

WaveSimC

0.8

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Chapter 1

README

1.1 COMSW4995 Final Project: WaveSimC

This is the repository for our final project for the discipline COMSW4995: Design in C++ at Columbia University during the Fall of 2022.

This project aims to implement in modern C++ a wave equation solver for geophysical application.

In addition, a custom implementation of numpy in modern C++ is also included as a header library. That library aims to make c++ more pythonic and easier to use for scientific computing. Instead of numpy n-dimensional arrays the library uses `boost::multi_array` and contains many utilities to expand the functionality of the library.

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1.1.2 Acknowledgments

We would like to thank Professor Bjarne Stroustrup for his guidance and support during the development of this project.

1.2 Theory

1.2.1 Wave solving

Wave equation is a partial differential equation that describes the propagation of waves in a medium. The wave equation is given by:

1.2.2 Design Philosophy

1.2.2.1 Numpy implementation

We have noticed that many users are very familiar with python and use it extensively with libraries such as numpy and scipy. However their code is often slow and not very low-level friendly. Even with numpy and scipy's low-level optimizations, there could still be margin for improvement by converting everything to C++, which would allow users to unleash even more optimizations and exert more control over how their code runs. This could also allow the code to run on less powerful devices that often don't support python.

With that in mind we decided to find a way to make transferring that numpy, scipy, etc code to C++ in an easy way, while keeping all of the high level luxuries of python. We decided to implement a numpy-like library in C++ that would allow users to write code in a similar way to python, but with the performance of C++.

We started with the implementation of the functions used in the python version of the wave solver and plan to expand the library to include more functions and features in the future.

The library is contained in a header library format for easy of use.

1.2.3 Multi Arrays how math is done on them

Representing arrays with more than one dimensions is a difficult task in any programming language, specially in a language like C++ that implements strict type checking. To implement that in a flexible and typesafe way, we chose to build our code around the `boost::multi_array`. This library provides a container that can be used to represent arrays with any number of dimensions. The library is very flexible and allows the user to define the type of the array and the number of dimensions at compile time. The library is sadly not very well documented but the documentation can be found here: https://www.boost.org/doc/libs/1_75_0/libs/multi_array/doc/index.html

We decided to build the math functions in a pythonic way, so we implemented numpy functions into our C++ library in a way that they would accept n-dimensions through a template parameters and act accordingly while enforcing dimensional consistency at compile time. We also used concepts and other modern C++ concepts to make sure that, for example, a python call such as `np.max(my_n_dimensional_array)` would be translated to `np::max(my_n_dimensional_array)` in C++.

To perform operations on an n-dimensional array we choose to iterate over it and convert the pointers to indexes using a simple arithmetic operation with one division. This is somewhat time consuming since we don't have O(1) time access to any point in the array, instead having O(n) where n is the amount of elements in the multi array. This is the tradeoff necessary to have n-dimensions represented in memory, hopefully in modern cpus this overhead won't be too high. Better solutions could be investigated further.

We also implemented simple arithmetic operators with multi arrays to make them more arithmetic friendly such as they are in python.

Only one small subset of numpy functions were implemented, but the library is easily extensible and more functions can be added in the future.

1.3 Building

1.3.1 Install the boost library

It is important to install the boost library before building the project. The boost library is used for data structures and algorithms. The boost library can be installed using the following command on ubuntu:

```
sudo apt-get install libboost-all-dev
```

For Mac:

```
brew install boost
```

1.3.2 Build the project

```
mkdir build
cd build
cmake ..
make Main
```

1.3.3 Running

```
./Main
```

1.3.4 Building the documentation

Docs building script:

```
./compileDocs.sh
doxygen dconfig
cd documentation/latex
pdflatex refman.tex
cp refman.pdf ../WaveSimC-0.8-doc.pdf
```


Chapter 2

Module Index

2.1 Modules

Here is a list of all modules:

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Chapter 3

Namespace Index

3.1 Namespace List

Here is a list of all documented namespaces with brief descriptions:

np	Custom implementation of numpy in C++	15
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Chapter 4

File Index

4.1 File List

Here is a list of all documented files with brief descriptions:

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Chapter 5

Module Documentation

5.1 Np

Namespaces

- namespace `np`
Custom implementation of numpy in C++.

Functions

- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator* (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs)`
Multiplication operator between two multi arrays, element-wise.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator* (T const &lhs, boost::multi_array< T, ND > const &rhs)`
Multiplication operator between a multi array and a scalar.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator* (boost::multi_array< T, ND > const &lhs, T const &rhs)`
Multiplication operator between a multi array and a scalar.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator+ (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs)`
Addition operator between two multi arrays, element wise.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator+ (T const &lhs, boost::multi_array< T, ND > const &rhs)`
Addition operator between a multi array and a scalar.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator+ (boost::multi_array< T, ND > const &lhs, T const &rhs)`
Addition operator between a scalar and a multi array.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator- (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs)`
Minus operator between two multi arrays, element-wise.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator- (T const &lhs, boost::multi_array< T, ND > const &rhs)`

Minus operator between a scalar and a multi array, element-wise.

- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator- (boost::multi_array< T, ND > const &lhs, T const &rhs)`

Minus operator between a multi array and a scalar, element-wise.

- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator/ (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs)`

Division between two multi arrays, element wise.

- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator/ (T const &lhs, boost::multi_array< T, ND > const &rhs)`

Division between a scalar and a multi array, element wise.

- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > operator/ (boost::multi_array< T, ND > const &lhs, T const &rhs)`

Division between a multi array and a scalar, element wise.

5.1.1 Detailed Description

5.1.2 Function Documentation

5.1.2.1 `operator*()` [1/3]

```
template<class T, long unsigned int ND>
boost::multi_array< T, ND > operator* (
    boost::multi_array< T, ND > const & lhs,
    boost::multi_array< T, ND > const & rhs ) [inline]
```

Multiplication operator between two multi arrays, element-wise.

Definition at line 447 of file [np.hpp](#).

```
00448 {
00449     std::function<T(T, T)> func = std::multiplies<T>();
00450     return np::element_wise_duo_apply(lhs, rhs, func);
00451 }
```

5.1.2.2 `operator*()` [2/3]

```
template<class T, long unsigned int ND>
boost::multi_array< T, ND > operator* (
    boost::multi_array< T, ND > const & lhs,
    T const & rhs ) [inline]
```

Multiplication operator between a multi array and a scalar.

Definition at line 463 of file [np.hpp](#).

```
00464 {
00465     return rhs * lhs;
00466 }
```


5.1.2.3 operator*() [3/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator* (
    T const & lhs,
    boost::multi_array< T, ND > const & rhs ) [inline]
```

Multiplication operator between a multi array and a scalar.

Definition at line 455 of file [np.hpp](#).

```
00456 {
00457     std::function<T(T)> func = [lhs](T item)
00458     { return lhs * item; };
00459     return np::element_wise_apply(rhs, func);
00460 }
```

5.1.2.4 operator+() [1/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator+ (
    boost::multi_array< T, ND > const & lhs,
    boost::multi_array< T, ND > const & rhs )
```

Addition operator between two multi arrays, element wise.

Definition at line 471 of file [np.hpp](#).

```
00472 {
00473     std::function<T(T, T)> func = std::plus<T>();
00474     return np::element_wise_duo_apply(lhs, rhs, func);
00475 }
```

5.1.2.5 operator+() [2/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator+ (
    boost::multi_array< T, ND > const & lhs,
    T const & rhs ) [inline]
```

Addition operator between a scalar and a multi array.

Definition at line 488 of file [np.hpp](#).

```
00489 {
00490     return rhs + lhs;
00491 }
```

5.1.2.6 operator+() [3/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator+ (
    T const & lhs,
    boost::multi_array< T, ND > const & rhs ) [inline]
```

Addition operator between a multi array and a scalar.

Definition at line 479 of file [np.hpp](#).

```
00480 {
00481     std::function<T(T)> func = [lhs](T item)
00482     { return lhs + item; };
00483     return np::element_wise_apply(rhs, func);
00484 }
```

5.1.2.7 operator-() [1/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator- (
    boost::multi_array< T, ND > const & lhs,
    boost::multi_array< T, ND > const & rhs )
```

Minus operator between two multi arrays, element-wise.

Definition at line 496 of file [np.hpp](#).

```
00497 {
00498     std::function<T(T, T)> func = std::minus<T>();
00499     return np::element_wise_duo_apply(lhs, rhs, func);
00500 }
```

5.1.2.8 operator-() [2/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator- (
    boost::multi_array< T, ND > const & lhs,
    T const & rhs ) [inline]
```

Minus operator between a multi array and a scalar, element-wise.

