
Dynamic Heterogeneous Panels

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1 Asymptotic bias of LS estimator

1.1 Brief the source of bias

Consider the dynamic heterogeneous panels data model:

$$y_{i,t} = \phi_i y_{i,t-1} + \beta_i x_{i,t} + u_{i,t}, \text{ for } i = 1, \dots, N; t = 1, \dots, T, \quad (1)$$

In above model, we assume the number of regressor is one and the model can be rewritten as

$$\Delta y_{i,t} = -(1 - \phi_i) (y_{i,t} - \pi_i x_{i,t}) + \varepsilon_{i,t}, \quad (2)$$

where $\pi_i = \frac{\beta_i}{1-\phi_i}$. And we defined $\theta_i = (1 - \phi_i)$. Then we suppose

$$\begin{aligned} \theta_i &= \theta + \eta_{i1}, \\ \pi_i &= \pi + \eta_{i3}. \end{aligned} \quad (3)$$

Therefore, we know

$$\beta_i = \pi_i \theta_i = (\pi + \eta_{i3}) (\theta + \eta_{i1}) = \pi\theta + \pi\eta_{i1} + \theta\eta_{i3} + \eta_{i1}\eta_{i3} \quad (4)$$

And we defined $\eta_{i2} = \pi\eta_{i1} + \theta\eta_{i3} + \eta_{i1}\eta_{i3}$. Then, we know $\beta_i = \pi\theta + \eta_{i2}$. Therefore, from equation (1), we have

$$\begin{aligned} y_{i,t} &= \phi_i y_{i,t-1} + (\pi\theta + \pi\eta_{i1} + \theta\eta_{i3} + \eta_{i1}\eta_{i3}) x_{i,t} + u_{i,t} \\ &= (1 - \theta_i) y_{i,t-1} + \beta_i x_{i,t} + \eta_{i2} x_{i,t} + u_{i,t} \\ &= (1 - \theta) y_{i,t-1} - \eta_{i1} y_{i,t-1} + \beta_i x_{i,t} + \eta_{i2} x_{i,t} + u_{i,t} \\ &= \phi y_{i,t-1} + \beta x_{i,t} + (u_{i,t} - \eta_{i1} y_{i,t-1} + \eta_{i2} x_{i,t}) \\ &= \phi y_{i,t-1} + \beta x_{i,t} + v_{i,t}, \end{aligned} \quad (5)$$

where $v_{i,t} = (\varepsilon_{i,t} - \eta_{i1} y_{i,t-1} + \eta_{i2} x_{i,t})$. Then, we can see that $y_{i,t-1}$ and $x_{i,t}$ are correlated with $v_{i,t}$.

1.2 Asymptotic bias of LS estimator

Based on heterogeneous dynamic panel data model (1), and we assume the $x_{i,t}$ generate from stationary AR(1) process, as

$$x_{i,t} = \mu_i (1 - \rho) = \rho x_{i,t-1} + \mu_{i,t}, \quad (6)$$

where $\mu_i \stackrel{i.i.d.}{\sim} (0, \tau_i^2)$. And, from equation (5), we can obtain fixed effect estimator as

$$\begin{pmatrix} \hat{\phi}_i \\ \hat{\beta}_i \end{pmatrix} = \begin{pmatrix} \frac{\mathbf{y}'_{i,-1} \mathbf{y}_{i,-1}}{T} & \frac{\mathbf{y}'_{i,-1} \mathbf{x}_i}{T} \\ \frac{\mathbf{x}'_i \mathbf{y}_{i,-1}}{T} & \frac{\mathbf{x}'_i \mathbf{x}_i}{T} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\mathbf{y}'_{i,-1} \mathbf{y}_i}{T} \\ \frac{\mathbf{x}'_i \mathbf{y}_i}{T} \end{pmatrix}, \quad (7)$$

where $y_i = (y_{i,1}, \dots, y_{i,T})'$, $y_{i,-1} = (y_{i,0}, \dots, y_{i,T-1})'$ and $x_i = (x_{i,0}, \dots, x_{i,T})'$ are $T \times 1$ vectors. Therefore, under equation (5), we have

$$\begin{pmatrix} \hat{\phi}_i - \phi_i \\ \hat{\beta}_i - \beta_i \end{pmatrix} = \begin{pmatrix} \frac{y'_{i,-1} y_{i,-1}}{T} & \frac{y'_{i,-1} x_i}{T} \\ \frac{x'_i y_{i,-1}}{T} & \frac{x'_i x_i}{T} \end{pmatrix}^{-1} \begin{pmatrix} \frac{y'_{i,-1} v_i}{T} \\ \frac{x'_i v_i}{T} \end{pmatrix}, \quad (8)$$

