WaveSimPP 1.0

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1	Main Page	1
	1.1 COMSW4995 Final Project: WaveSimPP	1
	1.1.1 Authors	1
	1.1.2 License	1
2	Tutorial	3
	2.1 Using the executable	3
	2.2 Creating a wave simulation and solving it	3
	2.3 Obtaining the results	4
	2.3.1 Plotting the results using the wavePlotter::Plotter class	4
	2.3.2 Exporting the results to csv files	5
3	README	7
	3.1 COMSW4995 Final Project: WaveSimPP	7
	3.1.1 Authors	7
	3.1.2 Acknowledgments	7
	3.2 Theory	8
	3.2.1 Wave simulation	8
	3.2.2 References	8
	3.2.3 Design Philosophy	8
	3.2.3.1 Numpy implementation	8
	3.2.4 Multi Arrays and how math is done on them	9
	3.3 Building	9
	3.3.1 Install the boost library	9
	3.3.2 Install Matplotplusplus	9
	3.3.3 Building the project	10
	3.3.4 Running the executable	10
	3.3.5 Building the documentation	10
4	Module Index	11
	4.1 Modules	11
5	Namespace Index	13
	5.1 Namespace List	13
6	Class Index	15
	6.1 Class List	15
7	File Index	17
	7.1 File List	17
8	Module Documentation	19
	8.1 WaveSimCore	19
	8.1.1 Detailed Description	20
	8.1.2 Function Documentation	20

8.1.2.1 d2fdx2()	20
8.1.2.2 d2fdz2()	20
8.1.2.3 dfdx()	20
8.1.2.4 dfdz()	21
8.1.2.5 divergence()	21
8.1.2.6 get_profile()	21
8.1.2.7 get_sigma_1()	22
8.1.2.8 get_sigma_2()	23
8.1.2.9 ricker()	23
8.1.2.10 wave_solver()	24
8.1.2.11 wave_solver_complex()	25
8.2 Np	26
8.2.1 Detailed Description	27
8.2.2 Function Documentation	27
8.2.2.1 convert_to_matplot()	27
8.2.2.2 operator*() [1/3]	28
8.2.2.3 operator*() [2/3]	28
8.2.2.4 operator*() [3/3]	28
8.2.2.5 operator+() [1/3]	29
8.2.2.6 operator+() [2/3]	29
8.2.2.7 operator+() [3/3]	29
8.2.2.8 operator-() [1/3]	30
8.2.2.9 operator-() [2/3]	
8.2.2.10 operator-() [3/3]	
8.2.2.11 operator/() [1/3]	31
8.2.2.12 operator/() [2/3]	31
8.2.2.13 operator/() [3/3]	31
8.3 WavePlotter	31
8.3.1 Detailed Description	32
8.3.2 Function Documentation	32
8.3.2.1 animate()	32
8.3.2.2 exportAllFrames()	33
8.3.2.3 exportFrame()	33
8.3.2.4 Plotter()	33
8.3.2.5 renderAllFrames()	34
8.3.2.6 renderFrame()	34
8.3.2.7 setSaveDirectory()	34
9 Namespace Documentation	35
9.1 np Namespace Reference	35
9.1.1 Detailed Description	37
9.1.2 Typedef Documentation	37

9.1.2.1 ndArrayValue	37
9.1.3 Enumeration Type Documentation	37
9.1.3.1 indexing	38
9.1.4 Function Documentation	38
9.1.4.1 abs()	38
9.1.4.2 element_wise_apply()	
9.1.4.3 element_wise_duo_apply()	39
9.1.4.4 exp() [1/2]	39
9.1.4.5 exp() [2/2]	39
9.1.4.6 for_each() [1/4]	
9.1.4.7 for_each() [2/4]	40
9.1.4.8 for_each() [3/4]	
9.1.4.9 for_each() [4/4]	41
9.1.4.10 getIndex()	41
9.1.4.11 getIndexArray()	
9.1.4.12 gradient()	
9.1.4.13 linspace()	
9.1.4.14 log() [1/2]	
9.1.4.15 log() [2/2]	
9.1.4.16 max() [1/2]	
9.1.4.17 max() [2/2]	
9.1.4.18 meshgrid()	
9.1.4.19 min() [1/2]	
9.1.4.20 min() [2/2]	
9.1.4.21 pow() [1/2]	
9.1.4.22 pow() [2/2]	
9.1.4.23 slice()	
9.1.4.24 sqrt() [1/2]	
9.1.4.25 sqrt() [2/2]	
9.1.4.26 zeros()	
9.2 wavePlotter Namespace Reference	
9.2.1 Detailed Description	
9.3 waveSimCore Namespace Reference	
9.3.1 Detailed Description	49
10 Class Documentation	51
10.1 wavePlotter::Plotter Class Reference	51
10.1.1 Detailed Description	51
11 File Documentation	53
11.1 coeff.hpp	
11.2 computational.hpp	
11.3 helper_func.hpp	54

11.4 solver.hpp	55
11.5 solver_complex.hpp	56
11.6 source.hpp	57
11.7 wave.cpp	58
11.8 np.hpp	58
11.9 np_to_matplot.hpp	65
11.10 wavePlotter.hpp	65
11.11 wave_solver_with_animation.cpp	66
11.12 main.cpp	67
11.13 Benchmarks.cpp	69
11.14 CoreTests.cpp	70
11.15 MatPlotTest.cpp	72
11.16 variadic cop	72

Main Page

1.1 COMSW4995 Final Project: WaveSimPP

This is the repository for our final project for the discpline COMSW4995: Design in C++ at Columbia University during the Fall of 2022.

This project aims to implement in modern C++ a wave equation solver for geophysical application.

In addition, a custom implementation of numpy in modern C++ is also included as a header library. That library aims to make c++ more pythonic and easier to use for scientific computing. Instead of numpy n-dimensional arrays the library use boost::multi_array and contains many utilities to expand the functionality of the library.

Please check the Readme file for more information.

1.1.1 Authors

Victor Barros - Undergradute Student - Mechanical Engineering - Columbia University

Yan Cheng - PhD Candidate - Applied Mathematics - Columbia University

1.1.2 License

This proejct is licensed under the MIT License - see the LICENSE.md file for details

2 Main Page

Tutorial

2.1 Using the executable

If all you want is to run one simulation with custom parameters, you can use the executable that is generated when you build the project. The executable is located in the build folder.

To build the executable you can use the following commands:

```
mkdir build
cd build
cmake ..
make WaveSimPPExec
```

And then run it with the following command:

./WaveSimPPExec

The executable will ask you for the parameters of the simulation. A complete description of the parameters can be found in the the help menu of the executable. To see the help menu you can run the executable with the -h flag: ./WaveSimPPExec -h

2.2 Creating a wave simulation and solving it

The source code of this tutorial can be found in src/examples/wave solver with animation.cpp

The first part of creating the wave simulation is to define the constants for the simulation. These constants are the number of grid points in the x and z directions, the number of time steps, the differentiation values, the domain, the source parameters, and the source location.

It is important to pay attention to the fact that the plane is XZ not XY due to geophysical conventions.

```
// Define the constants for the simulation
// Number of x and z grid points
int nx = 100;
int nz = 100;
// Number of time steps
int nt = 1000;
// Differentiation values
double dx = 0.01;
double dz = 0.01;
double dt = 0.001;
// Define the domain
double xmin = 0.0;
double xmax = nx * dx;
double zmin = 0.0;
double zmax = nz * dz;
double tmin = 0.0;
double tmax = nt * dt;
// Define the source parameters
```

4 Tutorial

```
double f_M = 10.0;
double amp = 1e0;
double shift = 0.1;
// Source location
int source_is = 50;
int source_is = 50;
```

The next step is to create the source object and the velocity profile

Then we can proceed to solve the wave equation using the wave solver.

```
// Solve the wave equation boost::multi_array<double, 3 > u = waveSimPPCore::wave_solver(vel, dt, dx, dz, nt, nx, nz, f);
```

u is the multi_array that contains the result of the simulation. It has the shape (nt, nx, nz). That means that the first index is the time step, the second index is the x grid point, and the third index is the z grid point.

You can access the result of a specific time step by simply using:

```
boost::multi_array<double, 2> u_at_time_20 = u[20];
```

2.3 Obtaining the results

To see the results you have two main options:

- Use the wavePlotter::Plotter class to plot the results and/or create an animation.
- Export all the frames to a series of csv files for processing in another program.

2.3.1 Plotting the results using the wavePlotter::Plotter class

You should do some light processing to conver the domain from a boost::multi_array to a matplot::vector_2d. This is done using the np::convert_to_matplot function.

After that you can convert the result of each frame to a matplot::vector_2d and plot it using the wavePlotter::Plotter

```
// Define the number of different levels for the contour plot
int num_levels = 100;
// Create the levels for the contour plot based on the min and max values of u
double min_u = np::min(u);
double max_u = np::max(u);
std::vector<double> levels = matplot::linspace(min_u, max_u, num_levels);
// Create the x and z axis for the contour plot and convert them to matplot format
boost::multi_array<double, 1> x = np::linspace(xmin, xmax, nx);
boost::multi_array<double, 1> z = np::linspace(zmin, zmax, nz);
const boost::multi_array<double, 1> axis[2] = {x, z};
std::vector<boost::multi_array<double, 2> XcZ = np::meshgrid(axis, false, np::xy);
matplot::vector_2d Xp = np::convert_to_matplot(XcZ[0]);
matplot::vector_2d Zp = np::convert_to_matplot(XcZ[1]);
```

From this point on you can plot it as you wish, for example, if you want a filled contour plot you can do:

```
matplot::vector_2d Up = np::convert_to_matplot(this->u[frame_index]);
matplot::contourf(this->Xp, this->2p, Up, this->levels);
matplot::show();
```

Another option is to pass the data to the wavePlotter:Plotter class and use the plot function to render all frames

```
// Create the plotter object and animate the results
wavePlotter::Plotter my_plotter(u, Xp, Zp, num_levels, nt);
// If you want to render a specific frame, use this:
// my_plotter.renderFrame(int frame_index);
// Renders the entire animation from start_frame to end_frame
int start_frame = 20;
int end_frame = nt - 1;
int fps = 30;
my_plotter.animate("example-wave.mp4", start_frame, end_frame, fps);
```

The animation will be saved in . and the frames will be saved to ./output

2.3.2 Exporting the results to csv files

You can export the results to a series of individual csv files using the wavePlotter::Plotter class.

```
// Create the plotter object and animate the results
wavePlotter::Plotter my_plotter(u, Xp, Zp, num_levels, nt);
// If you want to export a specific frame, use this:
// my_plotter.exportFrame(int frame_index);
// Exports the entire simulation from start_frame to end_frame int start_frame = 20;
int end_frame = nt - 1;
my_plotter.exportAllFrames(start_frame, end_frame);
```

The frames will be saved to ./output

Each frame is saved as a csv file with the following format:

```
data at 0_0, data at 0_1, data at 0_2, ..., data at 0_nz data at 1_0, data at 1_1, data at 1_2, ..., data at 1_nz data at 2_0, data at 2_1, data at 2_2, ..., data at 2_nz ...
```

If you want to import the data into a python program you can use the following code:

```
frames = []
for i in range(999):
    filename = "output/frame_" + f'{i:08}' +".csv"
    frames.append(pd.read_csv(filename))
```

Please refer to src/examples/PythonLoadingExample.ipynb for a complete example on how to load the data into a python program and render it using matplotlib (of course you can use any other library to render the data).

6 Tutorial

README

3.1 COMSW4995 Final Project: WaveSimPP

Click Here to Access the Github Repo

This is the repository for our final project for the discpline COMSW4995: Design in C++ at Columbia University during the Fall of 2022.

This project aims to implement in modern C++ a wave equation solver for geophysical application.

In addition, a custom implementation of numpy in modern C++ is also included as a header library. That library aims to make c++ more pythonic and easier to use for scientific computing. Instead of numpy n-dimensional arrays the library use boost::multi_array and contains many utilities to expand the functionality of the library.

Detailed documentation

3.1.1 Authors

Victor Barros - Undergradute Student - Mechanical Engineering - Columbia University

Yan Cheng - PhD Candidate - Applied Mathematics - Columbia University

3.1.2 Acknowledgments

We would like to thank Professor Bjarne Stroustrup for his guidance and support during the development of this project.

8 README

3.2 Theory

3.2.1 Wave simulation

When waves travel in an inhomogeneous medium, they may be delayed, reflected, and refracted, and the wave data encodes information about the medium—this is what makes geophysical imaging possible. The propagation of waves in a medium is described by a partial differential equation known as the wave equation. In two dimension, the wave equation is given by:

```
\begin{align*} $$ \left(1_{v^2}\frac{2}{\frac{1}{v^2}}\right) - \frac{1_{v^2}}{\frac{1}{v^2}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}} + \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}} + \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}} + \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}} - \frac{1_{v^2}}{\frac{1_{v^2}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}}} - \frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}{\frac{1_{v^2}}}}}}}}}}}
```

In our simulation, the numerical scheme we use is the finite difference method with the perfectly matched layers [1]:

3.2.2 References

[1] Johnson, Steven G. (2021). Notes on perfectly matched layers (PMLs). arXiv preprint arXiv:2108.05348.

3.2.3 Design Philosophy

3.2.3.1 Numpy implementation

We have noticed that many users are very familiar with python and use it extensively with libraries such as numpy and scipy. However their code is often slow and not very low-level friendly. Even with numpy and scipy's low-level optimizations, there could still be margin for improvement by converting everything to C++, which would allow users to unleash even more optimizations and exert more control over how their code runs. This could also allow the code to run on less powerful devices that often don't support python.

With that in mind we decided to find a way to make transferring that numpy, scipy, etc code to C++ in an easy way, while keeping all of the high level luxuries of python. We decided to implement a numpy-like library in C++ that would allow users to write code in a similar way to python, but with the performance of C++.

We started with the implementation of the functions used in the python version of the wave solver and plan to expand the library to include more functions and features in the future.

The library is contained in a header library format for easy of use.

3.3 Building 9

3.2.4 Multi Arrays and how math is done on them

Representing arrays with more than one dimensions is a difficult task in any programming language, specially in a language like C++ that implements strict type checking. To implement that in a flexible and typesafe way, we chose to build our code around the boost::multi_array. This library provides a container that can be used to represent arrays with any number of dimensions. The library is very flexible and allows the user to define the type of the array and the number of dimensions at compile time. The library is sadly not very well documented but the documentation can be found here: $https://www.boost.org/doc/libs/1_75_0/libs/multi_\leftrightarrow array/doc/index.html$

We decided to build the math functions in a pythonic way, so we implemented numpy functions into our C++ library in a way that they would accept n-dimensions through a template parameters and act accordingly while enforcing dimensional conistency at compile time. We also used concepts and other modern C++ concepts to make sure that, for example, a python call such as $np.max(my_n_dimensional_array)$ would be translated to $np::max(my_n_dimensional_array)$ in C++.

To perform operations on an n-dimensional array we choose to iterate over it and convert the pointers to indexes using a simple arithmetic operation with one division. This is somewhat time consuming since we don't have O(1) time access to any point in the array, instead having O(n) where n is the amount of elements in the multi array. This is the tradeoff necessary to have n-dimensions represented in memory, hopefully in modern cpus this overhead won't be too high. Better solutions could be investigated further.

We also implemented simple arithmetic operators with multi arrays to make them more arithmetic friendly such as they are in python.

Only one small subset of numpy functions were implemented, but the library is easily extensible and more functions can be added in the future.

3.3 Building

Please be aware that since this library uses a few C++ 20 features it is only been tested on gcc-11 and above. It is possible that it will work on other compilers but it is not guaranteed.

3.3.1 Install the boost library

It is important to install the boost library before building the project. The boost library is used for data structures and algorithms. The boost library can be installed using the following command on ubuntu:

```
sudo apt-get install libboost-all-dev
```

For Mac:

brew install boost

3.3.2 Install Matplotplusplus

This is the library used to generate graphics in the project. To be able to compile this project you must have it installed in your system. First install its dependencies:

```
sudo apt-get install gnuplot
```

or in Mac:

brew install gnuplot

Then install the library itself by cloning from source:

```
cd src/ExternalLibraries
git clone https://github.com/alandefreitas/matplotplusplus
cd matplotplusplu
mkdir build
cd build
cmake .. -DCMAKE_BUILD_TYPE=Release -DCMAKE_CXX_FLAGS="-02" -DBUILD_EXAMPLES=OFF -DBUILD_TESTS=OFF
sudo cmake --build . --parallel 2 --config Release
sudo cmake --install .
```

If you are using clang on mac, make sure to force CMAKE to use gcc by adding the following flag to the first cmake command:

```
-DCMAKE_C_COMPILER=/usr/bin/gcc -DCMAKE_CXX_COMPILER=/usr/bin/g++
```

(or equivalent paths depending on where your gcc is installed)

10 **README**

3.3.3 Building the project

mkdir build cd build cmake .. make all

You can also build only the executable by running:

make WaveSimPPExec

3.3.4 Running the executable

./WaveSimPPExec

Use the help flag -h to see the available runtime options and the full list and description of the parameters.

3.3.5 Building the documentation

Docs building script:

./compileDocs.sh

Manually: doxygen dconfig cd documentation/latex
pdflatex refman.tex
cp refman.pdf ../WaveSimPP-1.0-doc.pdf

Module Index

4.1 Modules

Here is a list of all modules:

WaveSimCore	19
Np	26
WavePlotter	31

12 Module Index

Namespace Index

5.1 Namespace List

Here is a list of all documented namespaces with brief descriptions:

np		
	Custom implementation of numpy in C++	35
wavePlo	rer	
	Custom plotter class	48
waveSim	Core	49

14 Namespace Index

Class Index

6.1 Class List

Here are the class	es, structs	. unions	and interfaces	with brief	descriptions

wavePlotter::Plotter	
This class is used to plot the wave field TODO: make it multithreaded	51

16 Class Index

File Index

7.1 File List

Here is a list of all documented files with brief descriptions:

src/main.cpp	7
src/CoreAlgorithm/coeff.hpp	3
src/CoreAlgorithm/computational.hpp	1
src/CoreAlgorithm/helper_func.hpp	1
src/CoreAlgorithm/solver.hpp	5
src/CoreAlgorithm/solver_complex.hpp	3
src/CoreAlgorithm/source.hpp	7
src/CoreAlgorithm/wave.cpp	3
src/CustomLibraries/np.hpp	3
src/CustomLibraries/np_to_matplot.hpp	5
src/CustomLibraries/wavePlotter.hpp	5
src/examples/wave_solver_with_animation.cpp	3
src/tests/Benchmarks.cpp	9
src/tests/CoreTests.cpp)
src/tests/MatPlotTest.cpp	2
src/tests/variadic.cop	5

18 File Index

Module Documentation

8.1 WaveSimCore

Namespaces

• namespace waveSimCore

Functions

- boost::multi_array< double, 2 > waveSimCore::get_sigma_1 (boost::multi_array< double, 1 > x, double dx, int nx, int nz, double c_max, int n=15, double R=1e-3, double m=2.0)
- boost::multi_array< double, 2 > waveSimCore::get_sigma_2 (boost::multi_array< double, 1 > z, double dz, int nx, int nz, double c_max, int n=10, double R=1e-3, double m=2.0)
- boost::multi_array< double, 2 > waveSimCore::get_profile (double xmin, double xmax, double zmin, double zmax, int nx, int nx, double r)

Get the velocity profile of the model as a 2D Array.

