
CS 161: Fundamentals of Artificial Intelligence

Fall 2022 – Assignment 5

Questions

This will be a non-coding homework. Please upload a digital copy of your solution on BruinLearn. Submitted files can be in either PDF or plain text. You may also submit a scanned PDF of a handwritten solution, but please ensure that the scanned file is *clearly legible*.

1. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

- (a) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
- (b) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$
- (c) $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

Justify your answer using truth tables.

2. Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is mortal and it is a mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

- (a) Represent the above information using a propositional logic knowledge base (a set of sentences in propositional logic). Make sure to clearly define all propositional symbols (variables) first.
- (b) Convert the knowledge base into CNF.
- (c) (i) Is it possible to derive from the knowledge base that the unicorn is **mythical**? (ii) How about **magical**? (iii) **Horned**? Justify your answers using resolution.

3. Consider the following:

An oil well may be drilled on Mr. Y's farm in Texas. Based on what has happened to similar farms, we judge the probability of only oil being present to be .5, the probability of only natural gas being present to be .2, and the probability of neither being present to be .3. Oil and gas never occur together. If oil is present, a geological test will give a positive result with probability .9; if natural gas is present, it will give a positive result with probability .3; and if neither are present, the test will be positive with probability .1.

Suppose the test comes back positive. What's the probability that oil is present?

1. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

- $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
- $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$
- $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

Justify your answer using truth tables.

Let S denote smoke, F denote fire, and H denote heat.

a)	S	F	$S \Rightarrow F$	$\neg S \Rightarrow \neg F$	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
	T	T	T	T	T
	T	F	F	T	T
	F	T	T	F	F → invalid ⇒ Neither
	F	F	T	T	T

b)	S	F	H	$S \Rightarrow F$	$S \vee H$	$(S \vee H) \Rightarrow F$	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
	T	T	T	T	T	T	T
	T	T	F	T	T	T	T
	T	F	T	F	T	F	T
	T	F	F	F	T	F	T
	F	T	T	T	T	T	T
	F	T	F	T	F	T	T
	F	F	T	T	T	F	F → invalid ⇒ Neither
	F	F	F	T	F	T	T

c)	S	F	H	$S \wedge H$	$(S \wedge H) \Rightarrow F$	$S \Rightarrow F$	$H \Rightarrow F$	$(S \Rightarrow F) \vee (H \Rightarrow F)$	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
	T	T	T	T	T	T	T	T	T
	T	T	F	F	T	T	T	T	T
	T	F	T	F	F	F	F	F	T
	T	F	F	F	T	F	T	T	T
	F	T	T	F	T	T	T	T	T
	F	T	F	F	T	T	T	T	T
	F	F	T	F	T	F	T	T	T
	F	F	F	F	T	T	T	T	T

} all T → valid

2. Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is mortal and it is a mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

- Represent the above information using a propositional logic knowledge base (a set of sentences in propositional logic). Make sure to clearly define all propositional symbols (variables) first.
- Convert the knowledge base into CNF.
- (i) Is it possible to derive from the knowledge base that the unicorn is mythical? (ii) How about magical? (iii) Horned? Justify your answers using resolution.

a)

$$P_1 : \text{mythical} \Rightarrow \neg \text{mortal}$$

$$P_2 : \neg \text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal})$$

$$P_3 : (\neg \text{mortal} \vee \text{mammal}) \Rightarrow \text{horned}$$

$$P_4 : \text{horned} \Rightarrow \text{magical}$$

b) $P_1 : \text{mythical} \Rightarrow \neg \text{mortal} \rightarrow \neg \text{mythical} \vee \neg \text{mortal}$

$$P_2 : \neg \text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal}) \rightarrow (\text{mythical} \vee \text{mammal}) \wedge (\neg \text{mythical} \vee \text{mortal})$$

$$P_3 : (\neg \text{mortal} \vee \text{mammal}) \Rightarrow \text{horned} \rightarrow (\text{horned} \vee \text{mortal}) \wedge (\text{horned} \vee \neg \text{mammal})$$

$$P_4 : \text{horned} \Rightarrow \text{magical} \rightarrow \neg \text{horned} \vee \text{magical}$$

↓

CNF :

$$P_1 : \neg \text{mythical} \vee \neg \text{mortal}$$

$$P_2 : (\text{mythical} \vee \text{mammal}) \wedge (\neg \text{mythical} \vee \text{mortal})$$

$$P_3 : (\text{horned} \vee \text{mortal}) \wedge (\text{horned} \vee \neg \text{mammal})$$

$$P_4 : \neg \text{horned} \vee \text{magical}$$

c) Given the set from (b), we have the following knowledge base

$$KB = P_1 \wedge P_2 \wedge P_3 \wedge P_4$$

i) mortal $\Rightarrow \neg \text{mythical}$ from P_1

$$\text{mortal} \Rightarrow \text{mortal} \wedge \text{mortal}$$
 from P_2

$$\neg \text{mortal} \vee \text{mortal} \wedge \text{mortal}$$

$$\neg \text{mortal} \vee \text{mortal}$$

$$\neg \text{mortal} \vee \text{mammal} \Rightarrow \text{horned} P_3$$

$$\text{horned} \Rightarrow \text{magical} P_4$$

Can derive magical and mortal from KB above,

but not mythical

3. Consider the following:

An oil well may be drilled on Mr. Y's farm in Texas. Based on what has happened to similar farms, we judge the probability of only oil being present to be .5, the probability of only natural gas being present to be .2, and the probability of neither being present to be .3. Oil and gas never occur together. If oil is present, a geological test will give a positive result with probability .9; if natural gas is present, it will give a positive result with probability .3; and if neither are present, the test will be positive with probability .1.

Suppose the test comes back positive. What's the probability that oil is present?

$$P(\text{only oil}) = 0.5$$

$$P(\text{positive oil}) = 0.9$$

$$P(\text{only natural gas}) = 0.2$$

$$P(\text{positive gas}) = 0.3$$

$$P(\text{neither}) = 0.3$$

$$P(\text{positive neither}) = 0.1$$

$$\begin{aligned} \text{Probability oil is present is } P_{\text{oil}} &= \frac{P_{\text{positive oil}}}{P_{\text{positive}}} = \frac{P(\text{only oil}) \cdot P(\text{positive oil})}{P(\text{only oil}) \cdot P(\text{positive oil}) + P(\text{only gas}) \cdot P(\text{positive gas}) + P(\text{neither}) \cdot P(\text{positive neither})} \\ &= \frac{0.5 \cdot 0.9}{(0.5 \cdot 0.9) + (0.2 \cdot 0.3) + (0.3 \cdot 0.1)} \\ &= \frac{0.45}{0.54} \\ &= 0.833 \end{aligned}$$

$$\text{Probability oil is present} = 0.833$$