Definition at line 513 of file [np.hpp](#).

```
00514 {
00515     return rhs - lhs;
00516 }
```

5.1.2.9 operator-() [3/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator- (
    T const & lhs,
    boost::multi_array< T, ND > const & rhs ) [inline]
```

Minus operator between a scalar and a multi array, element-wise.

Definition at line 504 of file [np.hpp](#).

```
00505 {
00506     std::function<T(T)> func = [lhs](T item)
00507     { return lhs - item; };
00508     return np::element_wise_apply(rhs, func);
00509 }
```

5.1.2.10 operator/() [1/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator/ (
    boost::multi_array< T, ND > const & lhs,
    boost::multi_array< T, ND > const & rhs )
```

Division between two multi arrays, element wise.

Definition at line 521 of file [np.hpp](#).

```
00522 {
00523     std::function<T(T, T)> func = std::divides<T>();
00524     return np::element_wise_duo_apply(lhs, rhs, func);
00525 }
```

5.1.2.11 operator/() [2/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator/ (
    boost::multi_array< T, ND > const & lhs,
    T const & rhs ) [inline]
```

Division between a multi array and a scalar, element wise.

Definition at line 538 of file [np.hpp](#).

```
00539 {
00540     return rhs / lhs;
00541 }
```

5.1.2.12 operator/() [3/3]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > operator/ (
    T const & lhs,
    boost::multi_array< T, ND > const & rhs ) [inline]
```

Division between a scalar and a multi array, element wise.

Definition at line 529 of file [np.hpp](#).

```
00530 {
00531     std::function<T(T)> func = [lhs](T item)
00532     { return lhs / item; };
00533     return np::element_wise_apply(rhs, func);
00534 }
```


Chapter 6

Namespace Documentation

6.1 np Namespace Reference

Custom implementation of numpy in C++.

Typedefs

- typedef double [ndArrayValue](#)

Enumerations

- enum **indexing** { **xy** , **ij** }

Functions

- template<std::size_t ND>
boost::multi_array< ndArrayValue, ND >::index [getIndex](#) (const boost::multi_array< ndArrayValue, ND > &m, const ndArrayValue *requestedElement, const unsigned short int direction)
Gets the index of one element in a multi_array in one axis.
- template<std::size_t ND>
boost::array< typename boost::multi_array< ndArrayValue, ND >::index, ND > [getIndexArray](#) (const boost::multi_array< ndArrayValue, ND > &m, const ndArrayValue *requestedElement)
Gets the index of one element in a multi_array.
- template<typename Array , typename Element , typename Functor >
void [for_each](#) (const boost::type< Element > &type_dispatch, Array A, Functor &xform)
- template<typename Element , typename Functor >
void [for_each](#) (const boost::type< Element > &, Element &Val, Functor &xform)
Function to apply a function to all elements of a multi_array.
- template<typename Element , typename Iterator , typename Functor >
void [for_each](#) (const boost::type< Element > &type_dispatch, Iterator begin, Iterator end, Functor &xform)
Function to apply a function to all elements of a multi_array.
- template<typename Array , typename Functor >
void [for_each](#) (Array &A, Functor xform)

- `template<long unsigned int ND>`
`constexpr std::vector< boost::multi_array< double, ND > > gradient (boost::multi_array< double, ND >`
`inArray, std::initializer_list< double > args)`
- `boost::multi_array< double, 1 > linspace (double start, double stop, long unsigned int num)`
Implements the numpy linspace function.
- `template<long unsigned int ND>`
`std::vector< boost::multi_array< double, ND > > meshgrid (const boost::multi_array< double, 1`
`>(&cinput)[ND], bool sparsing=false, indexing indexing_type=xy)`
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > element_wise_apply (const boost::multi_array< T, ND > &input_array, std::function< T(T)> func)`
Creates a new array and fills it with the values of the result of the function called on the input array element-wise.
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > sqrt (const boost::multi_array< T, ND > &input_array)`
- `template<class T >`
`T sqrt (const T input)`
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > exp (const boost::multi_array< T, ND > &input_array)`
- `template<class T >`
`T exp (const T input)`
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > log (const boost::multi_array< T, ND > &input_array)`
- `template<class T >`
`T log (const T input)`
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > pow (const boost::multi_array< T, ND > &input_array, const T exponent)`
- `template<class T >`
`T pow (const T input, const T exponent)`
- `template<class T, long unsigned int ND>`
`boost::multi_array< T, ND > element_wise_duo_apply (boost::multi_array< T, ND > const &lhs, boost::multi_array< T, ND > const &rhs, std::function< T(T, T)> func)`

6.1.1 Detailed Description

Custom implementation of numpy in C++.

6.1.2 Typedef Documentation

6.1.2.1 ndarrayValue

```
typedef double np::ndArrayValue
```

Definition at line 22 of file [np.hpp](#).

6.1.3 Enumeration Type Documentation

6.1.3.1 indexing

```
enum np::indexing
```

Definition at line 172 of file [np.hpp](#).

```
00173     {
00174         xy,
00175         ij
00176     };
```

6.1.4 Function Documentation

6.1.4.1 element_wise_apply()

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > np::element_wise_apply (
    const boost::multi_array< T, ND > & input_array,
    std::function< T(T)> func ) [inline]
```

Creates a new array and fills it with the values of the result of the function called on the input array element-wise.

Definition at line 243 of file [np.hpp](#).

```
00244     {
00245
00246         // Create output array copying extents
00247         using arrayIndex = boost::multi_array<double, ND>::index;
00248         using ndIndexArray = boost::array<arrayIndex, ND>;
00249         boost::detail::multi_array::extent_gen<ND> output_extents;
00250         std::vector<size_t> shape_list;
00251         for (std::size_t i = 0; i < ND; i++)
00252         {
00253             shape_list.push_back(input_array.shape()[i]);
00254         }
00255         std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00256         boost::multi_array<T, ND> output_array(output_extents);
00257
00258         // Looping through the elements of the output array
00259         const T *p = input_array.data();
00260         ndIndexArray index;
00261         for (std::size_t i = 0; i < input_array.num_elements(); i++)
00262         {
00263             index = getIndexArray(input_array, p);
00264             output_array(index) = func(input_array(index));
00265             ++p;
00266         }
00267         return output_array;
00268     }
```

6.1.4.2 element_wise_duo_apply()

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > np::element_wise_duo_apply (
    boost::multi_array< T, ND > const & lhs,
    boost::multi_array< T, ND > const & rhs,
    std::function< T(T, T)> func )
```

Creates a new array in which the value at each index is the the result of the input function applied to an element of the left hand side array and one on the right hand side array in the same index Outputs a copy of the result

Definition at line 324 of file [np.hpp](#).

```

00325     {
00326         // Create output array copying extents
00327         using arrayIndex = boost::multi_array<double, ND>::index;
00328         using ndIndexArray = boost::array<arrayIndex, ND>;
00329         boost::detail::multi_array::extent_gen<ND> output_extents;
00330         std::vector<size_t> shape_list;
00331         for (std::size_t i = 0; i < ND; i++)
00332         {
00333             shape_list.push_back(lhs.shape()[i]);
00334         }
00335         std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00336         boost::multi_array<T, ND> output_array(output_extents);
00337
00338         // Looping through the elements of the output array
00339         const T *p = lhs.data();
00340         ndIndexArray index;
00341         for (std::size_t i = 0; i < lhs.num_elements(); i++)
00342         {
00343             index = getIndexArray(lhs, p);
00344             output_array(index) = func(lhs(index), rhs(index));
00345             ++p;
00346         }
00347         return output_array;
00348     }

```

6.1.4.3 exp() [1/2]

```

template<class T , long unsigned int ND>
boost::multi_array< T, ND > np::exp (
    const boost::multi_array< T, ND > & input_array ) [inline]

```

Definition at line 285 of file [np.hpp](#).

```

00286     {
00287         std::function<T(T)> func = (T(*) (T))std::exp;
00288         return element_wise_apply(input_array, func);
00289     }

```

6.1.4.4 exp() [2/2]

```

template<class T >
T np::exp (
    const T input ) [inline]

```

Definition at line 291 of file [np.hpp](#).

```

00292     {
00293         return std::exp(input);
00294     }

```

6.1.4.5 for_each() [1/4]

```

template<typename Array , typename Functor >
void np::for_each (
    Array & A,
    Functor xform ) [inline]

```

Function to apply a function to all elements of a multi_array Simple overload

Definition at line 80 of file [np.hpp](#).

```

00081     {
00082         // Dispatch to the proper function
00083         for_each(boost::type<typename Array::element>(), A.begin(), A.end(), xform);
00084     }

```


6.1.4.6 for_each() [2/4]

```
template<typename Element , typename Functor >
void np::for_each (
    const boost::type< Element > & ,
    Element & Val,
    Functor & xform ) [inline]
```

Function to apply a function to all elements of a multi_array.

