NARAYANA IIT ACADEMY INDIA

40+ YEARS OF EXCELLENCE

Sec: SR.IIT_*CO-SC(MODEL-A&B) PTA-1 Date: 18-09-22
Time: 3HRS 2018_P1 Max. Marks: 180

KEY SHEET

PHYSICS

1	**AC**	2	**ABD**	3	**BD**	4	**CD**	5	**ABCD**
6	**AC**	7	**2.25**	8	**4**	9	**2.5**	10	**6**
11	**60**	12	**9**	13	**0.6**	14	**3.05**	15	**D**
16	**B**	17	**A**	18	**B**				

CHEMISTRY

19	**ABCD**	20	**BCD**	21	**ABD**	22	**CD**	23	**BD**
24	**ABD**	25	**3**	26	**9**	27	**0**	28	**1**
29	**4**	30	**4**	31	**2**	32	**6**	33	**B**
34	**C**	35	**C**	36	**B**				

MATHAMATICS

37	**BC**	38	**BC**	39	**AC**	40	**AB**	41	**BC**
42	**ABD**	43	**16**	44	**2**	45	**2**	46	**6**
47	**4**	48	**6.33**	49	**84**	50	**4**	51	**D**
52	**A**	53	**C**	54	**B**				

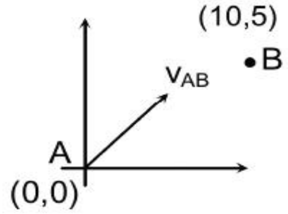
SOLUTIONS
PHYSICS

1. $\vec{v}_{AB} = (3-a)\hat{i} + (3-b)\hat{j}$

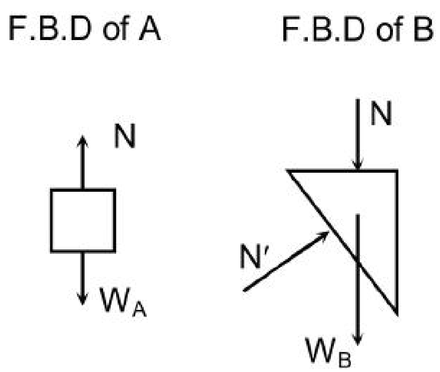
$\vec{a}_{AB} = \vec{0}$

$(3-a)\times 2 = 10 \;\; and \;\; (3-b)\times 2 = 5$

$a = -2$ and $b = \dfrac{1}{2}$



2. So, acceleration of A is always vertically downward and acceleration of B is along the incline more than $g\sin\theta$. Further we can also say that there is no horizontal force on the system. So, $m_B a_{Bx} + m_C a_{Cx} = 0$.

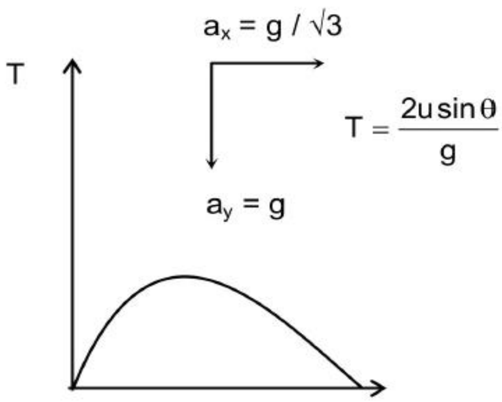


3. $R = u\cos\theta \dfrac{2u\sin\theta}{g} + \dfrac{1}{2}\dfrac{g}{\sqrt{3}}\left(\dfrac{2u\sin\theta}{g}\right)^2$

$= \dfrac{2u^2}{g}\left[\cos\theta\sin\theta + \dfrac{1}{\sqrt{3}}\sin^2\theta\right]$

$\dfrac{dR}{d\theta} = 0, \tan 2\theta = -\sqrt{3}$

$\theta = 60^0$



6. Let vertical velocity be v

$a_y = 0$

$a_x = kv$

$t = \dfrac{y}{v}$

$$\frac{dx}{dt}=ky=kvt$$

$$x=\frac{kvt^2}{2}$$

$$x=\left(\frac{k}{2v}\right)y^2$$

7. From the equation of trajectory $y=10\tan\theta-\frac{20}{9}\sec^2\theta$

$\frac{dy}{d\theta}=0$ gives, $\tan\theta=\left(\frac{9}{4}\right)$

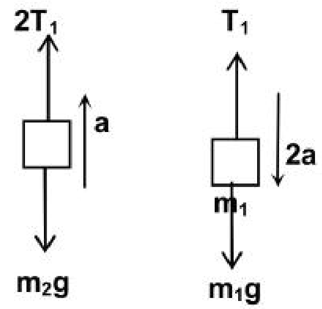
8. $m_1g-T_1=2m_1a$

$2T_1-m_2g=m_2a$

$2m_1g-m_2g=\left(4m_1+m_2\right)a$

$a=\left(\frac{2m_1-m_2}{4m_1+m_2}\right)g=\left(\frac{2\times1-1}{4\times1+1}\right)\times10=2\ m/s^2$

