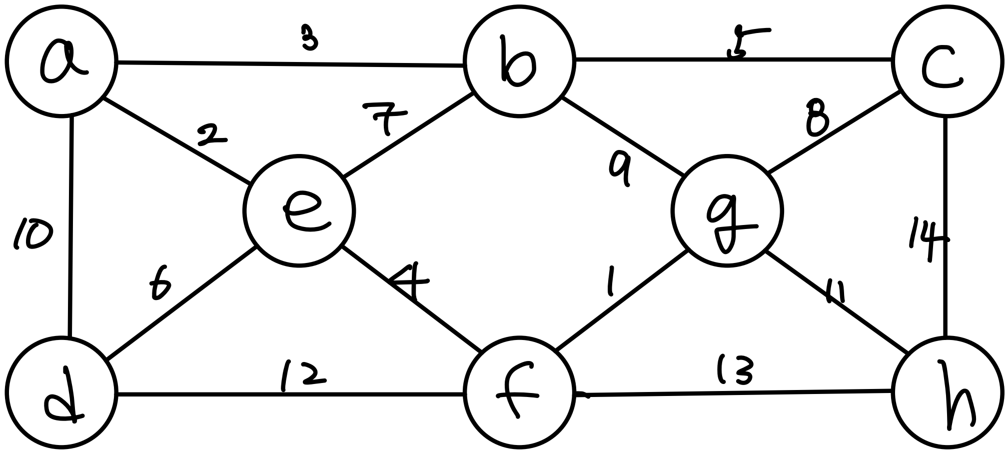
Assignment 7

2022-24670
GSDS
송종현

1.

e는 사이클 상에 존재하는 maximum weight edge이므로, G에 대한 임의의 cut에 대해서 light edge일 수 없다. 따라서 어떠한 MST의 부분집합에 대해서도 safe하지 않고, 이는 {e}를 포함한 minimum spanning tree를 구성할 수 없음을 뜻한다. 따라서 (V, E − {e})와 (V, E)가 같은 MST를 공유함을 뜻한다.

2. (a)



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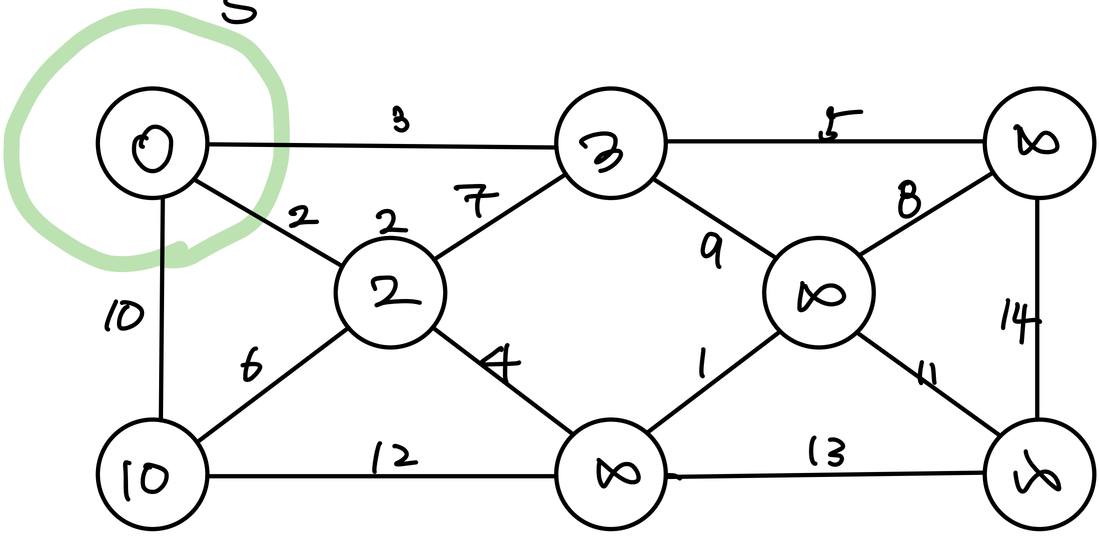

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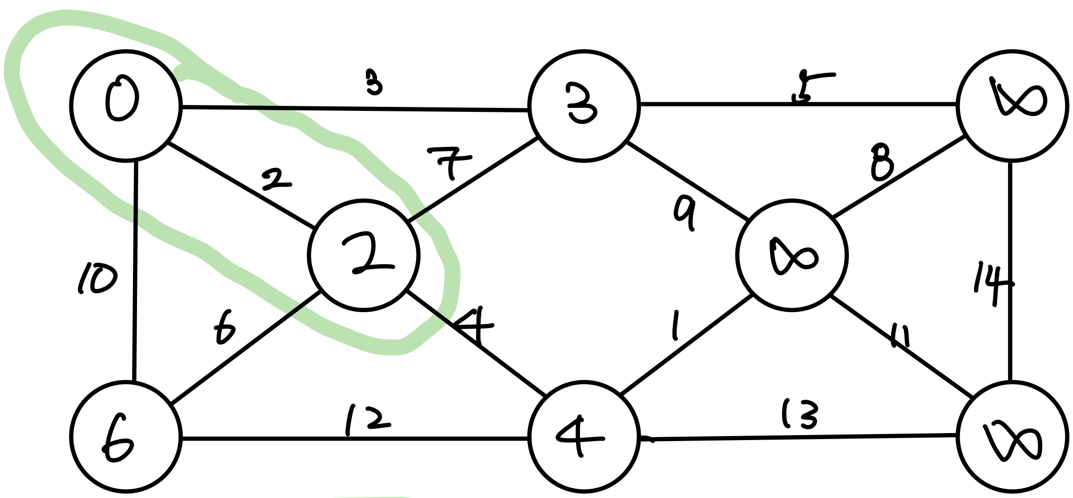

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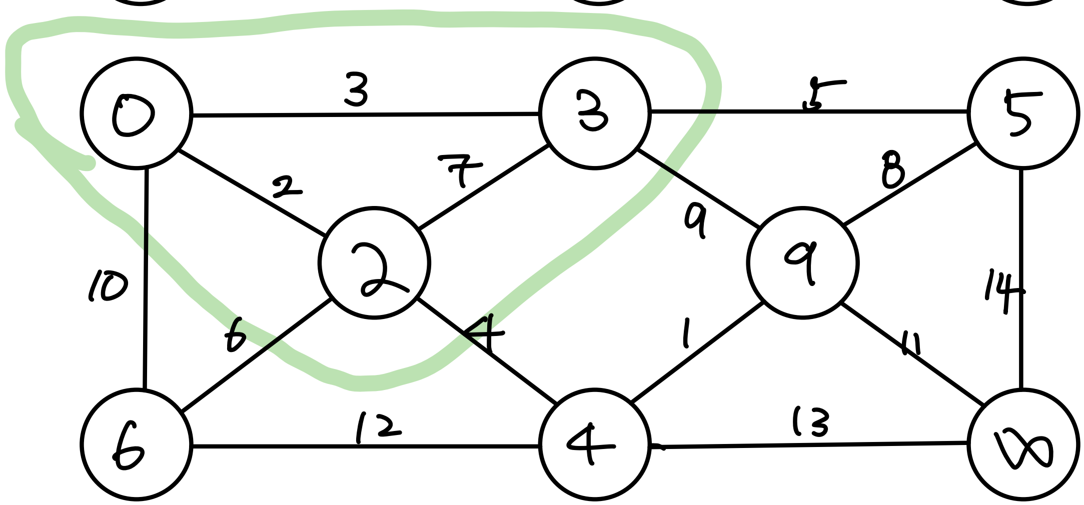