Definition at line 59 of file [np.hpp](#).

```
00060     {
00061         Val = xform(Val);
00062     }
```

6.1.4.7 for_each() [3/4]

```
template<typename Array , typename Element , typename Functor >
void np::for_each (
    const boost::type< Element > & type_dispatch,
    Array A,
    Functor & xform ) [inline]
```

Function to apply a function to all elements of a multi_array Simple overload

Definition at line 51 of file [np.hpp](#).

```
00053     {
00054         for_each(type_dispatch, A.begin(), A.end(), xform);
00055     }
```

6.1.4.8 for_each() [4/4]

```
template<typename Element , typename Iterator , typename Functor >
void np::for_each (
    const boost::type< Element > & type_dispatch,
    Iterator begin,
    Iterator end,
    Functor & xform ) [inline]
```

Function to apply a function to all elements of a multi_array.

Definition at line 66 of file [np.hpp](#).

```
00069     {
00070         while (begin != end)
00071         {
00072             for_each(type_dispatch, *begin, xform);
00073             ++begin;
00074         }
00075     }
```

6.1.4.9 getIndex()

```
template<std::size_t ND>
boost::multi_array< ndArrayValue, ND >::index np::getIndex (
    const boost::multi_array< ndArrayValue, ND > & m,
    const ndArrayValue * requestedElement,
    const unsigned short int direction ) [inline]
```

Gets the index of one element in a multi_array in one axis.

Definition at line 27 of file [np.hpp](#).

```
00028     {
00029         int offset = requestedElement - m.origin();
00030         return (offset / m.strides()[direction] % m.shape()[direction] + m.index_bases()[direction]);
00031     }
```

6.1.4.10 getIndexArray()

```
template<std::size_t ND>
boost::array< typename boost::multi_array< ndArrayValue, ND >::index, ND > np::getIndexArray
(
    const boost::multi_array< ndArrayValue, ND > & m,
    const ndArrayValue * requestedElement ) [inline]
```

Gets the index of one element in a multi_array.

Definition at line 36 of file [np.hpp](#).

```
00037     {
00038         using indexType = boost::multi_array<ndArrayValue, ND>::index;
00039         boost::array<indexType, ND> _index;
00040         for (unsigned int dir = 0; dir < ND; dir++)
00041         {
00042             _index[dir] = getIndex(m, requestedElement, dir);
00043         }
00044         return _index;
00045     }
00046 }
```

6.1.4.11 gradient()

```
template<long unsigned int ND>
constexpr std::vector< boost::multi_array< double, ND > > np::gradient (
    boost::multi_array< double, ND > inArray,
    std::initializer_list< double > args ) [inline], [constexpr]
```

Takes the gradient of a n-dimensional multi_array Todo: Actually implement the gradient calculation template <long unsigned int ND, typename... Args>

Definition at line 90 of file [np.hpp](#).

```
00091     {
00092         // static_assert(args.size() == ND, "Number of arguments must match the number of dimensions
of the array");
00093         using arrayIndex = boost::multi_array<double, ND>::index;
00094         using ndIndexArray = boost::array<arrayIndex, ND>;
00095         // constexpr std::size_t n = sizeof...(Args);
00096         std::size_t n = args.size();
00097         // std::tuple<Args...> store(args...);
```

```

00100         std::vector<double> arg_vector = args;
00101         boost::multi_array<double, ND> my_array;
00102         std::vector<boost::multi_array<double, ND> output_arrays;
00103         for (std::size_t i = 0; i < n; i++)
00104         {
00105             boost::multi_array<double, ND> dfdh = inArray;
00106             output_arrays.push_back(dfdh);
00107         }
00108
00109         ndArrayValue *p = inArray.data();
00110         ndIndexArray index;
00111         for (std::size_t i = 0; i < inArray.num_elements(); i++)
00112         {
00113             index = getIndexArray(inArray, p);
00114             /*
00115             std::cout << "Index: ";
00116             for (std::size_t j = 0; j < n; j++)
00117             {
00118                 std::cout << index[j] << " ";
00119             }
00120             std::cout << "\n";
00121             */
00122             // Calculating the gradient now
00123             // j is the axis/dimension
00124             for (std::size_t j = 0; j < n; j++)
00125             {
00126                 ndIndexArray index_high = index;
00127                 double dh_high;
00128                 if ((long unsigned int)index_high[j] < inArray.shape()[j] - 1)
00129                 {
00130                     index_high[j] += 1;
00131                     dh_high = arg_vector[j];
00132                 }
00133                 else
00134                 {
00135                     dh_high = 0;
00136                 }
00137                 ndIndexArray index_low = index;
00138                 double dh_low;
00139                 if (index_low[j] > 0)
00140                 {
00141                     index_low[j] -= 1;
00142                     dh_low = arg_vector[j];
00143                 }
00144                 else
00145                 {
00146                     dh_low = 0;
00147                 }
00148
00149                 double dh = dh_high + dh_low;
00150                 double gradient = (inArray(index_high) - inArray(index_low)) / dh;
00151                 // std::cout << gradient << "\n";
00152                 output_arrays[j](index) = gradient;
00153             }
00154             // std::cout << " value = " << inArray(index) << " check = " << *p << std::endl;
00155             ++p;
00156         }
00157         return output_arrays;
00158     }

```

6.1.4.12 linspace()

```

boost::multi_array< double, 1 > np::linspace (
    double start,
    double stop,
    long unsigned int num ) [inline]

```

Implements the numpy linspace function.

Definition at line 161 of file [np.hpp](#).

```

00162     {
00163         double step = (stop - start) / (num - 1);
00164         boost::multi_array<double, 1> output(boost::extents[num]);
00165         for (std::size_t i = 0; i < num; i++)
00166         {
00167             output[i] = start + i * step;
00168         }
00169         return output;
00170     }

```

6.1.4.13 log() [1/2]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > np::log (
    const boost::multi_array< T, ND > & input_array ) [inline]
```

Definition at line 297 of file [np.hpp](#).

```
00298     {
00299         std::function<T(T)> func = std::log<T>();
00300         return element_wise_apply(input_array, func);
00301     }
```

6.1.4.14 log() [2/2]

```
template<class T >
T np::log (
    const T input ) [inline]
```

Definition at line 303 of file [np.hpp](#).

```
00304     {
00305         return std::log(input);
00306     }
```

6.1.4.15 meshgrid()

```
template<long unsigned int ND>
std::vector< boost::multi_array< double, ND > > np::meshgrid (
    const boost::multi_array< double, 1 >(&) cinput[ND],
    bool sparsing = false,
    indexing indexing_type = xy ) [inline]
```

Implementation of meshgrid TODO: Implement sparsing=true If the indexing type is xx, then reverse the order of the first two elements of ci if the number of dimensions is 2 or 3 In accordance with the numpy implementation

Definition at line 184 of file [np.hpp](#).

```
00185     {
00186         using arrayIndex = boost::multi_array<double, ND>::index;
00187         using ndIndexArray = boost::array<arrayIndex, ND>;
00188         std::vector<boost::multi_array<double, ND> output_arrays;
00189         boost::multi_array<double, 1> ci[ND];
00190         // Copy elements of cinput to ci, do the proper inversions
00191         for (std::size_t i = 0; i < ND; i++)
00192         {
00193             std::size_t source = i;
00194             if (indexing_type == xy && (ND == 3 || ND == 2))
00195             {
00196                 switch (i)
00197                 {
00198                     case 0:
00199                         source = 1;
00200                         break;
00201                     case 1:
00202                         source = 0;
00203                         break;
00204                     default:
00205                         break;
00206                 }
00207             }
00208             ci[i] = boost::multi_array<double, 1>();
00209             ci[i].resize(boost::extents[cinput[source].num_elements()]);
00210             ci[i] = cinput[source];
00211         }
```

```

00212         // Deducing the extents of the N-Dimensional output
00213         boost::detail::multi_array::extent_gen<ND> output_extents;
00214         std::vector<size_t> shape_list;
00215         for (std::size_t i = 0; i < ND; i++)
00216         {
00217             shape_list.push_back(ci[i].shape()[0]);
00218         }
00219         std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00220
00221         // Creating the output arrays
00222         for (std::size_t i = 0; i < ND; i++)
00223         {
00224             boost::multi_array<double, ND> output_array(output_extents);
00225             ndArrayValue *p = output_array.data();
00226             ndIndexArray index;
00227             // Looping through the elements of the output array
00228             for (std::size_t j = 0; j < output_array.num_elements(); j++)
00229             {
00230                 index = getIndexArray(output_array, p);
00231                 boost::multi_array<double, 1>::index index_ld;
00232                 index_ld = index[i];
00233                 output_array(index) = ci[i][index_ld];
00234                 ++p;
00235             }
00236             output_arrays.push_back(output_array);
00237         }
00238         return output_arrays;
00239     }

```

6.1.4.16 pow() [1/2]

```

template<class T, long unsigned int ND>
boost::multi_array< T, ND > np::pow (
    const boost::multi_array< T, ND > & input_array,
    const T exponent ) [inline]

```

Definition at line 308 of file [np.hpp](#).

