Juicy Profits: Finding the Sweet Spot in Chilled Orange Juice Pricing

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Retailers face a classic dilemma: if they slash prices significantly, will it lead to enough extra sales to make up for the loss in revenue? To tackle this issue, we aim to build a model for sales in the refrigerated orange juice category, focusing on whether a big price drop would result in a proportional increase in quantity sold.

For a practical approach, we split the dataset of weekly store scanner data for each item in the category into training and validation samples, reserving 40% of the observations for comparing our models.

Retailers commonly use a cost-plus rule, a straightforward pricing strategy where the optimal price is calculated using a log-log demand model. In our case, we estimated this model, utilizing the natural logarithm of the price of each product to predict the natural logarithm of the total number of units (in ounces) sold of each product during the week (which we will refer to as movement).

Since elasticity is determined by dividing the percentage change in quantity by the percentage change in price, we can determine each product's price elasticity using this log-log model. We applied this model (ln(move) = b0 + b1 * ln(price) + e) (Appendix 1.1) to products 1, 4, 5, and 11 (Tropicana Premium, Tropicana, Minute Maid, and Dominicks), yielding price elasticities of -2.686801, -3.894563, -3.377983, and -3.375459, respectively (Appendix 1.2).

Subsequently, we computed the optimal prices for these products using the wholesale costs from store #2 in week #93. Based on the same log-log demand models (Appendix 1.1) and the given profit margin for each product, we can calculate the optimal price to be the wholesale cost divided by (1 + 1/b1). For instance, with b1 = -4, the optimal price is cost * (4/3), representing a 33% markup over cost.

The calculated optimal prices were $3.27, $2.06, $2.21, and $1.45, for each product 1, 4, 5, and 11 respectively (Appendix 1.3). Comparing these optimal prices with the actual prices charged during the same week at store #2 (Appendix 1.3) revealed varying percentages of difference (Appendix 1.3). For instance, product 1 had an 8.84% higher price than the calculated optimal, while product 4 was 17.32% higher. Product 5 was 7.66% higher and product 11 was 12.03% lower. These differences highlight the deviations between the cost-plus-derived optimal prices and the actual prices, raising questions about the effectiveness of our simple model. Thus, we decided to estimate five different predictive models for product #1 and evaluate them comparatively to find a better solution.

The first model, the “pooled” model (Appendix 2.1), aims to predict product 1 price movements by considering the prices of products 4, 5, and 11, the in-store promotion index, and features of product 1. These variables are taken into account as they are the closest substitutes for product 1 (i.e., they are all 64oz, and are mass-sold orange juices). The second and third models, the "store" (Appendix 2.2) and "mixed" (Appendix 2.3) models respectively, take into account the store-specific prices and indices. In addition, the decision tree and random forest models were built as potentially improved models. They are good at handling both numerical (i.e., price and demand movements) and non-numerical data (i.e., store numbers).

The mean error, root-mean-squared error, and mean-absolute deviation were computed to evaluate the model results, along with a plot of the histogram and time series of the residuals for each model (Appendix 3). In evaluating the model performance, the initial hypothesis was that the "mixed" model, which was a combination of the "pooled" and "store" models, would outperform both the models with the lowest errors. However, the "mixed" model has a higher error, and the residuals are not standard normal, indicating that the errors are neither random nor independent. Based on the model results, the "store" model generated the lowest error metrics, with the residuals randomly centered around 0, and following a normal distribution. The second and third best models would be the decision tree and random forest, respectively, due to the relatively lower error metrics and the residuals following a normal distribution. However, do note that the random forest generated an upward trending residual, indicating a problem as errors may not be random.

Our models seem adequate, but we need to check if they line up with economic expectations for validity. Specifically, they should show a negative correlation between price and quantity and confirm whether an increase in a competing product's price results in more sales for our product. These are crucial benchmarks for our models' evaluation.

When looking at each model's adequacy, we made assumptions about predictors and variables. We assumed that the only things affecting demand are the product price, the price of close competitors, and promotions. We also treated each product's prices equally. Based on these assumptions, the "store" model seems to be the best choice, backed by our previous evaluation of residuals and the sizes and signs of the coefficients from our models (Appendix 4.1). This choice fits with economic and marketing theories, where a price increase is expected to decrease quantity. The idea that products act as substitutes also mostly holds in this model - if the price of a competing product like Tropicana Premium 64oz goes up, it means more sales for Minute Maid 64oz. This supports the idea that customers tend to pick a store to buy orange juice and don't shop around much. This points to orange juice having low price elasticity, mainly competing with products in the same store.

We then solved for the optimal price, maximizing profit, for Tropicana Premium 64 Oz (product #1) using each of our five models. We chose to focus on the first five listed stores, store numbers 2, 5, 8, 9, and 12.

We used Excel Solver (Appendix 5.1) to find the “pooled,” “store,” and “mixed” models’ optimized prices for each store (Appendix 5.2). We used store number 2 as the baseline. The “pooled” model in Excel was a single model for all stores. The “store” model in Excel added store-specific coefficients to the baseline model to create a separate model for each store. The “mixed” model, a hierarchical model in which the intercept and log price variable is allowed to change, in Excel utilized the fixed coefficient and the random coefficient to calculate the store-specific coefficient to create a mixed model by store. We used grid search to find the optimized price for the decision tree model, as this was the better of the two improved models we tried based on the residuals. With this method, we went through a range of price values in small increments and found the corresponding sales quantity predicted from our fitted decision tree model. Then we used the price, wholesale cost, and quantity to calculate profit. We sorted for the highest profit and found the corresponding price.

We found that the optimal price for product 1, based on the “pooled” model was $3.85 ($0.06/oz), making the profit $121.73. The “store” model showed some variations among the stores, with stores 2, 5, 8, 9, and 12 having optimal prices of $3.85, $3.17, $2.84, $3.00, and $3.21 respectively. On the other hand, the “mixed” model showed very little variation among stores, with optimal prices for product 1 for all five stores around $3.37. Finally, our decision tree model found the profit to be increasing as we increased the price since the predicted quantity became constant after a certain value. (Appendix 5.3) We took another approach to improve our model. The evaluation scores indicated that the model is not superior compared to other models. This could be attributed to the demand model predominantly displaying a linear relationship, or it might be due to the dataset being too small for a complex model like DecisionTree, making it susceptible to overfitting.