- boost::multi_array< double, 2 > waveSimCore::dfdx (boost::multi_array< double, 2 > f, double dx)

 Takes the partial derivative of a 2D matrix f with respect to x.
- boost::multi_array< double, 2 > waveSimCore::dfdz (boost::multi_array< double, 2 > f, double dz)

 Takes the partial derivative of a 2D matrix f with respect to z.
- boost::multi_array< double, 2 > waveSimCore::d2fdx2 (boost::multi_array< double, 2 > f, double dx)

 Takes the second partial derivative of a 2D matrix f with respect to x.
- boost::multi_array< double, 2 > waveSimCore::d2fdz2 (boost::multi_array< double, 2 > f, double dz)

 Takes the second partial derivative of a 2D matrix f with respect to z.
- boost::multi_array< double, 2 > waveSimCore::divergence (boost::multi_array< double, 2 > f1, boost
 ::multi_array< double, 2 > f2, double dx, double dz)
- boost::multi_array< double, 3 > waveSimCore::wave_solver (boost::multi_array< double, 2 > c, double dt, double dx, double dz, int nt, int nx, int nx, boost::multi_array< double, 3 > f)
- boost::multi_array< double, 3 > waveSimCore::wave_solver_complex (boost::multi_array< double, 2 > c, double dt, double dx, double dz, int nt, int nx, int nz, boost::multi_array< double, 3 > f, boost::multi_array< double, 2 > sigma_1, boost::multi_array< double, 2 > sigma_2)
- boost::multi_array< double, 3 > waveSimCore::ricker (int i_s, int j_s, double f, double amp, double shift, double tmin, double tmax, int nt, int nx, int nz)

Get the Ricker wavelet as a 3D Array.

20 Module Documentation

8.1.1 Detailed Description

8.1.2 Function Documentation

8.1.2.1 d2fdx2()

```
boost::multi_array< double, 2 > waveSimCore::d2fdx2 ( boost::multi_array< double, 2 > f, double dx )
```

Takes the second partial derivative of a 2D matrix f with respect to x.

Definition at line 32 of file helper func.hpp.

8.1.2.2 d2fdz2()

```
boost::multi_array< double, 2 > waveSimCore::d2fdz2 ( boost::multi_array< double, 2 > f, double dz)
```

Takes the second partial derivative of a 2D matrix f with respect to z.

Definition at line 40 of file helper_func.hpp.

```
00041 {
00042 boost::multi_array<double, 2> df = dfdz(f, dz);
00043 boost::multi_array<double, 2> df2 = dfdz(df, dz);
00044 return df2;
00045 }
```

8.1.2.3 dfdx()

```
boost::multi_array< double, 2 > waveSimCore::dfdx ( boost::multi_array< double, 2 > f, double dx)
```

Takes the partial derivative of a 2D matrix f with respect to x.

Definition at line 18 of file helper_func.hpp.

8.1 WaveSimCore 21

8.1.2.4 dfdz()

```
boost::multi_array< double, 2 > waveSimCore::dfdz ( boost::multi_array< double, 2 > f, double dz)
```

Takes the partial derivative of a 2D matrix f with respect to z.

Definition at line 25 of file helper_func.hpp.

8.1.2.5 divergence()

```
boost::multi_array< double, 2 > waveSimCore::divergence ( boost::multi_array< double, 2 > f1, boost::multi_array< double, 2 > f2, double dx, double dz)
```

Takes the divergence of a 2D matrices fx,fz with respect to x and z Returns dfx/dx + dfz/dz

Definition at line 49 of file helper func.hpp.

8.1.2.6 get_profile()

Get the velocity profile of the model as a 2D Array.

Definition at line 16 of file computational.hpp.

```
00017
00018
                boost::multi_array<double, 2> c(boost::extents[nx][nz]);
00019
                boost::multi_array<double, 1> x = np::linspace(xmin, xmax, nx);
00020
00021
                boost::multi_array<double, 1> z = np::linspace(zmin, zmax, nz);
00022
                const boost::multi_array<double, 1> axis[2] = {x, z};
std::vector<boost::multi_array<double, 2> XZ = np::meshgrid(axis, false, np::ij);
00023
00024
00025
00026
               double x_0 = xmax / 2.0;
00027
               double z_0 = zmax / 2.0;
```

22 Module Documentation

```
00028
00029
              for (int i = 0; i < nx; i++)
00030
                  for (int j = 0; j < nz; j++)
00031
00032
                       if (np::pow(XZ[0][i][i] - x_0, 2.0) + np::pow(XZ[1][i][i] - z_0, 2.0) \le np::pow(r, x_0)
00033
       2.0))
00034
                          c[i][j] = 3.0;
00035
                      else
                           c[i][j] = 3.0;
00036
00037
                  }
00038
              }
00039
00040
              return c;
00041
```

8.1.2.7 get_sigma_1()

```
boost::multi_array< double, 2 > waveSimCore::get_sigma_1 ( boost::multi_array< double, 1 > x, double dx, int nx, int nz, double c\_max, int n=15, double R=1e-3, double m=2.0)
```

Returns the sigma 1 coefficient for the PML boundary conditions sigma_1 and sigma_2 are the coefficients for the x and z directions respectively Check Johnson, Steven G. (2021).

Definition at line 21 of file coeff.hpp.

```
00023
00024
                boost::multi_array<double, 2> sigma_1(boost::extents[nx][nz]);
                const double PML_width = n * dx;
00025
00026
00027
                const double sigma_max = -c_max * log(R) * (m + 1.0) / np::pow(PML_width, m + 1.0);
00028
00029
                const double x = np::max(x) - PML width:
00030
00031
                boost::multi_array<double, 1> polynomial(boost::extents[nx]);
00032
00033
                 for (int i = 0; i < nx; i++)</pre>
00034
00035
                     if (x[i] > x_0)
00036
                          polynomial[i] = sigma_max * np::pow(np::abs(x[i] - x_0), m); \\ polynomial[nx - 1 - i] = polynomial[i];
00037
00038
00039
00040
                     else
00041
                     {
                          polynomial[i] = 0;
00042
00043
                     }
00044
00045
                for (int i = 0; i < nx; i++)
    for (int j = 0; j < nz; j++)
        sigma_1[i][j] = polynomial[i];</pre>
00046
00047
00048
00049
00050
                return sigma_1;
00051
```

8.1 WaveSimCore 23

8.1.2.8 get_sigma_2()

Returns the sigma 2 coefficient for the PML boundary conditions sigma_1 and sigma_2 are the coefficients for the x and z directions respectively Check Johnson, Steven G. (2021).

Definition at line 56 of file coeff.hpp.

```
00058
00059
                  boost::multi_array<double, 2> sigma_2(boost::extents[nx][nz]);
                  const double PML_width = n * dz; const double sigma_max = -c_max * log(R) * (m + 1.0) / np::pow(PML_width, m + 1.0);
00060
00061
00062
00063
                  const double z_0 = np::max(z) - PML_width;
00064
                  boost::multi_array<double, 1> polynomial(boost::extents[nz]);
00065
00066
                  for (int j = 0; j < nz; j++)
00067
00068
                        if (z[j] > z_0)
00069
                             \label{eq:polynomial} $$ polynomial[j] = sigma_max * np::pow(np::abs(z[j] - z_0), m); $$ polynomial[nz - 1 - j] = polynomial[j]; $$ 
00070
00071
00072
00073
                       else
00074
                       {
00075
                             polynomial[j] = 0;
00076
                       }
00077
00078
                  for (int i = 0; i < nx; i++)
    for (int j = 0; j < nz; j++)
        sigma_2[i][j] = polynomial[j];</pre>
00079
08000
00081
00082
00083
                  return sigma_2;
00084
```

8.1.2.9 ricker()

```
boost::multi_array< double, 3 > waveSimCore::ricker (
    int i_s,
    int j_s,
    double f,
    double amp,
    double shift,
    double tmin,
    double tmax,
    int nt,
    int nx,
    int nz )
```

Get the Ricker wavelet as a 3D Array.

```
Definition at line 14 of file source.hpp.
```

```
00016 {
00017 const double pi = 3.141592654;
```

24 Module Documentation

```
00018
00019
               boost::multi_array<double, 1> t = np::linspace(tmin, tmax, nt);
00020
               boost::multi_array<double, 1> pft2 = np::pow(pi * f * (t - shift), 2.0);
               boost::multi_array<double, 1> r = amp * (1.0 - 2.0 * pft2) * np::exp(-1.0 * pft2);
00021
00022
00023
               int dimensions x[] = \{nx\};
               boost::multi_array<double, 1> x = np::zeros<double>(dimensions_x);
00025
00026
               int dimensions_z[] = {nz};
00027
               boost::multi_array<double, 1> z = np::zeros<double>(dimensions_z);
00028
00029
               x[i s] = 1.0;
               z[j_s] = 1.0;
00030
00031
               const boost::multi_array<double, 1> axis[3] = \{r, x, z\};
00032
               std::vector<boost::multi_array<double, 3> RXZ = np::meshgrid(axis, false, np::ij);
boost::multi_array<double, 3> source = RXZ[0] * RXZ[1] * RXZ[2];
00033
00034
00035
00036
               return source;
00037
```

8.1.2.10 wave_solver()

This function solves the wave equation using the methods explained in the readme The boundary conditions are so that waves bounce at the boundary

Definition at line 27 of file solver.hpp.

```
00030
00031
              const boost::multi_array<double, 2> CX = np::pow(c * dt / dx, 2.0); const boost::multi_array<double, 2> CZ = np::pow(c * dt / dz, 2.0);
00032
00033
00034
              const double Cf = np::pow(dt, 2.0);
00035
00036
              int dimensions_1[] = {nt, nx, nz};
00037
              boost::multi_array<double, 3> u = np::zeros<double>(dimensions_1);
              int dimensions_4[] = {nx, nz};
00038
00039
              boost::multi_array<double, 2> u_xx = np::zeros<double>(dimensions_4);
00040
              int dimensions_5[] = {nx, nz};
              boost::multi_array<double, 2> u_zz = np::zeros<double>(dimensions_5);
00041
00042
              for (int n = 1; n < nt - 1; n++)</pre>
00043
00044
00045
                  for (int i = 1; i < nx - 1; i++)
00046
                  {
00047
                       for (int j = 1; j < nz - 1; j++)
00048
                          00049
00050
00051
00052
                  }
00053
00054
                  // ! Update u
                  for (int i = 0; i < nx; i++)
00055
00056
00057
                       for (int j = 0; j < nz; j++)
00058
                          u[n + 1][i][j] = 2.0 * u[n][i][j] + CX[i][j] * u_xx[i][j] + CZ[i][j] * u_zz[i][j]
       + Cf * f[n][i][j] - u[n - 1][i][j];
00060
00061
                  }
00062
00063
                  // Dirichlet boundary condition
00064
                  for (int i = 0; i < nx; i++)</pre>
```

8.1 WaveSimCore 25

```
00065
                     {
                         u[n + 1][i][0] = 0.0;
u[n + 1][i][nz - 1] = 0.0;
00066
00067
00068
00069
                     for (int j = 0; j < nz; j++)
00070
00071
                         u[n + 1][0][j] = 0.0;
00072
                         u[n + 1][nx - 1][j] = 0.0;
00073
00074
00075
                return u:
00076
```

8.1.2.11 wave_solver_complex()

Solves the wave equation using a more complex solver compared to wave_solver Also has different boundary conditions. In this one the boundary extends to infinity

Definition at line 22 of file solver_complex.hpp.

```
00026
00027
00028
               const boost::multi_array<double, 2 > C1 = 1.0 + (dt * (sigma_1 + sigma_2) * 0.5);
00029
               const boost::multi_array<double, 2 \times C2 = (sigma_1 * sigma_2 * np::pow(dt, 2.0)) - 2.0;
00030
               const boost::multi_array<double, 2 > C3 = 1.0 - (dt * (sigma_1 + sigma_2) * 0.5);
00031
               const boost::multi_array<double, 2 > C4 = np::pow(dt * c, 2.0);
               const boost::multi_array<double, 2> C5 = 1.0 + (dt * sigma_1 * 0.5);
00032
00033
               const boost::multi_array<double, 2> C6 = 1.0 + (dt * sigma_2 * 0.5);
               const boost::multi_array<double, 2> C7 = 1.0 - (dt * sigma_1 * 0.5);
00034
00035
               const boost::multi_array<double, 2> C8 = 1.0 - (dt * sigma_2 * 0.5);
00036
00037
               int dimensions_1[] = {nt, nx, nz};
00038
              boost::multi_array<double, 3> u = np::zeros<double>(dimensions_1);
00039
00040
               int dimensions_2[] = {nx, nz};
00041
               boost::multi_array<double, 2> q_1 = np::zeros<double>(dimensions_2);
00042
               int dimensions_3[] = {nx, nz};
00043
               boost::multi_array<double, 2> q_2 = np::zeros<double>(dimensions_3);
00044
00045
               int dimensions 4[] = {nx, nz};
00046
               boost::multi_array<double, 2> u_xx = np::zeros<double>(dimensions_4);
00047
               int dimensions_5[] = {nx, nz};
00048
               boost::multi_array<double, 2> u_zz = np::zeros<double>(dimensions_5);
00049
               boost::multi_array<double, 2> f_n(boost::extents[nx][nz]);
boost::multi_array<double, 2> u_n(boost::extents[nx][nz]);
00050
00051
00052
               boost::multi_array<double, 2> u_n_1(boost::extents[nx][nz]);
00053
00054
               for (int n = 1; n < nt - 1; n++)
00055
00056
00057
                   for (int i = 0; i < nx; i++)
00058
00059
                        for (int j = 0; j < nz; j++)
00060
                           f_n[i][j] = f[n][i][j];
u_n[i][j] = u[n][i][j];
00061
00062
                            u_n_1[i][j] = u[n - 1][i][j];
00063
00064
00065
                   }
00066
```

26 Module Documentation

```
boost::multi_array<double, 2> div = divergence(q_1 * sigma_1, q_2 * sigma_2, dx, dz);
                      boost::multi_array<double, 2> dq_1dx = dfdx(q_1, dx);
boost::multi_array<double, 2> dq_2dz = dfdz(q_2, dz);
00068
00069
                      u_x = d2fdx2(u_n, dx); // (nx, nz)

u_z = d2fdz2(u_n, dz); // (nx, nz)
00070
00071
                      boost::multi_array<double, 2> dudx = dfdx(u_n, dx);
00072
                      boost::multi_array<double, 2> dudz = dfdz(u_n, dz);
00073
00074
00075
                                   for (int i = 1; i < nx-1; i++)
00076
00077
                                        for (int j = 1; j < nz-1; j++)
00078
                                            00079
00080
00081
00082
00083
                      // ! Update u
00084
00085
                      for (int i = 0; i < nx; i++)
00086
00087
                           for (int j = 0; j < nz; j++)
00088
         \begin{array}{c} u[n+1][i][j] = (C4[i][j] * ((u_xx[i][j] / np::pow(dx, 2.0)) + (u_zz[i][j] / np::pow(dz, 2.0)) - div[i][j] + sigma_2[i][j] * dq_1dx[i][j] + sigma_1[i][j] * dq_2dz[i][j] + f_n[i][j]) - (C2[i][j] * u_n[i][j]) - (C3[i][j] * u_n_1[i][j])) / C1[i][j]; \end{array} 
00089
00090
00091
00092
                      // ! Update q_1, q_2 for (int i = 0; i < nx; i++)
00093
00094
00095
00096
                           for (int j = 0; j < nz; j++)
00097
00098
                                q_1[i][j] = (dt * dudx[i][j] + C7[i][j] * q_1[i][j]) / C5[i][j];
                                q_2[i][j] = (dt * dudz[i][j] + C8[i][j] * q_2[i][j]) / C6[i][j];
00099
00100
00101
00102
00103
                      // Dirichlet boundary condition
00104
                      for (int i = 0; i < nx; i++)</pre>
00105
                           u[n + 1][i][0] = 0.0;
u[n + 1][i][nz - 1] = 0.0;
00106
00107
00108
00109
                      for (int j = 0; j < nz; j++)
00110
00111
                           u[n + 1][0][j] = 0.0;
00112
                           u[n + 1][nx - 1][j] = 0.0;
00113
                      }
00114
00115
                 return u;
00116
            }
```

8.2 Np

Namespaces

namespace np

Custom implementation of numpy in C++.

Functions

template < class T , long unsigned int ND>
 boost::multi_array < T, ND > operator* (boost::multi_array < T, ND > const &lhs, boost::multi_array < T, ND > const &rhs)

Multiplication operator between two multi arrays, element-wise.

template < class T , long unsigned int ND>
 boost::multi_array < T, ND > operator* (T const &lhs, boost::multi_array < T, ND > const &rhs)
 Multiplication operator between a multi array and a scalar.

template < class T , long unsigned int ND>
 boost::multi_array < T, ND > operator* (boost::multi_array < T, ND > const &lhs, T const &rhs)

8.2 Np 27

Multiplication operator between a multi array and a scalar.

• template < class T , long unsigned int ND>

 $boost::multi_array < T, \ ND > operator + \ (boost::multi_array < T, \ ND > const \ \&lhs, \ boost::multi_array < T, \ ND > const \ \&rhs)$

Addition operator between two multi arrays, element wise.

• template < class T , long unsigned int ND>

```
boost::multi_array< T, ND > operator+ (T const &lhs, boost::multi_array< T, ND > const &rhs)
```

Addition operator between a multi array and a scalar.

