {}^8C_2.4 + {}^8C_3.6 + {}^8C_4.4 + {}^8C_5.1 = 792$

Explanation for case ${}^8C_2$, say two elements of set B are selected in ${}^8C_2$ is $\{-1, 0\}$.

Now, $x_1 + x_2 = 5$ $[x_1 \geq 1, x_2 \geq 1]$

Where, x_1 denotes number of elements of set A maps to –1 and x_2 denotes number of elements of set A maps to 0.

$\therefore$ Total number of solutions is ${}^4C_1$.

Similarly, explanation for case ${}^8C_3$ say three elements selected in ${}^8C_3$ is $\{-1, 0, 1\}$.

Now, $x_1 + x_2 + x_3 = 5$ $(x_1 \geq 1, x_2 \geq 1, x_3 \geq 1)$

$\therefore$ Total number of solutions is ${}^4C_2$ etc.

Onto functions from A to A such that $f(i) \neq i$ for all i.

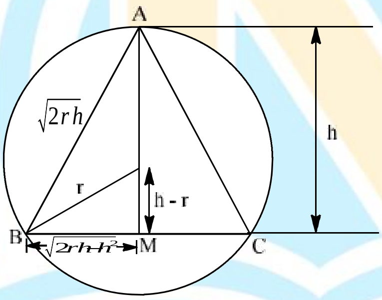
$$\Rightarrow \quad 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44.$$

67. $AB = \sqrt{h^2 + 2rh - h^2} = \sqrt{2rh}$

$$P = 2\sqrt{2rh} + 2\sqrt{2rh - h^2}$$

$$P = 2\left[\sqrt{2rh} + \sqrt{2rh - h^2}\right]$$

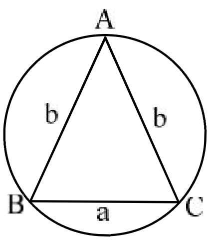
$$\Delta = \frac{2\sqrt{2rh - h^2}.h}{2} = h\sqrt{2rh - h^2}$$



$$\therefore \lim_{h\to 0} \frac{\Delta}{p^3} = \lim_{h\to 0} \frac{h\sqrt{2rh - h^2}}{8\left[\sqrt{2rh} - \sqrt{2rh - h^2}\right]^3} = \lim_{h\to 0} \frac{h^{3/2}\sqrt{2r - h}}{8h^{3/2}\left[\sqrt{2r} - \sqrt{2r - h}\right]^3}$$

$$= \frac{\sqrt{2r}}{8.8(2r)(2r)^{1/2}} = \frac{1}{128r} \Rightarrow (3)$$

Alternative : Note that as $h \to 0, b = \frac{a}{2}$ or $2b = a$



Hence, $\dfrac{\Delta}{p^3} = \dfrac{ab(b)}{4R.(a + 2b)^3}$ (Using $R = \dfrac{abc}{4\Delta}$)

$$=\frac{2b^3}{4R64b^3}=\frac{1}{128R}\Rightarrow(3).$$

68. $$p=\lim_{n\to\infty}\left(\frac{4}{4-\sqrt{3}+2\sin^{-}}\right)^n\left(\frac{1}{\left(4-\sqrt{3}+2\sin^{-}\right)^2}\right)$$

$$p=\lim_{n\to\infty}\left(\frac{2}{2+2\sin^{-}-\frac{\sqrt{3}}{2}}\right)^n\left(\frac{1}{\left(4-\sqrt{3}+2\sin^{-}\right)^2}\right)$$

$$If^{-}\in\left(0,\frac{\circledR}{3}\right)\Rightarrow\sin^{-}\in\left(0,\frac{\sqrt{3}}{2}\right)\Rightarrow p\ does\ not\ exist$$

$$If^{-}\in\left(\frac{\circledR}{3},\frac{\circledR}{3}\right)\Rightarrow\sin^{-}\in\left(\frac{\sqrt{3}}{2},1\right)\Rightarrow p=0$$