$a_A=4\ m/s^2$



9. $a=v\frac{dv}{dx}=\frac{25}{(x+2)^3},\ \frac{v^2}{2}=25\times\left[-\frac{1}{2(x+2)^2}\right]_0^x,\ v^2=25\left[\frac{1}{4}-\frac{1}{(x+2)^2}\right]$

$v=\sqrt{25\left[\frac{1}{4}-\frac{1}{(x+2)^2}\right]},\ v_{\max}=\frac{5}{2}=2.5\ m/s\ (at\ x=\infty)$

10. Let at certain moment its direction of motion makes an angle θ with the vertical as shown.

$5\cos\theta-5=ma_t$

$5-5\cos\theta=ma_y$

$\Rightarrow a_t=-a_y$

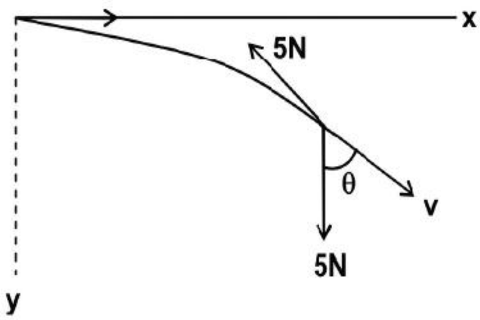
$\Rightarrow\frac{dv}{dt}=-\frac{dv_y}{dt}\Rightarrow v=-v_y+c$

When $v=9\ m/s,\ v_y=0$

$\Rightarrow c=9$

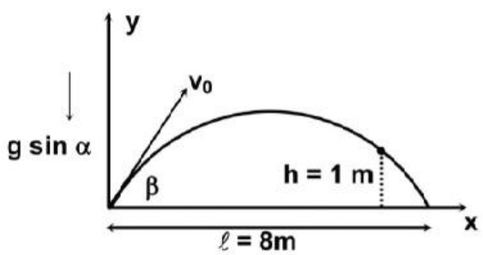
$v=-v\cos\theta+9$

$\Rightarrow v=\frac{9}{1+\cos\theta}=\frac{9}{1+\cos60^0}=6\ m/s$



12. $h=\ell\tan\beta-\frac{1}{2}\frac{g\sin\alpha\,\ell^2}{v_0^2\cos^2\beta}$

$v_0=\sqrt{\frac{g\sin\alpha\,\ell^2}{2\cos^2\beta\left(\ell\tan\beta-h\right)}}$



$$= \sqrt{\frac{10 \times \frac{4}{5} \times 8 \times 8}{2 \times \frac{4}{5} \times \frac{4}{5}\left(8 \times \frac{3}{4} - 1\right)}} = 9\ m/s$$

13. If they meet a height h after time T of the projection of the second.

Then, $h = u(T) - \frac{1}{2}g(T)^2 = v(T-t) - \frac{1}{2}g(T-t)^2$ .... (i)

$$T = \frac{5t^2 + 3t}{10t - 2}$$

For minimum $T, \frac{dT}{dt} = 0$

$$50t^2 - 20t - 6 = 0$$

$$\Rightarrow t = 0.6 = \frac{6}{g}$$

14. If S_1 stretches by x_1, then P_1 will move down by $\frac{x_1}{2}$,

Since tension in S_2 is double of that in S_1, the second spring will stretch by $2x_1$

Therefore, P_2 will descend by $\frac{2x_1 + \frac{x_2}{2}}{2} = \frac{5x_1}{4}$

[Since P_1 moves down by $\frac{x_1}{2}$].

Now, the extension in S_3 will be $4x_1$

Thus, P_3 will move down by a distance

$$\frac{4x_1 + \frac{5x_1}{4}}{2} = \frac{21x_1}{8}$$

Given : $\frac{21x_1}{8} = x \Rightarrow x_1 = \frac{8x}{21}$

Tension in $S_3 = k(4x_1) = \frac{32\ kx}{21}$

$$\therefore\ F = \frac{64kx}{21}$$

15. & 16.