```

00309     {
00310         std::function<T(T)> pow_func = [exponent](T input)
00311         { return std::pow(input, exponent); };
00312         return element_wise_apply(input_array, pow_func);
00313     }

```

6.1.4.17 pow() [2/2]

```

template<class T >
T np::pow (
    const T input,
    const T exponent ) [inline]

```

Definition at line 315 of file [np.hpp](#).

```

00316     {
00317         return std::pow(input, exponent);
00318     }

```

6.1.4.18 `sqrt()` [1/2]

```
template<class T , long unsigned int ND>
boost::multi_array< T, ND > np::sqrt (
    const boost::multi_array< T, ND > & input_array ) [inline]
```

Definition at line 273 of file [np.hpp](#).

```
00274     {
00275         std::function<T(T)> func = (T(*) (T))std::sqrt;
00276         return element_wise_apply(input_array, func);
00277     }
```

6.1.4.19 `sqrt()` [2/2]

```
template<class T >
T np::sqrt (
    const T input ) [inline]
```

Definition at line 279 of file [np.hpp](#).

```
00280     {
00281         return std::sqrt(input);
00282     }
```

Chapter 7

File Documentation

7.1 coeff.hpp

```
00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_COEFF_HPP
00006 #define WAVESIMC_COEFF_HPP
00007
00008 #include "CustomLibraries/np.hpp"
00009 #include <math.h>
00010
00011
00012 boost::multi_array<double, 2> get_sigma_1(boost::multi_array<double, 1> x, double dx, int nx, int nz,
00013 double c_max, int n=10, double R=1e-3, int m=2)
00014 {
00015     boost::multi_array<double, 2> sigma_1 = np::zeros(nx, nz);
00016     const double PML_width = n * dx;
00017     const double sigma_max = - c_max * log(R) * (m+1) / (PML_width**(m+1));
00018
00019     // TODO: max: find the maximum element in 1D array
00020     const double x_0 = max(x) - PML_width;
00021
00022     // each column of sigma_1 is a 1D array named "polynomial"
00023     boost::multi_array<double, 1> polynomial = np::zeros(nx);
00024     for (int i=0; i<nx; i++)
00025     {
00026         if (x[i] > x_0)
00027         {
00028             // TODO: Does math.h have an absolute value function?
00029             polynomial[i] = sigma_max * abs(x[i] - x_0)**m;
00030             polynomial[nx-i] = polynomial[i];
00031         }
00032         else
00033         {
00034             polynomial[i] = 0;
00035         }
00036     }
00037
00038     // Copy 1D array into each column of 2D array
00039     for (int i=0; i<nx; i++)
00040         for (int j=0; j<nz; j++)
00041             sigma_1[i][j] = polynomial[i];
00042
00043     return sigma_1;
00044 }
00045
00046
00047
00048 boost::multi_array<double, 2> get_sigma_2(boost::multi_array<double, 1> z, double dz, int nx, int nz,
00049 double c_max, int n=10, double R=1e-3, int m=2)
00050 {
00051     boost::multi_array<double, 2> sigma_2 = np::zeros(nx, nz);
00052     const double PML_width = n * dz;
00053     const double sigma_max = - c_max * log(R) * (m+1) / (PML_width**(m+1));
00054
00055     // TODO: max: find the maximum element in 1D array
00056     const double z_0 = max(z) - PML_width;
00057
00058     // each column of sigma_1 is a 1D array named "polynomial"
```

```

00059     boost::multi_array<double, 1> polynomial = np::zeros(nz);
00060     for (int j=0; j<nz; j++)
00061     {
00062         if (z[j] > z_0)
00063         {
00064             // TODO: Does math.h have an absolute value function?
00065             polynomial[j] = sigma_max * abs(z[j] - z_0)**m;
00066             polynomial[nz-j] = polynomial[j];
00067         }
00068         else
00069         {
00070             polynomial[j] = 0;
00071         }
00072     }
00073
00074     // Copy 1D array into each column of 2D array
00075     for (int i=0; i<nz; i++)
00076         for (int j=0; j<nz; j++)
00077             sigma_1[i][j] = polynomial[j];
00078
00079     return sigma_2;
00080 }
00081
00082 #endif //WAVESIMC_COEFF_HPP

```

7.2 computational.hpp

```

00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_COMPUTATIONAL_HPP
00006 #define WAVESIMC_COMPUTATIONAL_HPP
00007
00008 boost::multi_array<double, 2> get_profile()
00009 {
00010
00011 }
00012
00013 #endif //WAVESIMC_COMPUTATIONAL_HPP

```

7.3 helper_func.hpp

```

00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_HELPER_FUNC_HPP
00006 #define WAVESIMC_HELPER_FUNC_HPP
00007
00008 #include "CustomLibraries/np.hpp"
00009
00010 boost::multi_array<double, 2> dfdx(boost::multi_array<double, 2> f, double dx)
00011 {
00012     std::vector<boost::multi_array<double, 2>> grad_f = np::gradient(f, {dx, dx});
00013     return grad_f[0];
00014 }
00015
00016 boost::multi_array<double, 2> dfdz(boost::multi_array<double, 2> f, double dz)
00017 {
00018     std::vector<boost::multi_array<double, 2>> grad_f = np::gradient(f, {dz, dz});
00019     return grad_f[1];
00020 }
00021
00022 boost::multi_array<double, 2> d2fdx2(boost::multi_array<double, 2> f, double dx)
00023 {
00024     boost::multi_array<double, 2> f_x = dfdx(f, dx);
00025     boost::multi_array<double, 2> f_xx = dfdx(f_x, dx);
00026     return f_xx;
00027 }
00028
00029 boost::multi_array<double, 2> d2fdz2(boost::multi_array<double, 2> f, double dz)
00030 {
00031     boost::multi_array<double, 2> f_z = dfdz(f, dz);
00032     boost::multi_array<double, 2> f_zz = dfdz(f_z, dz);
00033     return f_zz;
00034 }
00035

```



```

00036 boost::multi_array<double, 2> divergence(boost::multi_array<double, 2> f1, boost::multi_array<double,
    2> f2,
00037                                     double dx, double dz)
00038 {
00039     boost::multi_array<double, 2> f_x = dfdx(f1, dx);
00040     boost::multi_array<double, 2> f_z = dfdz(f2, dz);
00041     // TODO: use element-wise add
00042     div = f1 + f2;
00043     return div;
00044 }
00045
00046
00047 #endif //WAVESIMC_HELPER_FUNC_HPP

```

7.4 solver.hpp

