CS M51A, Fall 2022, Assignment 1
(Total Mark: 100 points, 10%)

Due: Wed Oct 12, 10:00 AM Pacific Time

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Note: You must complete the assignments entirely on your own, without discussing with others.

1. (2 points) (a) Briefly outline the primary differences between digital and analog systems.

   Digital systems take discrete, non-continuous signals which allow it to process numerical and non-numerical information in a cheaper, easier way. Analog take continuous signals and don't have the mentioned uses.

   (2 points) (b) What are the two types of digital systems?

   1) Combinational system 2) sequential system

2. (6 points) Given the 8-bit binary number 1010 0110, give its decimal equivalent if these eight bits are interpreted as

   (a) an 8-bit unsigned number. (show your steps)

   1 0 1 0 0 1 1 0
   128 64 32 16 8 4 2 1

   $128 + 32 + 4 + 2 = 166$

   (b) an 8-bit signed magnitude number. (show your steps)

   1 0 1 0 0 1 1 0
   − 64 32 16 8 4 2 1

   $-(32+4+2) = -38$

   (c) an 8-bit 2's complement number.

   1010 0110
   0101 1001 flip
   0101 1010 +1
   1101 1010 msb = 1
   (−) 64 32 16 8 4 2 1

   $-(64+16+8+2) = -90$

3. (4 points) Two's Complement

   (a) Write 48 in two's complement representation.

   0110000

   (b) Sign extend the number in part (a) to 8 bits.

   0011 0000

   (c) Write -23 in two's complement representation.

   10111 → 01000 → 01001 → 101001

   (d) Sign extend the number in part (c) to 8 bits.

   1110 1001

4. (8 points) Add the following pairs of 8-bit two's complement binary numbers, giving a 8-bit result (i.e., throw away the carry-out). Also give the signed decimal value of the the 8-bit result. Note whether or not an overflow occurred for any addition.

```
                          1011 1111    -65
                         +0100 0001     65
                         ----------
2's Complement Binary:    00000000
       Signed Decimal:    0
            Overflow?     N
 Is the sum correct?      Y
```

```
                          0100 1110    78
                         +0010 1111    47
                         ----------
2's Complement Binary:    01111101
       Signed Decimal:    125
            Overflow?     N
 Is the sum correct?      Y
```

```
                          1010 1100    -84
                         +1011 1101    -67
                         ----------
2's Complement Binary:    01101001
       Signed Decimal:    105
            Overflow?     Y
 Is the sum correct?      N
```

```
                          0011 1001    57
                         +1011 0101    -75
                         ----------
2's Complement Binary:    11101110
       Signed Decimal:    -18
            Overflow?     N
 Is the sum correct?      Y
```

5. (4 points) For the following high-level specification, determine the output in both decimal and 5-bits binary.

   ↖ unsigned for $x \geq 0$

   - Input $x \in \{0, 1, 2, 3\}$
   - Function $y(x) = x^3 + 2$

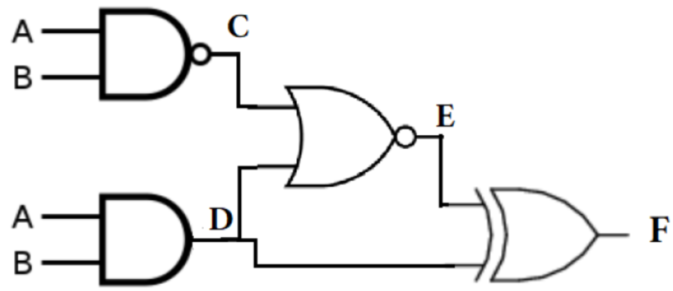
   $y(0) = 2 \longrightarrow 00010$

   $y(1) = 3 \longrightarrow 00011$

   $y(2) = 10 \longrightarrow 01010$

   $y(3) = 29 \longrightarrow 11101$

6. (a) (8 points) Derive the truth table for the following system, filling in the intermediate gate outputs in the table below.