Now, we can see the asymptotic bias by taking the probability limit as

$$\text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} v_i}{T} \end{pmatrix} = \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} \varepsilon_i}{T} \end{pmatrix} - \eta_{1i} \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} y_{i,-1}}{T} \end{pmatrix} + \eta_{2i} \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} x_i}{T} \end{pmatrix}. \quad (9)$$

We assume $y_{i,t}$ has started from a long time period in the past, so we have

$$y_i = \sum_{s=0}^{\infty} x_{i,-s} \beta_i \phi_i^s + \sum_{s=0}^{\infty} \varepsilon_{i,-s} \phi_i^s, \quad (10)$$

where $x_{i,-s}$ and $\varepsilon_{i,-s}$ is $T \times 1$ vector on s th lags of x_i and ε_i . On the first term of equation (9), we have

$$\begin{aligned} \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} \varepsilon_i}{T} \end{pmatrix} &= \sum_{s=0}^{\infty} \beta_i \phi_i^s \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{x'_{i,-s-1} H \varepsilon_i}{T} \end{pmatrix} + \sum_{s=0}^{\infty} \phi_i^s \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{\varepsilon'_{i,-s-1} H \varepsilon_i}{T} \end{pmatrix} \\ &= 0, \end{aligned} \quad (11)$$

and

$$\begin{aligned} \text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} y_{i,-1}}{T} \end{pmatrix} &= \left(\sum_{s=0}^{\infty} x_{i,-s-1} \beta_i \phi_i^s + \sum_{s=0}^{\infty} \varepsilon_{i,-s-1} \phi_i^s \right)' \times \left(\sum_{s=0}^{\infty} x_{i,-s-1} \beta_i \phi_i^s + \sum_{s=0}^{\infty} \varepsilon_{i,-s-1} \phi_i^s \right) \\ &= \sum_{s=0}^{\infty} \phi_i^s \beta_i (\rho x_{i,-s-2} + \mu_i)' (\rho x_{i,-s-2} + \mu_i) \phi_i^s \beta_i + \sum_{s=0}^{\infty} \phi_i^{2s} \varepsilon'_{i,-s-1} \varepsilon_{i,-s-1} + \\ &\sum_{s=0}^{\infty} \phi_i^s \beta_i (\rho x_{i,-s-2} + \mu_i)' (\rho x_{i,-s-2} + \mu_i) \phi_i^s \beta_i = \left(\frac{\sigma_i^2}{1 - \phi_i^2} \right) + \sum_{s=0}^{\infty} \sum_{j=0}^{\infty} \left(\frac{\beta_i^2 \phi_i^{s+j} \rho^{|s-j|} \tau_i^2}{1 - \rho^2} \right). \end{aligned} \quad (12)$$

and

$$\text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} x_i}{T} \end{pmatrix} = \left(\sum_{s=0}^{\infty} (\rho x_{i,-s-2} + \mu_{i,-1}) \beta_i \phi_i^s + \sum_{s=0}^{\infty} \varepsilon_{i,-s-1} \phi_i^s \right)' \quad (13)$$

$$(\rho (\rho x_{i,-s-2} + \mu_{i,-1}) + \mu_i) = \sum_{s=0}^{\infty} \left(\frac{\phi_i^s \rho^{s+1} \beta_i \tau_i^2}{1 - \rho^2} \right). \quad (14)$$