$\therefore$ For existence of limit $^{-}=\frac{\circledR}{3}\Rightarrow p=\frac{1}{4^2}=\frac{1}{16}$

Hence, $\frac{p+\cos^{-}}{p}=1+\frac{\cos^{-}}{p}=1+\frac{16}{2}=9$

69. $$f(x)=\frac{1}{|\sin x|}+\frac{1}{|\cos x|}$$

Using $AM\ge GM$, we get

$$\frac{\frac{1}{|\sin x|}+\frac{1}{|\cos x|}}{2}\ge\left(\frac{1}{|\sin x||\cos x|}\right)^{1/2}\Rightarrow\frac{1}{|\sin x|}+\frac{1}{|\cos x|}\ge 2\left(2|\operatorname{cosec}2x|\right)^{1/2}$$

[where $(\operatorname{cosec}2x)\ge 1$]

$$\therefore\quad\frac{1}{|\sin x|}+\frac{1}{|\cos x|}\ge 2\sqrt{2}$$

$\therefore$ Range of $f(x)\in\left[2\sqrt{2},\infty\right)$. Hence, (1) is the correct answer.

70. $$f(x)=\frac{x(\sin x+\tan x)}{\left[\frac{x+\circledR}{\circledR}\right]-\frac{1}{2}}=\frac{x(\sin x+\tan x)}{\left[\frac{x}{\circledR}\right]+1-\frac{1}{2}}=\frac{x(\sin x+\tan x)}{\left[\frac{x}{\circledR}\right]+0.5}$$

$$\Rightarrow f(-x)=\frac{-x(\sin(-x)+\tan(-x))}{\left[-\frac{x}{\circledR}\right]+0.5}$$

$$\Rightarrow f(-x)=\begin{cases}\dfrac{x(\sin x+\tan x)}{-1-\left[\frac{x}{\circledR}\right]+0.5}, & x\ne n\circledR\\ 0, & x=n\circledR\end{cases}$$

Hence, $f(-x)=-\left(\frac{x(\sin x+\tan x)}{\left[\frac{x}{\circledR}\right]+0.5}\right)$ and $f(-x)=0$

$f(-x) = -f(x)$

Hence, $f(x)$ is an odd function (if $x \neq n$®).

71. $x^2 + x + a = 0$ has a real solution

$\Rightarrow 1 - 4a \geq 0$

72. Here, $f(x)$ is onto

$\therefore \quad \frac{®}{6} \leq \sin^{-1}\left(\frac{x^2 - a}{x^2 + 1}\right) < \frac{®}{2}$

$\Rightarrow \quad \frac{1}{2} \leq \frac{x^2 - a}{x^2 + 1} < 1$

$\Rightarrow \quad \frac{1}{2} \leq 1 - \frac{(a+1)}{x^2 + 1} < 1, \forall x \in R$

$\Rightarrow \quad a + 1 > 0$

$\Rightarrow \quad a \in (-1, \infty)$. Hence (3) is the correct answer.

73. $$\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2} = \lim_{x\to 0} \frac{x\left[2x + \frac{2^3x^3}{3} + 2\frac{2^5x^5}{15}....\right] - 2x\left[x + \frac{x^3}{3} + 2\frac{x^5}{15}......\right]}{\left(2\sin^2 x\right)^2}$$

$$= \lim_{x\to 0} \frac{x^4\left(\frac{8}{3} - \frac{2}{3}\right) + x^6\left(\frac{64}{15} - \frac{4}{15}\right) + ...}{4\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....\right)^4}$$

$$= \lim_{x\to 0} \frac{2 + 4x^2 + ...}{4\left[1 - \frac{x^2}{3!} + ...\right]} = \frac{2}{4} = \frac{1}{2}.$$