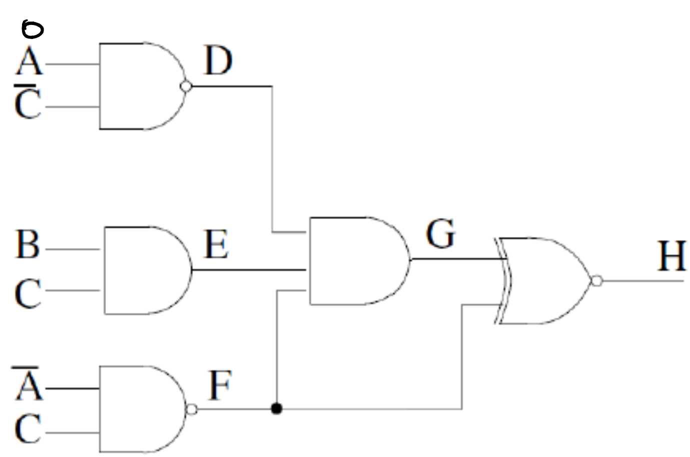
A	B	C	D	E	F
0	0	1	0	0	0
0	1	1	0	0	0
1	0	1	0	0	0
1	1	0	1	0	1

(b) (4 points) State a logical expression for F in terms of A and B. Simplify it as much as possible.

upon seeing the inputs and outputs, we can deduce

$F = AB$

7. (a) (10 points) Complete the truth table table for H in the following system, giving the truth table values for the internal variables D, E, F, and G as intermediate steps.



A	B	C	D	E	F	G	H
0	0	0	1	0	1	0	0
0	0	1	1	0	0	0	1
0	1	0	1	0	1	0	0
0	1	1	1	1	0	0	1
1	0	0	0	0	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

(b) (4 points) Give boolean formulas for G and H in terms of A, B, and C. Then try to reduce and simplify these functions as much as possible.

$$F = ((AC')'(BC)(A'C)')(A'C) + ((AC')'(BC)(A'C)')'(A'C)'$$

$$= (A'C)((A'+C)(BC)(A+C')) + ((A'+C)(BC)(A+C'))'(A+C')$$

$$= (A'C)((A'BC+BCC)(A'BC+BCC)') + ((A'BC+BCC)(A+C'))'(A+C')$$

$$= (ABC')(BC)(A\cancel{B}C)'(A'C) + (A'B\cancel{C}CA + A'B\cancel{C}C'C + BCCA + BC\cancel{C}C')'(A+C')$$

$$= A\cancel{A'}B\cancel{B'}C\cancel{C'}C + (ABC)'(A+C')$$

$$= 0 + (A'+B'+C')(A+C')$$

$$= \cancel{A'A} + B'A + C'A + A'C' + B'C' + C'C' = C'(\underbrace{1+A+A'}_{1}) + AB' = C'+AB'$$

$F = C'+AB'$

8. Using boolean algebra, Simplify the following expression as much as possible.

(a) (4 points) F= A + B + B'C' + A'B'+ A'B'C'

$$= A+B + B'C' + (A+B)' + A'B'C'$$

$$= A+B + (A+B)' + B'C'(1+A')$$

$$= 1 + B'C'(1+A')$$

$$\uparrow$$

1 OR ____ $\Rightarrow$ $F=1$

(b) (4 points) F = A’B(D’ + C’D) + B(A + A’CD)

$= B(A'(D'+C'D) + A + A'CD)$

$= B(A'(D'+C') + A + CD)$

$= B(A'D' + A'C' + A + CD)$

$= B(A'(CD)' + A + CD)$

$= B(A + (CD)' + CD)$

$= B(A + \underbrace{1}_{1})$

$= B$

$F = B$

(c) (4 points) F= A’ + A(A’B + B’C)’

$= A' + A(A'B + B'C)'$

$= A' + A((A+B')(B+C'))$

$= A' + A(AB + AC' + \cancel{BB'}^{0} + B'C')$

$= A' + AAB + AAC' + AB'C'$

$= A' + AB + AC' + AB'C'$

$A' + A(B + B'C') + AC'$

$= A' + C' + A(B + C')$

$= A' + C' + AB + AC'$

$F = A' + B + C'$

(d) (4 points) F= (AB + C)(A’+B’)(B+AC)’