Therefore, we can obtain

$$\text{plim}_{T \rightarrow \infty} \begin{pmatrix} \frac{y'_{i,-1} v_i}{T} \end{pmatrix} = \left(\frac{\eta_{1i} \sigma_i^2}{1 - \phi_i^2} \right) + \sum_{s=0}^{\infty} \sum_{j=0}^{\infty} \left(\frac{\eta_{1i} \beta_i^2 \phi_i^{s+j} \rho^{|s-j|} \tau_i^2}{1 - \rho^2} \right) + \sum_{s=0}^{\infty} \left(\frac{\eta_{2i} \phi_i^s \rho^{s+1} \beta_i \tau_i^2}{1 - \rho^2} \right) \quad (15)$$

2 Estimation method on dynamic heterogeneous panel data model

For convenient, we assume the number of regressor is 1 and we express the model as

$$y_{i,t} = \phi_i y_{i,t-1} + \sum_{\ell=1}^k \beta_{\ell i} x_{\ell i,t} + u_{i,t}, \text{ for } i = 1, \dots, N; t = 1, \dots, T, \ell = 1, \dots, k. \quad (16)$$

We stack the T observations for each i yield

$$\mathbf{y}_i = \mathbf{y}_{i,-1} \phi_i + \sum_{\ell=1}^k \mathbf{x}_{\ell i} \beta_{\ell i} + \mathbf{u}_i, \quad (17)$$

where $\mathbf{y}_i = (y_{i,1}, \dots, y_{i,T})'$, $\mathbf{y}_{i,-1} = (y_{i,0}, \dots, y_{i,T-1})'$, $\mathbf{x}_{\ell i} = (x_{\ell i,1}, \dots, x_{\ell i,T})'$ and $\mathbf{u}_i = (u_{i,1}, \dots, u_{i,T})$. To be more compressive, the model can be expressed as

$$\mathbf{y}_i = \mathbf{W}_i \boldsymbol{\varphi}_i + \mathbf{u}_i, \quad (18)$$

where $\mathbf{W}_i = (\mathbf{y}_{i,-1}, \mathbf{X}_i)$ and $\boldsymbol{\varphi}_i = (\phi_i, \boldsymbol{\beta}_i')$

2.1 LSMG estimator

The LS (least square) estimator is defined as

$$\hat{\boldsymbol{\varphi}}_{LSi} = \left(\frac{\mathbf{W}_i' \mathbf{W}_i}{T} \right)^{-1} \left(\frac{\mathbf{W}_i' \mathbf{y}_i}{T} \right) \quad (19)$$

Follow [Pesaran and Smith \(1995\)](#), we define the LSMG (least square mean group) estimator as

$$\hat{\boldsymbol{\varphi}}_{LSMG} = \frac{1}{N} \sum_{i=1}^N \hat{\boldsymbol{\varphi}}_{LSi}. \quad (20)$$

2.2 IVMG estimator

We use current and lagged values of \mathbf{x}_i as instruments, as

$$\mathbf{Z}_i = (\mathbf{X}_i, \mathbf{X}_{i,-1}), \quad (21)$$

where \mathbf{Z}_i is $T \times k$ matrix.

The IV (instrument variable) estimator is defined as

$$\hat{\boldsymbol{\varphi}}_{IVi} = \left(\left(\frac{\mathbf{Z}_i' \mathbf{W}_i}{T} \right)' \left(\frac{\mathbf{Z}_i' \mathbf{Z}_i}{T} \right)^{-1} \left(\frac{\mathbf{Z}_i' \mathbf{W}_i}{T} \right) \right)^{-1} \left(\left(\frac{\mathbf{Z}_i' \mathbf{W}_i}{T} \right)' \left(\frac{\mathbf{Z}_i' \mathbf{Z}_i}{T} \right)^{-1} \left(\frac{\mathbf{Z}_i' \mathbf{y}_i}{T} \right) \right) \quad (22)$$

We also define the IVMG (instrument variable mean group) estimator as

$$\hat{\boldsymbol{\varphi}}_{IVMG} = \frac{1}{N} \sum_{i=1}^N \hat{\boldsymbol{\varphi}}_{IVi}. \quad (23)$$

3 Estimation method on dynamic heterogeneous panel data model with multifactor error structure

Consider the model (16), we drawn $x_{\ell i,t}$ as

$$x_{\ell i,t} = \gamma_{xi}^{0'} \mathbf{f}_{xt}^0 + \varepsilon_{xi,t} \quad (24)$$

and the idiosyncratic errors of the process for $y_{i,t}$ as

$$u_{i,t} = \gamma_{yi}^{0'} \mathbf{f}_{yt}^0 + \varepsilon_{yi,t}, \quad (25)$$

where γ_{yi}^0 and γ_{xi}^0 are $m_y \times 1$ and $m_x \times 1$ true factor loading respectively, \mathbf{f}_{yt}^0 and \mathbf{f}_{xt}^0 are $m_y \times 1$ and $m_x \times 1$ true vector of unobservable factors respectively.

