Assignment 3

Problem 1

Given Facts:

1. If the unicorn is mythical, then it is immortal:
   $Mytical \supset \neg Mortal$
2. If the unicorn is not mythical, then it is both mortal and a mammal:
   $\neg Mytical \supset (Mortal \land Mammal)$
3. If the unicorn is either immortal or a mammal, then it is horned:
   $(\neg Mortal \lor Mammal) \supset Horned$
4. If the unicorn is horned, then it is magical:
   $Horned \supset Magical$

Proof by Resolution:

1. Assume: $\neg Magical$
2. From $Horned \supset Magical$, $\neg Horned \lor Magical$, $\neg Horned$
3. With $(\neg Mortal \lor Mammal) \supset Horned$, $\neg Horned$ implies $\neg(\neg Mortal \lor Mammal)$, simplified to $Mortal \lor \neg Mammal$
4. From $\neg Mytical \supset (Mortal \land Mammal)$, $\neg Mytical$ implies $Mortal \land Mammal$, leading to a contradiction
5. Hence, $Magical$ must be true

Conclusion:
The unicorn is both horned and magical, but we cannot deduce if it is mythical without more information.

Problem 2

Define Vocabulary:

- Student(x): x is a student
- Person(x): x is a person
- Vegetarian(x): x is vegetarian
- Smart(x): x is smart
- Take(x,y): student x taking course y
- Failed(x,y): student x failed course y

- Homework(x,y): student x do homework for student y
- Like(x,y): person x likes person y

First-order logic:

a. Not all students take both History and Biology
   $\neg\forall x(Take(x, History) \land Take(x, Biology))$
b. Only one student failed History
   $\exists x(Fail(x, History) \land \forall y(Fail(y, History) \rightarrow y = x))$
c. Every person who dislikes all vegetarians is smart
   $\forall x(\forall y(Vegetarian(y) \supset \neg Like(x, y)) \supset Smart(x))$
d. No person likes a smart vegetarian
   $\neg\exists x[\exists y(Like(x, y) \land Vegetarian(y) \land Smart(y))]$
e. There is a student who does homework for those and only those who do not do homework for themselves
   $\exists x(Student(x) \land \forall y(Homework(x, y) \equiv \neg Homework(y, y)))$

Problem 3

By using GSCA Rule Learning,
Iteration 1:

- Choose attributes that separate positive from negative examples
- Derive rule $EXP \land GPA \supset HIRE$

Iteration 2:

- Remaining positive examples evaluated
- Derive rule $UST \land REC \supset HIRE$

Final Rules:

- $EXP \land GPA \supset HIRE$
- $UST \land REC \supset HIRE$

Problem 4

Let A=Alarm, B=Burglary, E=Earthquake, J=JohnCalls, M=MaryCalls

$$P(A) = \sum_{B,E} P(A|B, E)P(B)P(E)$$
$$= 0.001(0.002)(0.95) + 0.001(0.998)(0.94) + 0.999(0.002)(0.29) + 0.999(0.998)(0.001) = 0.0025$$

$$P(M) = P(M|A)P(A) + P(M|\neg A)P(\neg A) = 0.0117$$
$$P(J, M) = P(J, M, A) \times P(J, M, \neg A)$$
$$= P(J, M|A) \times P(A) + P(J, M|\neg A)P(\neg A) = 0.002$$
$$P(J|M) = \frac{P(J, M)}{P(M)} = \frac{0.002}{0.0117} = 0.17$$

Problem 5

By using resolution refutation, in CNF form:

$$elephant(Sam)$$
$$elephant(Clyde)$$
$$elephant(Oscar)$$
$$pink(Sam)$$
$$gray(Clyde)$$
$$likes(Clyde, Oscar)$$
$$pink(Oscar) \vee gray(Oscar)$$
$$likes(Oscar, Sam)$$

Refutation:

$$\neg elephant(x) \vee \neg gray(x) \vee \neg elephant(y) \vee \neg pink(y) \vee \neg likes(x, y)$$

Therefore:

$$gray(x) \wedge pink(y) \wedge likes(x, y)$$

Problem 6

Nash equilibrium is (Pol: expand, Fed: contract), for this matrix, it can be (3,3)

Problem 7

$N = \{1,2\}, A_1 = A_2 = \{1,2,3,4,5,6\}$

$$u_i(x_1, x_2) = \{\begin{matrix} 6 - x_i, & if\ agent\ wins \\ 0, & otherwise \end{matrix}$$

Nash Equilibria:

	1	2	3	4	5	6
1	2.5,2.5	0,4	0,3	0,2	0,1	0,0
2	4,0	2,2	0,3	0,2	0,1	0,0
3	3,0	3,0	1.5,1.5	0,2	0,1	0,0
4	2,0	2,0	2,0	1,1	0,1	0,0
5	1,0	1,0	1,0	1,0	0.5,0.5	0,0

6	0,0	0,0	0,0	0,0	0,0	0,0

Nash equilibria are (4,4), (5,5), (6,6)