
Monte Carlo simulation report

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1 Monte Carlo simulation design

1.1 dynamic heterogeneous panels data model without error factor structure

The data generating process:

$$\begin{aligned} y_{i,t} &= \alpha_i^* + \phi_i y_{i,t-1} + \sum_{\ell=1}^k \beta_{\ell i} x_{\ell i,t} + u_{i,t}, \text{ for } i = 1, \dots, N; t = -49, \dots, T, \\ x_{\ell i,t} &= \mu_{\ell i} + \sum_{\ell=1}^k \phi_{\ell i} x_{\ell i,t-1} + v_{\ell i,t}, \end{aligned} \quad (1)$$

where $u_{i,t} \sim \mathcal{N}(0, \sigma_u^2)$, and $v_{\ell i,t} = \rho_{v,\ell} v_{\ell i,t-1} + (1 - \rho_{v,\ell}^2)^{\frac{1}{2}} \varpi_{\ell i,t}$.

The slope coefficients are generated as

$$\phi_i = \phi + \eta_{\phi i}, \quad \beta_{1,i} = \beta_1 + \eta_{\beta_1 i} \text{ and } \beta_{2,i} = \beta_2 + \eta_{\beta_2 i}. \quad (2)$$

Here we consider $\phi \in \{0.5, 0.8\}$, $\beta_1 = 3$ and $\beta_2 = 1$. For the design of heterogeneous slopes, $\eta_{\phi i} \stackrel{i.i.d.}{\sim} U(-c, c)$, and

$$\eta_{\beta_{\ell} i} = [(2c)^2/12] \rho_{\beta} + (1 - \rho_{\beta}^2)^{1/2} \eta_{\phi i}. \quad (3)$$

Here, we set $c = 0.2$, $\rho_{\beta} = 0.4$ for $\ell = 1, 2$.

1.2 Dynamic heterogeneous panels data model with multi-factor error structure

This Monte Carlo simulation design same as [Norkute et al. \(2019\)](#). For convenience, we rewrite the data generating process as bellow

$$y_{i,t} = \alpha_i + \phi_i y_{i,t-1} + \sum_{\ell=1}^k \beta_{\ell i} x_{\ell i,t} + u_{i,t}, \text{ for } i = 1, \dots, N; t = -49, \dots, T. \quad (4)$$

(5)

We allow error factor structure in the model as

$$u_{i,t} = \sum_{s=1}^{m_y} \gamma_{si}^0 f_{s,t}^0 + \varepsilon_{i,t}, \quad (6)$$

where

$$f_{s,t}^0 = \rho_{s,t}^0 f_{s,t-1}^0 + (1 - \rho_{s,t}^2)^{1/2} \zeta_{s,t}, \quad (7)$$

with $\zeta_{s,t} \stackrel{i.i.d.}{\sim} N(0, 1)$ for $s = 1, \dots, m_y$. We assume $k = 2$ and $m_y = 1 + k = 3$ and set $\rho_{s,t}^0 = 0.5$ for all s . The error term, $\varepsilon_{i,t}$, setting as

$$\varepsilon_{i,t} = \varsigma_{\varepsilon} \sigma_{it} (\epsilon_{it} - 1) / \sqrt{2}, \quad (8)$$

where $\epsilon_{it} \stackrel{i.i.d.}{\sim} \chi_1^2$, $\sigma_{it}^2 = \eta_i \varphi_t$, $\eta_i \stackrel{i.i.d.}{\sim} \chi_2^2/2$, and $\varphi_t = t/T$ for $t = 0, \dots, T$. And we set

$$\varsigma_\varepsilon = \frac{\pi_\mu}{1 - \pi_\mu} m_y. \quad (9)$$

we set $\pi_\mu \in \{1/4, 3/4\}$.

The process of regressors is

$$x_{lit} = \mu_{li} + \sum_{\ell=1}^k \phi_{\ell i} x_{li,t-1} + \sum_{s=1}^{m_x} \gamma_{\ell si}^0 f_{s,t}^0 + v_{lit}, \text{ for } i = 1, \dots, N; t = -49, \dots, T; \ell = 1, 2. \quad (10)$$

We set number of factor, m_x , is 2. Therefore, $\mathbf{f}_{y,t}^0 = (f_{1t}^0, f_{2t}^0, f_{3t}^0)'$ and $\mathbf{f}_{x,t}^0 = (f_{1t}^0, f_{2t}^0)'$. We set

$$v_{li,t} = \rho_{v,\ell} v_{li,t-1} + (1 - \rho_{v,\ell}^2)^{\frac{1}{2}} \varpi_{li,t}, \text{ for } \ell = 1, 2, \quad (11)$$

where $\rho_{v,\ell} = 0.5$ for all ℓ . The individual effect is

$$\alpha_i^* \stackrel{i.i.d.}{\sim} N(0, (1 - \rho_i)^2), \mu_{li}^* = \rho_{\mu,\ell} \alpha_i^* + (1 - \rho_{\mu,\ell}^2)^{1/2} \omega_{li}, \quad (12)$$

where $\omega \stackrel{i.i.d.}{\sim} N(0, (1 - \rho_i)^2)$ and $\rho_{\mu,\ell} = 0.5$.