• template < class T , long unsigned int ND>

```
boost::multi_array < T, ND > operator+ (boost::multi_array < T, ND > const &lhs, T const &rhs)
```

Addition operator between a scalar and a multi array.

template < class T , long unsigned int ND>

boost::multi_array< T, ND > operator- (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs)

Minus operator between two multi arrays, element-wise.

• template<class T , long unsigned int ND>

```
boost::multi_array < T, ND > operator- (T const &lhs, boost::multi_array < T, ND > const &rhs)
```

Minus operator between a scalar and a multi array, element-wise.

• template < class T , long unsigned int ND>

```
boost::multi array < T, ND > operator- (boost::multi array < T, ND > const &lhs, T const &rhs)
```

Minus operator between a multi array and a scalar, element-wise.

• template < class T , long unsigned int ND>

```
boost::multi_array< T, ND > operator/ (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs)
```

Division between two multi arrays, element wise.

• template < class T , long unsigned int ND>

```
boost::multi_array< T, ND > operator/ (T const &lhs, boost::multi_array< T, ND > const &rhs)
```

Division between a scalar and a multi array, element wise.

• template < class T , long unsigned int ND>

```
boost::multi_array< T, ND > operator/ (boost::multi_array< T, ND > const &lhs, T const &rhs)
```

Division between a multi array and a scalar, element wise.

matplot::vector_2d np::convert_to_matplot (const boost::multi_array< double, 2 > &arr)

Convert a 2D boost::multi_array to a matplot::vector_2d.

8.2.1 Detailed Description

8.2.2 Function Documentation

8.2.2.1 convert_to_matplot()

Convert a 2D boost::multi_array to a matplot::vector_2d.

```
Definition at line 16 of file np_to_matplot.hpp.
```

```
00017
00018 std::vector<double> x = matplot::linspace(0, 0, arr.shape()[0]);
00019 std::vector<double> y = matplot::linspace(00, 0, arr.shape()[1]);
00020 matplot::vector_2d result = std::get<0>(matplot::meshgrid(x, y));
```

28 Module Documentation

```
00021
              // std::cout « "arr.shape()[0] = " « arr.shape()[0] « " arr.shape()[1] = " « arr.shape()[1] «
       std::endl;
00022
               for (size_t i = 0; i < arr.shape()[0]; i++)</pre>
00023
              {
00024
                   for (size_t j = 0; j < arr.shape()[1]; j++)</pre>
00025
00026
                       result[i][j] = arr[i][j];
00027
00028
00029
               return result;
00030
```

8.2.2.2 operator*() [1/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator* (
                boost::multi_array< T, ND > const & lhs,
                boost::multi_array< T, ND > const & rhs ) [inline]
```

Multiplication operator between two multi arrays, element-wise.

```
Definition at line 504 of file np.hpp.
```

8.2.2.3 operator*() [2/3]

Multiplication operator between a multi array and a scalar.

```
Definition at line 520 of file np.hpp.
```

```
00521 {
00522     return rhs * lhs;
00523 }
```

8.2.2.4 operator*() [3/3]

Multiplication operator between a multi array and a scalar.

Definition at line 512 of file np.hpp.

8.2 Np 29

8.2.2.5 operator+() [1/3]

Addition operator between two multi arrays, element wise.

Definition at line 528 of file np.hpp.

8.2.2.6 operator+() [2/3]

Addition operator between a scalar and a multi array.

Definition at line 545 of file np.hpp.

8.2.2.7 operator+() [3/3]

Addition operator between a multi array and a scalar.

Definition at line 536 of file np.hpp.

30 Module Documentation

8.2.2.8 operator-() [1/3]

Minus operator between two multi arrays, element-wise.

Definition at line 553 of file np.hpp.

```
00554 {
00555     std::function<T(T, T)> func = std::minus<T>();
00556     return np::element_wise_duo_apply(lhs, rhs, func);
00557 }
```

8.2.2.9 operator-() [2/3]

Minus operator between a multi array and a scalar, element-wise.

Definition at line 570 of file np.hpp.

8.2.2.10 operator-() [3/3]

Minus operator between a scalar and a multi array, element-wise.

Definition at line 561 of file np.hpp.

8.3 WavePlotter 31

8.2.2.11 operator/() [1/3]

Division between two multi arrays, element wise.

Definition at line 578 of file np.hpp.

```
00579 {
00580     std::function<T(T, T)> func = std::divides<T>();
00581     return np::element_wise_duo_apply(lhs, rhs, func);
00582 }
```

8.2.2.12 operator/() [2/3]

Division between a multi array and a scalar, element wise.

Definition at line 595 of file np.hpp.

8.2.2.13 operator/() [3/3]

Division between a scalar and a multi array, element wise.

Definition at line 586 of file np.hpp.

8.3 WavePlotter

Namespaces

· namespace wavePlotter

Custom plotter class.

32 Module Documentation

Classes

· class wavePlotter::Plotter

This class is used to plot the wave field TODO: make it multithreaded.

Functions

 wavePlotter::Plotter (const boost::multi_array< double, 3 > &u, const matplot::vector_2d &Xp, const matplot::vector_2d &Zp, int num_levels, int nt)

Constructor.

void wavePlotter::Plotter::renderFrame (int index)

Renders a frame of the wave field to a image on disk.

void wavePlotter::Plotter::renderAllFrames (int begin frame index, int end frame index)

Renders all frames of the wave field to form an animation to be saved on disk.

void wavePlotter::Plotter::animate (std::string output_file_name, int begin_frame_index, int end_frame_index, int frame_rate)

Renders a complete video animation of the wave field.

void wavePlotter::Plotter::exportFrame (int index)

Export a frame of the wave field to a .csv format for external use.

void wavePlotter::Plotter::exportAllFrames (int begin frame index, int end frame index)

Export all frames of the wave field to a .csv format for external use.

void wavePlotter::Plotter::setSaveDirectory (std::string save_directory)

Set the save directory for the rendered frames.

8.3.1 Detailed Description

8.3.2 Function Documentation

8.3.2.1 animate()

```
void wavePlotter::Plotter::animate (
    std::string output_file_name,
    int begin_frame_index,
    int end_frame_index,
    int frame_rate ) [inline]
```

Renders a complete video animation of the wave field.

Definition at line 56 of file wavePlotter.hpp.

8.3 WavePlotter 33

8.3.2.2 exportAllFrames()

Export all frames of the wave field to a .csv format for external use.

Definition at line 82 of file wavePlotter.hpp.

8.3.2.3 exportFrame()

```
void wavePlotter::Plotter::exportFrame (
    int index ) [inline]
```

Export a frame of the wave field to a .csv format for external use.

Definition at line 63 of file wavePlotter.hpp.

```
00064
00065
                   matplot::vector_2d Up = np::convert_to_matplot(this->u[index]);
00066
                    std::ofstream outfile;
                   outfile.open(save_directory + "/frame_" + format_num(index) + ".csv");
00067
00068
                    for (std::size_t i = 0; i < Up.size(); i++)</pre>
00069
00070
                        for (std::size_t j = 0; j < Up[i].size(); j++)</pre>
00071
                            outfile « Up[i][j];
00072
                            if (j != Up[i].size() - 1)
    outfile « ",";
00073
00074
00075
                        outfile « "\n";
00076
00077
                   outfile.close();
00078
00079
```

8.3.2.4 Plotter()

Constructor.

Definition at line 25 of file wavePlotter.hpp.

```
00026
00027
                    this->u.resize(boost::extents[u.shape()[0]][u.shape()[1]][u.shape()[2]]);
00028
                    this -> u = u;
                    this->Xp = Xp;
this->Zp = Zp;
00029
00030
00031
                    this->num_levels = num_levels;
00032
                    this->nt = nt;
00033
                    double min_u = np::min(u);
                    double max_u = np::max(u);
std::cout « "min_u = " « min_u « " max_u = " « max_u « "\n";
00034
00035
00036
                    this->levels = matplot::linspace(min_u, max_u, num_levels);
00037
```

34 Module Documentation

8.3.2.5 renderAllFrames()

Renders all frames of the wave field to form an animation to be saved on disk.

Definition at line 47 of file wavePlotter.hpp.

8.3.2.6 renderFrame()

```
void wavePlotter::Plotter::renderFrame (
    int index ) [inline]
```

Renders a frame of the wave field to a image on disk.

Definition at line 39 of file wavePlotter.hpp.

8.3.2.7 setSaveDirectory()

Set the save directory for the rendered frames.

Definition at line 91 of file wavePlotter.hpp.

```
00092 {
00093 this->save_directory = save_directory;
00094 }
```

Chapter 9

Namespace Documentation

9.1 np Namespace Reference

Custom implementation of numpy in C++.

Typedefs

• typedef double ndArrayValue

Enumerations

enum indexing { xy , ij }

Functions

template<std::size_t ND>

boost::multi_array< ndArrayValue, ND >::index getIndex (const boost::multi_array< ndArrayValue, ND > &m, const ndArrayValue *requestedElement, const unsigned short int direction)

Gets the index of one element in a multi_array in one axis.

template<std::size t ND>

Gets the index of one element in a multi_array.

template<typename Array , typename Element , typename Functor > void for_each (const boost::type< Element > &type_dispatch, Array A, Functor &xform)

template<typename Element , typename Functor >
 void for_each (const boost::type< Element > &, Element &Val, Functor &xform)

Function to apply a function to all elements of a multi_array.

template<typename Element , typename Iterator , typename Functor >
 void for_each (const boost::type< Element > &type_dispatch, Iterator begin, Iterator end, Functor &xform)

Function to apply a function to all elements of a multi_array.

template<typename Array , typename Functor > void for_each (Array &A, Functor xform)

input)[ND])

```
• template<typename T , long unsigned int ND>
  requires std::is floating point<T>
  ::value constexpr std::vector< boost::multi array< T, ND >> gradient (boost::multi array< T, ND > inArray,
  std::initializer_list< T > args)

    boost::multi_array< double, 1 > linspace (double start, double stop, long unsigned int num)

      Implements the numpy linspace function.
• template<typename T , long unsigned int ND>
  requires std::is_arithmetic<T>
  ::value constexpr std::vector< boost::multi_array< T, ND >> meshgrid (const boost::multi_array< T, 1
  >(&cinput)[ND], bool sparsing=false, indexing indexing_type=xy)
• template < class T , long unsigned int ND>
  requires std::is arithmetic<T>
  ::value constexpr boost::multi array< T, ND > element wise apply (const boost::multi array< T, ND >
  &input array, std::function < T(T) > func)
      Creates a new array and fills it with the values of the result of the function called on the input array element-wise.
• template < class T , long unsigned int ND>
  requires std::is_arithmetic<T>
  ::value constexpr boost::multi_array< T, ND > sqrt (const boost::multi_array< T, ND > &input_array)
      Implements the numpy sgrt function on multi arrays.
• template<class T >
  requires std::is_arithmetic<T>
  ::value constexpr T sqrt (const T input)
      Implements the numpy sqrt function on scalars.
• template < class T , long unsigned int ND>
  requires std::is_arithmetic<T>
  ::value constexpr boost::multi_array< T, ND > exp (const boost::multi_array< T, ND > &input_array)
      Implements the numpy exp function on multi arrays.
• template<class T >
  requires std::is_arithmetic<T>
  ::value constexpr T exp (const T input)
      Implements the numpy exp function on scalars.
• template < class T , long unsigned int ND>
  requires std::is_arithmetic<T>
  ::value constexpr boost::multi_array< T, ND > log (const boost::multi_array< T, ND > &input_array)
      Implements the numpy log function on multi arrays.
• template<class T >
  requires std::is arithmetic<T>
  ::value constexpr T log (const T input)
      Implements the numpy log function on scalars.

    template < class T , long unsigned int ND>

  requires std::is_arithmetic<T>
  ::value constexpr boost::multi_array < T, ND > pow (const boost::multi_array < T, ND > &input_array, const
  T exponent)
      Implements the numpy pow function on multi arrays.
template<class T >
  requires std::is arithmetic<T>
  ::value constexpr T pow (const T input, const T exponent)
      Implements the numpy pow function on scalars.
• template < class T , long unsigned int ND>
  constexpr boost::multi_array< T, ND > element_wise_duo_apply (boost::multi_array< T, ND > const &lhs,
  boost::multi array < T, ND > const &rhs, std::function < T(T, T)> func)
• template<typename T , typename inT , long unsigned int ND>
  requires std::is_integral<inT>
  ::value &&std::is_arithmetic< T >::value constexpr boost::multi_array< T, ND > zeros (inT(&dimensions_ \infty
```

Implements the numpy zeros function for an n-dimensionl multi array. • template<typename T , long unsigned int ND> requires std::is_arithmetic<T> ::value constexpr T max (boost::multi_array< T, ND > const &input_array) Implements the numpy max function for an n-dimensionl multi array. • template<class T , class... Ts, class = std::enable_if_t<(std::is_same_v<T, Ts> && ...)>> requires std::is arithmetic<T> ::value constexpr T max (T input1, Ts... inputs) Implements the numpy max function for an variadic number of arguments. • template<typename T , long unsigned int ND> requires std::is_arithmetic<T> ::value constexpr T min (boost::multi_array< T, ND > const &input_array) Implements the numpy min function for an n-dimensionl multi array. • template<class T , class... Ts, class = std::enable_if_t<(std::is_same_v<T, Ts> && ...)>> requires std::is_arithmetic<T> constexpr T min (T input1, Ts... inputs) Implements the numpy min function for an variadic number of arguments. • template<typename T >requires std::is_arithmetic<T> ::value constexpr T abs (T input) Implements the numpy abs function for a scalar. • template<typename T , long unsigned int ND> requires std::is_arithmetic<T> ::value constexpr boost::multi_array< T, ND - 1 > slice (boost::multi_array< T, ND > const &input_array,

Slices the array through one dimension and returns a ND - 1 dimensional array.

• matplot::vector_2d convert_to_matplot (const boost::multi_array< double, 2 > &arr)

Convert a 2D boost::multi_array to a matplot::vector_2d.

9.1.1 Detailed Description

std::size_t slice_index)

Custom implementation of numpy in C++.

9.1.2 Typedef Documentation

9.1.2.1 ndArrayValue

typedef double np::ndArrayValue

Definition at line 22 of file np.hpp.

9.1.3 Enumeration Type Documentation

9.1.3.1 indexing

```
enum np::indexing
```

Definition at line 172 of file np.hpp.

```
00173 {
00174 xy,
00175 ij
00176 };
```

9.1.4 Function Documentation

9.1.4.1 abs()

Implements the numpy abs function for a scalar.

Definition at line 463 of file np.hpp.

9.1.4.2 element wise apply()

Creates a new array and fills it with the values of the result of the function called on the input array element-wise.

Definition at line 243 of file np.hpp.

```
00244
00245
00246
              // Create output array copying extents
              using arrayIndex = boost::multi_array<double, ND>::index;
00247
00248
              using ndIndexArray = boost::array<arrayIndex, ND>;
00249
              boost::detail::multi_array::extent_gen<ND> output_extents;
00250
              std::vector<size_t> shape_list;
00251
              for (std::size_t i = 0; i < ND; i++)</pre>
00252
              {
00253
                  shape_list.push_back(input_array.shape()[i]);
00254
00255
              std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00256
              boost::multi_array<T, ND> output_array(output_extents);
00257
00258
              // Looping through the elements of the output array
00259
              const T *p = input_array.data();
00260
              ndIndexArray index;
00261
              for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00262
              {
00263
                  index = getIndexArray(input_array, p);
00264
                  output_array(index) = func(input_array(index));
00265
                  ++p;
00266
00267
              return output_array;
00268
```

9.1.4.3 element_wise_duo_apply()

Creates a new array in which the value at each index is the the result of the input function applied to an element of the left hand side array and one on the righ hand side array in the same index Outputs a copy of the result

Definition at line 337 of file np.hpp.

```
00338
               // Create output array copying extents
00339
00340
               using arrayIndex = boost::multi_array<double, ND>::index;
               using ndIndexArray = boost::array<arrayIndex, ND>;
00341
00342
               boost::detail::multi_array::extent_gen<ND> output_extents;
               std::vector<size_t> shape_list;
for (std::size_t i = 0; i < ND; i++)</pre>
00343
00344
00345
00346
                   shape list.push back(lhs.shape()[i]);
00347
00348
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00349
               boost::multi_array<T, ND> output_array(output_extents);
00350
00351
               // Looping through the elements of the output array
00352
               const T *p = lhs.data();
ndIndexArray index;
00353
00354
               for (std::size_t i = 0; i < lhs.num_elements(); i++)</pre>
00355
00356
                   index = getIndexArray(lhs, p);
00357
                   output_array(index) = func(lhs(index), rhs(index));
00358
                   ++p;
00359
00360
               return output_array;
00361
```

9.1.4.4 exp() [1/2]

Implements the numpy exp function on multi arrays.

```
Definition at line 289 of file np.hpp.
```

9.1.4.5 exp() [2/2]

Implements the numpy exp function on scalars.

```
Definition at line 297 of file np.hpp.
```

9.1.4.6 for_each() [1/4]

Function to apply a function to all elements of a multi array Simple overload

Definition at line 80 of file np.hpp.

9.1.4.7 for_each() [2/4]

Function to apply a function to all elements of a multi_array.

Definition at line 59 of file np.hpp.

9.1.4.8 for each() [3/4]

Function to apply a function to all elements of a multi_array Simple overload

Definition at line 51 of file np.hpp.

```
for_each(type_dispatch, A.begin(), A.end(), xform);
00055 }
```

9.1.4.9 for_each() [4/4]

Function to apply a function to all elements of a multi_array.

Definition at line 66 of file np.hpp.

9.1.4.10 getIndex()

Gets the index of one element in a multi_array in one axis.

Definition at line 27 of file np.hpp.

```
00028 {
00029     int offset = requestedElement - m.origin();
00030     return (offset / m.strides()[direction] % m.shape()[direction] + m.index_bases()[direction]);
00031 }
```

9.1.4.11 getIndexArray()

Gets the index of one element in a multi array.

Definition at line 36 of file np.hpp.

9.1.4.12 gradient()

Takes the gradient of a n-dimensional multi_array Uses ij indexing Todo: Implement xy indexing

Definition at line 90 of file np.hpp.

