Monte Carlo simulation report

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1 Monte Carlo simulation design

1.1 dynamic heterogeneous panels data model without error factor structure

The data generating process:

$$\begin{aligned} y_{i,t} &= \phi_i y_{i,t-1} + \sum_{\ell=1}^{k} \beta_{\ell i} x_{\ell i,t} + u_{i,t},\ \ for\ i = 1, \dots N;\ t = -49, \dots, T\,, \\ x_{\ell i,t} &= \sum_{\ell=1}^{k} \phi_{\ell i} x_{\ell i,t-1} + v_{\ell i,t}, \end{aligned} \tag{1}$$

where $u_{i,t} \sim \mathcal{N}(0,\, 1)\,$, and $v_{\ell i,t} = \rho_{v,\ell} v_{\ell i,t-1} + \left(1 - \rho_{v,\ell}^2\right)^{\frac{1}{2}} \varpi_{\ell i,t}, \varpi_{\ell i,t} \overset{i.i.d.}{\sim} U(0.5, 1.5)\,.$

The slope coefficients are generated as

$$\phi_i = \phi + \eta_{\phi i},\ \ \beta_{1,i} = \beta_1 + \eta_{\beta_1 i}\, and\, \beta_{2,i} = \beta_2 + \eta_{\beta_2 i}. \tag{2}$$

Here we consider $\phi \in \{0.5, 0.8\}$, $\beta_1 = 3$ and $\beta_2 = 1$. For the design of heterogenous slopes, $\eta_{\phi i} \overset{i.i.d.}{\sim} U\left(-c, c\right)$, and

$$\eta_{\beta_\ell i} = \left(1 - \rho_\beta^2\right)^{1/2} \eta_{\phi i}. \tag{3}$$

Here, we set $c = 0.2$, $\rho_\beta = 0.4$ for $\ell = 1, 2$.

1.2 Dynamic heterogeneous panels data model with multi-factor error structure

This Monte Carlo simulation design same as Norkute et al. (2019). For convenience, we rewrite the data generating process as bellow

$$y_{i,t} = \alpha_i + \phi_i y_{i,t-1} + \sum_{\ell=1}^{k} \beta_{\ell i} x_{\ell i,t} + u_{i,t},\ \ for\ i = 1, \dots N;\ t = -49, \dots, T\,. \tag{4}$$

$$\tag{5}$$

We allow error factor structure in the model as

$$u_{i,t} = \sum_{s=1}^{m_y} \gamma_{si}^0 f_{s,t}^0 + \varepsilon_{i,t}, \tag{6}$$

where

$$f_{s,t}^0 = \rho_{s,t}^0 f_{s,t-1}^0 + \left(1 - \rho_{fs}^2\right)^{1/2} \zeta_{s,t}, \tag{7}$$

with $\zeta_{s,t} \overset{i.i.d.}{\sim} N(0, 1)$ for $s = 1, \dots m_y$. We assume $k = 2$ and $m_y = 1 + k = 3$ and set $\rho_{s,t}^0 = 0.5$ for all s. The error term, $\varepsilon_{i,t}$, setting as

$$\varepsilon_{i,t} = \varsigma_\varepsilon \sigma_{it} \left(\epsilon_{it} - 1\right) / \sqrt{2}, \tag{8}$$

where $\epsilon_{it} \overset{i.i.d.}{\sim} \chi_1^2$, $\sigma_{it}^2 = \eta_i \varphi_t$, $\eta_i \overset{i.i.d.}{\sim} \chi_2^2/2$, and $\varphi_t = t/T \; for \; t = 0, \ldots, T$. And we set

$$\varsigma_\varepsilon = \frac{\pi_\mu}{1-\pi_\mu} m_y. \tag{9}$$

we set $\pi_\mu \in \{1/4, 3/4\}$.

The process of regressors is

$$x_{\ell it} = \mu_{\ell i} + + \sum_{\ell=1}^{k} \phi_{\ell i} x_{\ell i,t-1} + \sum_{s=1}^{m_x} \gamma_{\ell si}^0 f_{s,t}^0 + v_{\ell it}, \; for \; i = 1, \ldots N; \; t = -49, \ldots, T; \; \ell = 1, 2. \tag{10}$$

We set number of factor, m_x, is 2. Therefore, $\boldsymbol{f}_{y,t}^0 = (f_{1t}^0, f_{2t}^0, f_{3t}^0)'$ and $\boldsymbol{f}_{x,t}^0 = (f_{1t}^0, f_{2t}^0)'$. We set

$$v_{\ell i,t} = \rho_{v,\ell} v_{\ell i,t-1} + \left(1 - \rho_{v,\ell}^2\right)^{\frac{1}{2}} \varpi_{\ell i,t}, for \; \ell = 1, 2, \tag{11}$$

where $\rho_{v,\ell} = 0.5$ for all ℓ. The individual effect is

$$\alpha_i^* \overset{i.i.d.}{\sim} N\left(0, (1-\rho_i)^2\right), \; \mu_{\ell i}^* = \rho_{\mu,\ell} \alpha_i^* + \left(1 - \rho_{\mu,\ell}^2\right)^{1/2} \omega_{\ell i}, \tag{12}$$

where $\omega \overset{i.i.d.}{\sim} N\left(0, (1-\rho_i)^2\right)$ and $\rho_{\mu,\ell} = 0.5$.