74. We must have $x^4 - 21x^2 \geq 0$ and $10 - \sqrt{x^4 - 21x^2} \geq 0$

$\Rightarrow x^2\left(x^2 - 21\right) \geq 0 \qquad ----(1)$

and $100 \geq x^4 - 21x^2$ -------- (2)

(1) gives $x = 0$ *or* $x \leq -\sqrt{21}$ *or* $x \geq \sqrt{21}$

(2)$\Rightarrow x^4 - 21x^2 - 100 \leq 0$

$\Rightarrow \left(x^2 - 25\right)\left(x^2 + 4\right) \leq 0$

$\Rightarrow x^2 - 25 \leq 0$ (as $x^2 + 4 > 0$ always)

$\Rightarrow -5 \leq x \leq 5$

Domain is given by $\left[-5, -\sqrt{21}\right] \cup \left[\sqrt{21}, 5\right]$ and $x = 0$.

75. $$f(x) = \ell n\left(\frac{x^2 + e}{x^2 + 1}\right) = \ell n\left(\frac{x^2 + 1 - 1 + e}{x^2 + 1}\right) = \ell n\left(1 + \frac{e - 1}{x^2 + 1}\right)$$

Clearly range is (0, 1]

Hence (2) is correct answer.

76. Period of f(x) is 2® , but f(x) is not defined for x ∈ (® /2, 3® /2). Hence it suffices to consider x∈ [–® /2, ® /2].

Further since f(x) is even, we consider x ∈ [0, ® /2].

Now $\sqrt{\cos(\sin x)}$ and $\sqrt{\sin(\cos x)}$ are decreasing functions for x ∈ [® ,® /2].

$\Rightarrow R_f = [f(\pi/2), f(0)] = [\sqrt{\cos 1}, 1+\sqrt{\sin 1}]$

77. Given $f(x)=\log_e\{\text{sgn}(9-x^2)\}+\sqrt{[x]^3-4[x]}=y_1+y_2(say)$

Now, y_1 is defined if $\text{sgn}(9-x^2)>0$

But sgn x = 1(i.e. > 0) if x > 0

$\therefore \text{sgn}(9-x^2)>0 \Rightarrow 9-x^2>0 \Rightarrow x^2-9<0 \Rightarrow (x-3)(x+3)<0 \Rightarrow -3<x<3$ …(A)

Again, y_2 is defined if $[x]^3-4[x]\ge 0 \Rightarrow [x]\{[x]^2-4\}\ge 0 \Rightarrow [x]([x]-2)([x]+2)\ge 0$.

Following the wavy curve method, we find

Thus $[x]\ge 2$ *or* $[x]$ lies between – 2 and 0, i.e. [x] = - 2, - 1 or 0

Now, $[x]\ge 2 \Rightarrow x\ge 2$ …(B)

[x] = - 2 $\Rightarrow$ - 2 $\le$ x < 1

[x] = - 1 $\Rightarrow$ - 1 $\le$ x < 0

[x] = 0 $\Rightarrow$ 0 $\le$ x < 1.

Hence [x] = - 2, - 1, 0 $\Rightarrow -2\le x<1$

$\therefore (B)\cup(C)=(x\ge 2)$ *or* $(-2\le x<1)$ …(C)

Hence $D_f=(A)\cup(C)=[-2,1)\cup[2,3)$.

78. 3 does not belong to the range of *f* implies 2 also cannot belong to range of *f* because, if $f(x)=2$ for some $x\in R$. Then $f(x+p)=\frac{2-5}{2-3}=3$ which is not in the range of *f*. Hence 2 and 3 are not in the range of *f*. If $f(x+2p)=f(x)$, this implies

$$f(x)=f(x+p+p)$$
$$=\frac{f(x+p)-5}{f(x+p)-3}$$
$$=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}$$
$$=\frac{-4f(x)+10}{-2f(x)+4}=\frac{2f(x)-5}{f(x)-2}$$