```
00091
00092
               // static_assert(args.size() == ND, "Number of arguments must match the number of dimensions
       of the array");
00093
              using arrayIndex = boost::multi_array<T, ND>::index;
00094
               using ndIndexArray = boost::array<arrayIndex, ND>;
00095
00096
00097
               // constexpr std::size_t n = sizeof...(Args);
00098
               std::size_t n = args.size();
00099
               // std::tuple<Args...> store(args...);
00100
               std::vector<T> arg_vector = args;
00101
               boost::multi_array<T, ND> my_array;
00102
               std::vector<boost::multi array<T, ND» output arrays;
00103
               for (std::size_t i = 0; i < n; i++)</pre>
00105
                   boost::multi_array<T, ND> dfdh = inArray;
00106
                   output_arrays.push_back(dfdh);
00107
00108
00109
               ndArrayValue *p = inArray.data();
00110
               ndIndexArray index;
               for (std::size_t i = 0; i < inArray.num_elements(); i++)</pre>
00111
00112
00113
                   index = getIndexArray(inArray, p);
00114
                   std::cout « "Index: ";
00115
                   for (std::size_t j = 0; j < n; j++)
00116
00117
00118
                       std::cout « index[j] « " ";
00119
                   std::cout « "\n";
00120
00121
00122
                   // Calculating the gradient now
00123
                      j is the axis/dimension
00124
                       (std::size_t j = 0; j < n; j++)
00125
00126
                       ndIndexArray index_high = index;
00127
                       T dh high;
                       if ((long unsigned int)index_high[j] < inArray.shape()[j] - 1)</pre>
00128
00129
00130
                            index_high[j] += 1;
00131
                            dh_high = arg_vector[j];
00132
00133
                       else
00134
                       {
00135
                            dh_high = 0;
00136
00137
                       ndIndexArray index_low = index;
00138
                       T dh low;
                       if (index_low[j] > 0)
00139
00140
00141
                            index_low[j] -= 1;
00142
                            dh_low = arg_vector[j];
00143
00144
                       else
00145
                       {
00146
                            dh low = 0;
00147
00148
00149
                       T dh = dh_high + dh_low;
                       T gradient = (inArray(index_high) - inArray(index_low)) / dh;
// std::cout « gradient « "\n";
output_arrays[j](index) = gradient;
00150
00151
00152
00153
                   .
// std::cout « " value = " « inArray(index) « " check = " « *p « std::endl;
                   ++p;
00155
00156
00157
               return output_arrays;
00158
```

9.1.4.13 linspace()

Implements the numpy linspace function.

Definition at line 161 of file np.hpp.

9.1.4.14 log() [1/2]

Implements the numpy log function on multi arrays.

Definition at line 304 of file np.hpp.

9.1.4.15 log() [2/2]

Implements the numpy log function on scalars.

Definition at line 312 of file np.hpp.

```
00313 {
00314 return std::log(input);
00315 }
```

9.1.4.16 max() [1/2]

Implements the numpy max function for an n-dimensionl multi array.

Definition at line 384 of file np.hpp.

```
00385
00386
               T \max = 0;
00387
               bool max_not_set = true;
const T *data_pointer = input_array.data();
00388
00389
               for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00390
00391
                    T element = *data_pointer;
00392
                    if (max_not_set || element > max)
00393
00394
                        max = element;
00395
                        max_not_set = false;
00396
00397
                    ++data_pointer;
00398
00399
               return max:
00400
```

9.1.4.17 max() [2/2]

Implements the numpy max function for an variadic number of arguments.

Definition at line 404 of file np.hpp.

```
00405
00406
00407
00407
00408
00408
00409
00410
00411
00411
00412
00413
00414
00415
}
T max = input1;
(input > max)

(if (input > max)

(max = input;

)

out 10

return max;

)
```

9.1.4.18 meshgrid()

Implementation of meshgrid TODO: Implement sparsing=true If the indexing type is xx, then reverse the order of the first two elements of ci if the number of dimensions is 2 or 3 In accordance with the numpy implementation

Definition at line 184 of file np.hpp.

```
00185
00186
               using arrayIndex = boost::multi_array<T, ND>::index;
               using oneDArrayIndex = boost::multi_array<T, 1>::index;
00187
00188
               using ndIndexArray = boost::array<arrayIndex, ND>;
               std::vector<boost::multi_array<T, ND» output_arrays; boost::multi_array<T, 1> ci[ND];
00189
00190
00191
               // Copy elements of cinput to ci, do the proper inversions
               for (std::size_t i = 0; i < ND; i++)</pre>
00192
00193
00194
                   std::size_t source = i;
00195
                    if (indexing_type == xy && (ND == 3 || ND == 2))
00196
                        if (i == 0)
00197
00198
                            source = 1;
00199
                        else if (i == 1)
00200
                           source = 0;
00201
                        else
00202
                            source = i;
00203
00204
                   ci[i] = boost::multi_array<T, 1>();
00205
                   ci[i].resize(boost::extents[cinput[source].num_elements()]);
00206
                   ci[i] = cinput[source];
00207
00208
               // Deducing the extents of the N-Dimensional output
00209
               boost::detail::multi_array::extent_gen<ND> output_extents;
               std::vector<size_t> shape_list;
for (std::size_t i = 0; i < ND; i++)</pre>
00210
00211
00212
               {
00213
                   shape_list.push_back(ci[i].shape()[0]);
00214
00215
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00216
00217
               \ensuremath{//} Creating the output arrays
00218
               for (std::size_t i = 0; i < ND; i++)</pre>
00219
               {
00220
                   boost::multi_array<T, ND> output_array(output_extents);
00221
                   ndArrayValue *p = output_array.data();
00222
                   ndIndexArray index;
00223
                   \ensuremath{//} Looping through the elements of the output array
00224
                   for (std::size_t j = 0; j < output_array.num_elements(); j++)</pre>
00225
00226
                        index = getIndexArray(output_array, p);
00227
                        oneDArrayIndex index_1d;
00228
                        index_1d = index[i];
                        output_array(index) = ci[i][index_ld];
00229
00230
                        ++p;
00231
00232
                   output_arrays.push_back(output_array);
00233
00234
               if (indexing_type == xy && (ND == 3 || ND == 2))
00235
00236
                   std::swap(output_arrays[0], output_arrays[1]);
00237
00238
               return output arrays;
00239
          }
```

9.1.4.19 min() [1/2]

Implements the numpy min function for an n-dimensionl multi array.

Definition at line 419 of file np.hpp.

```
00425
                {
00426
                    T element = *data_pointer;
00427
                    if (min_not_set || element < min)</pre>
00428
                        min = element;
min_not_set = false;
00429
00430
00431
00432
                    ++data_pointer;
00433
00434
                return min;
00435
```

9.1.4.20 min() [2/2]

Implements the numpy min function for an variadic number of arguments.

Definition at line 439 of file np.hpp.

```
00440
00441
               T min = input1;
00442
               for (T input : {inputs...})
00443
                   if (input < min)</pre>
00444
00445
                   {
00446
                       min = input;
00447
                   }
00448
00449
               return min;
00450
          }
00451
00453
          template <typename T, long unsigned int ND>
00454
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND>
       abs(boost::multi_array<T, ND> const &input_array)
00455
               std::function<T(T)> abs_func = [](T input)
00456
              { return std::abs(input); };
return element_wise_apply(input_array, abs_func);
00457
00458
00459
```

9.1.4.21 pow() [1/2]

Implements the numpy pow function on multi arrays.

Definition at line 319 of file np.hpp.

9.1.4.22 pow() [2/2]

Implements the numpy pow function on scalars.

```
Definition at line 328 of file np.hpp.
```

```
00329 {
00330          return std::pow(input, exponent);
00331 }
```

9.1.4.23 slice()

Slices the array through one dimension and returns a ND - 1 dimensional array.

```
Definition at line 470 of file np.hpp.
```

```
00471
00472
00473
                // Deducing the extents of the N-Dimensional output
00474
               boost::detail::multi_array::extent_gen<ND - 1> output_extents;
00475
               std::vector<size_t> shape_list;
00476
                for (std::size_t i = 1; i < ND; i++)</pre>
00477
00478
                    shape_list.push_back(input_array.shape()[i]);
00479
00480
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00481
00482
               boost::multi_array<T, ND - 1> output_array(output_extents);
00483
               const T *p = input_array.data();
00484
               boost: apy induction; taket();
boost: array<std::size_t, ND> index;
for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00485
00486
00487
00488
                    index = getIndexArray(input_array, p);
00489
                    output_array(index) = input_array[slice_index](index);
00490
                    p++;
00491
00492
               return output_array;
00493
```

9.1.4.24 sqrt() [1/2]

Implements the numpy sqrt function on multi arrays.

```
Definition at line 274 of file np.hpp.
```

9.1.4.25 sqrt() [2/2]

Implements the numpy sqrt function on scalars.

```
Definition at line 282 of file np.hpp.
```

```
00283 {
00284          return std::sqrt(input);
00285 }
```

9.1.4.26 zeros()

Implements the numpy zeros function for an n-dimensionl multi array.

Definition at line 365 of file np.hpp.

```
// Deducing the extents of the N-Dimensional output
00368
               boost::detail::multi_array::extent_gen<ND> output_extents;
               std::vector<size_t> shape_list;
for (std::size_t i = 0; i < ND; i++)</pre>
00369
00370
00371
00372
                   shape list.push back(dimensions input[i]);
00374
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00375
               // Applying a function to return zero always to all of its elements
00376
               boost::multi_array<T, ND> output_array(output_extents);
00377
               std::function<T(T)> zero_func = [](T input)
00378
               { return 0; };
return element_wise_apply(output_array, zero_func);
00380
```

9.2 wavePlotter Namespace Reference

Custom plotter class.

Classes

class Plotter

This class is used to plot the wave field TODO: make it multithreaded.

9.2.1 Detailed Description

Custom plotter class.

9.3 waveSimCore Namespace Reference

Functions

- boost::multi_array< double, 2 > get_sigma_1 (boost::multi_array< double, 1 > x, double dx, int nx, int nz, double c max, int n=15, double R=1e-3, double m=2.0)
- boost::multi_array< double, 2 > get_sigma_2 (boost::multi_array< double, 1 > z, double dz, int nx, int nz, double c max, int n=10, double R=1e-3, double m=2.0)
- boost::multi_array< double, 2 > get_profile (double xmin, double xmax, double zmin, double zmax, int nx, int nz, double r)

Get the velocity profile of the model as a 2D Array.

• boost::multi_array< double, 2 > dfdx (boost::multi_array< double, 2 > f, double dx)

Takes the partial derivative of a 2D matrix f with respect to x.

• boost::multi_array< double, 2 > dfdz (boost::multi_array< double, 2 > f, double dz)

Takes the partial derivative of a 2D matrix f with respect to z.

boost::multi_array< double, 2 > d2fdx2 (boost::multi_array< double, 2 > f, double dx)

Takes the second partial derivative of a 2D matrix f with respect to x.

• boost::multi array< double, 2 > d2fdz2 (boost::multi array< double, 2 > f, double dz)

Takes the second partial derivative of a 2D matrix f with respect to z.

- boost::multi_array< double, 2 > divergence (boost::multi_array< double, 2 > f1, boost::multi_array< double, 2 > f2, double dx, double dz)
- boost::multi_array< double, 3 > wave_solver (boost::multi_array< double, 2 > c, double dt, double dx, double dz, int nt, int nz, boost::multi_array< double, 3 > f)
- boost::multi_array< double, 3 > wave_solver_complex (boost::multi_array< double, 2 > c, double dt, double dx, double dz, int nt, int nx, int nz, boost::multi_array< double, 3 > f, boost::multi_array< double, 2 > sigma = _1, boost::multi_array< double, 2 > sigma_2)
- boost::multi_array< double, 3 > ricker (int i_s, int j_s, double f, double amp, double shift, double tmin, double tmax, int nt, int nx, int nz)

Get the Ricker wavelet as a 3D Array.

9.3.1 Detailed Description

This namespace contains all the core algorithm for solving wave equation

For the core algorithm, we need six functionalities:

- 1) create the computational domain,
- 2) create a velocity profile (1 & 2 can be put together)
- 3) create attenuation coefficients,
- 4) create source functions,
- 5) helper functions to compute eg. df/dx
- 6) use all above to create a solver function for wave equation

Chapter 10

Class Documentation

10.1 wavePlotter::Plotter Class Reference

This class is used to plot the wave field TODO: make it multithreaded.

#include <wavePlotter.hpp>

Public Member Functions

Plotter (const boost::multi_array< double, 3 > &u, const matplot::vector_2d &Xp, const matplot::vector_2d &Zp, int num_levels, int nt)

Constructor.

• void renderFrame (int index)

Renders a frame of the wave field to a image on disk.

void renderAllFrames (int begin_frame_index, int end_frame_index)

Renders all frames of the wave field to form an animation to be saved on disk.

• void animate (std::string output_file_name, int begin_frame_index, int end_frame_index, int frame_rate)

Renders a complete video animation of the wave field.

void exportFrame (int index)

Export a frame of the wave field to a .csv format for external use.

void exportAllFrames (int begin_frame_index, int end_frame_index)

Export all frames of the wave field to a .csv format for external use.

void setSaveDirectory (std::string save_directory)

Set the save directory for the rendered frames.

10.1.1 Detailed Description

This class is used to plot the wave field TODO: make it multithreaded.

Plotter class

Definition at line 21 of file wavePlotter.hpp.

The documentation for this class was generated from the following file:

• src/CustomLibraries/wavePlotter.hpp

52 Class Documentation

Chapter 11

File Documentation

11.1 coeff.hpp

```
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC COEFF HPP
00006 #define WAVESIMC_COEFF_HPP
00008 #include "CustomLibraries/np.hpp"
00009 #include <math.h>
00010
00016 namespace waveSimCore
00017 {
00021
          boost::multi_array<double, 2> get_sigma_1(boost::multi_array<double, 1> x, double dx, int nx, int
       nz,
00022
                                                       double c_max, int n = 15, double R = 1e-3, double m = 1e-3
       2.0)
00023
00024
               boost::multi_array<double, 2> sigma_1(boost::extents[nx][nz]);
00025
               const double PML_width = n * dx;
00026
00027
               const double sigma_max = -c_max * log(R) * (m + 1.0) / np::pow(PML_width, m + 1.0);
00028
00029
               const double x_0 = np::max(x) - PML_width;
00030
00031
               boost::multi_array<double, 1> polynomial(boost::extents[nx]);
00032
00033
               for (int i = 0; i < nx; i++)
00034
00035
                   if (x[i] > x_0)
00036
                       polynomial[i] = sigma_max * np::pow(np::abs(x[i] - x_0), m); \\ polynomial[nx - 1 - i] = polynomial[i];
00037
00038
00039
                   else
00040
00041
00042
                       polynomial[i] = 0;
00043
                   }
00044
00045
00046
               for (int i = 0; i < nx; i++)
                   for (int j = 0; j < nz; j++)
00047
                       sigma_1[i][j] = polynomial[i];
00048
00049
00050
              return sigma 1;
00051
00052
00056
          boost::multi_array<double, 2> get_sigma_2(boost::multi_array<double, 1> z, double dz, int nx, int
       nz,
00057
                                                       double c max, int n = 10, double R = 1e-3, double m = 10
00058
00059
               boost::multi_array<double, 2> sigma_2(boost::extents[nx][nz]);
               const double PML_width = n * dz;
const double sigma_max = -c_max * log(R) * (m + 1.0) / np::pow(PML_width, m + 1.0);
00060
00061
00062
00063
               const double z = np::max(z) - PML width:
00064
00065
               boost::multi_array<double, 1> polynomial(boost::extents[nz]);
```

```
for (int j = 0; j < nz; j++)
00067
00068
                     if (z[j] > z_0)
00069
                         polynomial[j] = sigma_max * np::pow(np::abs(z[j] - z_0), m);
polynomial[nz - 1 - j] = polynomial[j];
00070
00071
00072
00073
00074
00075
                         polynomial[j] = 0;
00076
                    }
00077
                }
00078
00079
                for (int i = 0; i < nx; i++)
08000
                    for (int j = 0; j < nz; j++)</pre>
00081
                        sigma_2[i][j] = polynomial[j];
00082
00083
                return sigma 2;
00084
           }
00085 }
00086 #endif // WAVESIMC_COEFF_HPP
```

11.2 computational.hpp

```
00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_COMPUTATIONAL_HPP
00006 #define WAVESIMC_COMPUTATIONAL_HPP
00007
00013 namespace waveSimCore
00014 {
00016
           boost::multi_array<double, 2> get_profile(double xmin, double xmax, double zmin, double zmax, int
        nx, int nz, double r)
00017
00018
                boost::multi_array<double, 2> c(boost::extents[nx][nz]);
00019
               boost::multi_array<double, 1> x = np::linspace(xmin, xmax, nx);
boost::multi_array<double, 1> z = np::linspace(zmin, zmax, nz);
00020
00021
00022
                const boost::multi_array<double, 1> axis[2] = {x, z};
std::vector<boost::multi_array<double, 2> XZ = np::meshgrid(axis, false, np::ij);
00023
00024
00025
00026
                double x_0 = xmax / 2.0;
00027
                double z_0 = zmax / 2.0;
00028
00029
                for (int i = 0; i < nx; i++)
00030
00031
                     for (int j = 0; j < nz; j++)
00032
                         if (np::pow(XZ[0][i][j] - x_0, 2.0) + np::pow(XZ[1][i][j] - z_0, 2.0) <= np::pow(r,
00033
        2.0))
00034
                             c[i][j] = 3.0;
00035
                         else
                             c[i][j] = 3.0;
00036
00037
00038
                }
00039
00040
                return c;
00041
           }
00042 }
00043 #endif // WAVESIMC_COMPUTATIONAL_HPP
```

11.3 helper func.hpp

11.4 solver.hpp 55