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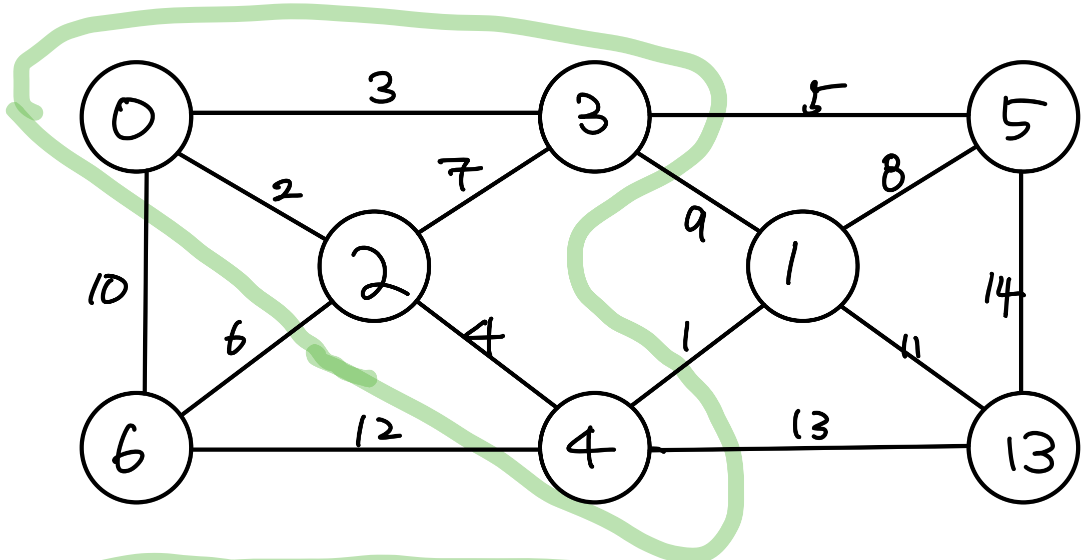
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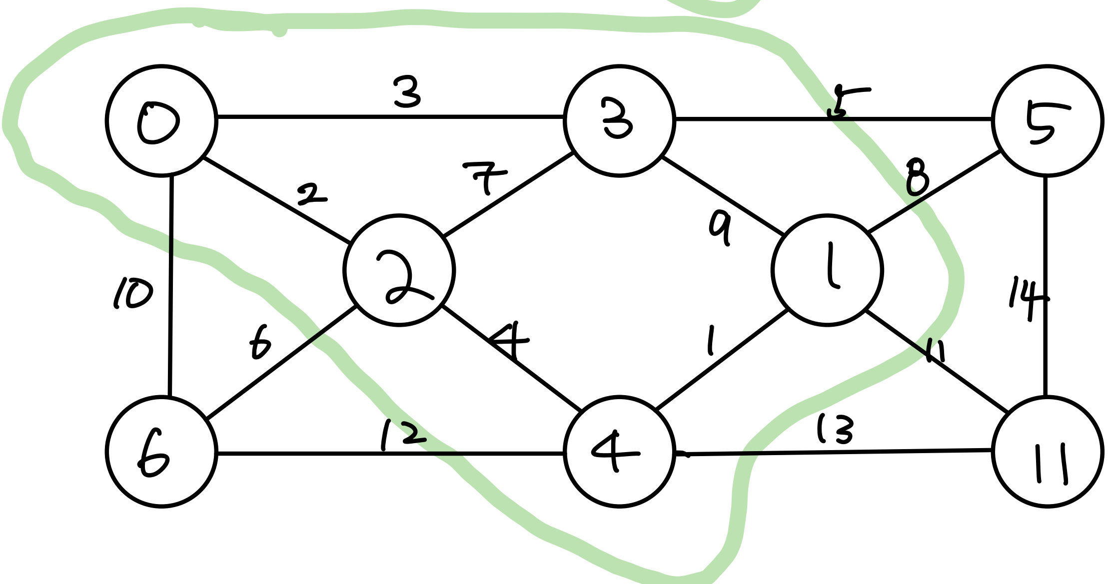
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