Now, we define the factor loading in $u_{i,t}$ are generated as $\gamma_{si}^{0*} \overset{i.i.d.}{\sim} N(0,1)$, for $s = 1, \ldots, m_y = 3$, and the factor loading in x_{1it} and x_{2it} are drawn as

$$\begin{aligned} \gamma_{1si}^{0*} &= \rho_{\gamma,1s} \gamma_{3i}^{0*} + \left(1 - \rho_{\gamma,1s}^2\right)^{1/2} \xi_{1si}; \; \xi_{1si} \overset{i.i.d.}{\sim} N(0,1); \\ \gamma_{2si}^{0*} &= \rho_{\gamma,2s} \gamma_{si}^{0*} + \left(1 - \rho_{\gamma,2s}^2\right)^{1/2} \xi_{2si}; \; \xi_{2si} \overset{i.i.d.}{\sim} N(0,1); \end{aligned} \tag{13}$$

for $s = 1, \ldots, m_x = 2$. We set $\rho_{\gamma,11} = \rho_{\gamma,12} \in \{0, 0.5\}$ and $\rho_{\gamma,21} = \rho_{\gamma,22} = 0.5$. The factor loading are generated as

$$\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^0 + \boldsymbol{\Gamma}_i^{0*} \tag{14}$$

where

$$\boldsymbol{\Gamma}_i^0 = \begin{bmatrix} \gamma_{1i}^0 & \gamma_{11i}^0 & \gamma_{21i}^0 \\ \gamma_{2i}^0 & \gamma_{12i}^0 & \gamma_{22i}^0 \\ \gamma_{3i}^0 & 0 & 0 \end{bmatrix} \tag{15}$$

and

$$\boldsymbol{\Gamma}_i^{0*} = \begin{bmatrix} \gamma_{1i}^{0*} & \gamma_{11i}^{0*} & \gamma_{21i}^{0*} \\ \gamma_{2i}^{0*} & \gamma_{12i}^{0*} & \gamma_{22i}^{0*} \\ \gamma_{3i}^{0*} & 0 & 0 \end{bmatrix}. \tag{16}$$

We set

$$\boldsymbol{\Gamma}^0 = \begin{bmatrix} 1/4 & 1/4 & -1 \\ 1/2 & -1 & 1/4 \\ 1/2 & 0 & 0 \end{bmatrix}. \tag{17}$$

And

$$\alpha_i = \alpha + \alpha_i^*,\ \mu_{\ell i} = \mu_\ell + \mu_{\ell i}^*, \tag{18}$$

where $\alpha = 1/2$, $\mu_1 = 1$, $\mu_2 = -1/2$.

The slope coefficients are generated as

$$\phi_i = \phi + \eta_{\phi i},\ \ \beta_{1,i} = \beta_1 + \eta_{\beta_1 i}\, and\, \beta_{2,i} = \beta_2 + \eta_{\beta_2 i}. \tag{19}$$

Here we consider $\phi \in \{0.5, 0.8\}$, $\beta_1 = 3$ and $\beta_2 = 1$. For the design of heterogenous slopes, $\eta_{\phi i} \overset{i.i.d.}{\sim} U(-c, c)$, and

$$\eta_{\beta_\ell i} = \left[(2c)^2/12\right]\rho_\beta \xi_{\beta\ell i} + \left(1 - \rho_\beta^2\right)^{1/2}\eta_{\phi i}, \tag{20}$$

where

$$\xi_{\beta\ell i} = \frac{\bar{v_{\ell i}^2} - \bar{v_\ell^2}}{\left[N^{-1}\sum_{i=1}^N \left(\bar{v_{\ell i}^2} - \bar{v_\ell^2}\right)^2\right]^{1/2}}, \tag{21}$$

with $\bar{v_{elli}^2} = T^{-1}\sum_{t=1}^T v_{\ell it}^2$, $\bar{v_\ell^2} = N^{-1}\sum_{i=1}^N \bar{v_{\ell i}^2}$, for $\ell = 1, 2$. Here, we set $c = 0.2$, $\rho_\beta = 0.4$ for $\ell = 1, 2$. And

$$\varsigma_v^2 = \varsigma_\varepsilon^2\left[SNR - \frac{\rho_v^2}{1 - \rho_v^2}\right]\left(\frac{\beta_1^2 + \beta_2^2}{1 - \rho_v^2}\right)^{-1}, \tag{22}$$

where $SNR = 4$. For the (T, N), we consider $T \in \{25, 50, 100, 200\}$ and $N \in \{25, 50, 100, 200\}$.

2 Monte Carlo simulation results

References

Norkute, M., V. Sarafidis, T. Yamagata, and G. Cui (2019). Instrumental variable estimation of dynamic linear panel data models with defactored regressors and a multifactor error structure. ISER Discussion Paper No. 1019.