Now, we define the factor loading in $u_{i,t}$ are generated as $\gamma_{si}^{0*} \stackrel{i.i.d.}{\sim} N(0, 1)$, for $s = 1, \dots, m_y = 3$, and the factor loading in x_{1it} and x_{2it} are drawn as

$$\begin{aligned} \gamma_{1si}^{0*} &= \rho_{\gamma,1s} \gamma_{3i}^{0*} + (1 - \rho_{\gamma,1s}^2)^{1/2} \xi_{1si}; \xi_{1si} \stackrel{i.i.d.}{\sim} N(0, 1); \\ \gamma_{2si}^{0*} &= \rho_{\gamma,2s} \gamma_{3i}^{0*} + (1 - \rho_{\gamma,2s}^2)^{1/2} \xi_{2si}; \xi_{2si} \stackrel{i.i.d.}{\sim} N(0, 1); \end{aligned} \quad (13)$$

for $s = 1, \dots, m_x = 2$. We set $\rho_{\gamma,11} = \rho_{\gamma,12} \in \{0, 0.5\}$ and $\rho_{\gamma,21} = \rho_{\gamma,22} = 0.5$. The factor loading are generated as

$$\mathbf{\Gamma} = \mathbf{\Gamma}^0 + \mathbf{\Gamma}_i^{0*} \quad (14)$$

where

$$\mathbf{\Gamma}_i^0 = \begin{bmatrix} \gamma_{1i}^0 & \gamma_{11i}^0 & \gamma_{21i}^0 \\ \gamma_{2i}^0 & \gamma_{12i}^0 & \gamma_{22i}^0 \\ \gamma_{3i}^0 & 0 & 0 \end{bmatrix} \quad (15)$$

and

$$\mathbf{\Gamma}_i^{0*} = \begin{bmatrix} \gamma_{1i}^{0*} & \gamma_{11i}^{0*} & \gamma_{21i}^{0*} \\ \gamma_{2i}^{0*} & \gamma_{12i}^{0*} & \gamma_{22i}^{0*} \\ \gamma_{3i}^{0*} & 0 & 0 \end{bmatrix}. \quad (16)$$

We set

$$\mathbf{\Gamma}^0 = \begin{bmatrix} 1/4 & 1/4 & -1 \\ 1/2 & -1 & 1/4 \\ 1/2 & 0 & 0 \end{bmatrix}. \quad (17)$$

And

$$\alpha_i = \alpha + \alpha_i^*, \mu_{\ell i} = \mu_\ell + \mu_{\ell i}^*, \quad (18)$$

where $\alpha = 1/2$, $\mu_1 = 1$, $\mu_2 = -1/2$.

The slope coefficients are generated as

$$\phi_i = \phi + \eta_{\phi i}, \beta_{1,i} = \beta_1 + \eta_{\beta_1 i} \text{ and } \beta_{2,i} = \beta_2 + \eta_{\beta_2 i}. \quad (19)$$

Here we consider $\phi \in \{0.5, 0.8\}$, $\beta_1 = 3$ and $\beta_2 = 1$. For the design of heterogenous slopes, $\eta_{\phi i} \stackrel{i.i.d.}{\sim} U(-c, c)$, and

$$\eta_{\beta_{\ell} i} = [(2c)^2/12] \rho_\beta \xi_{\beta_{\ell} i} + (1 - \rho_\beta^2)^{1/2} \eta_{\phi i}, \quad (20)$$

where

$$\xi_{\beta_{\ell} i} = \frac{v_{\ell i}^2 - v_\ell^2}{\left[N^{-1} \sum_{i=1}^N (v_{\ell i}^2 - v_\ell^2)^2 \right]^{1/2}}, \quad (21)$$

with $v_{\ell i}^2 = T^{-1} \sum_{t=1}^T v_{\ell i t}^2$, $v_\ell^2 = N^{-1} \sum_{i=1}^N v_{\ell i}^2$, for $\ell = 1, 2$. Here, we set $c = 0.2$, $\rho_\beta = 0.4$ for $\ell = 1, 2$. And

$$\varsigma_v^2 = \varsigma_\varepsilon^2 \left[SNR - \frac{\rho_v^2}{1 - \rho_v^2} \right] \left(\frac{\beta_1^2 + \beta_2^2}{1 - \rho_v^2} \right)^{-1}, \quad (22)$$

where $SNR = 4$. For the (T, N) , we consider $T \in \{25, 50, 100, 200\}$ and $N \in \{25, 50, 100, 200\}$.

2 Monte Carlo simulation results

References

Norkute, M., V. Sarafidis, T. Yamagata, and G. Cui (2019). Instrumental variable estimation of dynamic linear panel data models with defactored regressors and a multifactor error structure. ISER Discussion Paper No. 1019.