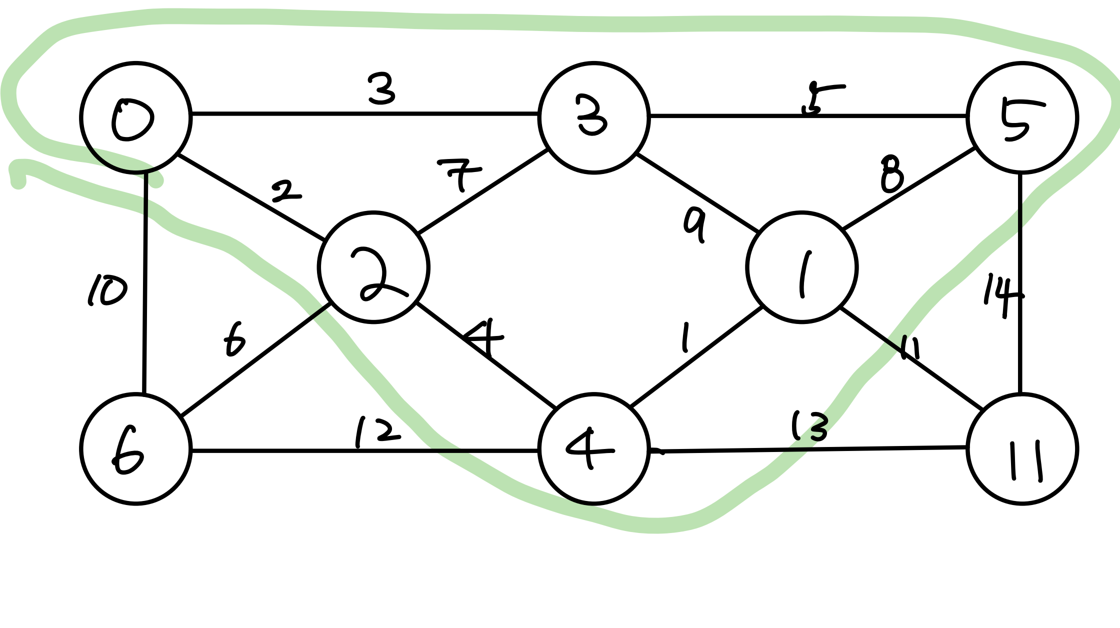
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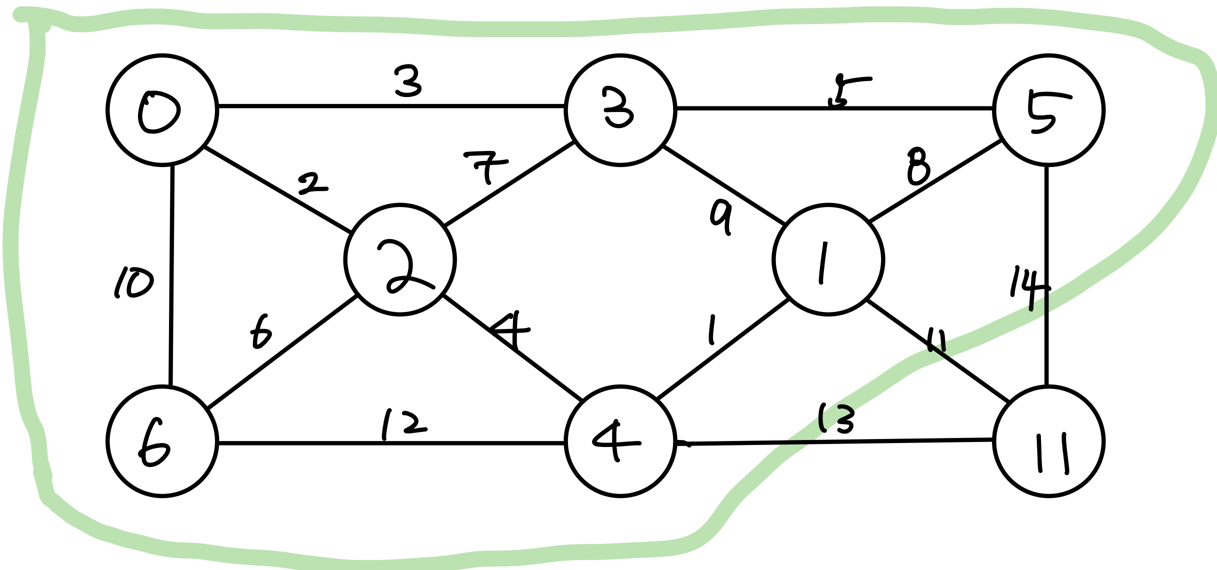
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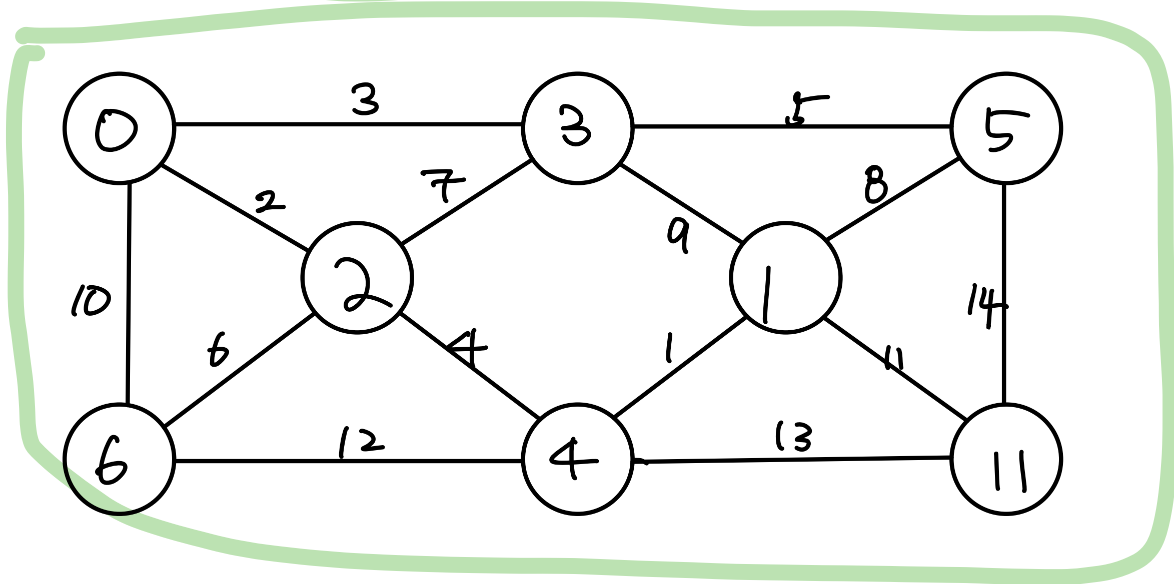