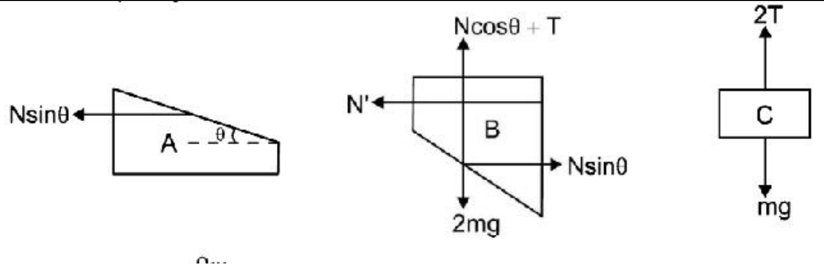
Let the acceleration of B downwards be $a_B = a$

From constraint; acceleration of A and C are

$a_A = a \cot^{-} = \frac{4a}{3}$ towards left

$a_C = \frac{a}{2}$ upwards

free body diagram of A, B and C are



$$N\sin\theta = \frac{9m}{64}\left(a\cot\theta\right) \qquad \text{……. (1)}$$

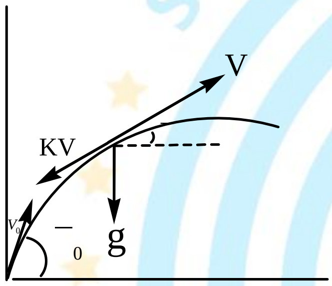
$$2mg - T - N\cos\theta = 2ma \qquad \text{…….. (2)}$$

$$2T - mg = m\frac{a}{2} \qquad \text{…….. (3)}$$

Solving we get

$a_C = \frac{a}{2} = 3\,m/s^2$ and 2T = 13 N

17.



$$a_y = \frac{dV_y}{dt} = -\left[\frac{KV\sin\theta}{m} + g\right] = -\left[\frac{KV_y}{m} + g\right]$$

$$\int_{V_0\sin\theta_0}^{V_y} \frac{dV_y}{g + \frac{k}{m}V_y} = -\int_0^t dt \Rightarrow \frac{m}{k}\left[\ln\left(g + \frac{k}{m}V_y\right)\right]_{V_0\sin\theta_0}^{V_y} = -t$$

$$\Rightarrow \frac{\frac{k}{m}V_y + g}{\frac{k}{m}V_0\sin\theta_0 + g} = e^{-\frac{k}{m}t} \Rightarrow V_y = \frac{m}{k}\left[\left(\frac{k}{m}V_0\sin\theta_0 + g\right)e^{-\frac{kt}{m}} - g\right]$$

18. $a_x = \frac{dV_x}{dt} = -\frac{k}{m}V_x \Rightarrow \int_{V_0\cos\theta_0}^{V_x} \frac{dV_x}{V_x} = -\frac{k}{m}\int_0^t dt \quad \Rightarrow V_x = V_0\cos\theta_0 e^{-\frac{k}{m}t}$

$$dx = V_x dt$$

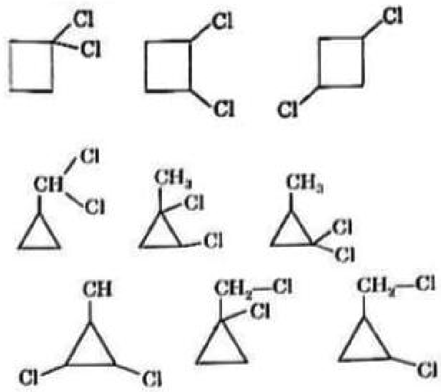
$$\Rightarrow \int_0^x dx = V_0\cos\theta_0 \int_0^t e^{-\frac{k}{m}t}dt \Rightarrow x = \frac{mV_0\cos\theta_0}{k}\left[1 - e^{-\frac{k}{m}t}\right]$$

X is max at t = ∞

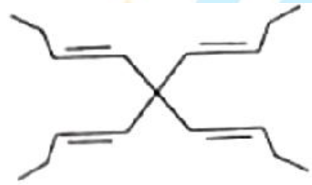
$$\Rightarrow x_{\max} = \frac{mV_0\cos\theta_0}{k}$$