$= (\cancel{AA'}^{0}B + CA' + \cancel{ABB'}^{0} + CB')(B+AC)'$

$= (CA' + CB')(B'(A'+C'))$

$= (CA' + CB')(B'A' + B'C')$

$= \cancel{AA'}^{0}B'C + \cancel{CA'B'C'}^{0} + CB'B'A' + \cancel{CC'B'B'}^{0}$

$= CB'B'A'$

$F = A'B'C$

9. (a) (4 points) For a system with two inputs (A,B), convert (A+B’)·(A’+B) to a sum of minterms expression.

$\cancel{AA'} + B'A' + AB + \cancel{BB'}$

$B'A' + AB$

(b) (4 points) For a system with three inputs (A,B,C) Convert A’B’C + A’BC + AB’C + ABC to a product of maxterms expression.

$A'B'C + A'BC + ABC' + ABC = A'(B'C + BC) + A(B'C + BC)$

$= (B'C + BC)(A + A')$

$= (C\underbrace{(B+B')}_{1})\underbrace{(A+A')}_{1} = C$

10. Design a system that takes as input 4 bits (A1 A0 B1 B0) that is treated as two separate 2 bit integer values A: A1A0 and B: B1B0. The output F of the circuit will

be a 1 if the value of A is equal or greater than the value of **B** (i.e. $A \geq B$) and 0 otherwise.

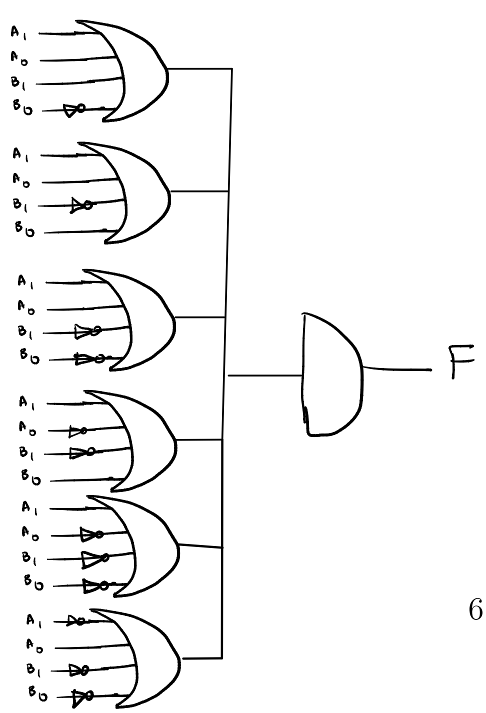
(a) (6 points) First fill in the truth table below and implement the output function F.

	A_1	A_0	B_1	B_0	F
min	0	0	0	0	1
max	0	0	0	1	0
	0	0	1	0	0
	0	0	1	1	0
min	0	1	0	0	1
	0	1	0	1	1
max	0	1	1	0	0
	0	1	1	1	0
min	1	0	0	0	1
	1	0	0	1	1
	1	0	1	0	1
max	1	0	1	1	0
min	1	1	0	0	1
	1	1	0	1	1
	1	1	1	0	1
	1	1	1	1	1

(b) (3 points) Give the product of maxterms expression for F

$$F = (A_1 + A_0 + B_1 + B_0') \cdot (A_1 + A_0 + B_1' + B_0) \cdot (A_1 + A_0 + B_1' + B_0') \cdot (A_1 + A_0' + B_1' + B_0) \cdot (A_1 + A_0' + B_1' + B_0') \cdot (A_1' + A_0 + B_1' + B_0)$$