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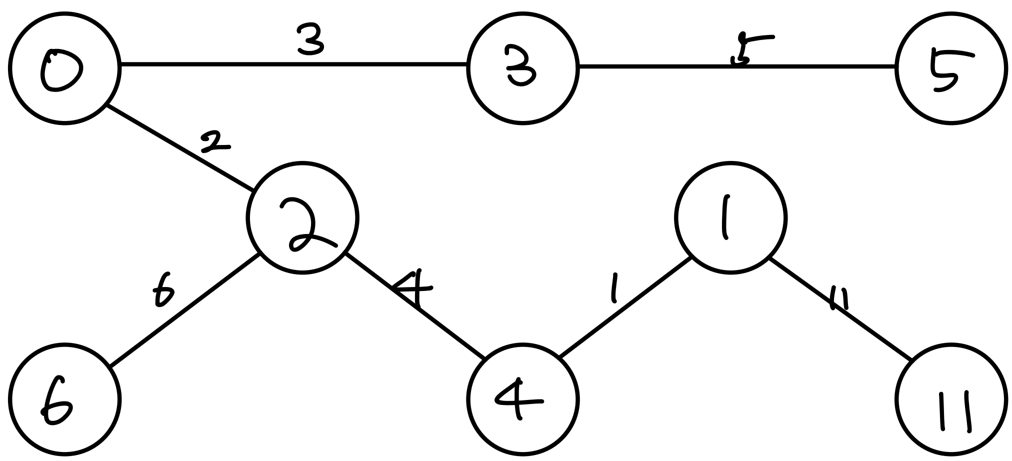
결과: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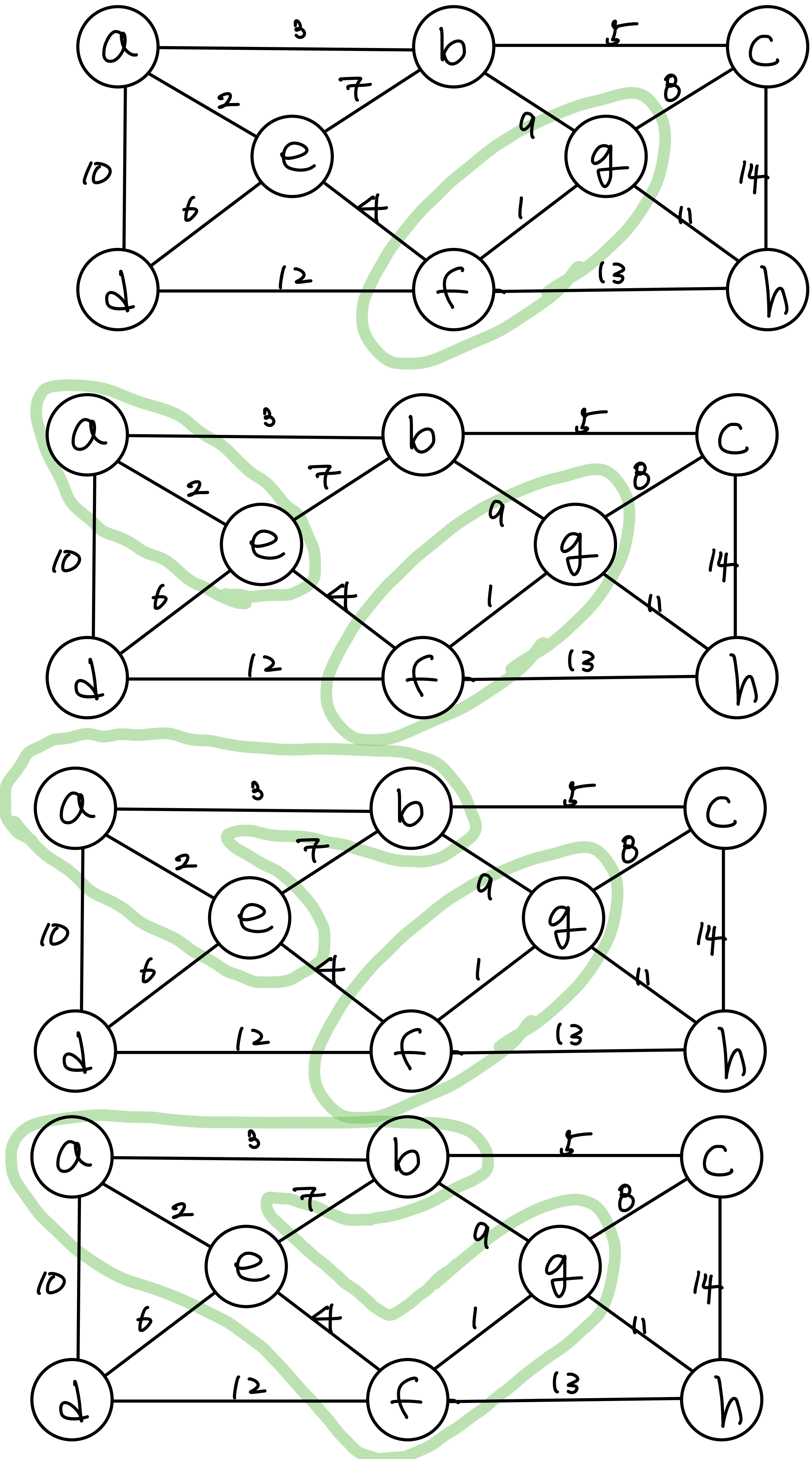
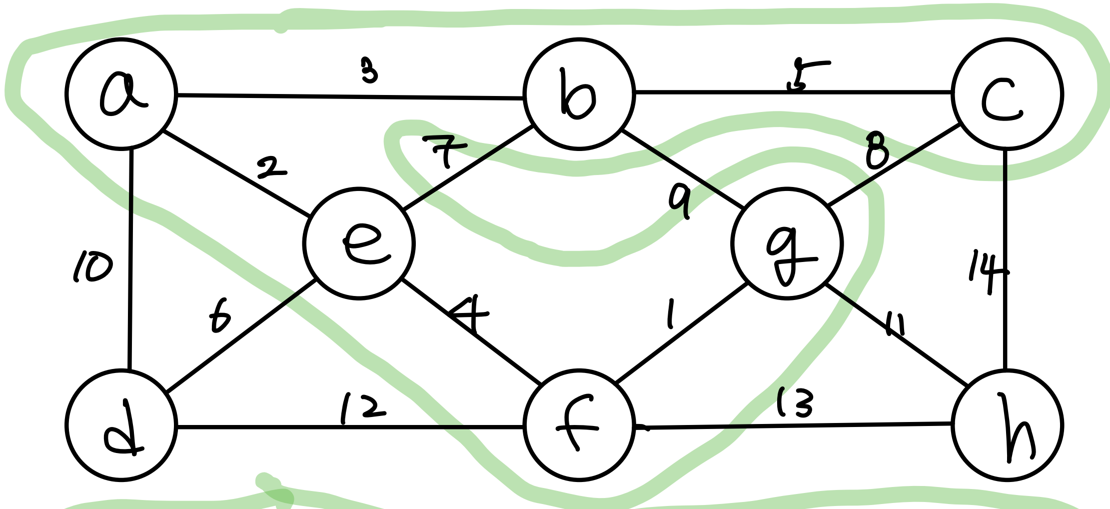
(b)

