MATHEMATICS

1. REAL NUMBERS

1. All rational and irrational numbers are called Real Numbers.
2. Real numbers of the form $\frac{p}{q}$, $q \neq 0$, $p, q \in I$ are called Rational numbers.
3. A lemma is a proved statement used for proving another statement.
4. Euclid's division lemma: Given two positive integers a and b, there exists unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$, where q and r can also be zero.
5. Euclid's division algorithm is a technique to compute the Highest common factor (HCF) of two given positive integers.
6. Fundamental theorem of Arithmetic: Every composite number can be expressed as product of prime factors and its factorisation is unique, apart from the order in which the prime factors occur.
7. The sum or difference of a rational and irrational number is an irrational.
8. The product or quotient of a non-zero rational number and an irrational number is irrational.
9. For any two positive integers a and b, H.C.F (a, b) x L.C.M (a, b) = a x b.
10. For a rational number $\frac{p}{q}$, if the denominator $q = 2^m \times 5^n$, where m and n are two non-negative integers, then its decimal expansion is terminating.
11. For a rational number $\frac{p}{q}$, if the denominator $q \neq 2^m \times 5^n$, where m and n are two non-negative integers, then its decimal expansion is non–terminating repeating.
12. Let 'p' be a prime number, if p divides a^2 then p divides a, where 'a' is a positive integer.
13. If the denominator of a rational number is of the form $2^m \times 5^n$ then it will terminate after n places if $n > m$ or m places if $m > n$.

2. POLYNOMIALS

1. A polynomial is an algebraic expression with powers of variables as non-negative integers. It's mathematical form is $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} ...... + a_2 x + a_1 x + a_o$ where n is a whole number and $a_n, a_{n-1}, ...a_0 \in R$.
2. If $p(x)$ is a polynomial in x, then the highest power of x in $p(x)$ is called the degree of the polynomial $p(x)$.
3. If $p(x)$ is a polynomial in x, and if k is any real number, then the value of $p(x)$ at $x = k$, is denoted by $p(k)$.

4. If $p(k)=0$, then 'k' is called zero of the polynomial $p(x)$.

5. If $ax+b$ is a given linear polynomial , then zero of the linear polynomial is –b/a.

i.e.. $-\frac{(\text{constant term})}{\text{coefficient of } x}$

5. A quadratic polynomial in x with real co-efficient is of the form ax^2+bx+c, with $a \neq 0$.

6. The zero of polynomial $p(x)$ are precisely the x coordinates of the points, where the graph of $y=p(x)$ intersect the x-axis.

7. A quadratic polynomial can have at most two zeroes and a cubic polynomial can have at most 3 zeroes.

8. If α, β are the zeroes of the polynomial ax^2+bx+c then

Sum of the zeroes $(\alpha+\beta)=\frac{-b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$ and

product of the zeroes $\alpha\beta=\frac{c}{a} .= \frac{\text{constant term}}{\text{coefficient of } x^2}$

9. If α, β, γ are the zeroes of the polynomial ax^3+bx^2+cx+d then

Sum of the zeroes $(\alpha+\beta+\gamma)=\frac{-b}{a} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$

Sum of product of zeroes taken two at a time $(\alpha\beta+\beta\gamma+\alpha\gamma)=\frac{c}{a}=\frac{\text{coefficient of } x}{\text{coefficient of } x^3}$

product of the zeroes $(\alpha\beta\gamma)=\frac{-d}{a}=-\frac{\text{constant term}}{\text{coefficient of } x^3}$

10. Division Algorithm for polynomial: For any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$, such that p(x) = g(x).q(x) + r(x), where r(x) = 0 or degree r(x) < degree g(x).

11. If $r(x)=0$, then $g(x)$ is a factor of $p(x)$.

12. If α, β are the zeroes, then the quadratic polynomial $p(x)$ is given by $p(x)=k\left(x^2-(\alpha+\beta)x+\alpha\beta\right)$, where $k \neq 0$ and k is any real number.

13. If α, β and γ are the zeroes, then the cubic polynomial $p(x)$ is given by $p(x)=k\left(x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma\right)$, where $k \neq 0$ and k is any real number.

14. Degree of a constant polynomial is zero.
15. Degree of zero polynomial is not defined.
16. A polynomial of degree ‘1’ is called a linear polynomial.
17. A polynomial of degree ‘2’ is called a quadratic polynomial.
18. A polynomial of degree ‘3’ is called a Cubic polynomial.

