NARAYANA IIT ACADEMY INDIA

40+ YEARS OF EXCELLENCE

Sec: JR_*CO-SC(MODEL-B) CTM-10 Date: 04-09-22

Time: 3 Hrs Max. Marks: 300

# KEY SHEET

## PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **A** | 2 | **C** | 3 | **D** | 4 | **C** | 5 | **C** |
| 6 | **C** | 7 | **D** | 8 | **B** | 9 | **A** | 10 | **B** |
| 11 | **D** | 12 | **A** | 13 | **D** | 14 | **B** | 15 | **A** |
| 16 | **B** | 17 | **D** | 18 | **C** | 19 | **B** | 20 | **B** |
| 21 | **4** | 22 | **3** | 23 | **6** | 24 | **9** | 25 | **5** |
| 26 | **6** | 27 | **105** | 28 | **21** | 29 | **6** | 30 | **144** |

## CHEMISTRY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 31 | **A** | 32 | **C** | 33 | **B** | 34 | **A** | 35 | **C** |
| 36 | **D** | 37 | **B** | 38 | **A** | 39 | **C** | 40 | **B** |
| 41 | **D** | 42 | **C** | 43 | **D** | 44 | **A** | 45 | **D** |
| 46 | **C** | 47 | **B** | 48 | **B** | 49 | **C** | 50 | **D** |
| 51 | **5** | 52 | **-3** | 53 | **5** | 54 | **3** | 55 | **3** |
| 56 | **2** | 57 | **2** | 58 | **3** | 59 | **6** | 60 | **8** |

## MATHEMATICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 61 | **C** | 62 | **C** | 63 | **C** | 64 | **A** | 65 | **C** |
| 66 | **C** | 67 | **D** | 68 | **C** | 69 | **A** | 70 | **A** |
| 71 | **D** | 72 | **A** | 73 | **C** | 74 | **D** | 75 | **A** |
| 76 | **A** | 77 | **C** | 78 | **C** | 79 | **C** | 80 | **D** |
| 81 | **21** | 82 | **0** | 83 | **5** | 84 | **1** | 85 | **8** |
| 86 | **2** | 87 | **0** | 88 | **4** | 89 | **1** | 90 | **0** |

## SOLUTIONS

## PHYSICS

1. $a_{cm} = \dfrac{m_1\overrightarrow{a_1} + m_2\overrightarrow{a_2}}{m_1 + m_2} = \dfrac{1(g)+1(g)}{1+1} = g$

2. $x_{cm} = \dfrac{A_1x_1 + A_2x_2}{A_1 + A_2}\left(\because\ m \sim A\right)$

3. Find $u_{cm}, a_{cm}$, then use $v_{cm}^2 - u_{cm}^2 = 2a_{cm}h_{cm}$
   Finally find $y_{cm} + h_{cm}$

4. $a_{cm} = \dfrac{m_1\overrightarrow{a_1} + m_2\overrightarrow{a_2}}{m_1 + m_2} \quad \left|\overrightarrow{a_1}\right| = \left|\overrightarrow{a_2}\right| = \dfrac{(m_1 - m_2)}{m_1 + m_2} = \dfrac{g}{3}$

   $= \dfrac{4\dfrac{g}{3} + 2\left(-\dfrac{g}{3}\right)}{4+2} = \dfrac{g}{9}$

   $S_{cm} = u_{cm}t + \dfrac{1}{2}a_{cm}t^2 = 0 + \dfrac{1}{2}\left(\dfrac{g}{9}\right)(3)^2 = \dfrac{g}{2}$

5. given $\dfrac{1x_1 + 2x_2 + 3x_3 + 4x_4}{1+2+3+4} = 0$

   $\Rightarrow 12 + 4x_4 = 0 \Rightarrow x_4 = -3$ $\quad$ $lly\ y_4 = -3$

   $z_4 = -3$

6. $\left|\overrightarrow{a_1}\right| = \left|\overrightarrow{a_2}\right| = \dfrac{mg\left(\sin 60^0 - \sin 60^0\right)}{2m} = \dfrac{g}{4}\left(\sqrt{3} - 1\right)$

   $a_{cm} = \dfrac{\sqrt{a_1^2 + a_2^2}}{2} = \dfrac{a}{\sqrt{2}}$

9. $m_1 = \circ\,\dfrac{\circledR R^2h}{3}, x_1 = \dfrac{h}{4}$

   $m_2 = \dfrac{2}{3}\circledR R^3,\ x_2 = \dfrac{3R}{8}$ use $m_1x_1 = m_2x_2$

10. $x_{cm} = \dfrac{\int dmx}{\int dmdm} = \text{ª}\, dx$

11. $v_{cm} = \dfrac{m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}}{m_1 + m_2}$

