Übungsblatt 4

PA 2

Im folgenden beschreibe $S_j, j \in \{0, \dots, 5\}$ das Ereignis, dass aus Urne U_j eine schwarze Kugel gezogen wird:

$\mathbb{P}(S_0) = 0, \mathbb{P}(S_1) = \frac{1}{5}, \mathbb{P}(S_2) = \frac{2}{5}, \mathbb{P}(S_3) = \frac{3}{5}, \mathbb{P}(S_4) = \frac{4}{5}, \mathbb{P}(S_5) = 1$

$A = \{\text{Aus einer zufälligen Urne wird eine schwarze Kugel gezogen}\}$

$$\mathbb{P}(A) = \sum_{j=1}^{5} \frac{1}{6} \cdot \mathbb{P}(S_j) = \frac{0+1+2+3+4+5}{6 \cdot 5} = \frac{15}{35} = \frac{3}{7}$$

HA 2

(a) $\mathbb{P}(G_1) = \dfrac{5}{12}$

(b) $\mathbb{P}(O_1) = \dfrac{3}{12} = \dfrac{1}{4}$

(c) $\mathbb{P}(G_2|R_1) = \dfrac{\mathbb{P}(G_2 \cap R_1)}{\mathbb{P}(R_1)} = \dfrac{\frac{5}{11} \cdot \frac{1}{12}}{\frac{1}{12}} = \dfrac{5}{132} \cdot \dfrac{12}{1} = \dfrac{60}{132} = \dfrac{5}{11}$

(d) $\mathbb{P}(O_2|O_1) = \dfrac{\mathbb{P}(O_2 \cap O_1)}{\mathbb{P}(O_1)} = \dfrac{\frac{2}{11} \cdot \frac{3}{12}}{\frac{3}{12}} = \dfrac{6}{132} \cdot \dfrac{12}{3} = \dfrac{72}{396} = \dfrac{2}{11}$

(e) $\mathbb{P}(W_2 \cap O_1) = \dfrac{3}{11} \cdot \dfrac{3}{12} = \dfrac{9}{132}$

(f) $\mathbb{P}(W_2) = \dfrac{5}{12} \cdot \dfrac{3}{11} + \dfrac{3}{12} \cdot \dfrac{3}{11} + \dfrac{3}{12} \cdot \dfrac{2}{11} + \dfrac{1}{12} \cdot \dfrac{3}{11} = \dfrac{33}{132}$

(g) $\mathbb{P}(O_1|W_2) = \dfrac{\mathbb{P}(O_1 \cap W_2)}{\mathbb{P}(W_2)} = \dfrac{\frac{3}{12} \cdot \frac{3}{11}}{\frac{33}{132}} = \dfrac{3}{11}$

HA 3

$$\mathbb{P}(A|R) = \frac{\mathbb{P}(A \cap R)}{\mathbb{P}(R)} = \frac{\frac{7}{10} \cdot \frac{1}{2}}{\frac{7}{10} \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{1}{2}} = \frac{\frac{7}{20}}{\frac{8}{20}} = \frac{7}{8}$$

HA 4

(a) $\mathbb{P}(F|\text{Mo}) = \mathbb{P}(F|\text{Di}) = \dfrac{3}{100}$

$\mathbb{P}(F^{\mathsf{c}}|\text{Mo}) = \mathbb{P}(F^{\mathsf{c}}|\text{Di}) = \dfrac{97}{100}$

$\mathbb{P}(F|\text{Mi}) = \mathbb{P}(F|\text{Do}) = \dfrac{2}{100}$

$\mathbb{P}(F^{\mathsf{c}}|\text{Mi}) = \mathbb{P}(F^{\mathsf{c}}|\text{Do}) = \dfrac{98}{100}$

$\mathbb{P}(F|\text{Fr})\dfrac{5}{100}$

$\mathbb{P}(F^{\mathsf{c}}|\text{Fr}) = \dfrac{95}{100}$

(b) $\mathbb{P}(F) = \mathbb{P}(\text{Mo} \cap F) + \mathbb{P}(\text{Di} \cap F) + \mathbb{P}(\text{Mi} \cap F) + \mathbb{P}(\text{Do} \cap F) + \mathbb{P}(\text{Fr} \cap F)$

$= \dfrac{1}{7} \cdot \dfrac{3}{100} + \dfrac{1}{7} \cdot \dfrac{3}{100} + \dfrac{1}{7} \cdot \dfrac{2}{100} + \dfrac{1}{7} \cdot \dfrac{2}{100} + \dfrac{1}{7} \cdot \dfrac{5}{100}$

$= \dfrac{3}{140}$

(c) $\mathbb{P}(\text{Fr}|F) = \dfrac{\mathbb{P}(\text{Fr} \cap F)}{\mathbb{P}(F)} = \dfrac{\frac{1}{7} \cdot \frac{3}{100}}{\frac{3}{140}} = \dfrac{420}{2100} = \dfrac{1}{5}$