3. PAIR OF LINEAR EQUATION IN TWO VARIABLES

1. A linear equation is an equation of straight line in the form of ax + by + c = 0 where $a \neq 0$, $b \neq 0$ and a,b,c are real numbers. In this equation a and b are coefficients and c is a constant.
2. Two linear equations having two same variables are called pair of linear equation in two variables. The general form of pair of linear equations is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers and $a_1 \neq 0, b_1 \neq 0$ and $a_2 \neq 0, b_2 \neq 0$.
3. Every solution of the equation is a point on the line representing it.
4. Each solution (x, y) of linear equation in two variables, ax +by+ c=0 corresponds to a point on the line representing the equation, and vice versa.
5. If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are the lines intersecting at a single point, then i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ ii) equations has unique solution iii) these equations are called consistent pair of equations.
6. If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are parallel lines, then i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ii) equations have no solution iii) these equations are called Inconsistent pair of equations.
7. If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are coincident lines, then i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ii) equations has infinitely many solutions iii) these equations are called consistent pair of equations (Dependent equations).
8. If the present age of a person is 'x' years, then (i) his age ‘a’ years ago will be $(x-a)$ years (ii) his age after ‘b’ years will be $(y+b)$ years.
9. If a person completes the work in x days then the amount of work done by him in one day is $\frac{1}{x}$.
10. The general form of a two digit number is $10x+y$, where x is in ten’s place and y is in unit’s place.

11. The general form of a two digit number when digits are reversed as per the above condition is $10y + x$.
12. If x Kmph is the speed of the boat and y Kmph is the speed of stream then Speed downstream (x+y) Kmph, speed upstream (x – y) Kmph.
13. $\text{Speed} = \frac{\text{distance}}{\text{time}}$

4. QUADRATIC EQUATIONS

1. The general form or standard form of quadratic equation is $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$.
2. Complete quadratic equation: $ax^2 + bx + c = 0$ where $a \neq 0$, $b \neq 0$, $c \neq 0$.
3. Pure quadratic equation: $ax^2 = 0$, where $a \neq 0, b = 0, c = 0$.
4. If α satisfies the quadratic equation $ax^2 + bx + c = 0$, such that $a\alpha^2 + b\alpha + c = 0$, then α is called as root of the quadratic equation.
5. Methods to solve the quadratic equations are Factorization method, Quadratic formula method or (Sridharacharya's rule) and Completing the square method.
6. The quadratic formula for finding out the roots of the equation $ax^2 + bx + c = 0$ is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b and c are real numbers.
7. From the quadratic formula the roots of the quadratic equation are $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b \pm \sqrt{D}}{2a}$ where $D \text{ or } \Delta = b^2 - 4ac$
8. For the quadratic equation $ax^2 + bx + c = 0$, $b^2 - 4ac$ is called Discriminant.
9. Nature of the roots of the quadratic equation $ax^2 + bx + c = 0$ depends upon the value of the discriminant.
10. If $b^2 - 4ac > 0$ then the nature of the roots of the equation $ax^2 + bx + c = 0$ are real and distinct.
11. If $b^2 - 4ac = 0$ then the nature of the roots of the equation $ax^2 + bx + c = 0$ are real and equal.
12. If $b^2 - 4ac < 0$ then the nature of the roots of the equation $ax^2 + bx + c = 0$ are not real or complex
12. The quadratic equations having the roots as $\alpha \text{ and } \beta$ is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

5. ARITHMETIC PROGRESSIONS

1. A succession of number formed and arranged in a definite order according to certain definite rule is called a progression