12. $\because$ net external force is zero
    Acceleration of centre of mass is zero

13. $m_1 = \circ\,\dfrac{4}{3}\circledR r^3,\ m_2 = \circ = \dfrac{4}{3}\circledR\,(2r)^3 = 8m_1$

    $d = 10r - (r + 2r) = 7r$

    Distance of COM from $m_2$ is $\dfrac{m_1d}{m_1 + m_2} = \dfrac{m_1 7r}{9m_1} = \dfrac{7r}{9}$

14. $\because F_{ext} = 0$, no shift of COM

    $0 = \dfrac{40(4+x) + 160(+x)}{40+160} \Rightarrow \quad x = \dfrac{-40\times 4}{200} = 0.8m$

15. $T_{\min} = mg\cos{}^{-} = mg\cos 37^0 = \frac{4mg}{5}$

$= mg + \frac{2mg}{5} = \frac{7mg}{5}$

17. $u = \sqrt{u^2 - 2gL}$

$v \perp u\ ,\ \left|\vec{v} - \vec{u}\right| = \sqrt{v^2 + u^2}$

18. $k = 2u \Rightarrow \frac{1}{2}m\left(2gh_1\right) = 2mg\left(h - h_1\right) \Rightarrow h_1 = \frac{2h}{3}$

$\frac{2h}{3} = \frac{1}{2}gt_0^2 \Rightarrow t_0 = \sqrt{\frac{4h}{3}}$ $\quad\therefore\ p = \frac{w}{t_0} = \frac{mgh}{t_0}$

19. $p = \frac{mgh}{t}$

21. $\int pdt = \Delta k$

$16 = \frac{1}{2} \times 10^{-1}\left(v^2 - 80\right) \Rightarrow v = 20m/s$

22. $u_{se}\quad x_{com}\quad y_{com}$

$d = \sqrt{x_{com}^2 + y_{com}^2}$

24. $a_t = \frac{du}{dt} = 6\ \ t\ , a_c = \frac{v^2}{R} = \frac{9\ \ {}^2t^2}{R}$

Use $\tan{}^{-} = \frac{a_c}{a_t}$

25. $« = \tan{}^{-}$

$a = -g\left(\sin{}^{-} + «\cos{}^{-}\right) = 2g\sin{}^{-}$

Use $v^2 - u^2 = 2as$

26. $v = \frac{ds}{dt} = 3t^2 - 12t + 18$

For $v$ to be min. or max $\frac{du}{dt} = 0$

$6t - 12 = 0 \Rightarrow t = 12\sec$

At $t = 2s$

$\frac{d^2u}{dt^2} = 6 > 0$. i.e $v$ is min. at $t = 2\sec$

27. Time of flight $\frac{2u\sin{}^{-}}{g} = 10 \Rightarrow u\sin{}^{-} = 50$

$y = u\sin{}^{-}t - \frac{1}{2}gt = 50 \times 3 - \frac{1}{2}(10)(3)^2 = 105m$

28. $F = \left(M + m\right)a$

F becomes max when $a = a_{\max} = «_s g$

$F_{\max} = «_s\left(M + m\right)g = 0.3(5+2)(10) = 21N$

29. $x_c = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

30. use $w = \Delta k = \frac{1}{2}mv^2 - 0$

# CHEMISTRY

31. Conceptual

32. $\left(P^H > 7\right)$ at neutralization for W.A & S.B

33. $K_{SP} = S^2 \Rightarrow S = 10^{-5} M$

$V_{H_2O} = \frac{1}{233} \times \frac{1}{10^{-5}} = 430L$

34. $I.P = \left(\frac{0.25 \times 100}{200}\right)^2 \left(\frac{0.015 \times 100}{200}\right) > K_{SP}$