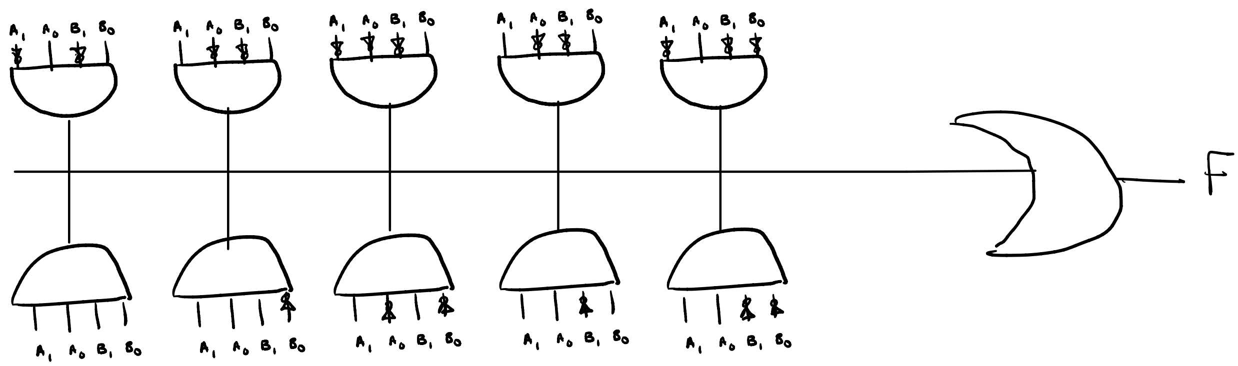
(c) (3 points) Draw the gate symbol design that implements F from part (b)



(d) (3 points) Give the sum of minterms expression for F.

$F = (A_1' A_0' B_1' B_0) + (A_1' A_0 B_1' B_0) + (A_1 A_0' B_1' B_0) + (A_1 A_0' B_1' B_0') + (A_1 A_0 B_1' B_0') + (A_1' A_0 B_1' B_0) + (A_1 A_0' B_1' B_0) + (A_1 A_0 B_1' B_0)$
$+ (A_1 A_0 B_1 B_0') + (A_1 A_0 B_1 B_0)$

(e) (3 points) Draw the gate symbol design that implements F from part (d)



(f) (3 points) Simplify the expression for F as much as possible (It does not need to be in the form of sum of minterms or product of maxterms).

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(g) (3 points) Draw the gate symbol design that implements F from part (f)

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10 f)

$F = (A_1 + A_0 + B_1 + B_0') \cdot (A_1 + A_0 + B_1' + B_0) \cdot (A_1 + A_0 + B_1' + B_0') \cdot (A_1 + A_0' + B_1' + B_0) \cdot (A_1 + A_0' + B_1' + B_0') \cdot$
$(A_1' + A_0 + B_1' + B_0)$

$= (\cancel{A_1 A_1} + \cancel{A_1 A_0} + A_1 \cancel{B_1'} + \cancel{A_1 B_0} + \cancel{A_0 A_1} + \cancel{A_0 A_0} + A_0 B_1' + A_0 B_0 + B_1 A_1 + B_1 A_0 + \cancel{B_1 B_1'} + B_1 B_0 + B_0' A_1 + B_0' A_0 + B_0' B_1' + \cancel{B_0' B_0})$

$\cdot (\cancel{A_1 A_1} + A_1 A_0' + A_1 B_1' + A_1 B_0 + A_0 A_1 + \cancel{A_0 A_0'} + A_0 B_1' + A_0 B_0 + B_1' A_1 + B_1' A_0 + \cancel{B_1' B_1} + B_1' B_0 + B_0' A_1 + B_0' A_0' + B_0' B_1' + \cancel{B_0' B_0})$

$\cdot (\cancel{A_1 A_1'} + A_1 A_0 + A_1 B_1' + A_1 B_0 + \cancel{A_0' A_1'} + \cancel{A_0' A_0} + A_0' B_1' + A_0' B_0 + B_1' A_1' + B_1' A_0 + \cancel{B_1' B_1} + B_1' B_0 + B_0' A_1' + B_0' A_0 + B_0' B_1' + \cancel{B_0' B_0})$

$= \ldots$

$= A_1 A_0 + A_1 B_1 + A_1 A_0' + A_1' B_1 + A_1' B_1' + A_0' B_0 + A_0' B_1' + A_1' A_0$

10 g)