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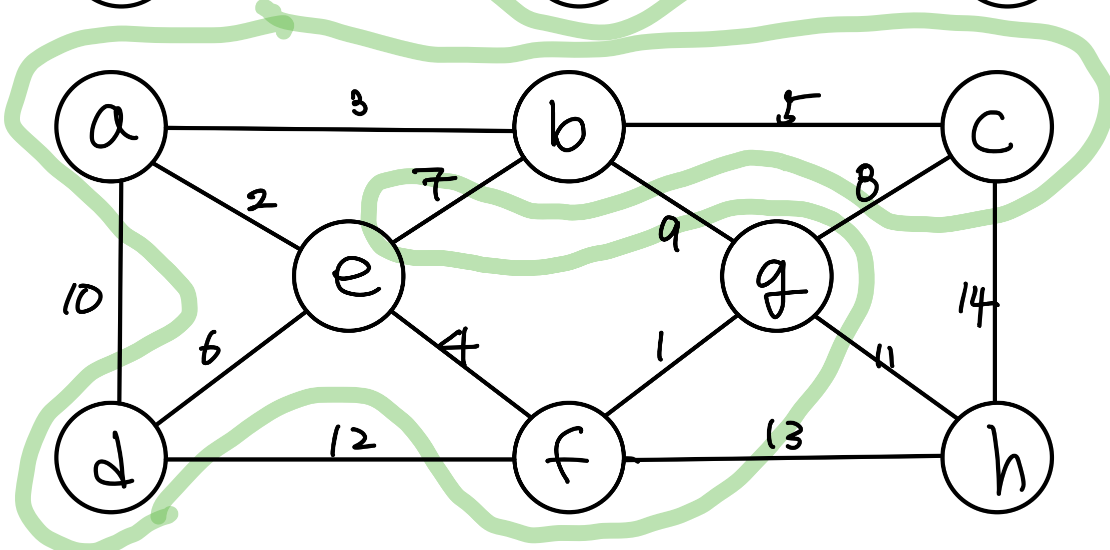
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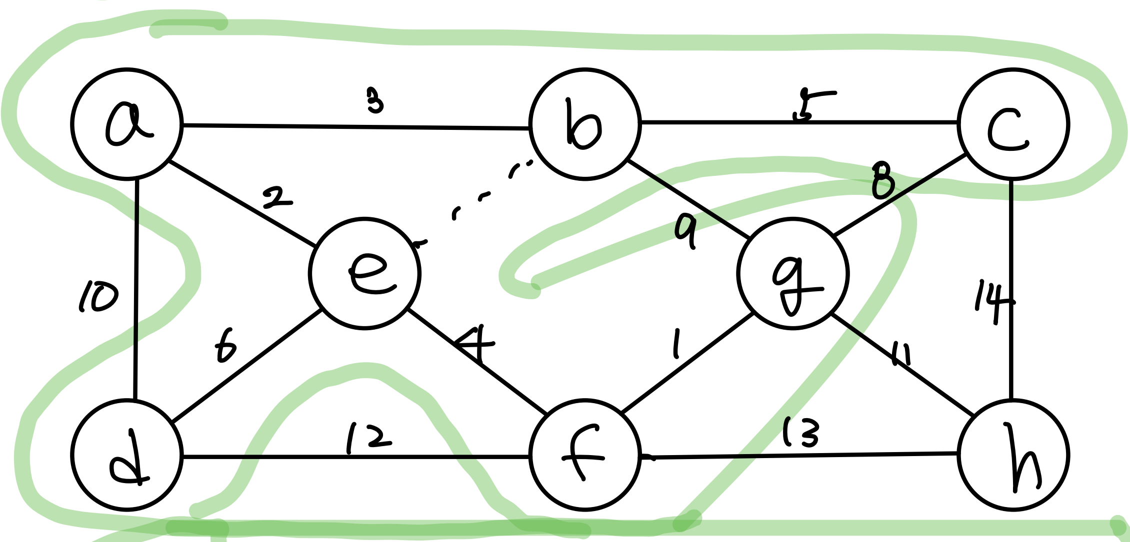
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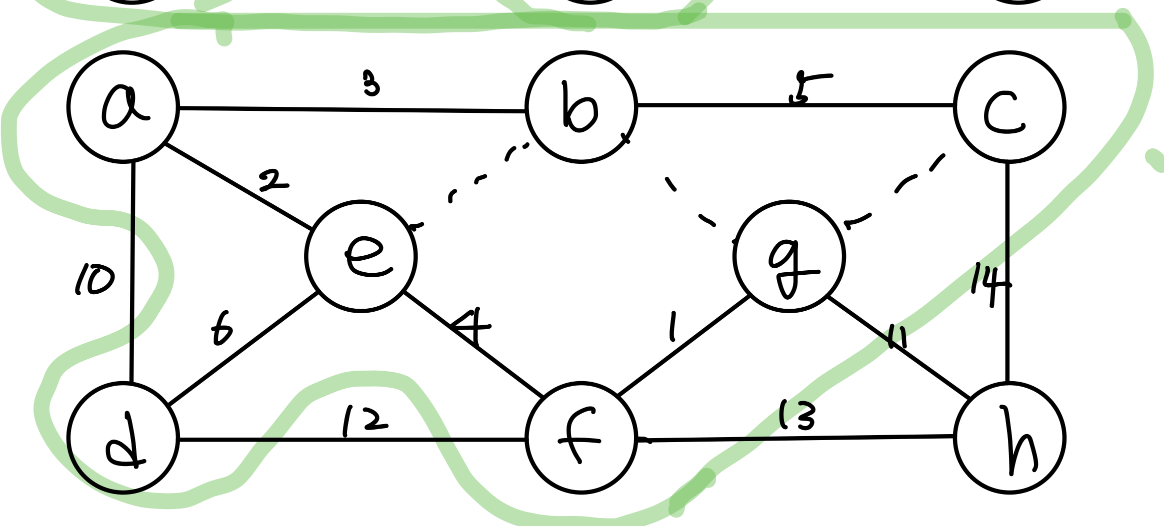
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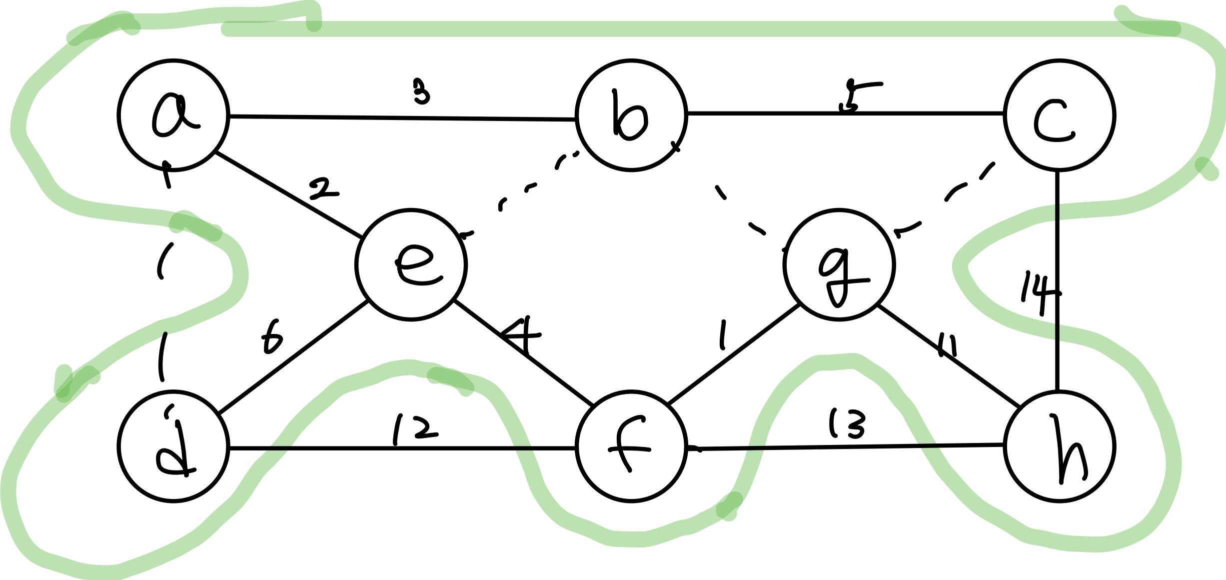
