Dynamic Heterogeneous Panels

Contents

1 Asymptotic bias of LS estimator

1.1 Brief the source of bias

Consider the dynamic heterogeneous panels data model:

$$y_{i,t} = \phi_i y_{i,t-1} + \beta_{1i} x_{i,t} + u_{i,t},\ for\ i = 1, \dots N;\ t = 1, \dots, T, \tag{1}$$

Form above model, the model can rewritten as

$$\Delta y_{i,t} = -\left(1-\phi_i\right)\left(y_{i,t} - \pi_i x_{i,t}\right) + u_{i,t}, \tag{2}$$

where $\pi_i = \frac{\beta_i}{1-\phi_i}$. And we defined $\theta_i = (1-\phi_i)$. Then we suppose

$$\begin{aligned} \theta_i &= \theta + \eta_{i1}, \\ \pi_i &= \pi + \eta_{i2}. \end{aligned} \tag{3}$$

Therefore, we know

$$\beta_i = \pi_i \theta_i = \left(\pi + \eta_{i2}\right)\left(\theta + \eta_{i1}\right) = \pi\theta + \pi\eta_{i,1} + \theta\eta_{i2} + \eta_{i1}\eta_{i2} \tag{4}$$

And we defined $\eta_{i3} = \pi\eta_{i,1} + \theta\eta_{i2} + \eta_{i1}\eta_{i2}$. Then,we know $\beta_i = \pi\theta + \eta_{i3}$. Therefore, from equation (1), we have

$$\begin{aligned} y_{i,t} &= \phi_i y_{i,t-1} + \left(\pi\theta + \pi\eta_{i,1} + \theta\eta_{i2} + \eta_{i1}\eta_{i2}\right) x_{i,t} + u_{i,t} \\ &= \left(1-\theta_i\right) y_{i,t-1} + \beta x_{i,t} + \eta_{i,3} x_{i,t} + u_{i,t} \\ &= (1-\theta) y_{i,t-1} - \eta_{i1} y_{i,t-1} + \beta x_{i,t} + \eta_{i3} x_{i,t} + u_{i,t} \\ &= \phi y_{i,t-1} + \beta x_{i,t} + \left(u_{i,t} - \eta_{i1} y_{i,t-1} + \eta_{i3} x_{i,t}\right) \\ &= \phi y_{i,t-1} + \beta x_{i,t} + v_{i,t}, \end{aligned} \tag{5}$$

where $v_{i,t} = (u_{i,t} - \eta_{i1} y_{i,t-1} + \eta_{i3} x_{i,t})$. Then,we can see that $y_{i,t-1}$ and $x_{i,t}$ are correlated with $v_{i,t}$.

1.2 Asymptotic bias of LS estimator

To be continue.

2 MC setting

2.1 dynamic heterogeneous panels data model without multi-factor error structure

Consider the dynamic heterogeneous panels data model:

$$\begin{aligned} &y_{i,t} = \phi_i y_{i,t-1} + \beta_{1i} x_{i,t} + u_{i,t},\ for\ i = 1, \dots N;\ t = 1, \dots, T, \\ &x_{i,t} = \rho x_{i,t-1} + \xi_{i,t}, \end{aligned} \tag{6}$$

where $u_{i,t} \sim \mathcal{N}(0,\, \sigma_u^2)$, and $\xi_{i,t} \sim \mathcal{N}(0,\, \sigma_\xi^2)$.

2.2 dynamic heterogeneous panels data model with multi-factor error structure

The generating process of y and x follow (6), but we assume

$$
\begin{aligned}
u_{i,t} &= \boldsymbol{\gamma}_{yi}^{'}\boldsymbol{f}_{yt} + \varepsilon_{yi,t}, \\
\xi_{i,t} &= \boldsymbol{\gamma}_{xi}^{'}\boldsymbol{f}_{xt} + \varepsilon_{xi,t},
\end{aligned}
\tag{7}
$$

where $\boldsymbol{\gamma}_{yi}$ and $\boldsymbol{\gamma}_{xi}$ are $m_y \times 1$ and $m_x \times 1$ factor loading respectively, $\boldsymbol{f}_{yt}$ and $\boldsymbol{f}_{xt}$ are $m_y \times 1$ and $m_x \times 1$ unobservable factors respectively. And, we set $\varepsilon_{yi,t} \sim \mathcal{N}(0,\, \sigma_u^2)$, , $\varepsilon_{xi,t} \sim \mathcal{N}(0,\, \sigma_\xi^2)$.

$$
\begin{aligned}
\phi_i &= \phi + \eta_{1i} \\
\beta_{1i} &= \beta_1 + \eta_{2i},
\end{aligned}
\tag{8}
$$

where $\eta_{1i} \sim \mathcal{IIDU}(0.5,\, 1)$, and $\eta_{2i} \sim \mathcal{IIDU}(0,\, 0.8)$ We also try ϕ_i is fixed across group, which means $\phi_i = \phi$.

We chose $\phi = \{0.25\}$ and $\beta_1 = 1 - \phi$ and $\rho = \{0, 0.5\}$. Also, we selected $\sigma_u^2 = 1$ and $\sigma_\xi^2 = 1$. The replication is 1000. We define LSMG (lest square mean group) estimator and instrument variable mean group (IVMG) estimator.

In the simulation results, we provide the bias and RMSE (root mean square errors) for LSMG estimator and IVMG estimator.

3 MC results

Report in excel file.
In Excel file:

3.1 Model without factor structure

Sheet 1: ARDL(1,0); ϕ_i ; β_i .
Sheet 2: ARDL(1,0); ϕ ; β_i .
Sheet 3: ARDL(1,1); ϕ_i ; β_i .
Sheet 4: ARDL(1,1); ϕ ; β_i .

3.2 Model have factor structure 1

We estimate the number of factor is 0, so we use traditional OLS estimation method and IV estimation method.
Sheet 1: ARDL(1,0); ϕ_i ; β_i.
Sheet 2: ARDL(1,0); ϕ ; β_i
Sheet 3: ARDL(1,1); ϕ_i ; β_i.
Sheet 4: ARDL(1,1); ϕ ; β_i .

3.3 Model have factor structure 2

We provide MC simulation results for IV estimator that is been provided by Norkute et al. (2019).
Sheet 1: ARDL(1,0); ϕ_i ; β_i.
Sheet 2: ARDL(1,0); ϕ ; β_i .
Sheet 3: ARDL(1,1); ϕ_i ; β_i .
Sheet 4: ARDL(1,1); ϕ ; β_i.

References

Norkute, M., V. Sarafidis, T. Yamagata, and G. Cui (2019). Instrumental variable estimation of dynamic linear panel data models with defactored regressors and a multifactor error structure. ISER Discussion Paper No. 1019.