```
std::vector<boost::multi_array<double, 2» grad_f = np::gradient(f, {dx, dx});</pre>
00021
               return grad f[0];
00022
00023
           boost::multi array<double, 2> dfdz(boost::multi array<double, 2> f, double dz)
00026
               std::vector<boost::multi_array<double, 2» grad_f = np::gradient(f, {dz, dz});</pre>
00028
               return grad_f[1];
00029
00030
           boost::multi array<double, 2> d2fdx2(boost::multi array<double, 2> f, double dx)
00032
00033
00034
               boost::multi_array<double, 2> df = dfdx(f, dx);
00035
               boost::multi_array<double, 2> df2 = dfdx(df, dx);
00036
               return df2;
00037
           }
00038
00040
           boost::multi array<double, 2> d2fdz2(boost::multi array<double, 2> f, double dz)
00041
00042
               boost::multi_array<double, 2> df = dfdz(f, dz);
00043
               boost::multi_array<double, 2> df2 = dfdz(df, dz);
00044
               return df2;
00045
           }
00046
          boost::multi_array<double, 2> divergence(boost::multi_array<double, 2> f1,
00049
       boost::multi_array<double, 2> f2,
00050
                                                         double dx, double dz)
00051
               boost::multi_array<double, 2> f_x = dfdx(f1, dx);
boost::multi_array<double, 2> f_z = dfdz(f2, dz);
boost::multi_array<double, 2> div = f_x + f_z;
00052
00053
00054
00055
               return div;
00056
00057
00058
00059 #endif // WAVESIMC HELPER FUNC HPP
```

11.4 solver.hpp

```
00002 // Created by Yan Cheng on 12/6/22.
00003 //
00004
00005 #ifndef CORETESTS CPP SOLVER2 HPP
00006 #define CORETESTS_CPP_SOLVER2_HPP
00008 #include "CustomLibraries/np.hpp"
00009
00023 namespace waveSimCore
00024 {
00027
          boost::multi array<double, 3> wave solver(boost::multi array<double, 2> c,
00028
                                                      double dt, double dx, double dz, int nt, int nx, int nz,
00029
                                                      boost::multi_array<double, 3> f)
00030
00031
              const boost::multi_array<double, 2> CX = np::pow(c * dt / dx, 2.0); const boost::multi_array<double, 2> CZ = np::pow(c * dt / dz, 2.0);
00032
00033
00034
              const double Cf = np::pow(dt, 2.0);
00035
00036
              int dimensions_1[] = {nt, nx, nz};
00037
              boost:: multi\_array < double, \ 3> \ u = np:: zeros < double > (dimensions\_1);
00038
              int dimensions_4[] = \{nx, nz\};
00039
              boost::multi_array<double, 2> u_xx = np::zeros<double>(dimensions_4);
00040
              int dimensions_5[] = {nx, nz};
00041
              boost::multi_array<double, 2> u_zz = np::zeros<double>(dimensions_5);
00042
00043
              for (int n = 1; n < nt - 1; n++)
00044
                   for (int i = 1; i < nx - 1; i++)</pre>
00045
00046
                   {
00047
                       for (int j = 1; j < nz - 1; j++)
00048
                           00049
00050
00051
00052
                  }
00053
                   // ! Update u
00054
00055
                   for (int i = 0; i < nx; i++)
00056
00057
                       for (int j = 0; j < nz; j++)
00058
                          u[n + 1][i][j] = 2.0 * u[n][i][j] + CX[i][j] * u_xx[i][j] + CZ[i][j] * u_zz[i][j]
00059
       + Cf * f[n][i][j] - u[n - 1][i][j];
```

```
00061
00062
                     \ensuremath{//} Dirichlet boundary condition
00063
                     for (int i = 0; i < nx; i++)</pre>
00064
00065
                          u[n + 1][i][0] = 0.0;
u[n + 1][i][nz - 1] = 0.0;
00067
00068
                     for (int j = 0; j < nz; j++)</pre>
00069
00070
                          u[n + 1][0][j] = 0.0;
00071
00072
                          u[n + 1][nx - 1][j] = 0.0;
00073
00074
00075
                return u;
00076
00077 }
00078 #endif // CORETESTS_CPP_SOLVER2_HPP
```

11.5 solver_complex.hpp

```
00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00005 #ifndef WAVESIMC_SOLVER_COMPLEX_HPP
00006 #define WAVESIMC_SOLVER_COMPLEX_HPP
00007
00008 #include "CustomLibraries/np.hpp" 00009 #include "helper_func.hpp"
00010
00011 #include <cmath>
00012
00018 namespace waveSimCore
00019 {
00022
           boost::multi_array<double, 3> wave_solver_complex(boost::multi_array<double, 2> c,
00023
                                                                        double dt, double dx, double dz, int nt, int nx,
00024
                                                                        boost::multi_array<double, 3> f,
00025
                                                                        boost::multi_array<double, 2> sigma_1,
        boost::multi_array<double, 2> sigma_2)
00026
00027
                const boost::multi_array<double, 2> C1 = 1.0 + (dt * (sigma_1 + sigma_2) * 0.5);
                const boost::multi_array<double, 2> C2 = (sigma_1 * sigma_2 * np::pow(dt, 2.0)) - 2.0;
const boost::multi_array<double, 2> C3 = 1.0 - (dt * (sigma_1 + sigma_2) * 0.5);
00029
00030
                const boost::multi_array<double, 2> C4 = np::pow(dt * c, 2.0);
00031
                const boost::multi_array<double, 2> C5 = 1.0 + (dt * sigma_1 * 0.5);
const boost::multi_array<double, 2> C6 = 1.0 + (dt * sigma_2 * 0.5);
00032
00033
                const boost::multi_array<double, 2> C7 = 1.0 - (dt * sigma_1 * 0.5);
00034
                const boost::multi_array<double, 2> C8 = 1.0 - (dt * sigma_2 * 0.5);
00036
00037
                int dimensions_1[] = {nt, nx, nz};
00038
                boost::multi_array<double, 3> u = np::zeros<double>(dimensions_1);
00039
00040
                 int dimensions_2[] = \{nx, nz\};
                boost::multi_array<double, 2> q_1 = np::zeros<double>(dimensions_2);
00041
00042
                 int dimensions_3[] = \{nx, nz\};
00043
                boost::multi_array<double, 2> q_2 = np::zeros<double>(dimensions_3);
00044
00045
                 int dimensions_4[] = \{nx, nz\};
00046
                boost::multi_array<double, 2> u_xx = np::zeros<double>(dimensions_4);
                 int dimensions_5[] = {nx, nz};
00048
                boost::multi_array<double, 2> u_zz = np::zeros<double>(dimensions_5);
00049
                boost::multi_array<double, 2> f_n(boost::extents[nx][nz]);
boost::multi_array<double, 2> u_n(boost::extents[nx][nz]);
boost::multi_array<double, 2> u_n_1(boost::extents[nx][nz]);
00050
00051
00052
00053
00054
                 for (int n = 1; n < nt - 1; n++)
00055
00056
00057
                      for (int i = 0; i < nx; i++)
00058
                          for (int j = 0; j < nz; j++)</pre>
00059
00060
00061
                               f_n[i][j] = f[n][i][j];
                               u_n[i][j] = u[n][i][j];
u_n_1[i][j] = u[n - 1][i][j];
00062
00063
00064
00065
                     }
00066
```

11.6 source.hpp 57

```
boost::multi_array<double, 2> div = divergence(q_1 * sigma_1, q_2 * sigma_2, dx, dz);
                  boost::multi_array<double, 2> dq_1dx = dfdx(q_1, dx);
boost::multi_array<double, 2> dq_2dz = dfdz(q_2, dz);
00068
00069
                  u_xx = d2fdx2(u_n, dx); // (nx, nz)
u_zz = d2fdz2(u_n, dz); // (nx, nz)
boost::multi_array<double, 2> dudx = dfdx(u_n, dx);
00070
00071
00072
                  boost::multi_array<double, 2> dudz = dfdz(u_n, dz);
00074
00075
                             for (int i = 1; i < nx-1; i++)
00076
00077
                   11
                                 for (int j = 1; j < nz-1; j++)
                   //
00078
                                     00079
00080
00081
00082
00083
                   // ! Update u
00084
00085
                   for (int i = 0; i < nx; i++)
00086
                       for (int j = 0; j < nz; j++)</pre>
00087
00088
       00089
00090
00091
00092
                  // ! Update q_1, q_2 for (int i = 0; i < nx; i++)
00093
00094
00095
00096
                       for (int j = 0; j < nz; j++)
00097
00098
                           q_1[i][j] = (dt * dudx[i][j] + C7[i][j] * q_1[i][j]) / C5[i][j];
                           q_2[i][j] = (dt * dudz[i][j] + C8[i][j] * q_2[i][j]) / C6[i][j];
00099
00100
00101
00102
00103
                   // Dirichlet boundary condition
00104
                   for (int i = 0; i < nx; i++)
00105
                      u[n + 1][i][0] = 0.0;
u[n + 1][i][nz - 1] = 0.0;
00106
00107
00108
00109
                   for (int j = 0; j < nz; j++)
00110
00111
                       u[n + 1][0][j] = 0.0;
00112
                       u[n + 1][nx - 1][j] = 0.0;
00113
00114
00115
              return u;
00116
00117 }
00118 #endif // WAVESIMC SOLVER HPP
```

11.6 source.hpp

```
00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_SOURCE_HPP
00006 #define WAVESIMC_SOURCE_HPP
00011 namespace waveSimCore
00012 {
00014
           boost::multi_array<double, 3> ricker(int i_s, int j_s, double f, double amp, double shift,
00015
                                                      double tmin, double tmax, int nt, int nx, int nz)
00016
               const double pi = 3.141592654;
00017
00018
00019
                boost::multi_array<double, 1> t = np::linspace(tmin, tmax, nt);
                boost::multi_array<double, 1> pft2 = np::pow(pi * f * (t - shift), 2.0); boost::multi_array<double, 1> r = amp * (1.0 - 2.0 * pft2) * np::exp(-1.0 * pft2);
00020
00021
00022
                int dimensions_x[] = \{nx\};
00023
00024
                boost::multi_array<double, 1> x = np::zeros<double>(dimensions_x);
00025
00026
                int dimensions_z[] = {nz};
00027
                boost::multi_array<double, 1> z = np::zeros<double>(dimensions_z);
00028
00029
                x[i_s] = 1.0;
00030
                z[j_s] = 1.0;
```

11.7 wave.cpp

```
00001 // Standard IO libraries
00002 #include <iostream>
00003 #include <fstream>
00004
00005 #include "CustomLibraries/np.hpp"
00006
00007 #include <math.h>
00008
00009 #include "solver.hpp"
00010 #include "computational.hpp"
00010 #Include "coeff.hpp"
00012 #include "source.hpp"
00013 #include "helper_func.hpp"
00014
00015 int main()
00016 {
00017
           double dx, dy, dz, dt;
          dx = 1.0;

dy = 1.0;
00018
00019
00020
          dz = 1.0;
00021
           dt = 1.0;
           std::vector<boost::multi_array<double, 4> my_arrays = np::gradient(A, {dx, dy, dz, dt});
00023
00024 }
```

11.8 np.hpp

```
00001 #ifndef NP_H_
00002 #define NP_H_
00003
00004 #include "boost/multi_array.hpp"
00005 #include "boost/array.hpp"
00006 #include "boost/cstdlib.hpp"
00007 #include <type_traits>
00008 #include <cassert>
00009 #include <iostream>
00010 #include <functional>
00011 #include <type_traits>
00012
00019 namespace np
00020 {
00021
00022
          typedef double ndArrayValue;
00023
00025
          template <std::size_t ND>
          inline boost::multi arrav<ndArravValue, ND>::index
00026
          getIndex(const boost::multi_array<ndArrayValue, ND> &m, const ndArrayValue *requestedElement,
00027
       const unsigned short int direction)
00028
00029
               int offset = requestedElement - m.origin();
00030
               return (offset / m.strides()[direction] % m.shape()[direction] + m.index_bases()[direction]);
00031
00032
00034
           template <std::size_t ND>
00035
           inline boost::array<typename boost::multi_array<ndArrayValue, ND>::index, ND>
00036
           getIndexArray(const boost::multi_array<ndArrayValue, ND> &m, const ndArrayValue *requestedElement)
00037
00038
               using indexType = boost::multi_array<ndArrayValue, ND>::index;
               boost::array<indexType, ND> _index;
for (unsigned int dir = 0; dir < ND; dir++)
00039
00040
00041
               {
00042
                   _index[dir] = getIndex(m, requestedElement, dir);
00043
               }
00044
00045
               return _index;
00046
          }
00050
           template <typename Array, typename Element, typename Functor>
```

11.8 np.hpp 59

```
inline void for_each(const boost::type<Element> &type_dispatch,
00052
                                Array A, Functor &xform)
00053
00054
              for_each(type_dispatch, A.begin(), A.end(), xform);
00055
00056
          template <typename Element, typename Functor>
00059
           inline void for_each(const boost::type<Element> &, Element &Val, Functor &xform)
00060
00061
              Val = xform(Val);
00062
00063
          template <typename Element, typename Iterator, typename Functor>
inline void for_each(const boost::type<Element> &type_dispatch,
00065
00066
00067
                                 Iterator begin, Iterator end,
00068
                                 Functor &xform)
00069
00070
              while (begin != end)
00071
00072
                   for_each(type_dispatch, *begin, xform);
00073
                   ++begin;
00074
              }
00075
          }
00076
00079
          template <typename Array, typename Functor>
00080
          inline void for_each (Array &A, Functor xform)
00081
00082
               // Dispatch to the proper function
00083
               for_each(boost::type<typename Array::element>(), A.begin(), A.end(), xform);
00084
          }
00085
00089
          template <typename T, long unsigned int ND>
          requires std::is_floating_point<T>::value inline constexpr std::vector<boost::multi_array<T, ND>
00090
       gradient(boost::multi_array<T, ND> inArray, std::initializer_list<T> args)
00091
              // static_assert(args.size() == ND, "Number of arguments must match the number of dimensions
00092
       of the array");
00093
              using arrayIndex = boost::multi_array<T, ND>::index;
00094
00095
              using ndIndexArray = boost::array<arrayIndex, ND>;
00096
00097
               // constexpr std::size_t n = sizeof...(Args);
00098
              std::size_t n = args.size();
00099
               // std::tuple<Args...> store(args...);
00100
               std::vector<T> arg_vector = args;
00101
              boost::multi_array<T, ND> my_array;
00102
               std::vector<boost::multi_array<T, ND» output_arrays;
00103
               for (std::size_t i = 0; i < n; i++)</pre>
00104
              {
00105
                   boost::multi_array<T, ND> dfdh = inArray;
00106
                   output_arrays.push_back(dfdh);
00107
00108
00109
              ndArrayValue *p = inArray.data();
              ndIndexArray index;
00110
               for (std::size_t i = 0; i < inArray.num_elements(); i++)</pre>
00111
00112
00113
                   index = getIndexArray(inArray, p);
00114
                   std::cout « "Index: ";
00115
                   for (std::size_t j = 0; j < n; j++)
00116
00117
00118
                       std::cout « index[j] « " ";
00119
00120
                   std::cout « "\n";
00121
                   // Calculating the gradient now
00122
00123
                   \ensuremath{//} j is the axis/dimension
00124
                   for (std::size_t j = 0; j < n; j++)
00125
00126
                       ndIndexArray index_high = index;
00127
                       T dh_high;
00128
                       if ((long unsigned int)index_high[j] < inArray.shape()[j] - 1)</pre>
00129
00130
                            index high[i] += 1;
00131
                           dh_high = arg_vector[j];
00132
00133
                       else
00134
00135
                           dh high = 0:
00136
00137
                       ndIndexArray index_low = index;
00138
                       T dh low;
00139
                       if (index_low[j] > 0)
00140
                           index_low[j] -= 1;
00141
00142
                           dh low = arg vector[i];
```

```
00143
00144
                        else
00145
00146
                            dh_low = 0;
00147
00148
00149
                        T dh = dh_high + dh_low;
00150
                        T gradient = (inArray(index_high) - inArray(index_low)) / dh;
00151
                        // std::cout « gradient « "n";
00152
                        output_arrays[j](index) = gradient;
00153
                    // std::cout « " value = " « inArray(index) « " check = " « *p « std::endl;
00154
00155
                    ++p;
00156
00157
               return output_arrays;
00158
          }
00159
           inline boost::multi_array<double, 1> linspace(double start, double stop, long unsigned int num)
00161
00162
00163
               double step = (stop - start) / (num - 1);
               boost::multi_array<double, 1> output(boost::extents[num]);
for (std::size_t i = 0; i < num; i++)</pre>
00164
00165
00166
                   output[i] = start + i * step;
00167
00168
00169
               return output;
00170
           }
00171
00172
           enum indexing
00173
           {
00174
               ΧV,
00175
               iή
00176
00177
          template <typename T, long unsigned int ND>
requires std::is_arithmetic<T>::value inline constexpr std::vector<boost::multi_array<T, ND>
00183
00184
       meshgrid(const boost::multi_array<T, 1> (&cinput)[ND], bool sparsing = false, indexing indexing_type
00185
00186
               using arrayIndex = boost::multi_array<T, ND>::index;
00187
               using oneDArrayIndex = boost::multi_array<T, 1>::index;
               using ndIndexArray = boost::array<arrayIndex, ND>;
00188
               std::vector<boost::multi_array<T, ND» output_arrays; boost::multi_array<T, 1> ci[ND];
00189
00190
00191
               // Copy elements of cinput to ci, do the proper inversions
00192
               for (std::size_t i = 0; i < ND; i++)</pre>
00193
00194
                    std::size_t source = i;
                    if (indexing_type == xy && (ND == 3 || ND == 2))
00195
00196
00197
                        if (i == 0)
00198
                            source = 1;
00199
                        else if (i == 1)
00200
                            source = 0;
00201
                        else
00202
                            source = i;
00203
00204
                   ci[i] = boost::multi_array<T, 1>();
00205
                    ci[i].resize(boost::extents[cinput[source].num_elements()]);
00206
                   ci[i] = cinput[source];
00207
               // Deducing the extents of the N-Dimensional output
00208
00209
               boost::detail::multi_array::extent_gen<ND> output_extents;
00210
               std::vector<size_t> shape_list;
00211
               for (std::size_t i = 0; i < ND; i++)</pre>
00212
               {
00213
                    shape_list.push_back(ci[i].shape()[0]);
00214
00215
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00216
00217
               // Creating the output arrays
00218
               for (std::size_t i = 0; i < ND; i++)</pre>
00219
               {
                   boost::multi_array<T, ND> output_array(output_extents);
00220
                   ndArrayValue *p = output_array.data();
ndIndexArray index;
00221
00222
00223
                    // Looping through the elements of the output array
00224
                    for (std::size_t j = 0; j < output_array.num_elements(); j++)</pre>
00225
                        index = getIndexArray(output_array, p);
00226
                        oneDArrayIndex index_ld;
index_ld = index[i];
00227
00228
                        output_array(index) = ci[i][index_1d];
00229
00230
                        ++p;
00231
00232
                   output_arrays.push_back(output_array);
00233
               }
```

11.8 np.hpp 61