2. An Arithmetic progression is a sequence of numbers in which each term is obtained by adding a particular number to the previous term, except the first term.
3. Each number in the sequence is called as a term.
4. The difference between each term and its preceding term is called as common difference (d), value of 'd' can be positive, negative or zero.
5. General form of Arithmetic progression is $a, a+d, a+2d, a+3d, \ldots\ldots\ldots\ldots$
   Where 'a' is the first term and 'd' is the common difference.
6. A given list of numbers $a_1, a_2, a_3, \ldots\ldots\ldots\ldots$ is in AP if $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 \ldots\ldots$
7. The Arithmetic progression having finite (countable) number of terms is called finite Arithmetic progression. Eg: 5,7,9,11,13,15
8. The Arithmetic progression having infinite (uncountable) number of terms is called infinite Arithmetic progression. Eg: -1,-3,-5,-7,-9,....................
9. The n^{th} term of the Arithmetic progression whose first term is a or a_1 and the common difference is d is given by $a_n = a + (n-1)d$
10. a_n is also called the general term of the AP. If there are n terms in the AP, then a_n represents the last term which is sometimes also denoted by l.
11. The arithmetic series is the sum of all the terms of an arithmetic sequence. Arithmetic series is in the form of {a + (a + d) + (a + 2d) + (a + 3d) +.........}
12. The sum of first n terms of the A.P is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, where a is the first term, d is the common difference and S_n denotes sum of n terms.
13. The sum of first n positive integers is given by $\frac{n(n+1)}{2}$
14. If 'a' is first term and 'l' denotes last term or n^{th} term then $S_n = \frac{n}{2}(a+l)$.
15. The sum of squares of first n positive integers is given by $\frac{n(n+1)(2n+1)}{6}$
16. The sum of cubes of first n positive integers is given by $\frac{n^2(n+1)^2}{4}$
17. If a,b,c are the terms of A.P then b = $\frac{(a+c)}{2}$, b is called the Arithmetic mean (A.M) of a & c.

18. If S_n denotes sum to n terms of an A.P then then n^{th} term of that A.P is given by $a_n = S_n - S_{n-1}$

19. If an A.P has n terms then the r^{th} term from the last = $(n - r + 1)^{th}$ term from the beginning.

6. TRIANGLES

1. A polygon which has three sides and three vertices is called as a Triangle.
2. Two polygons are said to be similar if their angles are equal and corresponding sides are in same ratio (or proportional).
3. Two polygons are said to be congruent if their angles are equal and corresponding sides are equal.
4. All congruent figures are similar but, all similar figures need not be congruent.
5. In ΔABC and ΔDEF, If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ then $\Delta ABC \sim \Delta DEF$.
6. If the corresponding angles of two triangles are same then they are called equiangular triangles.
7. If in a given triangle a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points then the other two sides are divided in the same ratio. (Basic Proportionality Theorem or Thales theorem).
8. If in a given triangle a straight line divides the two sides of a triangle in the same ratio then that straight line is parallel to third side.(Converse of Basic Proportionality theorem).
9. If in two triangles, the corresponding angles are equal, then their corresponding sides are proportional and hence the triangles are similar. (A.A.A criterion)
10. If two angles of one triangle are respectively equal to two angles of another triangle then two triangles are similar.
11. If the corresponding sides of two triangles are proportional then their corresponding angles are equal, and hence the two triangles are similar. (S.S.S criterion)
12. If one angle of a triangle is equal to one angle of other triangle and the sides including these angles are proportional then the two triangles are similar. (S.A.S criterion).
13. The ratio of areas of two similar triangles is equal to the ratio of squares of their sides, $\Delta ABC \sim \Delta PQR, then \ \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

14. The ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes and also their medians.
15. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (Baudhayan Theorem or Pythaogras theorem).
16. In a triangle, if square of one side is equal to the sum of the squares of other two sides then the angle opposite to the first side is a right angle.
17. The ratio of perimeters of two similar triangles is equal to the ratio of their corresponding sides i. e... $\frac{\text{perimeter}(\Delta ABC)}{\text{perimeter}(\Delta PQR)}=\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}$.
18. In any triangle the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side.
19. Sum of squares of the sides of the Rhombus is equal to sum of squares of its diagonals.
20. The sum of squares of the diagonals of a parallelogram is equal to sum of the squares of its sides.
21. In a right angled triangle, If we draw a perpendicular from the right angle to the hypotenuse of the triangle, then both the triangles will be similar to the whole triangle.

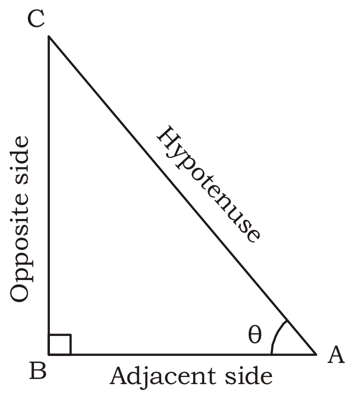
7. COORDINATE GEOMETRY

1. The point of intersection of x-axis and y-axis is called as the origin, denoted by 'O". The coordinates of Origin are (0, 0).
2. X – Coordinate of a point is called abscissa and y-coordinate of a point is called ordinate.
3. A plane is divided by axes into four quadrants.
4. The coordinates of the x- axis is (x, 0).
5. The coordinates of the y-axis is (0, y).
6. If $x \neq y$ then $(x, y) \neq (y, x)$ and if x = y then (x, y) = (y, x)
7. The distance between the two points $P(x_1, y_1)$, $Q(x_2, y_2)$ is given by $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
8. The distance of the point $P(x, y)$ from the Origin (0,0) is given by OP = $\sqrt{x^2+y^2}$.
9. If $P(x, y)$ be any point on the line segment AB, which divides AB in the ratio of $m_1 : m_2$ internally then coordinates of $P(x, y)$ will be $\left[\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$
10. The midpoint of the line segment joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ is given by $\left[\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right]$.