35. Solubility increases on removing OH- i.e of $P^H \downarrow$

36. Conceptual

37. Conceptual

38. $5.8 = P^{ka} + \log\frac{1}{2}$

$Ka = 7.94 \times 10^{-7}$

39. ¤ $= \frac{n}{\Delta P^H}$

$x = \frac{v \times 1}{0.1 \times 1} \Rightarrow V = 0.1 lit = 100 xml$

40. Due to common ion effect solubility $\downarrow$ resulting in $\left[Ag^+\right]\downarrow$

41. Even, if ‘S’ is same, $K_{SP}$ need not be same

42. $K = \left[NH_2^-\right]\left[H^+\right]$

$\left[NH_2^-\right] = \sqrt{10^{-30}} = 10^{-15}$

Mole $S = 10^{-18} mole$

No. of $NH_2^- = 10^{-18} \times 6 \times 10^{23} = 6 \times 10^5$

43. $Ka = \frac{\left[H^+\right]\left[CN^-\right]}{[HCN]}$

$\left[CN^-\right] = \frac{10^{-6} \times 10^{-2}}{10^{-1}} = 10^{-7}$

44. $H^+ = 0.1(from\ HCl), P^H = 1$

45. Conceptual

46. $\overset{2(-2)}{N_2}H_4 \rightarrow \overset{2(+2)}{(2N)}$

47. No.of Balloons $= \frac{(P_c - P_b)V_c}{P_b V_b} \times \frac{T_c}{T_b} = 180$

$(\therefore c = cylinder \quad b = balloon)$

48. Adiabatic expansion cooling effect observed $[T_3 < T_1]$

49. $\Delta H = \Delta E + \Delta ngRT$

$\Delta H / mole = 780.90$

For 39 g $\rightarrow$ 390.45kCal

50. Conceptual

51. $P^H = \frac{1}{2}\left(P^{ke} - P^{kb} - \log c\right)$

52. $\left(10^{-4}\right)\left(SO_4^{-2}\right) > 4\times10^{-11}$

$\left(SO_4^{--2}\right) > 4\times10^{-7}$

Moles$= 4\times10^{-7}\times0.5 = 2\times10^{-7}$

$\frac{y}{x} = -\frac{7}{2} = -3.5$

53. $P^H = P^{ka} + \log\frac{In^-}{HIn} = 5$

54. $K_{sp} = s(xs)^x = 27\times10^{-12} = 10^{-3}\left(x10^{-3}\right)^x \Rightarrow x^x = 3^3 \Rightarrow x = 3$

55. $\frac{\left[A^{+2}\right]\left[CO_3^{-2}\right]}{\left[A^{+2}\right]\left[SO_4^{-2}\right]} = \frac{1}{2} \Rightarrow \left[SO_4^{-2}\right] = 2\left[CO_3^{-2}\right]$

$\left[B^{+2}\right]\left[SO_4^{-2}\right] = 6\times10^{-10}$

$\left[B^{+2}\right]\left[CO_3^{-2}\right] = 3\times10^{-10}$

56. $P^H = 11, P^{OH} = 3, OH^- = 10^{-3}$

$OH^- = c\sim$

$\sim = \frac{OH^-}{C} = \frac{10^{-3}}{5\times10^{-2}} = 2\%$

57. $Ka = c\sim^2 = 0.1\left(10^{-2}\right)^2 = 10^{-5}$

$\sim = \sqrt{\frac{ka}{c}} = \sqrt{\frac{10^{-5}}{0.025}} = \frac{10^{-1}}{5} = x \Rightarrow 100x = 2$

58. $SO_2, SCl_4, PCl_3$ (having lone pairs and octet)

59. $q = nC_v dT \Rightarrow 12\frac{4.48}{22.4}\times C_v\times15 \Rightarrow C_v = 4$

$C_p = C_v + R \Rightarrow 4+2 = 6$

60. $\Delta S = nR\ln\frac{V_2}{V_1} = 18.424cal$

## MATHS

61. $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$\therefore I + 2A + 3A^2 + ......\infty = I + 2A$

62. $P^2 = I - P \Rightarrow P^2 - P - (I-P) = 2P - I$

$P^4 = 2P^3 - P$

$= 2P^2 - P$

$= 2(I-P) - P$

$= 2I - 3P$

$P^5 = 2P - 3P^2$

$= 2P - 3(I-P)$

$= 5P - 3I$

$P^6 = 5P^3 - 3P = 5(I-P) - 3P = 5I - 8P$

63. $A.adjA = |A|I$

$|A| = xyz\ 8x - 3(z-8) + 2(2-2y)$

$= xyz(8x+3z+4y)+28$

$= 60-20+28=68$

64. $BC = 1$

$t_r(A)+t_r(A/2)+t_r(A/2^2)$

$= t_r(A)+\frac{1}{2}t_r(A)+\frac{1}{2^2}t_r(A)$

$\frac{t_r(A)}{1-\frac{1}{2}}\ 2 = 2t_r(A) = 2(2+1) = 6$

65. $f^1(0) = \begin{vmatrix} 22 & 44 & 66 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 33 & 66 & 99 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 44 & 88 & 144 \end{vmatrix}$