CHEISTRY

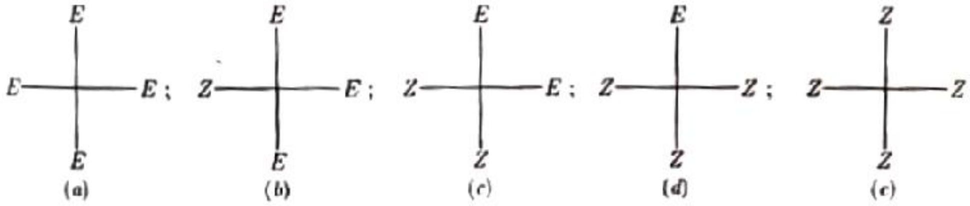
19. CONCEPTUAL
20. CONCEPTUAL
21. CONCEPTUAL
22. C) It exist in chair form (no plane of symmetry)
    D) Due to Gauche conformer it has non zero dipole moment
23. CONCEPTUAL
24. CONCEPTUAL
25. CONCEPTUAL
26.



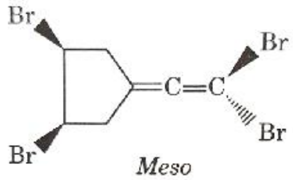
27. x $\rightarrow$ Chiral centre



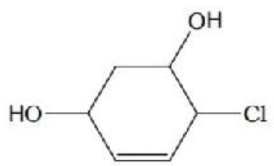
There are five stereoisomers and all of them are achiral and optically inactive.



28.



29. CONCEPTUAL
30. CONCEPTUAL
31. CONCEPTUAL
32.



6-chlorocyclohex-4-ene-1,3-diol

33. CONCEPTUAL
34. CONCEPTUAL

35. $\left[\sim^{o}\right]_{T}^{a} = \dfrac{\sim}{Cl} = \dfrac{-2.4}{\dfrac{1}{10}\times 2} = -12^{0}$

Optical purity $= \dfrac{[\sim]_{mix}}{[\sim]_{pure}} \times 100 = \dfrac{4}{12}\times 100 = 33.33\%$

Optical purity $= \dfrac{+6}{12}\times 100 = 50$

36.

50

$RM = 100 - 50 = 25\%$ $\quad$ $25\%1$

$\%\ d\ \text{form} = 50 + 25 = 75\%$

MATHS

37. $x^4 - 4x^2 = \log_2 y \Rightarrow x^2 = 2 + \sqrt{4 - \log_2 y}$ & $g(x) = 1 + \dfrac{6}{\sin x - 2} \Rightarrow A = [-5, -2]$

38. "O" should not exists in domain & Range

f: - odd, g- even $\Rightarrow h(-x) = \dfrac{-f(x)}{e^{g(x)}} - \dfrac{f(x)}{2} = -h(x)$

f: - odd, g- even $\Rightarrow a + b = 0 \ \ x^2 \ \& \dfrac{-a}{6} - \dfrac{b}{2} = 4$

39. $f(f(x)) = f(x) + x \ \Rightarrow$ If $f(x) = f(y) \Rightarrow x = y$

$f(f(x)) = f(y) + y \ \Rightarrow f(f(0)) = f(0) \Rightarrow f(0) = 0$

C & D : $f(x) + f(y) = f(x+y) \Rightarrow f(x) = kx \forall x \in R$ if f(x) is continuous at atleast one point.

40. $\underset{x\to 0}{Lt}\left\lfloor \dfrac{x}{\sin x} - 1 \right\rfloor = 0 \ \underset{x\to 0}{Lt}\ a\left(1 - \dfrac{x^2}{3!}\right) + b\left(1 - \dfrac{x^2}{2!}\right) = 4$

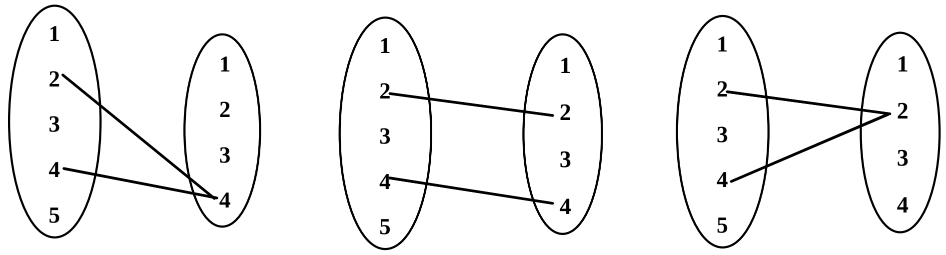
$\Rightarrow a + b = 0 \ \ x^2 \ \& \dfrac{-a}{6} - \dfrac{b}{2} = 4$