```

00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_SOLVER_HPP
00006 #define WAVESIMC_SOLVER_HPP
00007
00008 #include "CustomLibraries/np.hpp"
00009 #include "helper_func.hpp"
00010
00011 boost::multi_array<double, 3> wave_solver(boost::multi_array<double, 2> c,
00012                                     double dt, double dx, double dz, int nt, int nx, int nz,
00013                                     boost::multi_array<double, 3> f,
00014                                     boost::multi_array<double, 2> sigma_1,
00015                                     boost::multi_array<double, 2> sigma_2)
00016 {
00017     // TODO: "same shape" functionality of np::zeros
00018     boost::multi_array<double, 3> u = np::zeros(nt, nx, nz);
00019     boost::multi_array<double, 2> u_xx = np::zeros(nx, ny);
00020     boost::multi_array<double, 2> u_zz = np::zeros(nx, ny);
00021     boost::multi_array<double, 2> q_1 = np::zeros(nx, ny);
00022     boost::multi_array<double, 2> q_2 = np::zeros(nx, ny);
00023
00024     // TODO: make multiplication between scalar and boost::multi_array<double, 2> work
00025     // Basically we need to make * and ** work
00026     const boost::multi_array<double, 2> C1 = 1 + dt * (sigma_1 + sigma_2) / ((double) 2);
00027     // Question: Is ((double) 2) necessary?
00028     const boost::multi_array<double, 2> C2 = sigma_1 * sigma_2 * (dt**2) - 2;
00029     const boost::multi_array<double, 2> C3 = 1 - dt*(sigma_1 + sigma_2)/2;
00030     const boost::multi_array<double, 2> C4 = (dt*c)**2;
00031     const boost::multi_array<double, 2> C5 = 1 + dt*sigma_1/2;
00032     const boost::multi_array<double, 2> C6 = 1 + dt*sigma_2/2;
00033     const boost::multi_array<double, 2> C7 = 1 - dt*sigma_1/2;
00034     const boost::multi_array<double, 2> C8 = 1 - dt*sigma_2/2;
00035
00036     for (int n = 0; n < nt; n++)
00037     {
00038         u_xx = d2fdx2(u[n], dx);
00039         u_zz = d2fdz2(u[n], dz);
00040
00041         u[n+1] = (C4*(u_xx/(dx**2) + u_zz/(dz**2) - divergence(q_1*sigma_1, q_2*sigma_2, dx, dz)
00042             + sigma_2*dfdx(q_1, dx) + sigma_1*dfdz(q_2, dz) + f[n]) -
00043             C2 * u[n] - C3 * u[n-1]) / C1;
00044
00045         q_1 = (dt*dfdx(u[n], dx) + C7*q_1) / C5;
00046         q_2 = (dt*dfdz(u[n], dx) + C8*q_2) / C6;
00047
00048         // Dirichlet boundary condition
00049         for (int i = 0; i < nx; i++)
00050         {
00051             u[n+1][i][0] = 0;
00052             u[n+1][i][nx-1] = 0;
00053         }
00054         for (int j = 0; j < nz; j++)
00055         {
00056             u[n+1][0][j] = 0;
00057             u[n+1][nz-1][j] = 0;
00058         }
00059     }
00060     return u;
00061 }
00062 #endif //WAVESIMC_SOLVER_HPP

```

7.5 source.hpp

```

00001 //
00002 // Created by Yan Cheng on 11/28/22.
00003 //
00004
00005 #ifndef WAVESIMC_SOURCE_HPP
00006 #define WAVESIMC_SOURCE_HPP
00007
00008
00009 boost::multi_array<double, 3> ricker(int i_s, int j_s, double f=10, double amp=1e0, double shift=0.1)
00010 {
00011     const double pi = 3.141592654;
00012
00013     boost::multi_array<double, 1> t = np::linspace(tmin, tmax, nt);
00014
00015     // TODO: element-wise operators
00016     boost::multi_array<double, 1> pft2 = (pi * f * (t - shift))*2;
00017     boost::multi_array<double, 1> r = amp * (1 - 2 * pft2) * exp(-pft2);
00018
00019     boost::multi_array<double, 1> x = np.zeros(nx);
00020     boost::multi_array<double, 1> z = np.zeros(nz);
00021     x[i_s] = 1.0;
00022     z[j_s] = 1.0;
00023     boost::multi_array<double, 3> TXZ = np::meshgrid(r, x, z, sparse=True, indexing='ij');
00024
00025     return TXZ;
00026 }
00027
00028 #endif //WAVESIMC_SOURCE_HPP

```

7.6 wave.cpp

```

00001 // For the core algorithm, we need six functionalities:
00002 // 1) create the computational domain,
00003 // 2) create a velocity profile (1 & 2 can be put together)
00004 // 3) create attenuation coefficients,
00005 // 4) create source functions,
00006 // 5) helper functions to compute eg. df/dx
00007 // 6) use all above to create a solver function for wave equation
00008
00009 // Standard IO libraries
00010 #include <iostream>
00011 #include <fstream>
00012 #include "CustomLibraries/np.hpp"
00013
00014 #include <math.h>
00015
00016 #include "solver.hpp"
00017 #include "computational.hpp"
00018 #include "coeff.hpp"
00019 #include "source.hpp"
00020 #include "helper_func.hpp"
00021
00022
00023 int main()
00024 {
00025     double dx, dy, dz, dt;
00026     dx = 1.0;
00027     dy = 1.0;
00028     dz = 1.0;
00029     dt = 1.0;
00030     std::vector<boost::multi_array<double, 4>> my_arrays = np::gradient(A, {dx, dy, dz, dt});
00031     return 0;
00032 }

```

7.7 np.hpp

```

00001 #ifndef NP_H_
00002 #define NP_H_
00003
00004 #include "boost/multi_array.hpp"
00005 #include "boost/array.hpp"
00006 #include "boost/cstdlib.hpp"
00007 #include <type_traits>
00008 #include <cassert>
00009 #include <iostream>
00010 #include <functional>
00011 #include <type_traits>

```

```

00012
00019 namespace np
00020 {
00021
00022     typedef double ndArrayValue;
00023
00025     template <std::size_t ND>
00026     inline boost::multi_array<ndArrayValue, ND>::index
00027     getIndex(const boost::multi_array<ndArrayValue, ND> &m, const ndArrayValue *requestedElement,
const unsigned short int direction)
00028     {
00029         int offset = requestedElement - m.origin();
00030         return (offset / m.strides()[direction] % m.shape()[direction] + m.index_bases()[direction]);
00031     }
00032
00034     template <std::size_t ND>
00035     inline boost::array<typename boost::multi_array<ndArrayValue, ND>::index, ND>
00036     getIndexArray(const boost::multi_array<ndArrayValue, ND> &m, const ndArrayValue *requestedElement)
00037     {
00038         using indexType = boost::multi_array<ndArrayValue, ND>::index;
00039         boost::array<indexType, ND> _index;
00040         for (unsigned int dir = 0; dir < ND; dir++)
00041         {
00042             _index[dir] = getIndex(m, requestedElement, dir);
00043         }
00044
00045         return _index;
00046     }
00047
00050     template <typename Array, typename Element, typename Functor>
00051     inline void for_each(const boost::type<Element> &type_dispatch,
Array A, Functor &xform)
00052     {
00053         for_each(type_dispatch, A.begin(), A.end(), xform);
00054     }
00055
00056
00058     template <typename Element, typename Functor>
00059     inline void for_each(const boost::type<Element> &, Element &Val, Functor &xform)
00060     {
00061         Val = xform(Val);
00062     }
00063
00065     template <typename Element, typename Iterator, typename Functor>
00066     inline void for_each(const boost::type<Element> &type_dispatch,
Iterator begin, Iterator end,
Functor &xform)
00067     {
00068         while (begin != end)
00069         {
00070             for_each(type_dispatch, *begin, xform);
00071             ++begin;
00072         }
00073     }
00074
00075
00076
00079     template <typename Array, typename Functor>
00080     inline void for_each(Array &A, Functor xform)
00081     {
00082         // Dispatch to the proper function
00083         for_each(boost::type<typename Array::element>(), A.begin(), A.end(), xform);
00084     }
00085
00086
00089     template <long unsigned int ND>
00090     inline constexpr std::vector<boost::multi_array<double, ND> gradient(boost::multi_array<double,
ND> inArray, std::initializer_list<double> args)
00091     {
00092         // static_assert(args.size() == ND, "Number of arguments must match the number of dimensions
of the array");
00093         using arrayIndex = boost::multi_array<double, ND>::index;
00094
00095         using ndIndexArray = boost::array<arrayIndex, ND>;
00096
00097         // constexpr std::size_t n = sizeof...(Args);
00098         std::size_t n = args.size();
00099         // std::tuple<Args...> store(args...);
00100         std::vector<double> arg_vector = args;
00101         boost::multi_array<double, ND> my_array;
00102         std::vector<boost::multi_array<double, ND> output_arrays;
00103         for (std::size_t i = 0; i < n; i++)
00104         {
00105             boost::multi_array<double, ND> dfdh = inArray;
00106             output_arrays.push_back(dfdh);
00107         }
00108
00109         ndArrayValue *p = inArray.data();
00110         ndIndexArray index;
00111         for (std::size_t i = 0; i < inArray.num_elements(); i++)
00112     {

```