3.1 Norkutes' (2019) IVMG estimator

We asymptotically eliminate the common factor in \mathbf{x}_i by projecting matrix, $\mathbf{M}_{F_x^0}$.

$$\mathbf{M}_{F_x^0} = \mathbf{I}_T - \mathbf{F}_x^0 \left(\mathbf{F}_x^{0'} \mathbf{F}_x^0 \right)^{-1} \mathbf{F}_x^{0'}; \mathbf{M}_{F_{x,-1}^0} = \mathbf{I}_T - \mathbf{F}_{x,-1}^0 \left(\mathbf{F}_{x,-1}^{0'} \mathbf{F}_{x,-1}^0 \right)^{-1} \mathbf{F}_{x,-1}^{0'} \quad (26)$$

And using the defactored covariates as instruments, as

$$\mathbf{Z}_{IVi} = \left(\mathbf{M}_{F_x^0} \mathbf{x}_i, \mathbf{M}_{F_{x,-1}^0} \mathbf{x}_{i,-1} \right) \quad (27)$$

The first step IV estimator can be expressed as

$$\begin{aligned} \hat{\varphi}_{IVi} = & \left(\left(\frac{\mathbf{Z}_i' \mathbf{M}_{F_x^0} \mathbf{W}_i}{T} \right)' \left(\frac{\mathbf{Z}_i' \mathbf{M}_{F_x^0} \mathbf{Z}_i}{T} \right)^{-1} \left(\frac{\mathbf{Z}_i' \mathbf{M}_{F_x^0} \mathbf{W}_i}{T} \right) \right)^{-1} \\ & \left(\left(\frac{\mathbf{Z}_i' \mathbf{M}_{F_x^0} \mathbf{W}_i}{T} \right)' \left(\frac{\mathbf{Z}_i' \mathbf{M}_{F_x^0} \mathbf{Z}_i}{T} \right)^{-1} \left(\frac{\mathbf{Z}_i' \mathbf{M}_{F_x^0} \mathbf{y}_i}{T} \right) \right). \end{aligned} \quad (28)$$

4 Monte Carlo simulation design

4.1 dynamic heterogeneous panels data model without error factor structure

The data generating process:

$$\begin{aligned} y_{i,t} &= \phi_i y_{i,t-1} + \sum_{\ell=1}^k \beta_{\ell i} x_{\ell i,t} + u_{i,t}, \text{ for } i = 1, \dots, N; t = -49, \dots, T, \\ x_{\ell i,t} &= \sum_{\ell=1}^k \phi_{\ell i} x_{\ell i,t-1} + v_{\ell i,t}, \end{aligned} \quad (29)$$

where $u_{i,t} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, 1)$, and $v_{\ell i,t} = \rho_{v,\ell} v_{\ell i,t-1} + (1 - \rho_{v,\ell}^2)^{\frac{1}{2}} \varpi_{\ell i,t}$, $\varpi_{\ell i,t} \stackrel{i.i.d.}{\sim} U(0.5, 1.5)$, $\rho_{v,\ell} = 0.5$.

The slope coefficients are generated as

$$\phi_i = \phi + \eta_{\phi i}, \beta_{1,i} = \beta_1 + \eta_{\beta_1 i} \text{ and } \beta_{2,i} = \beta_2 + \eta_{\beta_2 i}. \quad (30)$$

Here we consider $\phi \in \{0.5\}$, $\beta_1 = 3$ and $\beta_2 = 1$. For the design of heterogenous slopes, $\eta_{\phi i} \stackrel{i.i.d.}{\sim} U(-c, c)$, and

$$\eta_{\beta_{\ell} i} = (1 - \rho_{\beta}^2)^{1/2} \eta_{\phi i}. \quad (31)$$

Here, we set $c = 0.2$, $\rho_{\beta} = 0.4$ for $\ell = 1, 2$.