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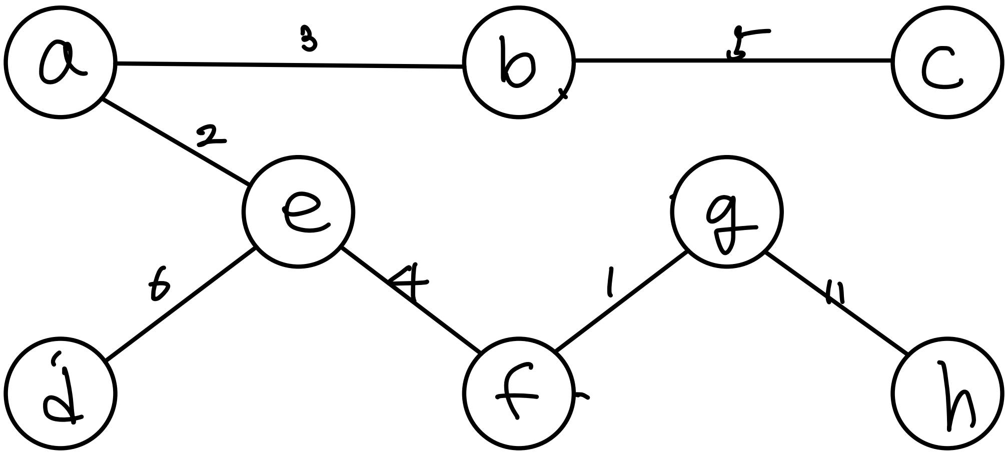
결과 :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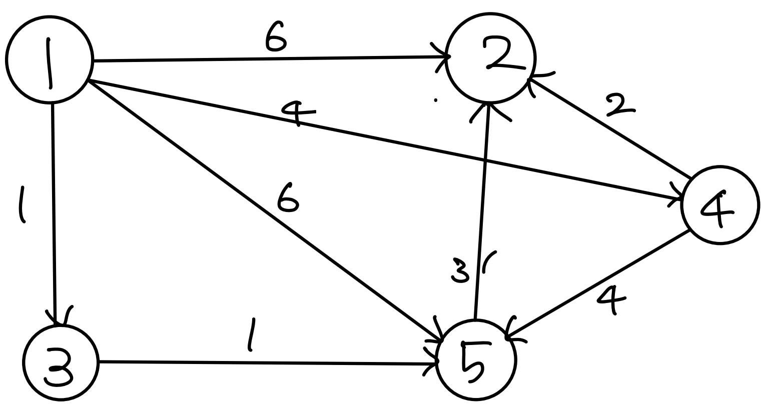
3.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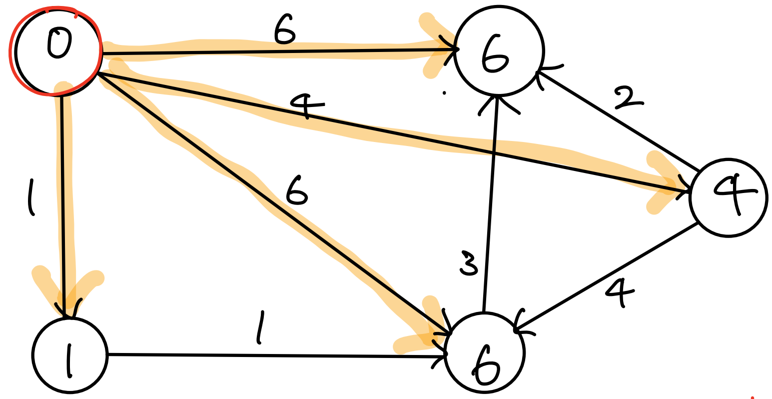
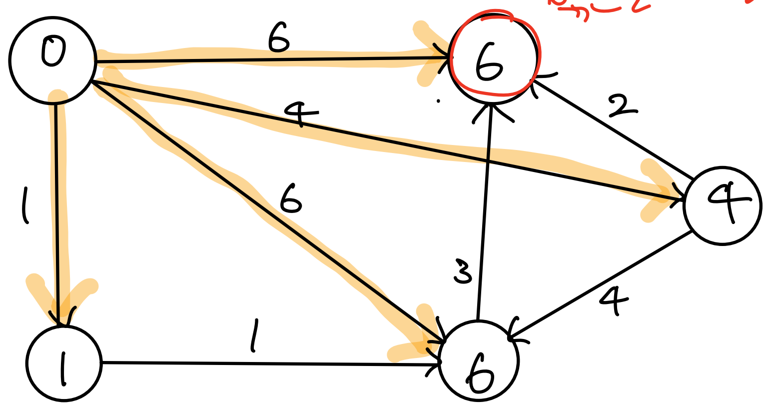
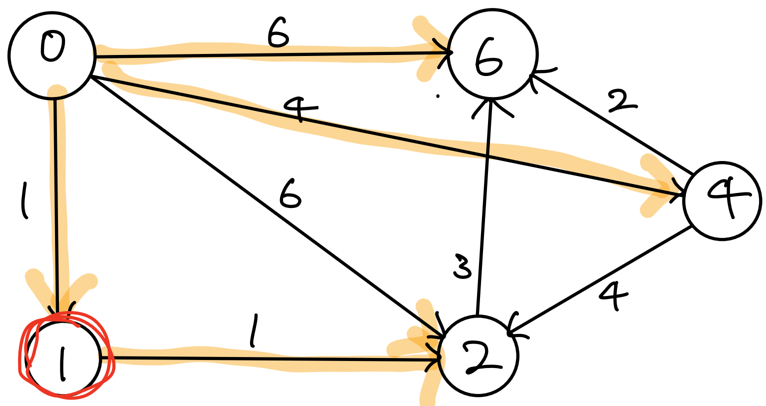
(a)