We ran a linear regression over all the variables to identify the most significant ones and then considered just those for our improved regression model. This model gives a huge drop in errors as compared to the pooled model, as well as a small drop from the store model (Appendix 5.4). The significant variables came out to be lpr1, lpr5, lpr12, afeat1, and adisp1. The optimal price using the improved linear regression for product 1, store 2 was 0.060127/oz equivalently $3.84/unit. (Appendix 5.5)

Based on this information, we suggest to managers that our "store" model is a better choice compared to the common cost-plus rule. This model offers more flexibility in modeling, making it a better fit. One key advantage is its ability to factor in store-specific elements. The model creates separate estimates for each store, allowing for the inclusion of factors that influence pricing and demand. This approach captures variations in consumer behavior, preferences, and local market dynamics, which a generalized cost-plus rule might miss. Different store locations face distinct market conditions, competition, and consumer demographics. The "store" model adapts to these differences, providing a more tailored and accurate representation of pricing dynamics within each specific market. Managers should consider this pragmatic and adaptive approach to better understand the factors shaping pricing strategies across different store locations. We have also shown quantitatively that the “store” model has a better mean error and root mean squared error compared with the cost-plus rule (Appendix 6.1). Additionally, the optimal price for store number two found by the store model is closer to the actual price than the optimal price for store number two found by the cost-plus rule (Appendix 6.2).

Finally, we applied our best model, the “store” model, for products 4, 5, and 11 (Tropicana, Minute Maid, and Dominick’s Store Brand, respectively) (Appendix 7.1). We optimized prices for each of these products at the remaining of our five chosen stores (Appendix 7.2). We also optimized the total profits for the category, using the same model. If we just consider store number two, we should set the prices to maximize the total profits for this category at $3.85, $2.30, $2.27, and $1.73 for products 1, 4, 5, and 11, respectively.

We can observe a few trends from our results. According to this model, Tropicana Premium should be the most expensive product across all stores, while Dominick’s Store Brand is the most budget-friendly, as one might expect for a store brand. Tropicana and Minute Maid, as substitute products, should be priced pretty similarly, as reflected in our model. Out of the five stores we analyzed, Store 8 was priced relatively lower across all products, while Stores 2 and 12 were priced relatively higher. As the pricing manager at Dominick's, our approach involves implementing store-specific pricing strategies based on observed trends and utilizing optimal predicted product values from the "store" model. Specifically, for store number two, our focus is on setting prices that maximize overall profits for the category. Dominick's primary concern lies in optimizing total profits for the store rather than individual brands. This ensures a holistic approach to pricing that aligns with Dominick's overarching financial objectives.

In conclusion, tailoring pricing decisions to store-specific dynamics enhances our ability to achieve the broader goal of maximizing total profits for Dominick's.

Appendices

Appendix 1.1

Simple linear models for predicting log movement based on log price (log-log models) for products 1, 4, 5, and 11 (Tropicana Premium, Tropicana, Minute Maid, and Dominicks, respectively).

```
(mdl1 <- lm(lmv1 ~ lpr1, data = df_rfjdata[lvec_trainsample, ]))
(mdl4 <- lm(lmv4 ~ lpr4, data = df_rfjdata[lvec_trainsample, ]))
(mdl5 <- lm(lmv5 ~ lpr5, data = df_rfjdata[lvec_trainsample, ]))
(mdl11 <- lm(lmv11 ~ lpr11, data = df_rfjdata[lvec_trainsample, ]))
```

Appendix 1.2

Products 1, 4, 5, and 11 (Tropicana Premium, Tropicana, Minute Maid, and Dominicks, respectively) and their calculated price elasticities:

Product	Elasticity
1	-2.686801
4	-3.894563
5	-3.377983
11	-3.375459

Appendix 1.3

Products 1, 4, 5, and 11 (Tropicana Premium, Tropicana, Minute Maid, and Dominicks, respectively) and their calculated optimal prices, actual prices, and the percentage difference between their optimal and actual prices for comparison:

Product	Optimal_Price	Actual_Price	Percentage_Difference
1	3.272483	3.59	8.844498
4	2.058718	2.49	17.320544
5	2.206889	2.39	7.661554
11	1.445179	1.29	-12.029381

Appendix 2.1

“Pooled” Model:
lmv1 = lpr1 + lpr4 + lpr5 + lpr11 + afeat1 + adisp1

Model coefficients:

```
Coefficients:
(Intercept)         lpr1         lpr4         lpr5        lpr11       afeat1       adisp1
    4.21785     -2.35948      0.33193      0.27245      0.15332      0.57595      0.03638
```

Appendix 2.2

“Store” Model:
lmv1 = store + store * lpr1 + store * lpr4 + store * lpr5 + store * lpr11 + store * afeat1 + store * adisp1

Appendix 2.3

“Mixed” Model:
lmv1 = lpr1 + (1 | store) + (lpr1 | store) + lpr4 + lpr5 + lpr11 + afeat1 + adisp1