4.2 Dynamic heterogeneous panels data model with multi-factor error structure

This Monte Carlo simulation design same as [Norkute et al. \(2019\)](#). For convenience, we rewrite the data generating process as bellow

$$y_{i,t} = \alpha_i + \phi_i y_{i,t-1} + \sum_{\ell=1}^k \beta_{\ell i} x_{\ell i,t} + u_{i,t}, \text{ for } i = 1, \dots, N; t = -49, \dots, T. \quad (32)$$

$$(33)$$

We allow error factor structure in the model as

$$u_{i,t} = \sum_{s=1}^{m_y} \gamma_{si}^0 f_{s,t}^0 + \varepsilon_{i,t}, \quad (34)$$

where

$$f_{s,t}^0 = \rho_{s,t}^0 f_{s,t-1}^0 + (1 - \rho_{s,t}^2)^{1/2} \zeta_{s,t}, \quad (35)$$

with $\zeta_{s,t} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, 1)$ for $s = 1, \dots, m_y$. We assume $k = 2$ and $m_y = 1 + k = 3$ and set $\rho_{s,t}^0 = 0.5$ for all s . The error term, $\varepsilon_{i,t}$, setting as

$$\varepsilon_{i,t} = \varsigma_{\varepsilon} \sigma_{it} (\epsilon_{it} - 1) / \sqrt{2}, \quad (36)$$

where $\epsilon_{it} \stackrel{i.i.d.}{\sim} \chi_1^2$, $\sigma_{it}^2 = \eta_i \varphi_t$, $\eta_i \stackrel{i.i.d.}{\sim} \chi_2^2/2$, and $\varphi_t = t/T$ for $t = 0, \dots, T$. And we set

$$\varsigma_{\varepsilon} = \frac{\pi_{\mu}}{1 - \pi_{\mu}} m_y. \quad (37)$$

we set $\pi_{\mu} \in \{3/4\}$.

The process of regressors is

$$x_{\ell i,t} = \mu_{\ell i} + \sum_{\ell=1}^k \phi_{\ell i} x_{\ell i,t-1} + \sum_{s=1}^{m_x} \gamma_{\ell si}^0 f_{s,t}^0 + v_{\ell i,t}, \text{ for } i = 1, \dots, N; t = -49, \dots, T; \ell = 1, 2. \quad (38)$$

We set number of factor, m_x , is 2. Therefore, $\mathbf{f}_{y,t}^0 = (f_{1t}^0, f_{2t}^0, f_{3t}^0)'$ and $\mathbf{f}_{x,t}^0 = (f_{1t}^0, f_{2t}^0)'$. We set

$$v_{\ell i,t} = \rho_{v,\ell} v_{\ell i,t-1} + (1 - \rho_{v,\ell}^2)^{\frac{1}{2}} \varpi_{\ell i,t}, \text{ for } \ell = 1, 2, \quad (39)$$

where $\rho_{v,\ell} = 0.5$ for all ℓ . The individual effect is

$$\alpha_i^* \overset{i.i.d.}{\sim} N(0, (1 - \rho_i)^2), \mu_{\ell i}^* = \rho_{\mu,\ell} \alpha_i^* + (1 - \rho_{\mu,\ell}^2)^{1/2} \omega_{\ell i}, \quad (40)$$

where $\omega \overset{i.i.d.}{\sim} N(0, (1 - \rho_i)^2)$ and $\rho_{\mu,\ell} = 0.5$.