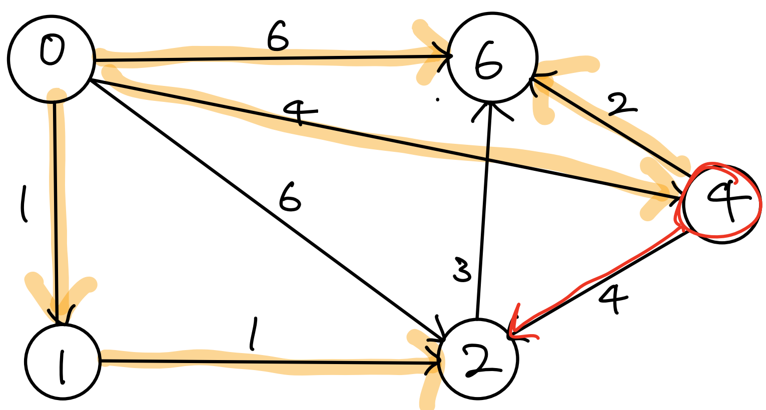


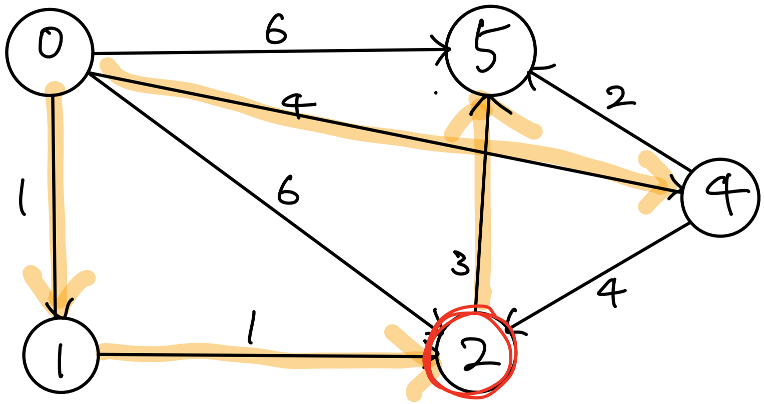
밖으로 나갈 수 x



1→5 순으로 순회







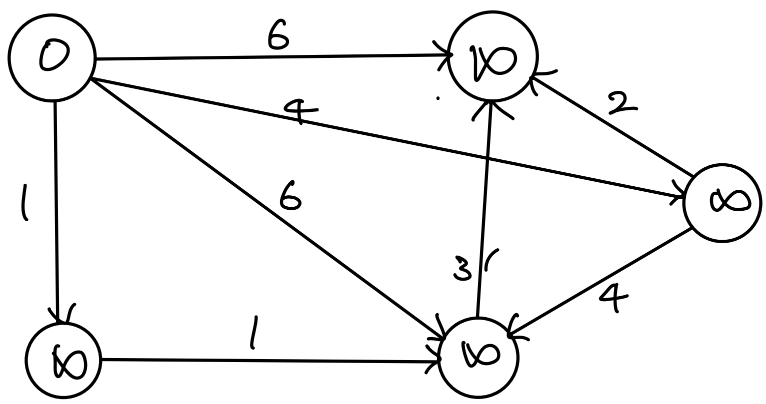
결과: $1 \to 3 \to 5 \to 2$

$1 \to 3$

$1 \to 4$

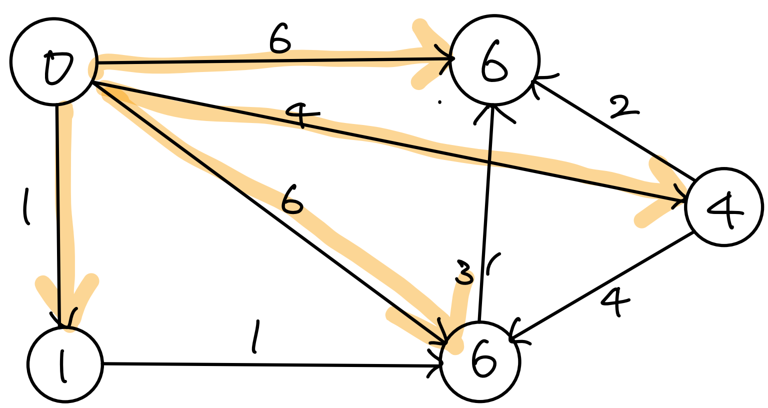
$1 \to 3 \to 5$

(b) 전제: Nonnegative weight일 것 (만족)