41. $r^2 - 1 < \left[r^2\right] \le r^2 \Rightarrow \underset{n\to\infty}{Lt}\sum_{r=0}^{n}\dfrac{\left[r^2\right]}{n^3} = \dfrac{1}{3},\ \underset{n\to\infty}{Lt}\sum_{r=0}^{n}\dfrac{\left[r^3\right]}{n^4} = \dfrac{1}{4}$

$0 \le \left\{\dfrac{r^3}{2}\right\} \le \dfrac{1}{2} \Rightarrow 0 \le \sum_{r=1}^{n}\left\{\dfrac{r^3}{2}\right\} \le \dfrac{n}{2}$

42. Period of f(x) = |sin2x| + |cos2x| is $\pi/4$
but f(x) = ln ([|sin2x| + |cos2x|])
Max. value of |sin2x| + |cos2x| = $\sqrt{2}$
f(x) = ln ([$\sqrt{2}$]) = ln (1) = 0
$\Rightarrow$ it is periodic function but fundamental period not defined.
f(x) is many one and into function.

43.



$f(3)=f(s)=4, f(1)$ has 4 values

$2\times3=6$ functions

$f(3)=2;\ 2\times3=6$ functions

44. $$\frac{(x-\sin x)\left(x^{999}+x^{998}(\sin x)+.....+(\sin x)^{999}\right)}{x^n}\Rightarrow n=1002$$

$$\&\ M=\frac{1}{6}\times1000\Rightarrow n-6M=2$$

45. $$\underset{x\to0}{\text{Lt}}(\cos x)^{\operatorname{cosec}^2}=e^{\underset{x\to0}{\text{Lt}}\frac{(\cos x-1)}{\sin^2 x}}=\frac{1}{\sqrt{e}}$$

$$\underset{x\to0}{\text{Lt}}\frac{\sin 2x+2\tan^{-1}3x+3x^2}{\ln\left(1+3x+\sin^2 x\right)+xe^x}=\underset{x\to0}{\text{Lt}}\frac{\sin 2x}{2x}.2x+\frac{6x\tan^{-1}3x}{3x}+3x^2$$

$$\frac{\ln\left(1+3x+\sin^2 x\right)}{3x+\sin^2 x}.\left(3x+\sin^2 x\right)+xe^2$$

$$=2$$

46. $$2\tan^{-1}\left(\operatorname{cosec}\left(\tan^{-1}x\right)-\tan\left(\cot^{-1}x\right)\right)=\tan^{-1}x;\ x\neq0$$

$$\underset{x\to0}{\text{Lt}}\frac{\frac{a}{2}\tan^{-1}x+\lambda x^4}{\tan x}=3\ \Rightarrow\frac{a}{2}=3\Rightarrow a=6$$

47. $(0,1),(1,0),(2,-1),(-1,2)$ are the solutions (By graph If (a, b) is a solution, then (b, a) is also a solution.

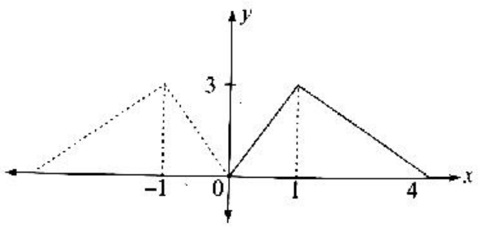
48. Dividie by $\sqrt{t}$ then apply limit. We get $\dfrac{\sqrt{x}}{\sqrt{x^2-3x+1}}=\sqrt{3}\ \sim\ =\dfrac{1}{3}\quad=3$.

49. $$T_r=\frac{1}{16}\left[4r^2+1+\frac{1}{2}\left(\frac{1}{(2r-1)}-\frac{1}{(2r+1)}\right)\right]$$

$$S_n=\frac{n^3}{12}+\frac{n^2}{8}+\frac{5n}{48}+\frac{1}{16}\left(\frac{n}{2n+1}\right)$$

50. $f(x-2)=f(x+6)\Rightarrow f(x)=f(x+8)$

$$f(x)=\begin{cases}3x, & 0\le x<1\\ 4-x, & 1\le x\le4\end{cases}$$



$$f(-89)-f(-67)+f(46)=f(-1)-f(-3)+f(-2)=3-1+2=4$$