So that $[f(x)-2]^2=-1$ which is absurd. Therefore, $2p$ is not a period. Again

$$f(x+3p)=\frac{2f(x+p)-5}{f(x+p)-2}$$
$$=\frac{3f(x)-5}{f(x)-1}\ne f(x).$$

Now $f(x+4p)=f(x+3p+p)$

$$=\frac{f(x+3p)-5}{f(x+3p)-3}$$

$$=\frac{\dfrac{3f(x)-5}{f(x)-1}-5}{\dfrac{3f(x)-5}{f(x)-1}-3}$$

$$=\frac{-2f(x)}{-2}=f(x).$$

Therefore $4p$ is a period.

79.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{f(2x)+f(2h)}{2}-f(x)}{h}$$

$f'(x)=-1$; $f(2x)=2f(x)\text{-}1$

$\Rightarrow f(x)=1-x$

80. $\cot^{-1}\left(r^2+\frac{3}{4}\right)=\tan^{-1}\left(\dfrac{1}{r^2+\frac{3}{4}}\right)$

$$=\tan^{-1}\left(\frac{1}{1+\left(r^2-\frac{1}{4}\right)}\right)$$

$$=\tan^{-1}\left(\frac{1}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$=\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+\left(r^2+\frac{1}{4}\right)}\right)$$

$$=\tan^{-1}\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)$$

81. $f(x)=\{x\}+\left\{x+\left[\dfrac{x}{1+x^2}\right]\right\}+\left\{x+\left[\dfrac{x}{1+2x^2}\right]\right\}+......+\left\{x+\left[\dfrac{x}{1+99x^2}\right]\right\}$

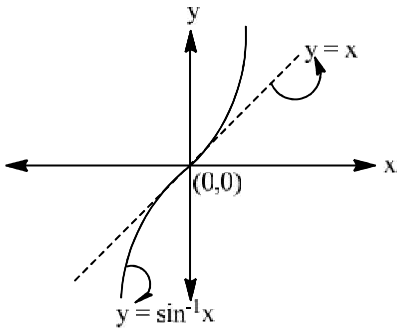
$$100\{x\}\ \left(\because \{x+m\}=\{x\},m\in I\right)$$

$$f\left(\sqrt{3}\right)=100(0.732)=73.2$$

$$\left[f\left(\sqrt{3}\right)\right]=73$$

82. In vicinity of $x=0,\left|\sin^{-1}x\right|>|x|$

$\Rightarrow \dfrac{\sin^{-1}x}{x}>1$, in vicinity of $x=0$.



$\therefore \quad l = 1^3 + 2^3 + 3^3 + ... + n^3$

$= \left(\frac{n(n+1)}{2}\right)^2 = 100 \quad \Rightarrow \quad n = 4$

83. Since, $f(x)$ is even so, $f(-x) = f(x)$

Thus, $x = \frac{x+1}{x+2}$ *or* $-x = \frac{x+1}{x+2}$

$\Rightarrow x^2 + 2x = x + 1$ *or* $-x^2 - 2x = x + 1$

$\Rightarrow x^2 + x - 1 = 0$ *or* $-x^2 - 3x - 1 = 0$

$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$ *or* $x = \frac{-3 \pm \sqrt{5}}{2}$

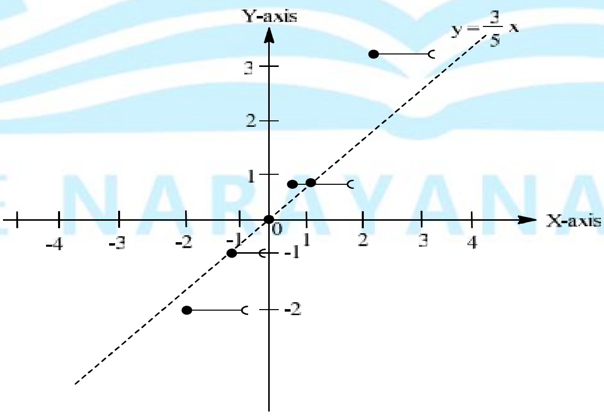
Thus, $x = \left\{\frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}\right\}$.