$S = \emptyset$

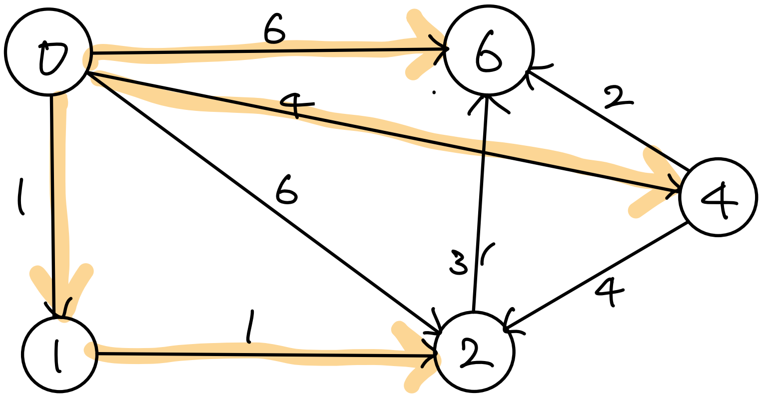
$Q = \{1,2,3,4,5\}$



$u=1$

$S = \{1\}$

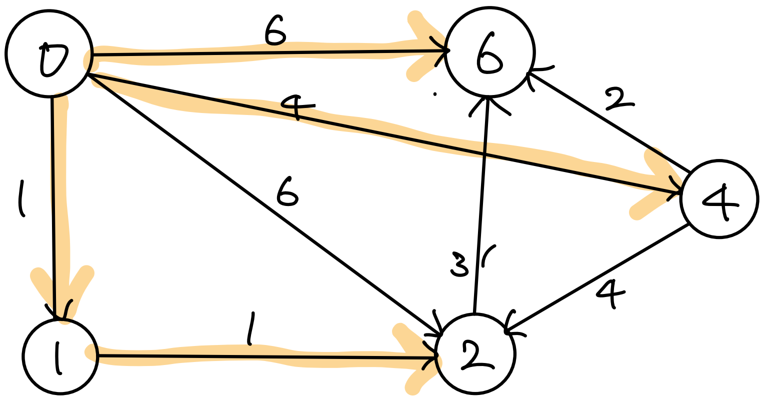
$Q = \{2,3,4,5\}$



$u=3$

$S=\{1,3\}$

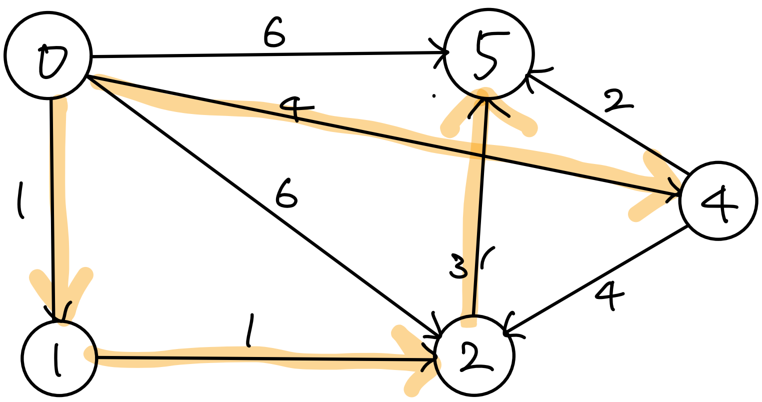
$Q=\{2,4,5\}$



$u=2$

$S=\{1,2,3\}$

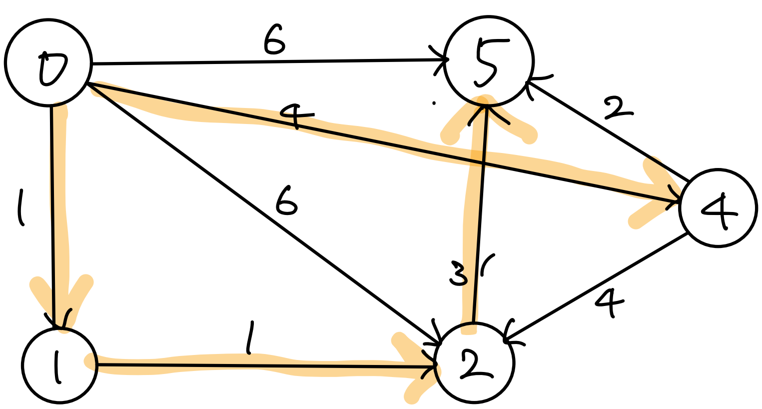
$Q=\{4,5\}$



$u=5$

$S=\{1,2,3,5\}$

$Q=\{4\}$



$u=4$

$S=\{1,2,3,4,5\}$

$Q=\emptyset$

결과: $1 \to 3 \to 5 \to 2$

$1 \to 4$

(c)

$$L^{(1)} = \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & 0 \end{pmatrix}$$

$$L^{(2)} = \begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$ ← 계속 ∞를 유지할 것임

$$L^{(3)} = \begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 5 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$L^{(4)} = \begin{pmatrix} 0 & 5 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 5 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

(d)

$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & 0 \end{pmatrix}$$

$$L^{(2)} = \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$L^{(4)} = \begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 5 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

(e)

$$d = \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad \pi = \begin{pmatrix} NIL & 1 & 1 & 1 & 1 \\ NIL & NIL & NIL & NIL & NIL \\ NIL & NIL & NIL & NIL & 3 \\ NIL & 4 & NIL & NIL & 4 \\ NIL & 5 & NIL & NIL & NIL \end{pmatrix}$$

$k=1$

$$d = \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad \pi = \begin{pmatrix} NIL & 1 & 1 & 1 & 1 \\ NIL & NIL & NIL & NIL & NIL \\ NIL & NIL & NIL & NIL & 3 \\ NIL & 4 & NIL & NIL & 4 \\ NIL & 5 & NIL & NIL & NIL \end{pmatrix}$$

$k=2$ $\quad (i,2)+(2,j)$

$$d = \begin{pmatrix} 0 & 6 & 1 & 4 & 6 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad \pi = \begin{pmatrix} NIL & 1 & 1 & 1 & 1 \\ NIL & NIL & NIL & NIL & NIL \\ NIL & NIL & NIL & NIL & 3 \\ NIL & 4 & NIL & NIL & 4 \\ NIL & 5 & NIL & NIL & NIL \end{pmatrix}$$

$(i,3)+(3,j)$

$k=3$

$$d = \begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad \pi = \begin{pmatrix} NIL & 1 & 1 & 1 & 3 \\ NIL & NIL & NIL & NIL & NIL \\ NIL & NIL & NIL & NIL & 3 \\ NIL & 4 & NIL & NIL & 4 \\ NIL & 5 & NIL & NIL & NIL \end{pmatrix}$$

$k=4$ $\quad (i,4)+(4,j)$

$$d=\begin{pmatrix} 0 & 6 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad \pi=\begin{pmatrix} NIL & 1 & 1 & 1 & 3 \\ NIL & NIL & NIL & NIL & NIL \\ NIL & NIL & NIL & NIL & 3 \\ NIL & 4 & NIL & NIL & 4 \\ NIL & 5 & NIL & NIL & NIL \end{pmatrix}$$

$k=5$ $\quad (i,5)+(5,j)$

$$d=\begin{pmatrix} 0 & 5 & 1 & 4 & 2 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 4 & 0 & \infty & 1 \\ \infty & 2 & \infty & 0 & 4 \\ \infty & 3 & \infty & \infty & 0 \end{pmatrix} \quad \pi=\begin{pmatrix} NIL & 5 & 1 & 1 & 3 \\ NIL & NIL & NIL & NIL & NIL \\ NIL & 5 & NIL & NIL & 3 \\ NIL & 4 & NIL & NIL & 4 \\ NIL & 5 & NIL & NIL & NIL \end{pmatrix}$$

4. EXTENED-BOTTOM-UP-CUT-ROD에 cut한 횟수를 포함하면 몇 번의 cut이 일어나 revenue가 감소하였는지 알 수 있다. 이를 pseudo-code로 나타내면 다음과 같다.

```
let r[0...n] and s[0...n] and t[0...n] be new arrays
        // array t는 cut한 횟수를 나타내는 array이다. 이것이 없어도 가격을
           제대로 구하는 데에는 상관이 없다.
r[0]=0
t[0]=0 // 0개의 cut 횟수는 0번이다.
int c; // cut하는 고정된 비용
for j=1 to n
    q=-∞
    for i=1 to j
        if q<p[i]+r[j-i]
            if j==i
                q=p[i]+r[j-i]
                t[j]=t[j-i] // cut이 발생하지 않음 (t[j]=0)
            else
                q=p[i]+r[j-i]-c // cut 비용 제거
                t[j]=t[j-i]+1 // (j-i)에서 발생한 cut에
                                 1회를 더함
            s[j]=i
        r[j]=q
return r,s
```

0	0,0,0
1	1,1,0
2	5,2,0
3	8,3,0
4	9,4,0
5	11,3,1
6	17,6,0
7	17,7,0
8	20,6,1
9	24,9,0
10	30,10,0

왼쪽 그림은 위 pseudo code를 cpp로 구현해 (r,s,t) 순으로 출력한 것이다. (cost=2라고 가정)

i= 4,7,9 에서 cost의 비용으로 인해 cut을 하지 않도록 결과가 변한 것을 볼 수 있다.