```

00113         index = getIndexArray(inArray, p);
00114         /*
00115         std::cout << "Index: ";
00116         for (std::size_t j = 0; j < n; j++)
00117         {
00118             std::cout << index[j] << " ";
00119         }
00120         std::cout << "\n";
00121         */
00122         // Calculating the gradient now
00123         // j is the axis/dimension
00124         for (std::size_t j = 0; j < n; j++)
00125         {
00126             ndIndexArray index_high = index;
00127             double dh_high;
00128             if ((long unsigned int)index_high[j] < inArray.shape()[j] - 1)
00129             {
00130                 index_high[j] += 1;
00131                 dh_high = arg_vector[j];
00132             }
00133             else
00134             {
00135                 dh_high = 0;
00136             }
00137             ndIndexArray index_low = index;
00138             double dh_low;
00139             if (index_low[j] > 0)
00140             {
00141                 index_low[j] -= 1;
00142                 dh_low = arg_vector[j];
00143             }
00144             else
00145             {
00146                 dh_low = 0;
00147             }
00148
00149             double dh = dh_high + dh_low;
00150             double gradient = (inArray(index_high) - inArray(index_low)) / dh;
00151             // std::cout << gradient << "\n";
00152             output_arrays[j](index) = gradient;
00153         }
00154         // std::cout << " value = " << inArray(index) << " check = " << *p << std::endl;
00155         ++p;
00156     }
00157     return output_arrays;
00158 }
00159
00160 inline boost::multi_array<double, 1> linspace(double start, double stop, long unsigned int num)
00161 {
00162     {
00163         double step = (stop - start) / (num - 1);
00164         boost::multi_array<double, 1> output(boost::extents[num]);
00165         for (std::size_t i = 0; i < num; i++)
00166         {
00167             output[i] = start + i * step;
00168         }
00169         return output;
00170     }
00171 }
00172
00173 enum indexing
00174 {
00175     xy,
00176     ij
00177 };
00178
00179 template <long unsigned int ND>
00180 inline std::vector<boost::multi_array<double, ND> meshgrid(const boost::multi_array<double, 1>
00181 (&cinput)[ND], bool sparsing = false, indexing indexing_type = xy)
00182 {
00183     {
00184         using arrayIndex = boost::multi_array<double, ND>::index;
00185         using ndIndexArray = boost::array<arrayIndex, ND>;
00186         std::vector<boost::multi_array<double, ND> output_arrays;
00187         boost::multi_array<double, 1> ci[ND];
00188         // Copy elements of cinput to ci, do the proper inversions
00189         for (std::size_t i = 0; i < ND; i++)
00190         {
00191             std::size_t source = i;
00192             if (indexing_type == xy && (ND == 3 || ND == 2))
00193             {
00194                 switch (i)
00195                 {
00196                     {
00197                         case 0:
00198                             source = 1;
00199                             break;
00200                         case 1:
00201                             source = 0;
00202                             break;
00203                         default:

```

```

00205         break;
00206     }
00207 }
00208 ci[i] = boost::multi_array<double, 1>();
00209 ci[i].resize(boost::extents[cinput[source].num_elements()]);
00210 ci[i] = cinput[source];
00211 }
00212 // Deducing the extents of the N-Dimensional output
00213 boost::detail::multi_array::extent_gen<ND> output_extents;
00214 std::vector<size_t> shape_list;
00215 for (std::size_t i = 0; i < ND; i++)
00216 {
00217     shape_list.push_back(ci[i].shape()[0]);
00218 }
00219 std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00220
00221 // Creating the output arrays
00222 for (std::size_t i = 0; i < ND; i++)
00223 {
00224     boost::multi_array<double, ND> output_array(output_extents);
00225     ndArrayValue *p = output_array.data();
00226     ndIndexArray index;
00227     // Looping through the elements of the output array
00228     for (std::size_t j = 0; j < output_array.num_elements(); j++)
00229     {
00230         index = getIndexArray(output_array, p);
00231         boost::multi_array<double, 1>::index index_ld;
00232         index_ld = index[i];
00233         output_array(index) = ci[i][index_ld];
00234         ++p;
00235     }
00236     output_arrays.push_back(output_array);
00237 }
00238 return output_arrays;
00239 }
00240
00242 template <class T, long unsigned int ND>
00243 inline boost::multi_array<T, ND> element_wise_apply(const boost::multi_array<T, ND> &input_array,
std::function<T(T)> func)
00244 {
00245     // Create output array copying extents
00246     using arrayIndex = boost::multi_array<double, ND>::index;
00247     using ndIndexArray = boost::array<arrayIndex, ND>;
00248     boost::detail::multi_array::extent_gen<ND> output_extents;
00249     std::vector<size_t> shape_list;
00250     for (std::size_t i = 0; i < ND; i++)
00251     {
00252         shape_list.push_back(input_array.shape()[i]);
00253     }
00254     std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00255     boost::multi_array<T, ND> output_array(output_extents);
00256
00257     // Looping through the elements of the output array
00258     const T *p = input_array.data();
00259     ndIndexArray index;
00260     for (std::size_t i = 0; i < input_array.num_elements(); i++)
00261     {
00262         index = getIndexArray(input_array, p);
00263         output_array(index) = func(input_array(index));
00264         ++p;
00265     }
00266     return output_array;
00267 }
00268
00269 // Complex operations
00270
00271 template <class T, long unsigned int ND>
00272 inline boost::multi_array<T, ND> sqrt(const boost::multi_array<T, ND> &input_array)
00273 {
00274     std::function<T(T)> func = (T(*) (T))std::sqrt;
00275     return element_wise_apply(input_array, func);
00276 }
00277
00278 template <class T>
00279 inline T sqrt(const T input)
00280 {
00281     return std::sqrt(input);
00282 }
00283
00284 template <class T, long unsigned int ND>
00285 inline boost::multi_array<T, ND> exp(const boost::multi_array<T, ND> &input_array)
00286 {
00287     std::function<T(T)> func = (T(*) (T))std::exp;
00288     return element_wise_apply(input_array, func);
00289 }
00290
00291 template <class T>
inline T exp(const T input)

```

```

00292     {
00293         return std::exp(input);
00294     }
00295
00296     template <class T, long unsigned int ND>
00297     inline boost::multi_array<T, ND> log(const boost::multi_array<T, ND> &input_array)
00298     {
00299         std::function<T(T)> func = std::log<T>();
00300         return element_wise_apply(input_array, func);
00301     }
00302     template <class T>
00303     inline T log(const T input)
00304     {
00305         return std::log(input);
00306     }
00307     template <class T, long unsigned int ND>
00308     inline boost::multi_array<T, ND> pow(const boost::multi_array<T, ND> &input_array, const T
exponent)
00309     {
00310         std::function<T(T)> pow_func = [exponent](T input)
00311         { return std::pow(input, exponent); };
00312         return element_wise_apply(input_array, pow_func);
00313     }
00314     template <class T>
00315     inline T pow(const T input, const T exponent)
00316     {
00317         return std::pow(input, exponent);
00318     }
00319
00320     template <class T, long unsigned int ND>
00321     boost::multi_array<T, ND> element_wise_duo_apply(boost::multi_array<T, ND> const &lhs,
boost::multi_array<T, ND> const &rhs, std::function<T(T, T)> func)
00322     {
00323         // Create output array copying extents
00324         using arrayIndex = boost::multi_array<double, ND>::index;
00325         using ndIndexArray = boost::array<arrayIndex, ND>;
00326         boost::detail::multi_array::extent_gen<ND> output_extents;
00327         std::vector<size_t> shape_list;
00328         for (std::size_t i = 0; i < ND; i++)
00329         {
00330             shape_list.push_back(lhs.shape()[i]);
00331         }
00332         std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00333         boost::multi_array<T, ND> output_array(output_extents);
00334
00335         // Looping through the elements of the output array
00336         const T *p = lhs.data();
00337         ndIndexArray index;
00338         for (std::size_t i = 0; i < lhs.num_elements(); i++)
00339         {
00340             index = getIndexArray(lhs, p);
00341             output_array(index) = func(lhs(index), rhs(index));
00342             ++p;
00343         }
00344         return output_array;
00345     }
00346
00347     template <typename T, typename inT, long unsigned int ND>
00348     inline constexpr boost::multi_array<T, ND> zeros(inT (&dimensions_input)[ND]) requires
std::is_integral<inT>::value && std::is_arithmetic<T>::value
00349     {
00350         // Deducing the extents of the N-Dimensional output
00351         boost::detail::multi_array::extent_gen<ND> output_extents;
00352         std::vector<size_t> shape_list;
00353         for (std::size_t i = 0; i < ND; i++)
00354         {
00355             shape_list.push_back(dimensions_input[i]);
00356         }
00357         std::copy(shape_list.begin(), shape_list.end(), output_extents.ranges_.begin());
00358         // Applying a function to return zero always to all of its elements
00359         boost::multi_array<T, ND> output_array(output_extents);
00360         std::function<T(T)> zero_func = [](T input)
00361         { return 0; };
00362         return element_wise_apply(output_array, zero_func);
00363     }
00364
00365     template <typename T, long unsigned int ND>
00366     inline constexpr T max(boost::multi_array<T, ND> const &input_array) requires
std::is_arithmetic<T>::value
00367     {
00368         T max = 0;
00369         bool max_not_set = true;
00370         const T *data_pointer = input_array.data();
00371         for (std::size_t i = 0; i < input_array.num_elements(); i++)
00372         {
00373             T element = *data_pointer;
00374             if (max_not_set || element > max)

```