Appendix 2.4

Decision Tree Model:
lmv1 = lpr1 + lpr4 + lpr5 + lpr11 + afeat1 + adisp1

Appendix 2.5

Random Forest Model:
lmv1 = lpr1 + lpr4 + lpr5 + lpr11 + afeat1 + adisp1

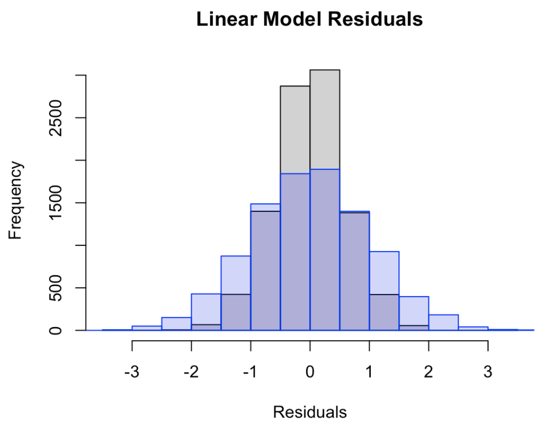
Appendix 3

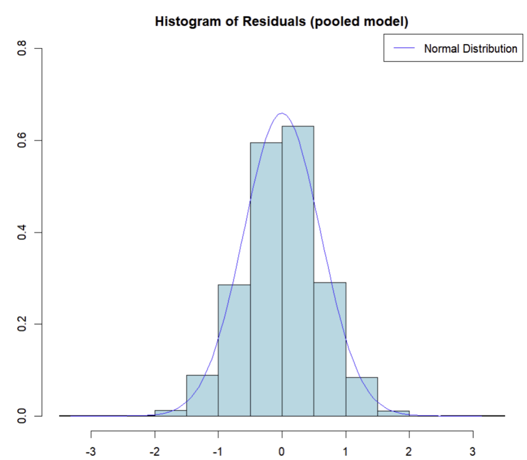
1) “Pooled” Model

a) Error metrics*

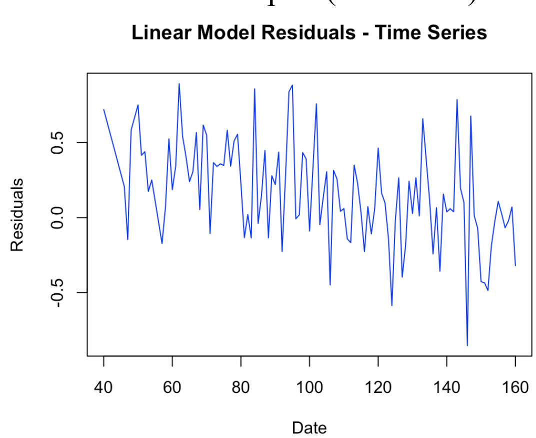
```
 Descriptive statistics by group
INDICES: 1
        vars    n mean   sd   min   max range   se
err1       1 5824 0.00 0.60 -3.32  3.14  6.46 0.01
sqerr1     2 5824 0.37 0.59  0.00 11.04 11.04 0.01
abserr1    3 5824 0.47 0.38  0.00  3.32  3.32 0.00
----------------------------------------------------------------
INDICES: 2
        vars    n mean   sd   min  max range   se
err1       1 3873 0.00 0.61 -2.22 2.22  4.44 0.01
sqerr1     2 3873 0.37 0.54  0.00 4.95  4.95 0.01
abserr1    3 3873 0.47 0.38  0.00 2.22  2.22 0.01
```

b) Residual histogram





c) Time-series residual plot (for store 2)

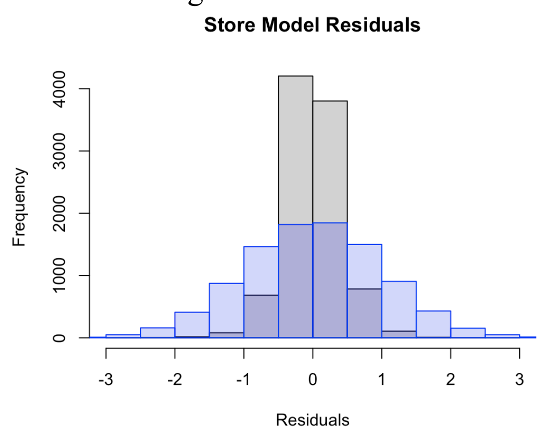


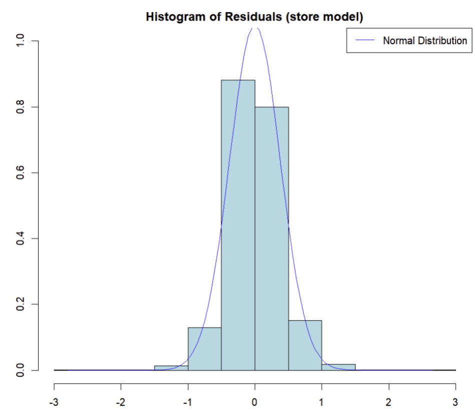
2) “Store” Model

a) Error metrics*

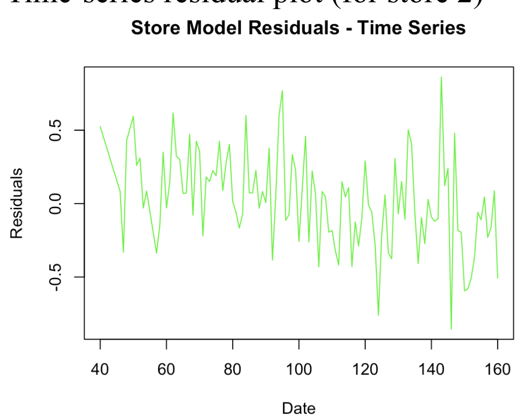
```
 Descriptive statistics by group
INDICES: 1
        vars    n mean   sd   min  max range se
err2       1 5824 0.00 0.38 -2.79 2.65  5.45  0
sqerr2     2 5824 0.14 0.32  0.00 7.79  7.79  0
abserr2    3 5824 0.28 0.26  0.00 2.79  2.79  0
----------------------------------------------------------------
INDICES: 2
        vars    n mean   sd   min  max range   se
err2       1 3873 0.00 0.42 -1.92 2.07  3.99 0.01
sqerr2     2 3873 0.18 0.33  0.00 4.27  4.27 0.01
abserr2    3 3873 0.31 0.28  0.00 2.07  2.07 0.00
```

b) Residual histogram





c) Time-series residual plot (for store 2)