```
00234
              if (indexing_type == xy && (ND == 3 || ND == 2))
00235
00236
                  std::swap(output_arrays[0], output_arrays[1]);
00237
00238
              return output arrays;
00239
          }
00240
00242
          template <class T, long unsigned int ND>
00243
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND>
       element_wise_apply(const boost::multi_array<T, ND> &input_array, std::function<T(T)> func)
00244
00245
00246
              // Create output array copying extents
00247
              using arrayIndex = boost::multi_array<double, ND>::index;
00248
              using ndIndexArray = boost::array<arrayIndex, ND>;
00249
              boost::detail::multi_array::extent_gen<ND> output_extents;
00250
              std::vector<size_t> shape_list;
              for (std::size_t i = 0; i < ND; i++)
00251
00252
00253
                  shape_list.push_back(input_array.shape()[i]);
00254
00255
              std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00256
              boost::multi_array<T, ND> output_array(output_extents);
00257
00258
              // Looping through the elements of the output array
00259
              const T *p = input_array.data();
00260
              ndIndexArray index;
00261
              for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00262
00263
                  index = getIndexArray(input_array, p);
00264
                  output_array(index) = func(input_array(index));
00265
                  ++p;
00266
00267
              return output_array;
00268
          }
00269
00270
          // Complex operations
00271
00273
          template <class T, long unsigned int ND>
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND> sqrt(const
00274
       boost::multi_array<T, ND> &input_array)
00275
          {
00276
              std::function < T(T) > func = (T(*)(T)) std::sart:
00277
              return element_wise_apply(input_array, func);
00278
          }
00279
00281
          template <class T>
00282
          requires std::is_arithmetic<T>::value inline constexpr T sqrt(const T input)
00283
00284
              return std::sgrt(input);
00285
          }
00286
00288
          template <class T, long unsigned int ND>
00289
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND> exp(const
       boost::multi_array<T, ND> &input_array)
00290
         {
00291
              std::function<T(T)> func = (T(*)(T))std::exp;
00292
              return element_wise_apply(input_array, func);
00293
          }
00294
00296
          template <class T>
00297
          requires std::is arithmetic<T>::value inline constexpr T exp(const T input)
00298
00299
              return std::exp(input);
00300
00301
00303
          template <class T, long unsigned int ND>
          requires std::is arithmetic<T>::value inline constexpr boost::multi array<T, ND> log(const
00304
       boost::multi_array<T, ND> &input_array)
00305
         {
00306
              std::function<T(T)> func = std::log<T>();
00307
              return element_wise_apply(input_array, func);
00308
          }
00309
00311
          template <class T>
00312
          requires std::is_arithmetic<T>::value inline constexpr T log(const T input)
00313
          {
00314
              return std::log(input);
00315
          }
00316
00318
          template <class T, long unsigned int ND>
00319
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND> pow(const
       boost::multi_array<T, ND> &input_array, const T exponent)
00320
00321
              std::function<T(T)> pow_func = [exponent](T input)
              { return std::pow(input, exponent); };
return element_wise_apply(input_array, pow_func);
00322
00323
```

```
00324
          }
00325
00327
          template <class T>
00328
          requires std::is_arithmetic<T>::value inline constexpr T pow(const T input, const T exponent)
00329
00330
               return std::pow(input, exponent);
00331
00332
00336
          template <class T, long unsigned int ND>
          inline constexpr boost::multi_array<T, ND> element_wise_duo_apply(boost::multi_array<T, ND> const
00337
       &lhs, boost::multi_array<T, ND> const &rhs, std::function<T(T, T)> func)
00338
00339
               // Create output array copying extents
00340
               using arrayIndex = boost::multi_array<double, ND>::index;
00341
               using ndIndexArray = boost::array<arrayIndex, ND>;
00342
               boost::detail::multi_array::extent_gen<ND> output_extents;
               std::vector<size_t> shape_list;
for (std::size_t i = 0; i < ND; i++)</pre>
00343
00344
00345
00346
                   shape_list.push_back(lhs.shape()[i]);
00347
00348
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00349
               boost::multi_array<T, ND> output_array(output_extents);
00350
00351
               // Looping through the elements of the output array
00352
               const T *p = lhs.data();
00353
               ndIndexArray index;
00354
               for (std::size_t i = 0; i < lhs.num_elements(); i++)</pre>
00355
00356
                   index = getIndexArray(lhs, p);
00357
                   output_array(index) = func(lhs(index), rhs(index));
00358
                   ++p;
00359
00360
               return output_array;
00361
          }
00362
00364
          template <typename T, typename inT, long unsigned int ND>
          requires std::is_integral<inT>::value && std::is_arithmetic<T>::value inline constexpr
00365
       boost::multi_array<T, ND> zeros(inT (&dimensions_input)[ND])
00366
00367
               // Deducing the extents of the N-Dimensional output
00368
               boost::detail::multi_array::extent_gen<ND> output_extents;
00369
               std::vector<size_t> shape_list;
               for (std::size_t i = 0; i < ND; i++)</pre>
00370
00371
               {
00372
                   shape_list.push_back(dimensions_input[i]);
00373
00374
               std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00375
               // Applying a function to return zero always to all of its elements
boost::multi_array<T, ND> output_array(output_extents);
00376
00377
               std::function<T(T)> zero_func = [](T input)
00378
               { return 0; };
00379
               return element_wise_apply(output_array, zero_func);
00380
          }
00381
00383
          template <typename T, long unsigned int ND>
          requires std::is_arithmetic<T>::value inline constexpr T max(boost::multi_array<T, ND> const
00384
       &input_array)
00385
               T \max = 0:
00386
00387
               bool max_not_set = true;
00388
               const T *data_pointer = input_array.data();
00389
               for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00390
00391
                   T element = *data_pointer;
00392
                   if (max_not_set || element > max)
00393
00394
                       max = element;
00395
                       max not set = false;
00396
00397
                   ++data_pointer;
00398
00399
               return max;
00400
          }
00401
          template <class T, class... Ts, class = std::enable_if_t<(std::is_same_v<T, Ts> && ...)>
00403
00404
           requires std::is_arithmetic<T>::value inline constexpr T max(T input1, Ts... inputs)
00405
               T max = input1;
00406
00407
               for (T input : {inputs...})
00408
00409
                   if (input > max)
00410
                   {
00411
                       max = input;
00412
                   }
00413
00414
               return max:
```

11.8 np.hpp 63

```
00415
00416
00418
          template <typename T, long unsigned int ND>
00419
          requires std::is_arithmetic<T>::value inline constexpr T min(boost::multi_array<T, ND> const
       &input_array)
00420
00421
              T \min = 0;
00422
              bool min_not_set = true;
00423
              const T *data_pointer = input_array.data();
00424
              for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00425
00426
                  T element = *data_pointer;
00427
                   if (min_not_set || element < min)</pre>
00428
00429
                       min = element;
00430
                      min_not_set = false;
00431
00432
                  ++data_pointer;
00433
              }
00434
              return min;
00435
00436
00438
          template <class T, class... Ts, class = std::enable_if_t<(std::is_same_v<T, Ts> && ...)>
          inline constexpr T min(T input1, Ts... inputs) requires std::is_arithmetic<T>::value
00439
00440
00441
              T min = input1;
00442
              for (T input : {inputs...})
00443
00444
                   if (input < min)</pre>
00445
                   {
00446
                       min = input:
00447
                  }
00448
00449
              return min;
00450
          }
00451
00453
          template <typename T, long unsigned int ND>
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND>
00454
       abs(boost::multi_array<T, ND> const &input_array)
00455
00456
              std::function<T(T)> abs_func = [](T input)
00457
              { return std::abs(input); };
00458
              return element_wise_apply(input_array, abs_func);
00459
          }
00460
00462
          template <typename T>
00463
          requires std::is_arithmetic<T>::value inline constexpr T abs(T input)
00464
00465
              return std::abs(input);
00466
00467
00469
          template <typename T, long unsigned int ND>
00470
          requires std::is_arithmetic<T>::value inline constexpr boost::multi_array<T, ND - 1>
       slice(boost::multi_array<T, ND> const &input_array, std::size_t slice_index)
00471
00472
00473
               // Deducing the extents of the N-Dimensional output
00474
              boost::detail::multi_array::extent_gen<ND - 1> output_extents;
00475
              std::vector<size_t> shape_list;
              for (std::size_t i = 1; i < ND; i++)</pre>
00476
00477
00478
                  shape_list.push_back(input_array.shape()[i]);
00479
00480
              std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00481
00482
              boost::multi_array<T, ND - 1> output_array(output_extents);
00483
00484
              const T *p = input_array.data();
              boost::array<std::size_t, ND> index;
for (std::size_t i = 0; i < input_array.num_elements(); i++)</pre>
00485
00486
00487
00488
                  index = getIndexArray(input_array, p);
00489
                  output_array(index) = input_array[slice_index](index);
00490
                  p++;
00491
00492
              return output_array;
00493
          }
00494
00495 }
00496
00497 // Override of operators in the boost::multi_array class to make them more np-like
00498 // Basic operators
00499 // All of the are element-wise
00500
00501 // Multiplication operator
00503 template <class T, long unsigned int ND>
00504 inline boost::multi array<T, ND> operator*(boost::multi array<T, ND> const &lhs, boost::multi array<T,
```

```
ND> const &rhs)
00505 {
00506
          std::function<T(T, T)> func = std::multiplies<T>();
00507
          return np::element_wise_duo_apply(lhs, rhs, func);
00508 }
00509
00511 template <class T, long unsigned int ND>
00512 inline boost::multi_array<T, ND> operator*(T const &lhs, boost::multi_array<T, ND> const &rhs)
00513 {
00514
          std::function<T(T)> func = [lhs](T item)
00515
          { return lhs * item; };
00516
          return np::element wise apply(rhs, func);
00517 }
00519 template <class T, long unsigned int ND>
00520 inline boost::multi_array<T, ND> operator*(boost::multi_array<T, ND> const &lhs, T const &rhs)
00521 {
00522
          return rhs * lhs:
00523 }
00525 // Plus operator
00527 template <class T, long unsigned int ND>
00528 boost::multi_array<T, ND> operator+(boost::multi_array<T, ND> const &lhs, boost::multi_array<T, ND>
       const &rhs)
00529 {
00530
          std::function<T(T, T)> func = std::plus<T>();
00531
          return np::element_wise_duo_apply(lhs, rhs, func);
00532 }
00533
00535 template <class T, long unsigned int ND>
00536 inline boost::multi_array<T, ND> operator+(T const &lhs, boost::multi_array<T, ND> const &rhs)
00537 {
00538
          std::function<T(T)> func = [lhs](T item)
00539
          { return lhs + item; };
00540
         return np::element_wise_apply(rhs, func);
00541 }
00542
00544 template <class T, long unsigned int ND>
00545 inline boost::multi_array<T, ND> operator+(boost::multi_array<T, ND> const &lhs, T const &rhs)
00546 {
00547
          return rhs + lhs;
00548 }
00549
00550 // Subtraction operator
00552 template <class T, long unsigned int ND>
00553 boost::multi_array<T, ND> operator-(boost::multi_array<T, ND> const &lhs, boost::multi_array<T, ND>
       const &rhs)
00554 {
00555
          \texttt{std::function} < \texttt{T(T, T)} > \texttt{func = std::minus} < \texttt{T>();}
00556
          return np::element_wise_duo_apply(lhs, rhs, func);
00557 }
00558
00560 template <class T, long unsigned int ND>
00561 inline boost::multi_array<T, ND> operator-(T const &lhs, boost::multi_array<T, ND> const &rhs)
00562 {
          std::function<T(T)> func = [lhs](T item)
00563
00564
          { return lhs - item; };
          return np::element_wise_apply(rhs, func);
00565
00566 }
00567
00569 template <class T, long unsigned int ND>
00570 inline boost::multi_array<T, ND> operator-(boost::multi_array<T, ND> const &lhs, T const &rhs)
00571 {
00572
          return rhs - lhs;
00573 }
00574
00575 // Division operator
00577 template <class T, long unsigned int ND> \,
00578 boost::multi_array<T, ND> operator/(boost::multi_array<T, ND> const &lhs, boost::multi_array<T, ND>
       const &rhs)
00579 {
00580
          std::function<T(T, T)> func = std::divides<T>();
00581
          return np::element_wise_duo_apply(lhs, rhs, func);
00582 }
00583
00585 template <class T, long unsigned int ND>
00586 inline boost::multi_array<T, ND> operator/(T const &lhs, boost::multi_array<T, ND> const &rhs)
00587 {
00588
          std::function<T(T)> func = [lhs](T item)
00589
          { return lhs / item; };
00590
          return np::element_wise_apply(rhs, func);
00591 }
00592
00594 template <class T, long unsigned int ND>
00595 inline boost::multi_array<T, ND> operator/(boost::multi_array<T, ND> const &lhs, T const &rhs)
00596 {
00597
          std::function<T(T)> func = [rhs](T item)
00598
         { return item / rhs; };
```

```
00599     return np::element_wise_apply(lhs, func);
00600 }
00601
00601 #endif
```

11.9 np_to_matplot.hpp

```
00001 #ifndef NPTOMATPLOT_H_
00002 #define NPTOMATPLOT_H_
00003
00004 #include <matplot/matplot.h>
00005 #include <thread>
00006 #include "boost/multi_array.hpp"
00007 #include "boost/array.hpp"
00013 namespace np
00014 {
00016
           inline matplot::vector_2d convert_to_matplot(const boost::multi_array<double, 2> &arr)
00017
00018
               std::vector<double> x = matplot::linspace(0, 0, arr.shape()[0]);
               std::vector<double> y = matplot::linspace(00, 0, arr.shape()[1]);
matplot::vector_2d result = std::get<0>(matplot::meshgrid(x, y));
00019
00021
               // std::cout « "arr.shape()[0] = " « arr.shape()[0] « " arr.shape()[1] = " « arr.shape()[1] «
       std::endl;
00022
               for (size_t i = 0; i < arr.shape()[0]; i++)</pre>
00023
               {
00024
                    for (size_t j = 0; j < arr.shape()[1]; j++)</pre>
00025
                    {
00026
                        result[i][j] = arr[i][j];
00027
                    }
00028
00029
               return result:
00030
           }
00031 }
00032 #endif
```

11.10 wavePlotter.hpp

```
00001 #ifndef WAVESOLVER H
00002 #define WAVESOLVER_H_
00003 #include <boost/multi_array.hpp>
00004 #include <boost/array.hpp>
00005 #include <fstream>
00006 #include <iostream>
00007 #include "CustomLibraries/np.hpp"
00008 #include "CustomLibraries/np_to_matplot.hpp"
00016 namespace wavePlotter
00017 {
00021
             class Plotter
00022
            public:
00023
                 Plotter(const boost::multi_array<double, 3> &u, const matplot::vector_2d &Xp, const
00025
        matplot::vector_2d &Zp, int num_levels, int nt)
00026
00027
                       this->u.resize(boost::extents[u.shape()[0]][u.shape()[1]][u.shape()[2]]);
00028
                       this->u = u;
00029
                       this->Xp = Xp;
                       this->Zp = Zp;
00030
00031
                       this->num_levels = num_levels;
00032
                       this->nt = nt;
                      double min_u = np::min(u);
double max_u = np::max(u);
std::cout « "min_u = " « min_u « " max_u = " « max_u « "\n";
this->levels = matplot::linspace(min_u, max_u, num_levels);
00033
00034
00035
00036
00037
00039
                  void renderFrame(int index)
00040
                      matplot::vector_2d Up = np::convert_to_matplot(this->u[index]);
matplot::contourf(this->Xp, this->Zp, Up, this->levels);
matplot::save(save_directory + "/contourf_" + format_num(index) + ".png");
00041
00042
00043
00044
00045
00047
                  void renderAllFrames(int begin_frame_index, int end_frame_index)
00048
00049
                       for (int i = begin_frame_index; i < end_frame_index; i++)</pre>
00050
00051
                            renderFrame(i);
00053
                  }
```

```
00054
                void animate(std::string output_file_name, int begin_frame_index, int end_frame_index, int
        frame_rate)
00057
              {
        renderAllFrames(begin_frame_index, end_frame_index);
std::string ffmpeg_render_command = "ffmpeg -framerate " + std::to_string(frame_rate) + "
-pattern_type glob -i '" + save_directory + "/*.png' -c:v libx264 -pix_fmt yuv420p " +
00058
00059
        output_file_name;
00060
                    std::system(ffmpeg_render_command.c_str());
00061
                void exportFrame(int index)
00063
00064
00065
                    matplot::vector_2d Up = np::convert_to_matplot(this->u[index]);
00066
                    std::ofstream outfile;
                    outfile.open(save_directory + "/frame_" + format_num(index) + ".csv");
00067
00068
                     for (std::size_t i = 0; i < Up.size(); i++)</pre>
00069
00070
                         for (std::size_t j = 0; j < Up[i].size(); j++)</pre>
00071
00072
                              outfile « Up[i][j];
                              if (j != Up[i].size() - 1)
    outfile « ",";
00073
00074
00075
00076
                         outfile « "\n";
00077
00078
                    outfile.close();
00079
00080
00082
                void exportAllFrames(int begin_frame_index, int end_frame_index)
00083
00084
                     for (int i = begin frame index; i < end frame index; i++)
00085
00086
                         exportFrame(i);
00087
                     }
00088
00089
00091
                void setSaveDirectory(std::string save directory)
00092
00093
                    this->save_directory = save_directory;
00094
00095
           private:
00096
00097
               std::string format num(int num, int length = 8)
00098
00099
                    std::string str_num = std::to_string(num);
00100
00101
                    int str_length = str_num.length();
                    for (int i = 0; i < length - str_length; i++)
    str_num = "0" + str_num;</pre>
00102
00103
00104
                    return str num:
00105
               }
00106
00107
                boost::multi_array<double, 3> u;
00108
                matplot::vector_2d Xp;
00109
                matplot::vector_2d Zp;
00110
                int num_levels;
                int nt;
                std::vector<double> levels;
00112
00113
                std::string save_directory = "output";
00114
           };
00115
00116
00117 #endif
```

11.11 wave solver with animation.cpp