Now, we define the factor loading in $u_{i,t}$ are generated as $\gamma_{si}^{0*} \overset{i.i.d.}{\sim} N(0, 1)$, for $s = 1, \dots, m_y = 3$, and the factor loading in x_{1it} and x_{2it} are drawn as

$$\begin{aligned} \gamma_{1si}^{0*} &= \rho_{\gamma,1s} \gamma_{3i}^{0*} + (1 - \rho_{\gamma,1s}^2)^{1/2} \xi_{1si}; \xi_{1si} \overset{i.i.d.}{\sim} N(0, 1); \\ \gamma_{2si}^{0*} &= \rho_{\gamma,2s} \gamma_{3i}^{0*} + (1 - \rho_{\gamma,2s}^2)^{1/2} \xi_{2si}; \xi_{2si} \overset{i.i.d.}{\sim} N(0, 1); \end{aligned} \quad (41)$$

for $s = 1, \dots, m_x = 2$. We set $\rho_{\gamma,11} = \rho_{\gamma,12} \in \{0.5\}$ and $\rho_{\gamma,21} = \rho_{\gamma,22} = 0.5$. The factor loading are generated as

$$\mathbf{\Gamma} = \mathbf{\Gamma}^0 + \mathbf{\Gamma}_i^{0*} \quad (42)$$

where

$$\mathbf{\Gamma}_i^0 = \begin{bmatrix} \gamma_{1i}^0 & \gamma_{11i}^0 & \gamma_{21i}^0 \\ \gamma_{2i}^0 & \gamma_{12i}^0 & \gamma_{22i}^0 \\ \gamma_{3i}^0 & 0 & 0 \end{bmatrix} \quad (43)$$

and

$$\mathbf{\Gamma}_i^{0*} = \begin{bmatrix} \gamma_{1i}^{0*} & \gamma_{11i}^{0*} & \gamma_{21i}^{0*} \\ \gamma_{2i}^{0*} & \gamma_{12i}^{0*} & \gamma_{22i}^{0*} \\ \gamma_{3i}^{0*} & 0 & 0 \end{bmatrix}. \quad (44)$$

We set

$$\mathbf{\Gamma}^0 = \begin{bmatrix} 1/4 & 1/4 & -1 \\ 1/2 & -1 & 1/4 \\ 1/2 & 0 & 0 \end{bmatrix}. \quad (45)$$

And

$$\alpha_i = \alpha + \alpha_i^*, \mu_{\ell i} = \mu_{\ell} + \mu_{\ell i}^*, \quad (46)$$

where $\alpha = 1/2$, $\mu_1 = 1$, $\mu_2 = -1/2$.

The slope coefficients are generated as

$$\phi_i = \phi + \eta_{\phi i}, \beta_{1,i} = \beta_1 + \eta_{\beta 1i} \text{ and } \beta_{2,i} = \beta_2 + \eta_{\beta 2i}. \quad (47)$$

Here we consider $\phi \in \{0.5\}$, $\beta_1 = 3$ and $\beta_2 = 1$. For the design of heterogenous slopes, $\eta_{\phi i} \overset{i.i.d.}{\sim} U(-c, c)$, and

$$\eta_{\beta \ell i} = [(2c)^2/12] \rho_{\beta} \xi_{\beta \ell i} + (1 - \rho_{\beta}^2)^{1/2} \eta_{\phi i}, \quad (48)$$

where

$$\xi_{\beta\ell i} = \frac{v_{\ell i}^2 - \bar{v}_\ell^2}{\left[N^{-1} \sum_{i=1}^N (v_{\ell i}^2 - \bar{v}_\ell^2)^2 \right]^{1/2}}, \quad (49)$$

with $v_{\ell i}^2 = T^{-1} \sum_{t=1}^T v_{\ell i t}^2$, $\bar{v}_\ell^2 = N^{-1} \sum_{i=1}^N v_{\ell i}^2$, for $\ell = 1, 2$. Here, we set $c = 0.2$, $\rho_\beta = 0.4$ for $\ell = 1, 2$. And

$$\varsigma_v^2 = \varsigma_\varepsilon^2 \left[SNR - \frac{\rho_v^2}{1 - \rho_v^2} \right] \left(\frac{\beta_1^2 + \beta_2^2}{1 - \rho_v^2} \right)^{-1}, \quad (50)$$

where $SNR = 4$. For the (T, N) , we consider $T \in \{25, 50, 100, 200\}$ and $N \in \{25, 50, 100, 200\}$.

5 Monte Carlo simulation results

5.1 Dynamic Heterogeneous Panels without multifactor error structure

We consider ARDL(1,0) model.

$\phi \in \{0.5\}$.