$\because co-eff$ of $x=0$

66. $\begin{vmatrix} r^2-s^2 & s^2-r^2 & 0 \\ s^2 & r^2 & s^2 \\ 0 & s^2-r^2 & r^2-s^2 \end{vmatrix}$

$=(r^2-s^2)(r^2+2s^2) = (a+b+c)^2(a^2+b^2+c^2-ab-bc-ca)^2$

67. $A^5(AB^2) = A^2BA$

$\Rightarrow B^2 = A^5BA.$

$\Rightarrow B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$\Rightarrow B^4 = A^2BA^2$

$\Rightarrow B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^2BA^3$

$\Rightarrow B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^4 = A^2(A^5BA)A^3 = A^2BA^4$

$A^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4 = A^2(A^5BA)A^4 = ABA^5$

$A^{54} = (ABA^5)(ABA^5) = AB^2A^5 = A(A^5BA)A^5 = B \Rightarrow A^{63} = I$

68. Conceptual

69. $A(\sim,\ \ )^{-1} = \frac{1}{e}\begin{bmatrix} e\ \cos\sim & -e\ \sin\sim & 0 \\ e\ \sin\sim & e\ \cos\sim & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= A(-\sim,-\ \ )$

70. From given data $|A| = 2^4$

$\Rightarrow |adj(adj\ A) = (2^4)^9 = 2^{36}$

$\Rightarrow \left\{\frac{\det(adj(adjA))}{7}\right\} = \left\{\frac{2^{36}}{7}\right\} = \left\{\frac{(7+1)}{7}\right\} = \frac{1}{7}$

71. $B = A^{-1}$

72. Conceptual

73. $$f(x) = \begin{vmatrix} 1-2\sin^2 x & \sin^2 x & 1-8\sin^2 x\left(1-\sin^2 x\right) \\ \sin^2 x & 1-2\sin^2 x & 1-\sin^2 x \\ 1-8\sin^2 x\left(1-\sin^2 x\right) & 1-\sin^2 x & 1-2\sin^2 x \end{vmatrix}$$

$\Rightarrow$ the required constant term is

$$f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = (0-1) = -1$$

74. $x^2 - 2x + 4 = -3\cos(ax+b)$

$(x-1)^2 + 3 = -3\cos(ax+b)$

L.H.S $\geq$ 3 & R.H.S $\leq 3$

$\therefore x = 1$ & $a + b = ®$

75. $a+b+c=0$ roots are $1, \dfrac{c}{a}$

76. $2^x = 3^y = 6^{-z} = k$

$\therefore x = \log_2 k,\ y = \log_3 k,\ z = -\log_6 k$

77. $ar^2 = 2$

$\therefore a^5 r^{10} = 2^5$

78. Conceptual

79. $\dfrac{2\sin 15^0 \cos 50^0}{2\cos 15^0 \cos 5^0} = \text{Tan}15^0 = 2 - \sqrt{3}$

80. $X^T = X, \qquad Y^T = -Y$

$\therefore (XY)^T = Y^T X^T = -YX$

81. $A = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$

82. $A - A^T$ is skew symmetric matrix

83. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\therefore \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Rightarrow a - b = -1 \,\&\, c - d = 2$

$A^2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow -a + 2b = 1 \,\&\, -c + 2d = 0$

By solving $a = -1, b = 0 c = 4, d = 2$

84. $|A|^4 = 16$

$\left(2\ \ ^2 + 1\right)^4 = 16$

$\therefore$ Real values of $\ \ = \pm \dfrac{1}{\sqrt{2}}$

85. $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$$= -(a+b+c)\left((a+b+c)^2 - 3(ab+bc+ca)\right)$$

86. $\log_{0.01}^{1000} = \frac{-3}{2}$

$\log_{0.1}^{0.0001} = 4$

87. $\sin^{4-} - 2\sin^{2-} - 1 = 0$

$\therefore \sin^{2-} = \frac{2 \pm 2\sqrt{2}}{2} = 1+\sqrt{2}, 1-\sqrt{2}$

Both are not possible

88. $\sin\left(\frac{\text{®} x^2}{3}\right) = 1 \Rightarrow \frac{\text{®} x^2}{3} = \frac{\text{®}}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}}$

y also get 2 values

$\therefore (x, y)$ has 4 values

89. $R_2 \to R_2 - (R_1 + R_3)$

$$\begin{vmatrix} x^2+x & x+1 & x+2 \\ -4 & 0 & 0 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = 12(2x-1)$$

90. Conceptual