11. Let P and Q be the points of trisection (dividing into three equal parts) of the line segment AB, then for finding the coordinates of P the ratio is 1:2 and for finding the coordinates of Q the ratio is 2: 1
12. The line drawn from the vertex of a triangle to the midpoint of the opposite side is called as median.
13. The point of concurrency of medians of a triangle is called as centroid.
14. The centroid divides the median in the ratio of 2:1
15. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ be the three vertices of a ΔABC, then the formula used for finding centroid of the triangle is $\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right]$.
16. The area of triangle formed by the points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is given by $\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$
17. Three points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are said to be collinear when area of the triangle formed by these three vertices is 0.

8. Introduction to Trigonometry

1. In a right angled triangle, the ratio of its sides and acute angles is the trigonometric ratios of the angles.
2. In ΔABC, $\angle B = 90^0$ then $\angle A + \angle C = 90^0$ ($\angle A$ and $\angle C$ are acute angles)
3. The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.
4. Trigonometric ratios of angle $\angle CAB$ i.e.. 'θ' in right angled triangle ABC is defined as:



i) Sine of $\angle CAB$ = $\sin\theta$ = $\frac{opposite\,side}{Hypotenuse} = \frac{BC}{AC}$

ii) Cosine of $\angle CAB$ = $\cos\theta$ $= \frac{adjacent}{hypotenuse} = \frac{AB}{AC}$

iii) Tangent of $\angle CAB$ = $\tan\theta = \dfrac{opposite\,side}{Adjacent\,side} = \dfrac{BC}{AB}$

iv) Cosecant of $\angle CAB$ = $\cos ec\,\theta = \dfrac{Hypotenuse}{opposite\,side} = \dfrac{AC}{BC}$

v) Secant of $\angle CAB$ = $\sec\theta = \dfrac{Hypotenuse}{Adjacent\,side} = \dfrac{AC}{AB}$

vi) Cosecant of $\angle CAB$ = $\cot\theta = \dfrac{Adjacent\,side}{opposite\,side} = \dfrac{AB}{BC}$

5. Recirpocal relation of trigonometric ratios: $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$

6. Quotient relation: i) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ ii) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

7. Trigonometric ratios for some specific angles

θ	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	(not defined)
$\text{cosec}\,\theta$	(not defined)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	(not defined)
$\cot\theta$	(not defined)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

8. If θ is an acute angle, then its complementary angle is $\left(90^0 - \theta\right)$. The following relations holds true for trigonometrci ratios of complementary angles.

i) $\sin(90^\circ - \theta) = \cos\theta$ ii) $\cos(90^\circ - \theta) = \sin\theta$ iii) $\tan(90^\circ - \theta) = \cot\theta$

iv) $\cos ec(90^\circ - \theta) = \sec\theta$ v) $\sec(90^\circ - \theta) = \cos ec\theta$ vi) $\cot(90^\circ - \theta) = \tan\theta$

9. An equation involving trigonometric ratios of an angle is called trigonometric identity.

10. i) $\sin^2\theta + \cos^2\theta = 1$; $(0^\circ \le \theta \le 90^\circ)$ ii) $1 + \tan^2\theta = \sec^2\theta$; $(0^\circ \le \theta < 90^\circ)$

iii) $1 + \cot^2\theta = \cos ec^2\theta$; $\left(0^0 < \theta \le 90^0\right)$

9. Some applications of Trigonometry

1. The line of sight is the line drawn from the observer to the point in the object viewed by the observer.
2. A horizontal line is the distance between the observer and the object.
3. The angle of elevation is the angle formed by the line of sight with the horizontal when it is above the horizontal level. i.e. the angle of elevation is made when we raise our head to look at the object.
4. The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal line when it is below the horizontal line, i.e. the case when we lower our head to look at the object.
5. Angle of elevation and angle of depression is measured with respect to the horizontal line.
6. The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.
7. The height of the object above the water surface is equal to the depth of its image below the water surface.