84. Here, $4\{x\} = x + [x]$

$\Rightarrow 4(x - [x]) = x + [x] \quad \Rightarrow \quad 4x = x + 5[x]$

$\Rightarrow 3x = 5[x] \quad \Rightarrow \quad [x] = \frac{3}{5}x$

To find their solution we plot the graph of both $y = [x]$ and $y = \frac{3}{5}x$. i.e.,



i.e., the two graphs intersects. When $[x] = 0$ and 1

$\Rightarrow \quad x = 0$ and $x = \frac{5}{3}$.

85. We know when $x \to 0$

$\Rightarrow \frac{x}{\tan x} < 1 \Rightarrow \frac{-x}{\tan x} > -1 \Rightarrow \frac{-2x}{\tan x} > -2$.

So, $\lim_{x\to 0}\left[\dfrac{-2x}{\tan x}\right]=-2$.

86. As we know, $\left(\dfrac{a+b+c+d}{4}\right)^2 \le \dfrac{a^2+b^2+c^2+d^2}{4}$ (using Tchebycheff's Inequality)

Where $a+b+c+d+e=8$

and $a^2+b^2+c^2+d^2+e^2=16$.

$\therefore$ Equation (i) reduces to,

$$\left(\frac{8-e}{4}\right)^2 \le \frac{16-e^2}{4}$$

$\Rightarrow$ $64+e^2-16e \le 4\left(16-e^2\right)$

$\Rightarrow$ $5e^2-16e \le 0$

$\Rightarrow$ $e\left(5e-16\right)\le 0$ (Using number line rule)

$\Rightarrow$ $0\le e\le \dfrac{16}{5}$

Thus range of $e\in\left[0,\dfrac{16}{5}\right]$.

87. Putting x = 1, y = 2, then

g(1) g(2) = g(1) + g(2) + g(2) – 2

$\Rightarrow$ 5g(1) = 8 + g(1)

$\therefore$ g(1) = 2

Also, replacing y by $\dfrac{1}{x}$ in the given relation, then

$$g(x)g\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)+g(1)-2$$

or $g(x)g\left(\dfrac{1}{x}\right)=g(x)+g\left(\dfrac{1}{x}\right)$

$\Rightarrow$ $g(x)=1\pm x^n$

$\Rightarrow$ $\pm 2^n=2^2$

Taking +ve sign

$2^n=2^2$

$\therefore$ n = 2

$\Rightarrow$ $g(x)=1+x^2$

$\therefore g(3)=1+3^2=10$.

88. At $x=11$

$3f(11)+2f(7)=140$.(1)

but $x=7$ to get

$3f(7)+2f(11)=100$ ..(2)

$\dfrac{(1)}{(2)} \Rightarrow \dfrac{3f(1)+2f(7)}{3f(7)+2f(11)}=\dfrac{7}{5}$

Using componendo and dividendo

$\dfrac{5(f(11)+f(7))}{(f(11)-f(7))}=\dfrac{6}{1} \Rightarrow \dfrac{(f(11)+f(7))}{(f(11)-f(7))}=\dfrac{6}{5}$

$$\frac{f(11)}{f(7)} = 11$$

89. Put $y = x \Rightarrow f\left(x + f\left(x\right)\right) = f\left(x\right) + x$

$$\Rightarrow f\left(t\right) = t \text{ (Identity function)}$$

90. $f(x) - x = (x-1)(x-2)(x-3)(x-\sim)$

$$f(-1) = 24(1+\sim)-1$$
$$f(0) = 6\sim$$
$$f(4) = 6(4-\sim)+4$$
$$f(5) = 24(5-\sim)+5$$

$$\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right] = \left[\frac{148}{28}\right] = 5$$