```
00001 #include <boost/multi_array.hpp>
00002 #include "CustomLibraries/np.hpp"
00004 #include "CustomLibraries/np_to_matplot.hpp"
00005 #include "CustomLibraries/wavePlotter.hpp"
00006 #include <matplot/matplot.h>
00007 #include <cassert>
00008 #include <iostream>
00009 #include <sstream>
00001 #include "CoreAlgorithm/helper_func.hpp"
00012 #include "CoreAlgorithm/source.hpp"
00013 #include "CoreAlgorithm/source.hpp"
00015 #include "CoreAlgorithm/solver.hpp"
00015 #include "CoreAlgorithm/solver.hpp"
00016 "CoreAlgorithm/solver.hpp"
00017 int main()
```

11.12 main.cpp 67

```
00018 {
00019
           // Define the constants for the simulation
00020
00021
           // Number of x and z grid points
00022
           int nx = 100;
int nz = 100;
00023
           // Number of time steps
00024
00025
           int nt = 1000;
00026
00027
           // Differentiation values
           double dx = 0.01;
double dz = 0.01;
00028
00029
00030
           double dt = 0.001;
00031
00032
           // Define the domain
           double xmin = 0.0;
double xmax = nx * dx;
00033
00034
           double zmin = 0.0;
00035
00036
           double zmax = nz * dz;
00037
           double tmin = 0.0;
00038
           double tmax = nt * dt;
00039
00040
           // Define the source parameters
00041
           double f M = 10.0;
00042
           double amp = 1e0;
00043
           double shift = 0.1;
00044
00045
           // Source location
00046
           int source_is = 50;
           int source_js = 50;
00047
00048
00049
           // Create the source
00050
           boost::multi_array<double, 3> f = waveSimCore::ricker(source_is, source_js, f_M, amp, shift, tmin,
        tmax, nt, nx, nz);
00051
00052
           // Create the velocity profile
00053
           double r = 150.0;
           boost::multi_array<double, 2> vel = waveSimCore::get_profile(xmin, xmax, zmin, zmax, nx, nz, r);
00054
00055
00056
           // Solve the wave equation
00057
           boost::multi_array<double, 3> u = waveSimCore::wave_solver(vel, dt, dx, dz, nt, nx, nz, f);
00058
00059
           // Define the number of different levels for the contour plot
00060
           int num_levels = 100;
00061
           // Create the levels for the contour plot based on the min and max values of \boldsymbol{u}
           double min_u = np::min(u);
double max_u = np::max(u);
00062
00063
           std::vector<double> levels = matplot::linspace(min_u, max_u, num_levels);
00064
00065
00066
           // Create the x and z axis for the contour plot and convert them to matplot format
           boost::multi_array<double, 1> x = np::linspace(xmin, xmax, nx);
boost::multi_array<double, 1> z = np::linspace(zmin, zmax, nz);
00067
00068
00069
           const boost::multi_array<double, 1 > axis[2] = \{x, z\};
00070
           std::vector<boost::multi_array<double, 2» XcZ = np::meshgrid(axis, false, np::xy);</pre>
00071
00072
           matplot::vector_2d Xp = np::convert_to_matplot(XcZ[0]);
matplot::vector_2d Zp = np::convert_to_matplot(XcZ[1]);
00073
00074
00075
           // Create the plotter object and animate the results
00076
           wavePlotter::Plotter my_plotter(u, Xp, Zp, num_levels, nt);
00077
00078
           // If you want to render a specific frame, use this:
00079
           // my_plotter.renderFrame(int frame_index);
08000
00081
           // Renders the entire animation from start_frame to end_frame
00082
           int start_frame = 20;
00083
           int end_frame = nt - 1;
00084
           int fps = 30:
00085
           my_plotter.animate("example-wave.mp4", start_frame, end_frame, fps);
00086
00087
           // The animation will be saved in .
           \ensuremath{//} Frames will be saved to ./output
00088
00089 }
```

11.12 main.cpp

```
00001 #include <iostream>
00002 #include <string>
00003 #include "ExternalLibraries/cxxopts.hpp"
00004
00005 #include <boost/multi_array.hpp>
00006 #include <boost/array.hpp>
00007 #include "CustomLibraries/np.hpp"
```

```
00008 #include "CustomLibraries/np_to_matplot.hpp"
00009 #include "CustomLibraries/wavePlotter.hpp
00010 #include <matplot/matplot.h>
00011 #include <cassert>
00012 #include <sstream>
00013
00014 #include "CoreAlgorithm/helper_func.hpp"
00015 #include "CoreAlgorithm/coeff.hpp'
00016 #include "CoreAlgorithm/source.hpp"
00017 #include "CoreAlgorithm/computational.hpp"
00018 #include "CoreAlgorithm/solver.hpp"
00019
00020 // Command line arguments
00021 cxxopts::Options options("WaveSimPP", "A wave propagation simulator written in C++.");
00022 int main(int argc, char *argv[])
00023 {
00024
              // Parse command line arguments
             options.add_options()("d,debug", "Enable debugging",
00025
         cxxopts::value<bool>() ->default_value("false"));
             options.add_options()("animate", "Render an animation at the end",
00026
         cxxopts::value<bool>()->default_value("true"));
00027
             options.add_options()("render", "Render each of the frames at the end",
         cxxopts::value<bool>()->default_value("false"));
         options.add_options()("export", "Export the data to a series of csv files", cxxopts::value<br/>
value("false"));
00028
             options.add_options()("o,output_dir", "Output directory path",
00029
         cxxopts::value<std::string>()->default_value("output"));
00030
             options.add_options()("output_filename", "Output filename",
         cxxopts::value<std::string>()->default_value("output.mp4"));
  options.add_options()("framerate", "Framerate of output video",
cxxopts::value<int>()->default_value("30"));
00031
00032
             options.add_options()("v,verbose", "Verbose output",
         cxxopts::value<bool>() ->default_value("false"));
00033
             options.add_options()("source_i", "Source i position",
         cxxopts::value<int>()->default_value("50"));
options.add_options()("source_j", "Source j position",
00034
         options.add_options()("source_j", "Source j position",
cxxopts::value<int>()->default_value("50"));
options.add_options()("nt", "Number of time steps", cxxopts::value<int>()->default_value("1000"));
options.add_options()("nx", "Number of x steps", cxxopts::value<int>()->default_value("100"));
options.add_options()("nz", "Number of z steps", cxxopts::value<int>()->default_value("100"));
options.add_options()("dt", "Time step size", cxxopts::value<double>()->default_value("0.001"));
options.add_options()("dx", "x step size", cxxopts::value<double>()->default_value("0.01"));
options.add_options()("dz", "z step size", cxxopts::value<double>()->default_value("0.01"));
options.add_options()("f, frequency", "Frequency of source",
options.add_options()("f, frequency", "Frequency of source",
00035
00036
00037
00038
00039
00040
00041
         cxxopts::value<double>() ->default_value("10.0"));
00042
             options.add_options()("a,amplitude", "Amplitude of source",
         cxxopts::value<double>()->default_value("1.0"));
options.add_options()("s,shift", "Shift of source",
cxxopts::value<double>()->default_value("1.0"));
00043
         cxxopts::value<double>()->default_value("0.1"));
             options.add_options()("r,radius", "Radius of velocity profile",
00044
         cxxopts::value()->default_value("150.0"));
options.add_options()("l,num_levels", "Number of levels in the filled contour plot",
00045
         cxxopts::value<int>()->default_value("100"));
00046
             options.add_options()("h,help", "Print help");
00047
             cxxopts::ParseResult result;
00048
00049
00050
                   result = options.parse(argc, argv);
00051
00052
             catch (const cxxopts::exceptions::exception &e)
00053
                   std::cerr \ll "WaveSimPP: " \ll e.what() \ll ' \n';
00054
00055
                   std::cerr « "usage: WaveSimPP [options] ...\n";
00056
                   return EXIT_FAILURE;
00057
00058
             if (result.count("help"))
00059
                   std::cout « options.help({"", "Group"}) « std::endl;
00060
00061
                   return true;
00062
             }
00063
00064
             // Define the constants for the simulation
00065
00066
             // Number of x and z grid points
00067
00068
             int nx = result["nx"].as<int>();
00069
00070
             int nz = result["nz"].as<int>();
              // Number of time steps
00071
             int nt = result["nt"].as<int>();
00072
00073
00074
             // Differentiation values
00075
             double dx = result["dx"].as<double>();
             double dz = result["dz"].as<double>();
00076
00077
             double dt = result["dt"].as<double>();
00078
00079
             // Define the domain
```

```
08000
           double xmin = 0.0;
           double xmax = nx * dx;
00081
           double zmin = 0.0;
00082
00083
           double zmax = nz * dz;
           double tmin = 0.0;
00084
00085
           double tmax = nt * dt;
00087
           double f_M = result["frequency"].as<double>();
double amp = result["amplitude"].as<double>();
00088
00089
00090
           double shift = result["shift"].as<double>();
00091
00092
           // Source location
00093
           int source_is = result["source_i"].as<int>();
00094
           int source_js = result["source_j"].as<int>();
00095
            // Create the source
           boost::multi_array<double, 3> f = waveSimCore::ricker(source_is, source_js, f_M, amp, shift, tmin,
00096
        tmax, nt, nx, nz);
00097
00098
            // Create the velocity profile
00099
           double r = result["radius"].as<double>();
00100
00101
           boost::multi_array<double, 2> vel = waveSimCore::get_profile(xmin, xmax, zmin, zmax, nx, nz, r);
00102
00103
            // Solve the wave equation
00104
           boost::multi_array<double, 3> u = waveSimCore::wave_solver(vel, dt, dx, dz, nt, nx, nz, f);
00105
00106
            // Define the number of different levels for the contour plot
00107
           int num_levels = result["num_levels"].as<int>();
00108
           // Create the levels for the contour plot based on the min and max values of \boldsymbol{u}
           double min_u = np::min(u);
double max_u = np::max(u);
00109
00110
00111
           std::vector<double> levels = matplot::linspace(min_u, max_u, num_levels);
00112
00113
            // Create the \boldsymbol{x} and \boldsymbol{z} axis for the contour plot and convert them to matplot format
           boost::multi_array<double, 1> x = np::linspace(xmin, xmax, nx);
boost::multi_array<double, 1> z = np::linspace(zmin, zmax, nz);
00114
00115
           const boost::multi_array<double, 1> axis[2] = {x, z};
00116
00117
           std::vector<boost::multi_array<double, 2» XcZ = np::meshgrid(axis, false, np::xy);</pre>
00118
           matplot::vector_2d Xp = np::convert_to_matplot(XcZ[0]);
matplot::vector_2d Zp = np::convert_to_matplot(XcZ[1]);
00119
00120
00121
00122
           // Create the plotter object and animate the results
           wavePlotter::Plotter my_plotter(u, Xp, Zp, num_levels, nt);
00123
00124
           my_plotter.setSaveDirectory(result["output_filename"].as<std::string>());
00125
00126
           // If you want to render a specific frame, use this:
00127
           // my_plotter.renderFrame(int frame_index);
00128
           // Renders the entire animation from start_frame to end_frame
00130
           int start_frame = 0;
00131
           int end_frame = nt;
           int fps = result["framerate"].as<int>();
if (result["export"].as<bool>())
00132
00133
00134
           {
                my_plotter.renderAllFrames(start_frame, end_frame);
00136
00137
           else if (result["animate"].as<bool>())
00138
00139
                my plotter.animate(result["output filename"].as<std::string>(), start frame, end frame, fps);
00140
00141
           else
00142
00143
                my_plotter.renderAllFrames(start_frame, end_frame);
00144
           }
00145 }
```

11.13 Benchmarks.cpp

```
00001 //
00002 // Created by Yan Cheng on 12/2/22.
00003 //
00004
00005 #include <boost/multi_array.hpp>
00006 #include <boost/array.hpp>
00007 #include "CustomLibraries/np.hpp"
00008 #include "CustomLibraries/np_to_matplot.hpp"
00009 #include "CustomLibraries/wavePlotter.hpp"
00010 #include <matplot/matplot.h>
00011 #include <cassert>
00012 #include <iostream>
00013 #include <stream>
```

```
00014 #include <chrono>
00016 #include "CoreAlgorithm/helper_func.hpp"
00017 #include "CoreAlgorithm/coeff.hpp"
00018 #include "CoreAlgorithm/source.hpp"
00019 #include "CoreAlgorithm/computational.hpp"
00020 #include "CoreAlgorithm/solver.hpp"
00021
00022 void benchmark_solver()
00023 {
00024
           int nx = 101:
00025
          int nz = 101:
          int nt = 750;
00026
00027
00028
          double dx = 10;
          double dz = 10;
double dt = 0.001;
00029
00030
00031
          double xmin = 0.0;
00033
          double xmax = nx * dx;
00034
           double zmin = 0.0;
00035
           double zmax = nz * dz;
          double tmin = 0.0;
00036
          double tmax = nt * dt;
00037
00038
00039
          double f_M = 10.0;
00040
           double amp = 1e0;
00041
          double shift = 0.1;
00042
00043
          boost::multi_array<double, 3> f = waveSimCore::ricker(50, 50, f_M, amp, shift, tmin, tmax, nt, nx,
      nz);
00044
00045
           double r = 150.0;
00046
          boost::multi_array<double, 2> vel = waveSimCore::get_profile(xmin, xmax, zmin, zmax, nx, nz, r);
00047
           auto start = std::chrono::high_resolution_clock::now();
          boost::multi_array<double, 3> u = waveSimCore::wave_solver(vel, dt, dx, dz, nt, nx, nz, f);
00048
00049
          auto stop = std::chrono::high_resolution_clock::now();
          auto duration = std::chrono::duration_cast<std::chrono::microseconds>(stop - start);
00051
           std::cout \mbox{``Time taken by the solving function: " <math>\mbox{``duration.count()} \mbox{``}
       std::endl;
00052 }
00053
00054 void benchmark gradient()
00055 {
00056
           int sizex = 100;
00057
           int sizey = 100;
00058
          int sizez = 100;
          int sizet = 10;
00059
00060
          double dx = 0.1;
00061
          double dv = 0.1;
00062
          double dz = 0.1;
00063
          double dt = 0.1;
00064
           int axis[4] = {sizex, sizey, sizez, sizet};
          boost::multi_array<double, 4> nd_array = np::zeros<double>(axis);
auto start = std::chrono::high_resolution_clock::now();
00065
00066
          auto grad = np::gradient(nd_array, {dx, dy, dz, dt});
auto stop = std::chrono::high_resolution_clock::now();
00067
00069
           auto duration = std::chrono::duration_cast<std::chrono::microseconds>(stop - start);
00070
           std::cout « "Time taken by the gradient function: " « duration.count() « " microseconds" «
       std::endl;
00071 }
00072
00073 int main()
00074 {
00075
          benchmark_solver();
00076
          benchmark_gradient();
00077 }
```

11.14 CoreTests.cpp

```
00001 //
00002 // Created by Yan Cheng on 12/2/22.
00003 //
00004
00005 #include <boost/multi_array.hpp>
00006 #include <boost/array.hpp>
00007 #include "CustomLibraries/np.hpp"
00008 #include "CustomLibraries/np_to_matplot.hpp"
00009 #include "CustomLibraries/wavePlotter.hpp"
00010 #include <matplot/matplot.h>
00011 #include <cassert>
00012 #include <iostream>
00013 #include <sstream>
```

11.14 CoreTests.cpp 71

```
00014
00015 #include "CoreAlgorithm/helper_func.hpp"
00016 #include "CoreAlgorithm/coeff.hpp"
00017 #include "CoreAlgorithm/source.hpp"
00018 #include "CoreAlgorithm/computational.hpp"
00019 #include "CoreAlgorithm/solver.hpp"
00021 std::string format_num(int num, int length = 8)
00022 {
00023
           std::string str_num = std::to_string(num);
00024
          int str_length = str_num.length();
for (int i = 0; i < length - str_length; i++)
    str_num = "0" + str_num;</pre>
00025
00026
00027
00028
           return str_num;
00029 }
00030
00031 void test ()
00032 {
00033
           int num_levels = 100;
           int nx = 100;
int nz = 100;
00034
00035
          int nt = 1000;
00036
00037
00038
          double dx = 0.01;
          double dz = 0.01;
00039
00040
          double dt = 0.001;
00041
00042
          double xmin = 0.0;
00043
          double xmax = nx * dx;
00044
          double zmin = 0.0;
00045
          double zmax = nz * dz;
00046
           double tmin = 0.0;
00047
           double tmax = nt * dt;
00048
          double f M = 10.0;
00049
00050
          double amp = 1e0;
00051
          double shift = 0.1;
00052
00053
          boost::multi_array<double, 3> f = waveSimCore::ricker(50, 50, f_M, amp, shift, tmin, tmax, nt, nx,
       nz);
00054
00055
           double r = 150.0:
00056
          boost::multi_array<double, 2> vel = waveSimCore::get_profile(xmin, xmax, zmin, zmax, nx, nz, r);
00057
00058
           boost::multi_array<double, 3> u = waveSimCore::wave_solver(vel, dt, dx, dz, nt, nx, nz, f);
          double min_u = np::min(u);
double max_u = np::max(u);
00059
00060
          std::cout « "min_u = " « min_u « " max_u = " « max_u « "\n";
00061
           std::vector<double> levels = matplot::linspace(min_u, max_u, num_levels);
00062
          std::cout « u[20][10][10] « "\n";
00063
00064
00065
           // boost::multi_array<double, 2> u20(boost::extents[nx][nz]);
          // for (int i = 0; i < nx; i++)
// {
00066
00067
00068
                  for (int j = 0; j < nz; j++)
00069
00070
                       u20[i][j] = u[10][i][j];
                      std::cout « u20[i][j] « " ";
00071
00072
           //
00073
                  std::cout « "\n";
00074
           // }
00075
           // std::cout « "\n";
00076
00077
          boost::multi_array<double, 1> x = np::linspace(xmin, xmax, nx);
00078
          boost::multi_array<double, 1> z = np::linspace(zmin, zmax, nz);
00079
          const boost::multi_array<double, 1> axis[2] = \{x, z\};
08000
          std::vector<boost::multi_array<double, 2» XcZ = np::meshgrid(axis, false, np::xy);</pre>
00081
00082
           matplot::vector_2d Xp = np::convert_to_matplot(XcZ[0]);
          matplot::vector_2d Zp = np::convert_to_matplot(Xcz[1]);
// matplot::contourf(Xp, Zp, vel, levels, "", num_levels);
00083
00084
00085
           // matplot::show();
00086
          wavePlotter::Plotter my_plotter(u, Xp, Zp, num_levels, nt);
// my_plotter.exportAllFrames(0, nt - 1);
00087
00088
00089
           my_plotter.animate("output-test.mp4", 20, nt - 1, 30);
00090
           // for (int i = 10; i < nt - 1; i++)
00091
00092
           // {
           11
00093
                  matplot::vector_2d Up = np::convert_to_matplot(u[i]);
                  matplot::contourf(Xp, Zp, Up, levels);
00094
                  matplot::save("output/contourf_" + format_num(i) + ".png");
00095
00096
00097 }
00098
00099 int main()
```

