Consider the Gaussian mixture model introduced in class (I follow the same notations therein). Write $\theta = \{p, \mu_1, \mu_2, \sigma\}$ and $\theta^{(t)} = \{p^{(t)}, \mu_1^{(t)}, \mu_2^{(t)}, \sigma^{(t)}\}$. Then, the E-step is

$$\begin{aligned} E_Z[\ell(X, Z, \theta)|X, \theta^{(t)}] &= \sum_i E_Z\left[Z_i \log p + (1 - Z_i)\log(1-p)\right. \\ &\quad \left. + Z_i \log f(x_i, \mu_2, \sigma) + (1 - Z_i)\log f(x_i, \mu_1, \sigma)|X, \theta^{(t)}\right] \\ &= \sum_i \left(E[Z_i|X, \theta^{(t)}]\log p + (1 - E[Z_i|X, \theta^{(t)}])\log(1-p)\right. \\ &\quad \left. + E[Z_i|X, \theta^{(t)}]\log f(x_i, \mu_2, \sigma) + (1 - E[Z_i|X, \theta^{(t)}])\log f(x_i, \mu_1, \sigma)\right). \end{aligned} \tag{1}$$

Now, note that $E[Z_i|X, \theta^{(t)}] = \frac{p^{(t)} f(X_i, \mu_2^{(t)}, \sigma^{(t)})}{(1-p^{(t)}) f(X_i, \mu_2^{(t)}, \sigma^{(t)}) + p^{(t)} f(X_i, \mu_2^{(t)}, \sigma^{(t)})}$. Define $\delta_i(\theta^{(t)}) := E[Z_i|X, \theta^{(t)}]$ Now, at the M-step, we must maximize

$$\begin{aligned} &\sum_i \left(\delta_i(\theta^{(t)})\log p + (1 - \delta_i(\theta^{(t)}))\log(1-p)\right. \\ &\quad \left. + \delta_i(\theta^{(t)})\log f(x_i, \mu_2, \sigma) + (1 - \delta_i(\theta^{(t)}))\log f(x_i, \mu_1, \sigma)\right), \end{aligned} \tag{2}$$

with respect to $\theta = (p, \mu_1, \mu_2, \sigma)$ (not $\theta^{(t)}$). The θ that maximizes this will now be $\theta^{(t+1)}$.

Note that only the first two terms depend on p, so we have $p^{(t+1)} = \Sigma\delta_i(\theta^{(t)})/n$. Likewise, only the last two terms depend on (μ_1, μ_2, σ) and this gives

$$\begin{aligned} \mu_1^{(t+1)} &= \frac{1}{\Sigma(1 - \delta_i(\theta^{(t)}))} \sum (1 - \delta_i(\theta^{(t)}))X_i \\ \mu_2^{(t+1)} &= \frac{1}{\Sigma\delta_i(\theta^{(t)})} \sum \delta_i(\theta^{(t)})X_i \\ \sigma^{2,(t+1)} &= \frac{1}{n} \sum \left[(1 - \delta_i(\theta^{(t)}))(X_i - \mu_1^{(t+1)})^2 + \delta_i(\theta^{(t)})(X_i - \mu_2^{(t+1)})^2\right]. \end{aligned} \tag{3}$$