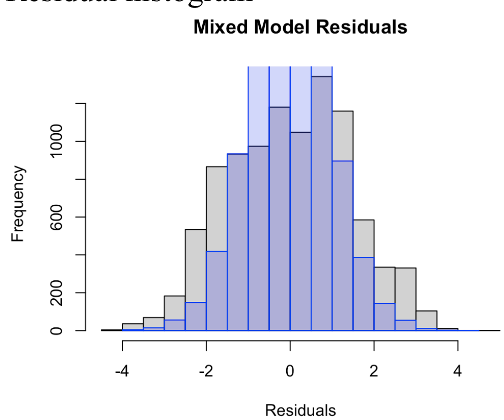


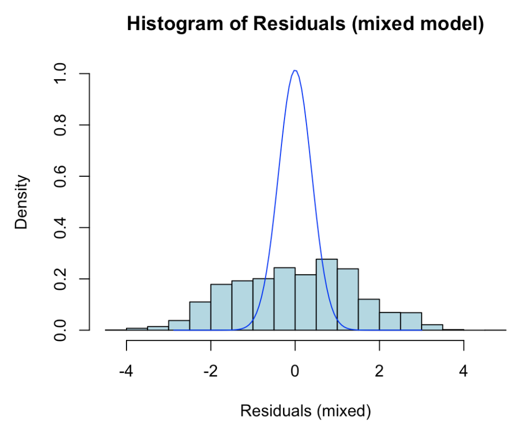
3) “Mixed” Model

a) Error metrics*

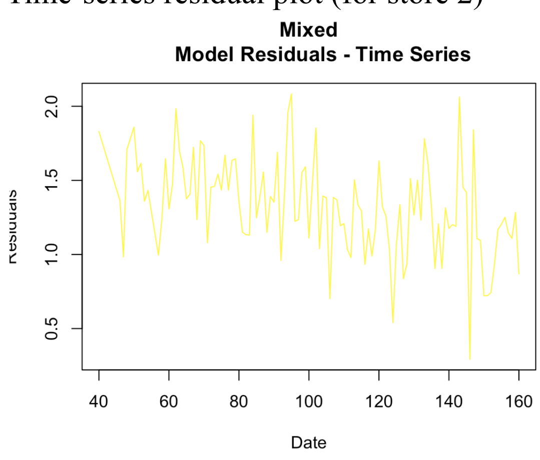
```
INDICES: 1
        vars    n mean   sd   min   max range   se
err3       1 5824 0.01 1.42 -4.37  4.63  9.00 0.02
sqerr3     2 5824 2.03 2.45  0.00 21.40 21.40 0.03
abserr3    3 5824 1.18 0.80  0.00  4.63  4.63 0.01
------------------------------------------------------
INDICES: 2
        vars    n  mean   sd   min   max range   se
err3       1 3873 -0.02 1.43 -4.27  3.74  8.00 0.02
sqerr3     2 3873  2.06 2.48  0.00 18.20 18.20 0.04
abserr3    3 3873  1.19 0.80  0.00  4.27  4.27 0.01
```

b) Residual histogram





c) Time-series residual plot (for store 2)

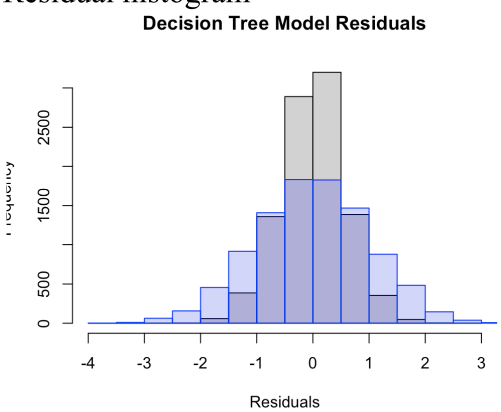


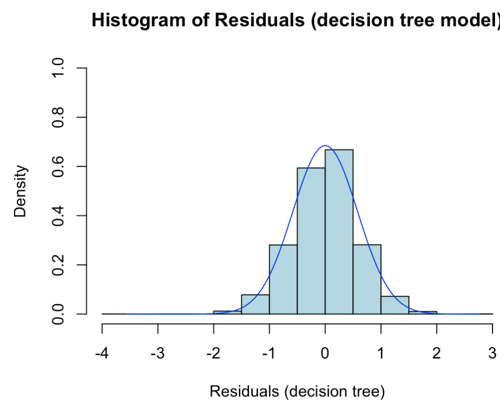
4) Decision Tree Model

a) Error metrics*

```
INDICES: 1
        vars    n mean   sd   min   max range   se
err4       1 5824 0.00 0.58 -3.56  2.77  6.33 0.01
sqerr4     2 5824 0.34 0.56  0.00 12.70 12.70 0.01
abserr4    3 5824 0.45 0.36  0.00  3.56  3.56 0.00
----------------------------------------------------
INDICES: 2
        vars    n mean   sd   min  max range   se
err4       1 3873 0.00 0.58 -2.36 2.26  4.62 0.01
sqerr4     2 3873 0.34 0.51  0.00 5.57  5.57 0.01
abserr4    3 3873 0.46 0.36  0.00 2.36  2.36 0.01
```

b) Residual histogram





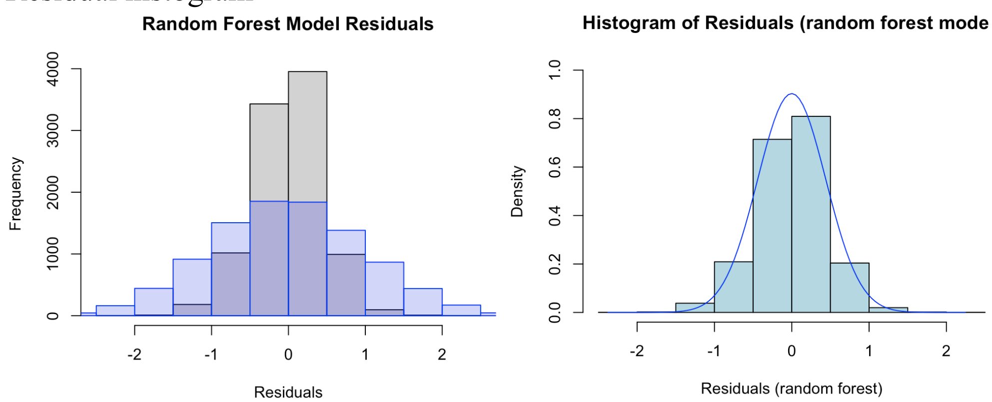
c) Time-series residual plot (for store 2)