```

00380         {
00381             max = element;
00382             max_not_set = false;
00383         }
00384         ++data_pointer;
00385     }
00386     return max;
00387 }
00388
00390 template <class T, class... Ts, class = std::enable_if_t<(std::is_same_v<T, Ts> && ...) >
00391 inline constexpr T max(T input1, Ts... inputs) requires std::is_arithmetic<T>::value
00392 {
00393     T max = input1;
00394     for (T input : {inputs...})
00395     {
00396         if (input > max)
00397         {
00398             max = input;
00399         }
00400     }
00401     return max;
00402 }
00403
00405 template <typename T, long unsigned int ND>
00406 inline constexpr T min(boost::multi_array<T, ND> const &input_array) requires
std::is_arithmetic<T>::value
00407 {
00408     T min = 0;
00409     bool min_not_set = true;
00410     const T *data_pointer = input_array.data();
00411     for (std::size_t i = 0; i < input_array.num_elements(); i++)
00412     {
00413         T element = *data_pointer;
00414         if (min_not_set || element < min)
00415         {
00416             min = element;
00417             min_not_set = false;
00418         }
00419         ++data_pointer;
00420     }
00421     return min;
00422 }
00423
00425 template <class T, class... Ts, class = std::enable_if_t<(std::is_same_v<T, Ts> && ...) >
00426 inline constexpr T min(T input1, Ts... inputs) requires std::is_arithmetic<T>::value
00427 {
00428     T min = input1;
00429     for (T input : {inputs...})
00430     {
00431         if (input < min)
00432         {
00433             min = input;
00434         }
00435     }
00436     return min;
00437 }
00438 }
00439
00440 // Override of operators in the boost::multi_array class to make them more np-like
00441 // Basic operators
00442 // All of the are element-wise
00443
00444 // Multiplication operator
00446 template <class T, long unsigned int ND>
00447 inline boost::multi_array<T, ND> operator*(boost::multi_array<T, ND> const &lhs, boost::multi_array<T,
ND> const &rhs)
00448 {
00449     std::function<T(T, T)> func = std::multiplies<T>();
00450     return np::element_wise_duo_apply(lhs, rhs, func);
00451 }
00452
00454 template <class T, long unsigned int ND>
00455 inline boost::multi_array<T, ND> operator*(T const &lhs, boost::multi_array<T, ND> const &rhs)
00456 {
00457     std::function<T(T)> func = [lhs](T item)
00458     { return lhs * item; };
00459     return np::element_wise_apply(rhs, func);
00460 }
00462 template <class T, long unsigned int ND>
00463 inline boost::multi_array<T, ND> operator*(boost::multi_array<T, ND> const &lhs, T const &rhs)
00464 {
00465     return rhs * lhs;
00466 }
00467
00468 // Plus operator
00470 template <class T, long unsigned int ND>
00471 boost::multi_array<T, ND> operator+(boost::multi_array<T, ND> const &lhs, boost::multi_array<T, ND>

```

```

    const &rhs)
00472 {
00473     std::function<T(T, T)> func = std::plus<T>();
00474     return np::element_wise_duo_apply(lhs, rhs, func);
00475 }
00476
00477 template <class T, long unsigned int ND>
00478 inline boost::multi_array<T, ND> operator+(T const &lhs, boost::multi_array<T, ND> const &rhs)
00479 {
00480     std::function<T(T)> func = [lhs](T item)
00481     { return lhs + item; };
00482     return np::element_wise_apply(rhs, func);
00483 }
00484
00485 template <class T, long unsigned int ND>
00486 inline boost::multi_array<T, ND> operator+(boost::multi_array<T, ND> const &lhs, T const &rhs)
00487 {
00488     return rhs + lhs;
00489 }
00490
00491 // Subtraction operator
00492 template <class T, long unsigned int ND>
00493 inline boost::multi_array<T, ND> operator-(boost::multi_array<T, ND> const &lhs, boost::multi_array<T, ND>
    const &rhs)
00494 {
00495     std::function<T(T, T)> func = std::minus<T>();
00496     return np::element_wise_duo_apply(lhs, rhs, func);
00497 }
00498
00499 template <class T, long unsigned int ND>
00500 inline boost::multi_array<T, ND> operator-(T const &lhs, boost::multi_array<T, ND> const &rhs)
00501 {
00502     std::function<T(T)> func = [lhs](T item)
00503     { return lhs - item; };
00504     return np::element_wise_apply(rhs, func);
00505 }
00506
00507 template <class T, long unsigned int ND>
00508 inline boost::multi_array<T, ND> operator-(boost::multi_array<T, ND> const &lhs, T const &rhs)
00509 {
00510     return rhs - lhs;
00511 }
00512
00513 // Division operator
00514 template <class T, long unsigned int ND>
00515 inline boost::multi_array<T, ND> operator/(boost::multi_array<T, ND> const &lhs, boost::multi_array<T, ND>
    const &rhs)
00516 {
00517     std::function<T(T, T)> func = std::divides<T>();
00518     return np::element_wise_duo_apply(lhs, rhs, func);
00519 }
00520
00521 template <class T, long unsigned int ND>
00522 inline boost::multi_array<T, ND> operator/(T const &lhs, boost::multi_array<T, ND> const &rhs)
00523 {
00524     std::function<T(T)> func = [lhs](T item)
00525     { return lhs / item; };
00526     return np::element_wise_apply(rhs, func);
00527 }
00528
00529 template <class T, long unsigned int ND>
00530 inline boost::multi_array<T, ND> operator/(boost::multi_array<T, ND> const &lhs, T const &rhs)
00531 {
00532     return rhs / lhs;
00533 }
00534
00535 #endif

```

7.8 main.cpp

```

00001 #include <iostream>
00002 #include <string>
00003 #include "ExternalLibraries/cxxopts.hpp"
00004 #include "CustomLibraries/np.hpp"
00005
00006 // Command line arguments
00007 cxxopts::Options options("WaveSimC", "A wave propagation simulator written in C++ for seismic data
    processing.");
00008 int main(int argc, char *argv[])
00009 {
00010     // Parse command line arguments
00011     options.add_options()("d,debug", "Enable debugging")("i,input_file", "Input file path",
    cxxopts::value<std::string>())("o,output_file", "Output file path",

```



```

        cxxopts::value<std::string>())("v,verbose", "Verbose output",
        cxxopts::value<bool>()->default_value("false"));
00012     auto result = options.parse(argc, argv);
00013
00014     std::cout << "Hello World"
00015               << "\n";
00016 }

```

7.9 variadic.cpp

```

00001 #include "boost/multi_array.hpp"
00002 #include "boost/array.hpp"
00003 #include "CustomLibraries/np.hpp"
00004 #include <cassert>
00005 #include <iostream>
00006
00007 void test_gradient()
00008 {
00009     // Create a 4D array that is 3 x 4 x 2 x 1
00010     typedef boost::multi_array<double, 4>::index index;
00011     boost::multi_array<double, 4> A(boost::extents[3][4][2][2]);
00012
00013     // Assign values to the elements
00014     int values = 0;
00015     for (index i = 0; i != 3; ++i)
00016         for (index j = 0; j != 4; ++j)
00017             for (index k = 0; k != 2; ++k)
00018                 for (index l = 0; l != 2; ++l)
00019                     A[i][j][k][l] = values++;
00020
00021     // Verify values
00022     int verify = 0;
00023     for (index i = 0; i != 3; ++i)
00024         for (index j = 0; j != 4; ++j)
00025             for (index k = 0; k != 2; ++k)
00026                 for (index l = 0; l != 2; ++l)
00027                     assert(A[i][j][k][l] == verify++);
00028
00029     double dx, dy, dz, dt;
00030     dx = 1.0;
00031     dy = 1.0;
00032     dz = 1.0;
00033     dt = 1.0;
00034     std::vector<boost::multi_array<double, 4> my_arrays = np::gradient(A, {dx, dy, dz, dt});
00035
00036     boost::multi_array<double, 1> x = np::linspace(0, 1, 5);
00037     std::vector<boost::multi_array<double, 1> gradf = np::gradient(x, {1.0});
00038     for (int i = 0; i < 5; i++)
00039     {
00040         std::cout << gradf[0][i] << ",";
00041     }
00042     std::cout << "\n";
00043     // np::print(std::cout, my_arrays[0]);
00044 }
00045
00046 void test_meshgrid()
00047 {
00048     boost::multi_array<double, 1> x = np::linspace(0, 1, 5);
00049     boost::multi_array<double, 1> y = np::linspace(0, 1, 5);
00050     boost::multi_array<double, 1> z = np::linspace(0, 1, 5);
00051     boost::multi_array<double, 1> t = np::linspace(0, 1, 5);
00052     const boost::multi_array<double, 1> axis[4] = {x, y, z, t};
00053     std::vector<boost::multi_array<double, 4> my_arrays = np::meshgrid(axis, false, np::xy);
00054     // np::print(std::cout, my_arrays[0]);
00055     int nx = 3;
00056     int ny = 2;
00057     boost::multi_array<double, 1> x2 = np::linspace(0, 1, nx);
00058     boost::multi_array<double, 1> y2 = np::linspace(0, 1, ny);
00059     const boost::multi_array<double, 1> axis2[2] = {x2, y2};
00060     std::vector<boost::multi_array<double, 2> my_arrays2 = np::meshgrid(axis2, false, np::xy);
00061     std::cout << "xv\n";
00062     for (int i = 0; i < ny; i++)
00063     {
00064         for (int j = 0; j < nx; j++)
00065         {
00066             std::cout << my_arrays2[0][i][j] << " ";
00067         }
00068         std::cout << "\n";
00069     }
00070     std::cout << "yv\n";
00071     for (int i = 0; i < ny; i++)
00072     {
00073         for (int j = 0; j < nx; j++)

```