```
00100 {
00101 test_();
00102 }
```

11.15 MatPlotTest.cpp

```
00001 #include <matplot/matplot.h>
00002 #include <thread>
00002 #Include "boost/multi_array.hpp"
00004 #include "boost/array.hpp"
00005 #include "CustomLibraries/np.hpp"
00006 #include "CustomLibraries/np_to_matplot.hpp"
00007
00008 using namespace matplot;
00009 void test_simple_plot()
00010 {
              std::vector<double> x = linspace(-2 * pi, 2 * pi);
std::vector<double> y = linspace(0, 4 * pi);
auto [X, Y] = meshgrid(x, y);
00011
00012
00013
              vector_2d Z =
00014
00015
                 transform(X, Y, [](double x, double y)
00016
                                  { return sin(x) + cos(y); });
00017
              contourf(X, Y, Z, 10);
00018
00019
              show():
00020 }
00021
00022 void test_conversion()
00023 {
              boost::multi_array<double, 1> x = np::linspace(0, 1, 100);
boost::multi_array<double, 1> y = np::linspace(0, 1, 100);
00024
00025
              // x = np::pow(x, 2.0);

// y = np::pow(y, 3.0);
00026
00028
00029
              const boost::multi_array<double, 1> axis[2] = {x, y};
              std::vector<boost::multi_array<double, 2» XcY = np::meshgrid(axis, false, np::xy);</pre>
00030
00031
              double dx, dy;
dx = 1.0 / 100.0;
dy = 1.0 / 100.0;
00032
00033
00034
00035
00036
              boost:: multi\_array < double, \ 2 > \ f = np::pow(XcY[0], \ 2.0) \ + \ XcY[0] \ \star \ np::pow(XcY[1], \ 1.0);
00037
              // g.push_back(np::gradient(XcY[0], {dx, dy}));
// g.push_back(np::gradient(XcY[1], {dx, dy}));
00038
00040
              std::vector<boost::multi_array<double, 2» gradf = np::gradient(f, {dx, dy});</pre>
              matplot::vector_2d Xp = np::convert_to_matplot(XcY[0]);
matplot::vector_2d Yp = np::convert_to_matplot(XcY[1]);
00041
00042
              matplot::vector_2d X = matplot::meshgrid(linspace(0, 1, 100), linspace(0, 1, 100)).first;
matplot::vector_2d Y = matplot::meshgrid(linspace(0, 1, 100), linspace(0, 1, 100)).second;
00043
00044
00045
              matplot::vector_2d Z = np::convert_to_matplot(gradf[1]);
std::cout « "X.size() = " « X.size() « std::endl;
std::cout « "Y.size() = " « Y.size() « std::endl;
00046
00047
00048
              std::cout « "Z.size() = " « Z.size() « std::endl;
00049
00050
              for (size_t i = 0; i < X.size(); i++)</pre>
00051
00052
                    for (size_t j = 0; j < X[i].size(); j++)</pre>
00053
          std::cout « "X[" « i « "][" « j « "] = " « X[i][j] « " | XP[" « i « "][" « j « "] = " « Xp[i][j] « " | YP[" « i « "][" « j « "] = " « Yp[i][j] « std::endl;
00054
00055
00056
              contourf(Xp, Yp, Z, 10);
00058
              show();
00059 }
00060
00061 int main()
00062 {
00063
               // test_simple_plot();
00064
              test_conversion();
00065
              return 0;
00066 }
```

11.16 variadic.cpp

```
00001 #include "boost/multi_array.hpp"
00002 #include "boost/array.hpp"
00003 #include "CustomLibraries/np.hpp"
```

11.16 variadic.cpp 73

```
00004 #include <cassert>
00005 #include <iostream>
00006
00007 void test_gradient()
} 80000
00009
            // Create a 4D array that is 3 \times 4 \times 2 \times 1
            typedef boost::multi_array<double, 4>::index index;
00010
00011
            boost::multi_array<double, 4> A(boost::extents[3][4][2][2]);
00012
00013
            // Assign values to the elements
00014
            int values = 0;
00015
            for (index i = 0; i != 3; ++i)
                for (index j = 0; j != 4; ++j)

for (index k = 0; k != 2; ++k)
00016
00017
                          for (index 1 = 0; 1 != 2; ++1)
00018
00019
                              A[i][j][k][1] = values++;
00020
00021
            // Verify values
00022
           int verify = 0;
            for (index i = 0; i != 3; ++i)
00023
                for (index j = 0; j != 4; ++j)
for (index k = 0; k != 2; ++k)
for (index l = 0; l != 2; ++1)
00024
00025
00026
                               assert(A[i][j][k][l] == verify++);
00027
00028
00029
            double dx, dy, dz, dt;
00030
            dx = 1.0;
00031
            dy = 1.0;
00032
            dz = 1.0;
00033
           dt = 1.0;
00034
           std::vector<boost::multi_array<double, 4> my_arrays = np::gradient(A, {dx, dy, dz, dt});
00035
00036
            boost::multi_array<double, 1> x = np::linspace(0, 1, 5);
00037
            std::vector<boost::multi_array<double, 1» gradf = np::gradient(x, {1.0});</pre>
00038
            for (int i = 0; i < 5; i++)
00039
00040
                std::cout « gradf[0][i] « ",";
00041
00042
            std::cout « "\n";
00043
            // np::print(std::cout, my_arrays[0]);
00044 }
00045
00046 void test mesharid()
00047 {
00048
            boost::multi_array<double, 1 > x = np::linspace(0, 1, 5);
00049
            boost::multi_array<double, 1> y = np::linspace(0, 1, 5);
           boost::multi_array<double, 1> z = np::linspace(0, 1, 5);
boost::multi_array<double, 1> t = np::linspace(0, 1, 5);
00050
00051
           const boost::multi_array<double, 1> axis[4] = {x, y, z, t};
std::vector<boost::multi_array<double, 4> my_arrays = np::meshgrid(axis, false, np::xy);
00052
00053
00054
            // np::print(std::cout, my_arrays[0]);
00055
            int nx = 3;
00056
            int ny = 2;
           boost::multi_array<double, 1> x2 = np::linspace(0, 1, nx);
boost::multi_array<double, 1> y2 = np::linspace(0, 1, ny);
00057
00058
            const boost::multi_array<double, 1> axis2[2] = {x2, y2};
std::vector<boost::multi_array<double, 2> my_arrays2 = np::meshgrid(axis2, false, np::xy);
00059
00060
00061
            std::cout « "xv\n";
00062
            for (int i = 0; i < ny; i++)</pre>
00063
                 for (int j = 0; j < nx; j++)
00064
00065
                {
00066
                     std::cout « my_arrays2[0][i][j] « " ";
00067
00068
                std::cout « "\n";
00069
00070
            std::cout « "yv\n";
            for (int i = 0; i < ny; i++)
00071
00072
00073
                 for (int j = 0; j < nx; j++)
00074
00075
                     std::cout « my_arrays2[1][i][j] « " ";
00076
00077
                std::cout « "\n";
00078
            }
00079 }
08000
00081 void test_complex_operations()
00082 {
00083
            int nx = 3:
            int ny = 2;
00084
00085
            boost::multi_array<double, 1> x = np::linspace(0, 1, nx);
            boost::multi_array<double, 1> y = np::linspace(0, 1, ny); const boost::multi_array<double, 1> axis[2] = {x, y};
00086
00087
00088
            std::vector<boost::multi_array<double, 2» my_arrays = np::meshgrid(axis, false, np::xy);</pre>
           boost::multi_array<double, 2> A = np::sqrt(my_arrays[0]);
std::cout « "sqrt\n";
00089
00090
```

```
for (int i = 0; i < ny; i++)
00092
00093
                 for (int j = 0; j < nx; j++)
00094
                 {
                      std::cout « A[i][j] « " ";
00095
00096
00097
                 std::cout « "\n";
00098
00099
            std::cout « "\n";
00100
            float a = 100.0;
            float sqa = np::sqrt(a);
std::cout « "sqrt of " « a « " is " « sqa « "\n";
00101
00102
            std::cout « "exp\n";
00103
00104
            boost::multi_array<double, 2> B = np::exp(my_arrays[0]);
00105
            for (int i = 0; i < ny; i++)</pre>
00106
                 for (int j = 0; j < nx; j++)
00107
00108
                 {
00109
                      std::cout « B[i][j] « " ";
00110
00111
                 std::cout « "\n";
00112
            }
00113
            std::cout « "Power\n";
00114
00115
            boost::multi_array<double, 1> x2 = np::linspace(1, 3, nx);
            boost::multi_array<double, 1> y2 = np::linspace(1, 3, ny);
00116
00117
            const boost::multi_array<double, 1> axis2[2] = {x2, y2};
00118
            std::vector<boost::multi_array<double, 2» my_arrays2 = np::meshgrid(axis2, false, np::xy);</pre>
            boost::multi_array<double, 2> C = np::pow(my_arrays2[1], 2.0);
for (int i = 0; i < ny; i++)</pre>
00119
00120
00121
            {
00122
                 for (int j = 0; j < nx; j++)
00123
00124
                      std::cout « C[i][j] « " ";
00125
                 std::cout « "\n";
00126
00127
            }
00128 }
00129
00130 void test_equal()
00131 {
00132
            boost::multi_array<double, 1 > x = np::linspace(0, 1, 5);
            boost::multi_array<double, 1> z = np::linspace(0, 1, 5);
boost::multi_array<double, 1> z = np::linspace(0, 1, 5);
00133
00134
            boost::multi_array<double, 1> t = np::linspace(0, 1, 5);
const boost::multi_array<double, 1> axis[4] = {x, y, z, t};
00135
00136
00137
            std::vector<boost::multi_array<double, 4> my_arrays = np::meshgrid(axis, false, np::xy);
            boost::multi_array<double, 1> x2 = np::linspace(0, 1, 5);
boost::multi_array<double, 1> y2 = np::linspace(0, 1, 5);
boost::multi_array<double, 1> y2 = np::linspace(0, 1, 5);
boost::multi_array<double, 1> z2 = np::linspace(0, 1, 5);
boost::multi_array<double, 1> t2 = np::linspace(0, 1, 5);
00138
00139
00140
00141
00142
            const boost::multi_array<double, 1> axis2[4] = {x2, y2, z2, t2};
00143
            std::vector<boost::multi_array<double, 4> my_arrays2 = np::meshgrid(axis2, false, np::xy);
00144
            std::cout « "equality test:\n";
            std::cout « (bool) (my_arrays == my_arrays2) « "\n";
00145
00146 }
00147 void test_basic_operations()
00148 {
00149
            int nx = 3;
00150
            int ny = 2;
            boost::multi_array<double, 1> x = np::linspace(0, 1, nx);
boost::multi_array<double, 1> y = np::linspace(0, 1, ny);
const boost::multi_array<double, 1> axis[2] = {x, y};
00151
00152
00153
00154
            std::vector<boost::multi_array<double, 2> my_arrays = np::meshgrid(axis, false, np::xy);
00155
00156
            std::cout « "basic operations:\n";
00157
00158
            std::cout « "addition:\n";
00159
            boost::multi_array<double, 2> A = my_arrays[0] + my_arrays[1];
00160
00161
            for (int i = 0; i < ny; i++)
00162
00163
                 for (int j = 0; j < nx; j++)
00164
                      std::cout « A[i][j] « " ";
00165
00166
00167
                 std::cout « "\n";
00168
            }
00169
            std::cout « "multiplication:\n";
00170
            boost::multi_array<double, 2> B = my_arrays[0] * my_arrays[1];
00171
00172
00173
            for (int i = 0; i < ny; i++)
00174
00175
                 for (int j = 0; j < nx; j++)
00176
00177
                      std::cout « B[i][j] « " ";
```

11.16 variadic.cpp 75

```
00178
00179
                        std::cout « "\n";
00180
00181
                 double coeff = 3;
                 boost::multi_array<double, 1> t = np::linspace(0, 1, nx);
boost::multi_array<double, 1> t_time_3 = coeff * t;
boost::multi_array<double, 1> t_time_2 = 2.0 * t;
00182
00183
00184
00185
                 std::cout « "t_time_3: ";
                 for (int j = 0; j < nx; j++)
00186
00187
                        std::cout « t_time_3[j] « " ";
00188
00189
                 std::cout « "\n";
std::cout « "t_time_2: ";
00190
00191
00192
                  for (int j = 0; j < nx; j++)
00193
                        std::cout « t_time_2[j] « " ";
00194
00195
00196
                 std::cout « "\n";
00197 }
00198
00199 void test_zeros()
00200 {
00201
                 int nx = 3;
00202
                 int ny = 2;
00203
                  int dimensions[] = {ny, nx};
00204
                 boost::multi_array<double, 2> A = np::zeros<double>(dimensions);
                 std::cout « "zeros:\n";
for (int i = 0; i < ny; i++)
00205
00206
00207
00208
                        for (int j = 0; j < nx; j++)
00209
                        {
00210
                               std::cout « A[i][j] « " ";
00211
                        std::cout « "\n";
00212
                 }
00213
00214 }
00215
00216 void test_min_max()
00217 {
00218
                 int nx = 24;
00219
                 int ny = 5;
                 boost::multi_array<double, 1> x = np::linspace(0, 10, nx);
boost::multi_array<double, 1> y = np::linspace(-1, 1, ny);
00220
00221
                 const boost::multi_array<double, 1> axis[2] = {x, y};
00222
00223
                 std::vector<boost::multi_array<double, 2> my_array = np::meshgrid(axis, false, np::xy);
                 std::cout « "min: " « np::min(my_array[0]) « "\n";
std::cout « "max: " « np::max(my_array[1]) « "\n";
std::cout « "max simple: " « np::max(1.0, 2.0, 3.0, 4.0, 5.0) « "\n";
std::cout « "min simple: " « np::min(1, -2, 3, -4, 5) « "\n";
00224
00225
00226
00227
00228 }
00229
00230 void test_toy_problem()
00231 {
00232
                 boost::multi_array<double, 1 > x = np::linspace(0, 1, 100);
00233
                 boost::multi_array<double, 1> y = np::linspace(0, 1, 100);
                 // x = np::pow(x, 2.0);
00234
00235
                 // y = np::pow(y, 3.0);
00236
00237
                 const boost::multi_array<double, 1> axis[2] = \{x, y\};
                 std::vector<boost::multi_array<double, 2» XcY = np::meshgrid(axis, false, np::xy);</pre>
00238
00239
00240
                 double dx, dy;
                 dx = 1.0 / 100.0;

dy = 1.0 / 100.0;
00241
00242
00243
00244
                 boost::multi_array<double, 2> f = np::pow(XcY[0], 2.0) + XcY[0] * np::pow(XcY[1], 1.0);
00245
                 // g.push_back(np::gradient(XcY[0], {dx, dy}));
00246
                  // g.push_back(np::gradient(XcY[1], {dx, dy}));
00247
00248
                 std::vector<boost::multi_array<double, 2» gradf = np::gradient(f, {dx, dy});</pre>
00249
                 // auto [gradfx_x, gradfx_y] = np::gradient(f, {dx, dy});
00250
00251
                 int i, j;
                 i = 10;
j = 20;
00252
00253
00254
                  std::cout   "df/dx of  f(x,y) = x^2 + xy  at  x =  "  x[i]    " and  y =  "  x[i]    " is equal to "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "   "  
            gradf[0][i][j];
00255
00256
                 std::cout « "\n":
00257 }
00258
00259 void test_abs()
00260 {
00261
                 int nx = 4;
00262
                 int ny = 4;
00263
                 boost::multi array<double, 1> x = np::linspace(-1, 1, nx);
```

```
boost::multi_array<double, 1> y = np::linspace(-1, 1, ny);
            const boost::multi_array<double, 1> axis[2] = {x, y};
std::vector<boost::multi_array<double, 2» XcY = np::meshgrid(axis, false, np::xy);</pre>
00265
00266
            boost::multi_array<double, 2> abs_f = np::abs(XcY[0]);
std::cout « "abs_f: \n";
for (int i = 0; i < ny; i++)</pre>
00267
00268
00269
00270
00271
                 for (int j = 0; j < nx; j++)
00272
                      std::cout < abs_f[i][j] < " ";
00273
00274
00275
                 std::cout « "\n";
00276
            }
00277 }
00278
00279 void test_slice()
00280 {
00281
            int nx = 4;
            int ny = 4;
00283
            boost::multi_array<double, 1> x = np::linspace(-1, 1, nx);
            boost::multi_array<double, 1> y = np::linspace(-1, 1, ny);
const boost::multi_array<double, 1> axis[2] = {x, y};
std::vector<boost::multi_array<double, 2» XcY = np::meshgrid(axis, false, np::xy);</pre>
00284
00285
00286
00287
            boost::multi_array<double, 2> f = np::pow(XcY[0], 2.0) + XcY[0] * np::pow(XcY[1], 1.0);
            std::cout « "f: \n";

for (int i = 0; i < ny; i++)
00288
00289
00290
00291
                 for (int j = 0; j < nx; j++)
00292
00293
                      std::cout « f[i][j] « " ";
00294
00295
                 std::cout « "\n";
00296
00297
            std::cout « "f[0]: \n";
            boost::multi_array<double, 1> f_slice = np::slice(f, 0);
for (int i = 0; i < nx; i++)</pre>
00298
00299
00300
                 std::cout « f_slice[i] « " ";
00302
00303
            std::cout « "\n";
00304
            std::cout « "f[1]: \n";
00305
            f_slice = np::slice(f, 1);
for (int i = 0; i < ny; i++)
00306
00307
00308
00309
                 std::cout « f_slice[i] « " ";
00310
            std::cout « "\n";
00311
00312 }
00313
00314 int main()
00315 {
00316
            test_gradient();
00317
            test_meshgrid();
00318
            test_complex_operations();
00319
            test equal();
00320
            test_basic_operations();
00321
            test_zeros();
00322
            test_min_max();
00323
            test_abs();
            test_toy_problem();
00324
00325
            test_slice();
00326 }
```