

Übungsblatt 4

PA 2

Im folgenden sei $S_j, j \in \{0, \dots, 5\}$ das Ereignis, dass aus Urne U_j eine schwarze Kugel gezogen wird.

$$\mathbb{P}(S_0) = 0, \mathbb{P}(S_1) = \frac{1}{5}, \mathbb{P}(S_2) = \frac{2}{5}, \mathbb{P}(S_3) = \frac{3}{5}, \mathbb{P}(S_4) = \frac{4}{5}, \mathbb{P}(S_5) = 1$$

$A = \{\text{Aus einer zufälligen Urne wird eine schwarze Kugel gezogen}\}$

$$\mathbb{P}(A) = \sum_{j=1}^5 \frac{1}{6} \cdot \mathbb{P}(S_j) = \frac{0 + 1 + 2 + 3 + 4 + 5}{6 \cdot 5} = \frac{15}{35} = \frac{3}{7}$$

HA 2

$$(a) \quad \mathbb{P}(G_1) = \frac{5}{12}$$

$$(b) \quad \mathbb{P}(O_1) = \frac{3}{12} = \frac{1}{4}$$

$$(c) \quad \mathbb{P}(G_2|R_1) = \frac{\mathbb{P}(G_2 \cap R_1)}{\mathbb{P}(R_1)} = \frac{\frac{5}{11} \cdot \frac{1}{12}}{\frac{1}{12}} = \frac{5}{132} \cdot \frac{12}{1} = \frac{60}{132} = \frac{5}{11}$$

$$(d) \quad \mathbb{P}(O_2|O_1) = \frac{\mathbb{P}(O_2 \cap O_1)}{\mathbb{P}(O_1)} = \frac{\frac{2}{11} \cdot \frac{3}{12}}{\frac{3}{12}} = \frac{6}{132} \cdot \frac{12}{3} = \frac{72}{396} = \frac{2}{11}$$

$$(e) \quad \mathbb{P}(W_2 \cap O_1) = \frac{3}{11} \cdot \frac{3}{12} = \frac{9}{132}$$

$$(f) \quad \mathbb{P}(W_2) = \frac{5}{12} \cdot \frac{3}{11} + \frac{3}{12} \cdot \frac{3}{11} + \frac{3}{12} \cdot \frac{2}{11} + \frac{1}{12} \cdot \frac{3}{11} = \frac{33}{132}$$

$$(g) \quad \mathbb{P}(O_1|W_2) = \frac{\mathbb{P}(O_1 \cap W_2)}{\mathbb{P}(W_2)} = \frac{\frac{3}{12} \cdot \frac{3}{11}}{\frac{33}{132}} = \frac{3}{11}$$

HA 3

$$\mathbb{P}(A|R) = \frac{\mathbb{P}(A \cap R)}{\mathbb{P}(R)} = \frac{\frac{7}{10} \cdot \frac{1}{2}}{\frac{7}{10} \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{1}{2}} = \frac{\frac{7}{20}}{\frac{8}{20}} = \frac{7}{8}$$

HA 4

$$(a) \quad \mathbb{P}(F|\text{Mo}) = \mathbb{P}(F|\text{Di}) = \frac{3}{100}$$

$$\mathbb{P}(F^c|\text{Mo}) = \mathbb{P}(F^c|\text{Di}) = \frac{97}{100}$$

$$\mathbb{P}(F|\text{Mi}) = \mathbb{P}(F|\text{Do}) = \frac{2}{100}$$

$$\mathbb{P}(F^c|\text{Mi}) = \mathbb{P}(F^c|\text{Do}) = \frac{98}{100}$$

$$\mathbb{P}(F|\text{Fr}) = \frac{5}{100}$$

$$\mathbb{P}(F^c|\text{Fr}) = \frac{95}{100}$$

$$(b) \quad \mathbb{P}(F) = \mathbb{P}(\text{Mo} \cap F) + \mathbb{P}(\text{Di} \cap F) + \mathbb{P}(\text{Mi} \cap F) + \mathbb{P}(\text{Do} \cap F) + \mathbb{P}(\text{Fr} \cap F)$$

$$= \frac{1}{7} \cdot \frac{3}{100} + \frac{1}{7} \cdot \frac{3}{100} + \frac{1}{7} \cdot \frac{2}{100} + \frac{1}{7} \cdot \frac{2}{100} + \frac{1}{7} \cdot \frac{5}{100}$$

$$= \frac{3}{140}$$

$$(c) \quad \mathbb{P}(\text{Fr}|F) = \frac{\mathbb{P}(\text{Fr} \cap F)}{\mathbb{P}(F)} = \frac{\frac{1}{7} \cdot \frac{5}{100}}{\frac{3}{140}} = \frac{420}{2100} = \frac{1}{5}$$