5) Random Forest Model

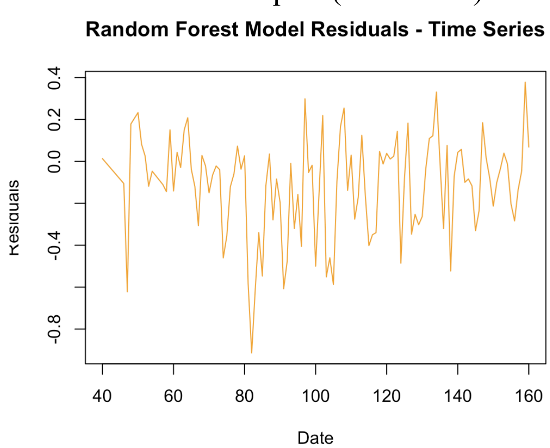
a) Error metrics*

```
 Descriptive statistics by group
INDICES: 1
        vars    n mean   sd   min  max range   se
err5       1 5824 0.00 0.40 -2.45 2.23  4.68 0.01
sqerr5     2 5824 0.16 0.29  0.00 6.00  6.00 0.00
abserr5    3 5824 0.31 0.25  0.00 2.45  2.45 0.00
----------------------------------------------------
INDICES: 2
        vars    n mean   sd  min  max range   se
err5       1 3873 0.00 0.50 -2.1 2.11  4.21 0.01
sqerr5     2 3873 0.25 0.37  0.0 4.45  4.45 0.01
abserr5    3 3873 0.39 0.31  0.0 2.11  2.11 0.00
```

b) Residual histogram



c) Time-series residual plot (for store 2)



* Note that “Indices 1” refers to the training data set and “Indices 2” refers to the hold-out data set.

Appendix 4.1

Model coefficients:

"Pooled"

```
Coefficients:
(Intercept)         lpr1         lpr4         lpr5        lpr11       afeat1       adisp1
    4.21785     -2.35948      0.33193      0.27245      0.15332      0.57595      0.03638
```

"Store"

```
Coefficients:
    (Intercept)         store5         store8         store9        store12
       4.170150      -1.360652      -3.261717      -1.632401      -3.165484
```

```
     store130       store131       store132       store134       store137           lpr1
    -2.930014      -0.257858      -4.894293      -1.314110      -1.116669      -2.147600
         lpr4           lpr5          lpr11         afeat1         adisp1    store5:lpr1
     0.305992       0.138121       0.064135       0.384805       0.070030      -0.702815
  store8:lpr1    store9:lpr1   store12:lpr1   store14:lpr1   store18:lpr1   store21:lpr1
    -1.495497      -1.039237      -0.638342       0.135317       0.259343      -1.295375

store132:lpr1  store134:lpr1  store137:lpr1    store5:lpr4    store8:lpr4    store9:lpr4
    -1.149660      -0.258429      -0.618372       0.019047       0.080519       0.247649
 store12:lpr4   store14:lpr4   store18:lpr4   store21:lpr4   store28:lpr4   store32:lpr4
    -0.320140      -0.410778      -0.087304       0.299889      -0.215638       0.049739
```

(showing just a few coefficients)

"Mixed"

```
  (Intercept)     lpr1      lpr4      lpr5     lpr11    afeat1       adisp1
5    3.105728 -2.67831 0.2365075 0.2187653 0.1129726 0.5298852 -0.004535555
```

No direct interpretable concept of coefficients for Decision Tree or Random Forest models

Appendix 5.1

Excel Screenshot of each model:

“Pooled” Model

Pooled Model

Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
4.21785	-2.35948	0.33193	0.27245	0.15332	0.57595	0.03638			
All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
	0.05582907	0.03494451	0.0344488	0.02646138	0.1596575	0.4273482	5144.53314	0.03216747	121.727868

“Store” model

Store Model										
	Store 2									
	Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
(baseline)	4.17015	-2.1476	0.305992	0.138121	0.064135	0.384805	0.07003			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.06019768	0.03586975	0.0354931	0.02699294	0.1620171	0.4807934	5378.20823	0.03216747	150.752297

	Store 5									
	Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
store specific	-1.360652	-0.702815	0.019047	0.284429	-0.07967	0.101399	-0.101648			
baseline	4.17015	-2.1476	0.305992	0.138121	0.064135	0.384805	0.07003			
	2.809498	-2.850415	0.325039	0.42255	-0.015535	0.486204	-0.031618			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.04955139	0.0357163	0.0351442	0.02655971	0.1548358	0.4277288	8058.57541	0.03216747	140.089633

	Store 8									
	Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
store specific	-3.261717	-1.495497	0.080519	-0.043682	0.334135	-0.147507	-0.163516			
baseline	4.17015	-2.1476	0.305992	0.138121	0.064135	0.384805	0.07003			
	0.908433	-3.643097	0.386511	0.094439	0.39827	0.237298	-0.093486			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.04433784	0.03332935	0.0330051	0.02561155	0.161606	0.4341253	9522.88234	0.03216747	115.897003

	Store 9									
	Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
store specific	-1.632401	-1.039237	0.247649	-0.228622	0.365483	-0.104795	-0.105095			
baseline	4.17015	-2.1476	0.305992	0.138121	0.064135	0.384805	0.07003			
	2.537749	-3.186837	0.553641	-0.090501	0.429618	0.28001	-0.035065			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.04687706	0.0347247	0.0342341	0.02613045	0.1548433	0.4351136	9872.23305	0.03216747	145.21647

	Store 12									
	Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
store specific	-3.165484	-0.638342	-0.32014	-0.187755	-0.00559	-0.221625	-0.221379			
baseline	4.17015	-2.1476	0.305992	0.138121	0.064135	0.384805	0.07003			
	1.004666	-2.785942	-0.014148	-0.049634	0.058545	0.16318	-0.151349			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.05017895	0.03699942	0.0360684	0.02777719	0.1563033	0.428385	10971.6007	0.03216747	197.614829

“Mixed” model:

Store 2 is the baseline model, and the fixed-coefficient + the random-coefficient gives the store-specific coefficient:

Mixed Model									
Store 2									
Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
2.904165805	-2.57087385	0.23001693	0.21513409	0.11332293	0.53375843	-0.00761731			
All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
	0.0526449	0.03494451	0.03444877	0.02646138	0.1596575	0.4273482	5696.20289	0.03216747	116.643628

Store 5									
Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
2.904165805	-2.57087385	0.23001693	0.21513409	0.11332293	0.53375843	-0.00761731			
0.05643115									
2.960596955									
All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
	0.0526449	0.03494451	0.03444877	0.02646138	0.1596575	0.4273482	6026.88879	0.03216747	123.415229

Store 8									
Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
2.904165805	-2.57087385	0.23001693	0.21513409	0.11332293	0.53375843	-0.00761731			
-0.03325136									
2.870914445									
All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
	0.0526449	0.03494451	0.03444877	0.02646138	0.1596575	0.4273482	5509.91066	0.03216747	112.828843