```

00074         {
00075             std::cout << my_arrays2[1][i][j] << " ";
00076         }
00077         std::cout << "\n";
00078     }
00079 }
00080
00081 void test_complex_operations()
00082 {
00083     int nx = 3;
00084     int ny = 2;
00085     boost::multi_array<double, 1> x = np::linspace(0, 1, nx);
00086     boost::multi_array<double, 1> y = np::linspace(0, 1, ny);
00087     const boost::multi_array<double, 1> axis[2] = {x, y};
00088     std::vector<boost::multi_array<double, 2> my_arrays = np::meshgrid(axis, false, np::xy);
00089     boost::multi_array<double, 2> A = np::sqrt(my_arrays[0]);
00090     std::cout << "sqrt\n";
00091     for (int i = 0; i < ny; i++)
00092     {
00093         for (int j = 0; j < nx; j++)
00094         {
00095             std::cout << A[i][j] << " ";
00096         }
00097         std::cout << "\n";
00098     }
00099     std::cout << "\n";
00100     float a = 100.0;
00101     float sq_a = np::sqrt(a);
00102     std::cout << "sqrt of " << a << " is " << sq_a << "\n";
00103     std::cout << "exp\n";
00104     boost::multi_array<double, 2> B = np::exp(my_arrays[0]);
00105     for (int i = 0; i < ny; i++)
00106     {
00107         for (int j = 0; j < nx; j++)
00108         {
00109             std::cout << B[i][j] << " ";
00110         }
00111         std::cout << "\n";
00112     }
00113
00114     std::cout << "Power\n";
00115     boost::multi_array<double, 1> x2 = np::linspace(1, 3, nx);
00116     boost::multi_array<double, 1> y2 = np::linspace(1, 3, ny);
00117     const boost::multi_array<double, 1> axis2[2] = {x2, y2};
00118     std::vector<boost::multi_array<double, 2> my_arrays2 = np::meshgrid(axis2, false, np::xy);
00119     boost::multi_array<double, 2> C = np::pow(my_arrays2[1], 2.0);
00120     for (int i = 0; i < ny; i++)
00121     {
00122         for (int j = 0; j < nx; j++)
00123         {
00124             std::cout << C[i][j] << " ";
00125         }
00126         std::cout << "\n";
00127     }
00128 }
00129
00130 void test_equal()
00131 {
00132     boost::multi_array<double, 1> x = np::linspace(0, 1, 5);
00133     boost::multi_array<double, 1> y = np::linspace(0, 1, 5);
00134     boost::multi_array<double, 1> z = np::linspace(0, 1, 5);
00135     boost::multi_array<double, 1> t = np::linspace(0, 1, 5);
00136     const boost::multi_array<double, 1> axis[4] = {x, y, z, t};
00137     std::vector<boost::multi_array<double, 4> my_arrays = np::meshgrid(axis, false, np::xy);
00138     boost::multi_array<double, 1> x2 = np::linspace(0, 1, 5);
00139     boost::multi_array<double, 1> y2 = np::linspace(0, 1, 5);
00140     boost::multi_array<double, 1> z2 = np::linspace(0, 1, 5);
00141     boost::multi_array<double, 1> t2 = np::linspace(0, 1, 5);
00142     const boost::multi_array<double, 1> axis2[4] = {x2, y2, z2, t2};
00143     std::vector<boost::multi_array<double, 4> my_arrays2 = np::meshgrid(axis2, false, np::xy);
00144     std::cout << "equality test:\n";
00145     std::cout << (bool)(my_arrays == my_arrays2) << "\n";
00146 }
00147 void test_basic_operations()
00148 {
00149     int nx = 3;
00150     int ny = 2;
00151     boost::multi_array<double, 1> x = np::linspace(0, 1, nx);
00152     boost::multi_array<double, 1> y = np::linspace(0, 1, ny);
00153     const boost::multi_array<double, 1> axis[2] = {x, y};
00154     std::vector<boost::multi_array<double, 2> my_arrays = np::meshgrid(axis, false, np::xy);
00155
00156     std::cout << "basic operations:\n";
00157
00158     std::cout << "addition:\n";
00159     boost::multi_array<double, 2> A = my_arrays[0] + my_arrays[1];
00160

```

```

00161     for (int i = 0; i < ny; i++)
00162     {
00163         for (int j = 0; j < nx; j++)
00164         {
00165             std::cout << A[i][j] << " ";
00166         }
00167         std::cout << "\n";
00168     }
00169
00170     std::cout << "multiplication:\n";
00171     boost::multi_array<double, 2> B = my_arrays[0] * my_arrays[1];
00172
00173     for (int i = 0; i < ny; i++)
00174     {
00175         for (int j = 0; j < nx; j++)
00176         {
00177             std::cout << B[i][j] << " ";
00178         }
00179         std::cout << "\n";
00180     }
00181     double coeff = 3;
00182     boost::multi_array<double, 1> t = np::linspace(0, 1, nx);
00183     boost::multi_array<double, 1> t_time_3 = coeff * t;
00184     boost::multi_array<double, 1> t_time_2 = 2.0 * t;
00185     std::cout << "t_time_3: ";
00186     for (int j = 0; j < nx; j++)
00187     {
00188         std::cout << t_time_3[j] << " ";
00189     }
00190     std::cout << "\n";
00191     std::cout << "t_time_2: ";
00192     for (int j = 0; j < nx; j++)
00193     {
00194         std::cout << t_time_2[j] << " ";
00195     }
00196     std::cout << "\n";
00197 }
00198
00199 void test_zeros()
00200 {
00201     int nx = 3;
00202     int ny = 2;
00203     int dimensions[] = {ny, nx};
00204     boost::multi_array<double, 2> A = np::zeros<double>(dimensions);
00205     std::cout << "zeros:\n";
00206     for (int i = 0; i < ny; i++)
00207     {
00208         for (int j = 0; j < nx; j++)
00209         {
00210             std::cout << A[i][j] << " ";
00211         }
00212         std::cout << "\n";
00213     }
00214 }
00215
00216 void test_min_max()
00217 {
00218     int nx = 24;
00219     int ny = 5;
00220     boost::multi_array<double, 1> x = np::linspace(0, 10, nx);
00221     boost::multi_array<double, 1> y = np::linspace(-1, 1, ny);
00222     const boost::multi_array<double, 1> axis[2] = {x, y};
00223     std::vector<boost::multi_array<double, 2> my_array = np::meshgrid(axis, false, np::xy);
00224     std::cout << "min: " << np::min(my_array[0]) << "\n";
00225     std::cout << "max: " << np::max(my_array[1]) << "\n";
00226     std::cout << "max simple: " << np::max(1.0, 2.0, 3.0, 4.0, 5.0) << "\n";
00227     std::cout << "min simple: " << np::min(1, -2, 3, -4, 5) << "\n";
00228 }
00229
00230 int main()
00231 {
00232     test_gradient();
00233     test_meshgrid();
00234     test_complex_operations();
00235     test_equal();
00236     test_basic_operations();
00237     test_zeros();
00238     test_min_max();
00239 }

```