$\beta_1 = 3$.

$\beta_2 = 1$.

$u_{i,t} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, 1)$.

$\varpi_{\ell i,t} \stackrel{i.i.d.}{\sim} U(0.5, 1.5)$.

$\rho_{v,\ell} = 0.5$.

$c = 0.2$.

$\rho_\beta = 0.4$.

$T \in \{25, 50, 100, 200\}$.

$N \in \{25, 50, 100, 200\}$.

LSMG estimator is provided in sheet 1 of MC.xlsx file.

IVMG estimator is provided in sheet 2 of MC.xlsx file.

5.2 Dynamic Heterogeneous Panels with multifactor error structure

We consider ARDL(1,0) model.

$\phi \in \{0.5\}$.

$\beta_1 = 3$.

$\beta_2 = 1$.

$k = 2$.

$m_y = 1 + k = 3$.

$m_x = k = 2$.

$\zeta_{s,t} \stackrel{i.i.d.}{\sim} N(0, 1)$

$$\begin{aligned}
\pi_\mu &\in \{3/4\}. \\
\rho_{s,t}^0 &= 0.5. \\
\rho_{v,\ell} &= 0.5. \\
\rho_{\mu,\ell} &= 0.5. \\
\gamma_{si}^{0*} &\overset{i.i.d.}{\sim} N(0, 1). \\
\xi_{1si} &\overset{i.i.d.}{\sim} N(0, 1). \\
\xi_{2si} &\overset{i.i.d.}{\sim} N(0, 1). \\
\rho_{\gamma,11} = \rho_{\gamma,12} &\in \{0.5\}. \\
\rho_{\gamma,21} = \rho_{\gamma,22} &= 0.5. \\
\mathbf{\Gamma}^0 &= \begin{bmatrix} 1/4 & 1/4 & -1 \\ 1/2 & -1 & 1/4 \\ 1/2 & 0 & 0 \end{bmatrix}. \\
\alpha &= 1/2. \\
\mu_1 &= 1. \\
\mu_2 &= -1/2. \\
c &= 0.2. \\
\rho_\beta &= 0.4. \\
SNR &= 4. \\
T &\in \{25, 50, 100, 200\}. \\
N &\in \{25, 50, 100, 200\}.
\end{aligned}$$

IVMG estimator is provided in sheet 3 of MC.xlsx file.

6 Short summary

6.1 Dynamic Heterogeneous Panels without multifactor error structure

1. The performance of IVMG estimator is better than LSMG estimator in bias and RMSE.

6.2 Dynamic Heterogeneous Panels with multifactor error structure

1. When N and T increase, the performance of IVMG estimator is good in bias and RMSE.

Related literature to dynamic Heterogeneous Panels with multifactor error structure: [Chudik and Pesaran \(2015\)](#) and [Norkute et al. \(2019\)](#).

Related literature to choosing number of instruments: [Donald and Newey \(2001\)](#), [Swanson \(2005\)](#), [Carrasco \(2012\)](#), [Bai and Ng \(2010\)](#) and [Kang \(2019\)](#).

References

- Bai, J. and S. Ng (2010). Instrumental variable estimation in a data rich environment. *Econometric Theory* 26, 1577–1606.
- Carrasco, M. (2012). A regularization approach to the many instruments problem. *Journal of Econometrics* 170, 383–398.
- Chudik, A. and M. H. Pesaran (2015). Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors. *Journal of Econometrics* 188, 393–420.
- Donald, S. G. and W. K. Newey (2001). Choosing the number of instruments. *Econometrica* 69, 1161–1191.
- Kang, B. (2019). Higher order approximation of iv estimators with invalid instruments.
- Norkute, M., V. Sarafidis, T. Yamagata, and G. Cui (2019). Instrumental variable estimation of dynamic linear panel data models with defactored regressors and a multifactor error structure. ISER Discussion Paper No. 1019.
- Pesaran, M. and R. Smith (1995). Estimating long-run relationships from dynamic heterogeneous panels. *Journal of Econometrics* 68, 79 – 113.
- Swanson, J. C. C. N. R. (2005). Consistent estimation with a large number of weak instruments. *Econometrica* 73, 1673–1692.