Store9									
Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
2.904165805	-2.5708738	0.23001693	0.21513409	0.11332293	0.53375843	-0.0076173			
0.06780926									
2.971975065									
All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
	0.05264491	0.03494451	0.03444877	0.02646138	0.1596575	0.4273482	6095.85341	0.03216747	124.827455

Store12									
Intercept	lpr1	lpr4	lpr5	lpr11	afeat1	adisp1			
2.904165805	-2.5708738	0.23001693	0.21513409	0.11332293	0.53375843	-0.0076173			
0.1942903									
3.098456105									
All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
	0.05264491	0.03494451	0.03444877	0.02646138	0.1596575	0.4273482	6917.74495	0.03216747	141.657683

Appendix 5.2

Optimized Price/oz

	simple model	pooled model	store model	mixed model
store 2	0.05109375	0.055829065	0.0601977	0.0526449
store 5	0.05109375	0.055829065	0.0495514	0.0526449
store 8	0.05109375	0.055829065	0.0443378	0.0526449
store 9	0.05109375	0.055829065	0.0468771	0.05264491
store 12	0.05109375	0.055829065	0.050179	0.05264491

Maximized Profit

	simple model	pooled model	store model	mixed model
store 2	114.112451	121.7278682	150.7523	116.643628
store 5	114.112451	121.7278682	140.08963	123.415229
store 8	114.112451	121.7278682	115.897	112.828843
store 9	114.112451	121.7278682	145.21647	124.827455
store 12	114.112451	121.7278682	197.61483	141.657683

Appendix 5.3

Optimal Price using the decision tree model

	decision tree									
Price 1/oz	**lpr1**	**lpr4**	**lpr5**	**lpr11**	**afeat1**	**adisp1**	**lmv1**	**quantity**	**Cost/oz**	**profit**
0.001	-6.90775528	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-563.7037
0.006	-5.11599581	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-473.2723
0.011	-4.50986001	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-382.8408
0.016	-4.13516656	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-292.4094
0.021	-3.86323284	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-201.978
0.026	-3.64965874	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-111.5466
0.031	-3.47376807	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	-21.11517
0.036	-3.32423634	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.8029091	18086.283	0.0321675	69.316246
0.041	-3.19418321	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	9.056103	8570.6859	0.0321675	75.700851
0.046	-3.07911388	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	83.307218
0.051	-2.97592965	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	113.42001
0.056	-2.88240359	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	143.53279
0.061	-2.79688141	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	173.64558
0.066	-2.71810054	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	203.75837
0.071	-2.6450754	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	233.87116
0.076	-2.57702194	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	263.98395
0.081	-2.51330612	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	294.09674
0.086	-2.45340798	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	324.20953
0.091	-2.39689577	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	354.32231
0.096	-2.34340709	-3.353994	-3.368282	-3.632069	0.1596575	0.4273482	8.7032673	6022.5577	0.0321675	384.4351

Appendix 5.4

Error Metrics for the Improved Linear Regression Model

```
 Descriptive statistics by group
INDICES: 1
        vars    n mean   sd   min  max range se
err5       1 5824 0.00 0.36 -2.53 2.55  5.08  0
sqerr5     2 5824 0.13 0.30  0.00 6.51  6.51  0
abserr5    3 5824 0.27 0.25  0.00 2.55  2.55  0
---------------------------------------------------
INDICES: 2
        vars    n mean   sd   min  max range   se
err5       1 3873 0.00 0.41 -1.83 2.02  3.86 0.01
sqerr5     2 3873 0.17 0.34  0.00 4.10  4.10 0.01
abserr5    3 3873 0.30 0.28  0.00 2.02  2.02 0.00
```

Appendix 5.5

Improved Linear Regression Optimal Price

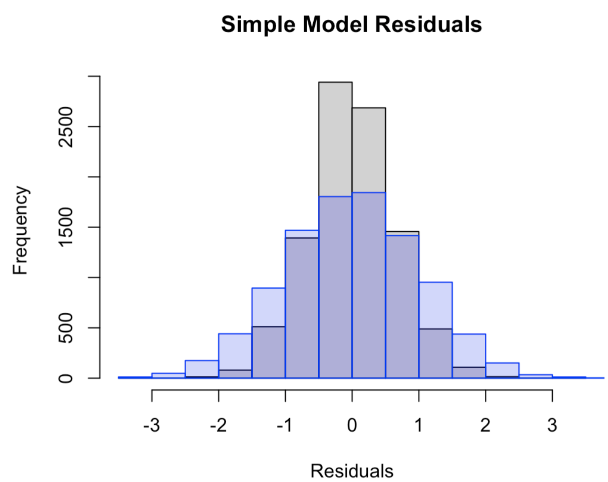
Improved Model								
Store 2								
Intercept	lpr1	lpr5	lpr12	afeat1	adisp1			
4.0688799	-2.1504976	-0.0472938	0.50207368	0.10006282	-0.0674176			
All in /oz	Priceprod1	lpr5	lpr12	afeat1	adisp1	Movement	Cost	Profit
	0.060127084	0.03549313	0.03059343	0.1620171	0.4807934	4942.02919	0.03216747	138.177229
	3.848133381							

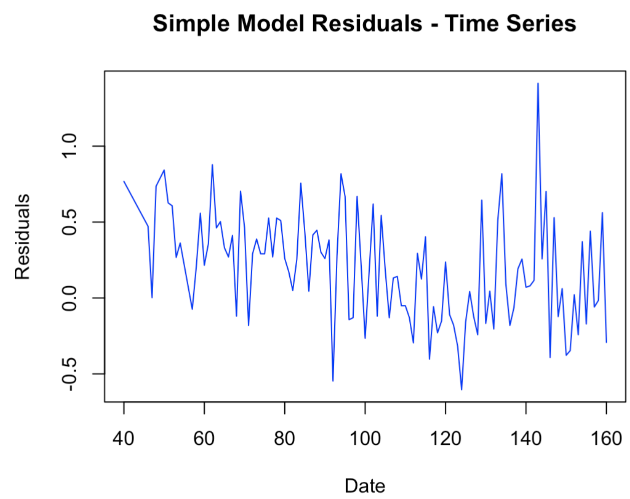
Appendix 6.1

Error Metrics for store 2: “Store” model has better RMSE and ME

Simple Model (log-log, cost plus-rule)

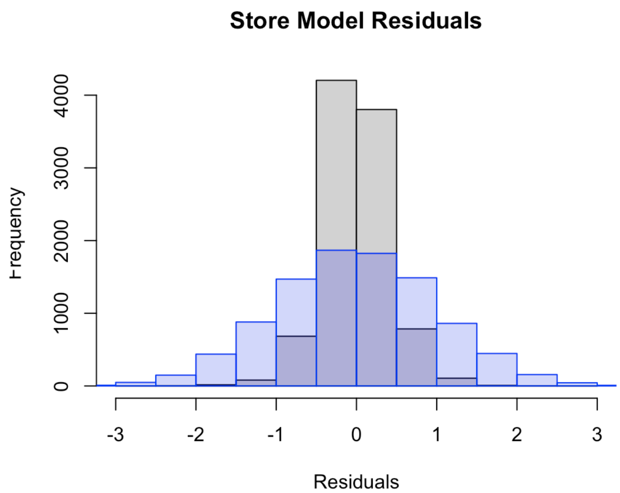
```
 Descriptive statistics by group
INDICES: 1
        vars    n mean   sd   min   max range   se
err0       1 5824 0.00 0.64 -3.35  3.02  6.37 0.01
sqerr0     2 5824 0.41 0.64  0.00 11.22 11.22 0.01
abserr0    3 5824 0.50 0.40  0.00  3.35  3.35 0.01
---------------------------------------------------
INDICES: 2
        vars    n mean   sd   min  max range   se
err0       1 3873 0.00 0.65 -2.19 2.45  4.64 0.01
sqerr0     2 3873 0.42 0.63  0.00 6.00  6.00 0.01
abserr0    3 3873 0.51 0.41  0.00 2.45  2.45 0.01
```

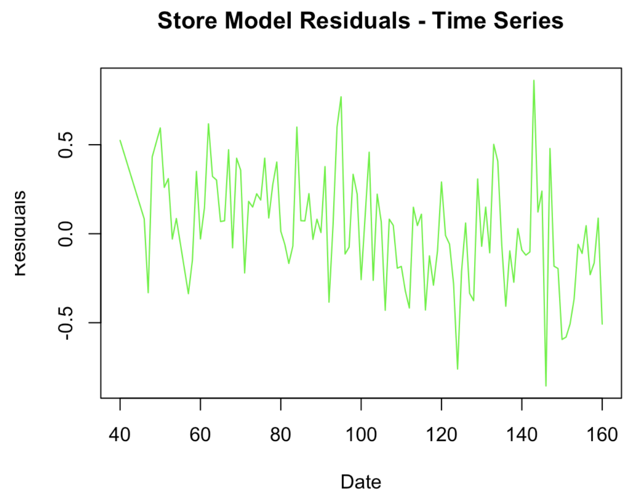




“Store” Model

```
 Descriptive statistics by group
INDICES: 1
        vars    n mean   sd   min  max range se
err2       1 5824 0.00 0.38 -2.79 2.65  5.45  0
sqerr2     2 5824 0.14 0.32  0.00 7.79  7.79  0
abserr2    3 5824 0.28 0.26  0.00 2.79  2.79  0
---------------------------------------------------
INDICES: 2
        vars    n mean   sd   min  max range   se
err2       1 3873 0.00 0.42 -1.92 2.07  3.99 0.01
sqerr2     2 3873 0.18 0.33  0.00 4.27  4.27 0.01
abserr2    3 3873 0.31 0.28  0.00 2.07  2.07 0.00
```


Store Model Residuals
Frequency
0
1000
2000
3000
4000
-3
-2
-1
0
1
2
3
Residuals


Store Model Residuals - Time Series
Residuals
-0.5
0.0
0.5
40
60
80
100
120
140
160
Date

Appendix 6.2

For Store 2, Product 1			
	Optimal Price	Actual Price	Percent Difference
Cost-plus Rule	$ 3.27	$ 3.59	8.84448468
"Store" Model	$ 3.85	$ 3.59	-7.316193004

Appendix 7.1

For Stores 2, 5, 8, 9, 12:

Product 4:

Product 4

Store Model

Store 2

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat4	adisp4			
baseline	2.3566671	0.7205046	-2.9047995	0.4708584	-0.0981808	0.6937944	0.3139254			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.06019768	0.03586975	0.0354931	0.02699294	0.1620171	0.4807934	8474.32187	0.03216747	237.537007
		3.85265133								

Store 5

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat4	adisp4			
store specific	6.053541	0.7108442	0.154546	1.192592	0.3388916	0.2001471	0.6569463			
baseline	2.3566671	0.7205046	-2.9047995	0.4708584	-0.0981808	0.6937944	0.3139254			
	8.4102081	1.4313488	-2.7502535	1.6634504	0.2407108	0.8939415	0.9708717			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.04955139	0.0357163	0.0351442	0.02655971	0.1548358	0.4277288	1610.33232	0.03216747	27.9938889
		3.17128891								

Store 8

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat4	adisp4			
	6.80447	0.6339636	0.154546	1.192592	0.3193785	0.09678307	0.4467101			
baseline	2.3566671	0.7205046	-2.9047995	0.4708584	-0.0981808	0.6937944	0.3139254			
	9.1611371	1.3544682	-2.7502535	1.6634504	0.2211977	0.79057747	0.7606355			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.04433784	0.03332935	0.0330051	0.02561155	0.161606	0.4341253	3898.60525	0.03216747	47.447469
		2.83762169								

Store 9

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat4	adisp4			
	12.36072	1.952444	-0.4788334	2.762484	-0.2742688	0.3023429	0.7024885			
baseline	2.3566671	0.7205046	-2.9047995	0.4708584	-0.0981808	0.6937944	0.3139254			
	14.7173871	2.6729486	-3.3836329	3.2333424	-0.3724496	0.9961373	1.0164139			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.04687706	0.0347247	0.0342341	0.02613045	0.1548433	0.4351136	7711.94862	0.03216747	113.439578
		3.00013156								

Store 12

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat4	adisp4			
	1.128781	0.9613643	-1.381502	0.4765116	0.1050488	-0.1514311	-0.0832739			
baseline	2.3566671	0.7205046	-2.9047995	0.4708584	-0.0981808	0.6937944	0.3139254			
	3.4854481	1.6818689	-4.2863015	0.94737	0.006868	0.5423633	0.2306515			
	All in /oz	Priceprod1	price prod 4	price prod 5	price prod 11	afeat1	adip1	Movement	Cost	Profit
		0.05017895	0.03699942	0.0360684	0.02777719	0.1563033	0.428385	14703.1744	0.03216747	264.826014
		3.21145308								

Product 5:

Store Model

Store 2

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat5	adisp5			
(baseline)	4.695637	0.417203	0.462071	-2.67204	0.533292	0.680437	0.13999			
	All in /oz	Priceprod1	price prod	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		53687092	0.03587	0.035493	0.026993	0.162017	0.480793	51401513	0.032167	2.7596E+15

Store 5

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat5	adisp5			
store specific	2.021727	0.324722	0.332372	0.034844	-0.11109	0.202141	-0.13681			
baseline	4.695637	0.417203	0.462071	-2.67204	0.533292	0.680437	0.13999			
total	6.717364	0.741925	0.794442	-2.6372	0.422204	0.882578	0.003179			
	All in /oz	Priceprod1	price prod	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		3231838	0.035716	0.032167	0.02656	0.154836	0.427729	8473216435	0.032167	2.73841E+16

Store 8

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat5	adisp5			
store specific	-0.54638	0.154651	0.470485	-1.09806	0.299074	0.560678	-0.31273			
baseline	4.695637	0.417203	0.462071	-2.67204	0.533292	0.680437	0.13999			
total	4.14926	0.571854	0.932556	-3.7701	0.832366	1.241115	-0.17274			
	All in /oz	Priceprod1	price prod	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.044338	0.033329	0.032167	0.025612	0.161606	0.434125	10175.9181	0.032167	123.8447006

Store 9

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat5	adisp5			
store specific	-3.87624	0.127812	0.087787	-1.23707	-0.10187	0.677634	-0.03103			
baseline	4.695637	0.417203	0.462071	-2.67204	0.533292	0.680437	0.13999			
total	0.819396	0.545014	0.549858	-3.90911	0.431423	1.358072	0.108959			
	All in /oz	Priceprod1	price prod	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.046877	0.034725	0.032167	0.02613	0.154843	0.435114	12381.7253	0.032167	182.1301187

Store 12

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat5	adisp5			
store specific	-0.30998	0.19257	0.076989	-0.32104	-0.15152	0.262011	-0.2144			
baseline	4.695637	0.417203	0.462071	-2.67204	0.533292	0.680437	0.13999			
total	4.385654	0.609772	0.53906	-2.99308	0.381768	0.942448	-0.07441			
	All in /oz	Priceprod1	price prod	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.050179	0.036999	0.032167	0.027777	0.156303	0.428385	18359.1515	0.032167	330.6755136

Product 11:

Store Model - Product 11

Store 2

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat11	adisp11			
(baseline)	4.52665	0.210499	0.9294679	0.337305	-2.54219	0.479907	0.263293			
	All in /oz	Priceprod1	price prod 4	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.045176	0.0358697	0.035493	0.032167	0.162017	0.480793	5414.436	0.032167	70.43368

Store 5

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat11	adisp11			
store specific	1.282607	-0.01177	0.2296709	0.358709	-0.12273	0.393525	-0.21322			
baseline	4.52665	0.210499	0.9294679	0.337305	-2.54219	0.479907	0.263293			
total	5.809257	0.198725	1.1591388	0.696014	-2.66492	0.873432	0.050068			
	All in /oz	Priceprod1	price prod 4	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.045176	0.0357163	0.035144	0.032167	0.154836	0.427729	4090.114	0.032167	53.20625

Store 8

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat11	adisp11			
store specific	3.167841	-0.0904	0.0459525	1.630632	-0.83789	0.161244	-0.18676			
baseline	4.52665	0.210499	0.9294679	0.337305	-2.54219	0.479907	0.263293			
total	7.694491	0.120103	0.9754204	1.967937	-3.38008	0.641152	0.076531			
	All in /oz	Priceprod1	price prod 4	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.044338	0.0333293	0.033005	0.032167	0.161606	0.434125	8460.897	0.032167	102.9723

Store 9

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat11	adisp11			
store specific	-3.57942	-0.16874	0.3095197	0.049998	-1.10101	0.640249	-0.45349			
baseline	4.52665	0.210499	0.9294679	0.337305	-2.54219	0.479907	0.263293			
total	0.947229	0.041762	1.2389876	0.387303	-3.64319	1.120157	-0.1902			
	All in /oz	Priceprod1	price prod 4	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.046877	0.0347247	0.034234	0.032167	0.154843	0.435114	2866.088	0.032167	42.15898

Store 12

	Intercept	lpr1	lpr4	lpr5	lpr11	afeat11	adisp11			
store specific	-0.17004	-0.46531	0.5729944	0.433372	-0.61659	0.782696	-0.45256			
baseline	4.52665	0.210499	0.9294679	0.337305	-2.54219	0.479907	0.263293			
total	4.356608	-0.25481	1.5024623	0.770677	-3.15877	1.262603	-0.18926			
	All in /oz	Priceprod1	price prod 4	price prod	price prod	afeat1	adip1	Movement	Cost	Profit
		0.050179	0.0369994	0.036068	0.032167	0.156303	0.428385	5311.076	0.032167	95.66034

Appendix 7.2

Optimal Prices of Products 1, 4, 5, 11 (Tropicana Premium, Tropicana, Minute Maid, and Dominicks, respectively) at Stores 2, 5, 8, 9, 12 using "Store" Model:

Optimal Price		Product (64oz)			
"Store" Model		1	4	5	11
Store	2	$ 3.85	$ 2.30	$ 2.27	$ 1.73
	5	$ 3.17	$ 2.29	$ 2.25	$ 1.70
	8	$ 2.84	$ 2.13	$ 2.11	$ 1.64
	9	$ 3.00	$ 2.22	$ 2.19	$ 1.67
	12	$ 3.21	$ 2.37	$ 2.31	$ 1.78