Assignment 2

Date . .

1. Ans:

(a) There is a car such that all cars are as fast as it.

(b) For every car, there is a car that is as expensive as it.

(c) There is a car such that all cars are as fast as it and as old as it.

(d) For every car, there is a car that is not as expensive as it then is as old as it.

2. Ans:

(a) $\exists q, \forall p \; A(p,q)$

(b) $\forall q, \exists p \; B(p,q)$

(c) $\exists q, \forall p \; [A(p,q) \land \neg B(p,q)]$

(d) $\forall q, \exists p \; [C(p,q) \land \neg A(p,q)]$

3. Ans:

(a) There is a boy ~~that~~ who is bad.

(b) All bats that weigh less than 50 lbs.

(c) There is a real number x that makes the equation $x^2 - 2x \leq 0$.

No.

Date . .

(d) There is a parent who doesn't have to change diapers.

(e) There is a flying saucer whose aim is not to conquer any galaxy.

(f) n^2 is not a perfect number holds for all integers n.

(g) For every house in Kansas, there is one who enters any houses doesn't go blind.

(h) There is a house that all doors are not white.

(i) All of people in New York City ~~doesn't~~ don't own any book published in 1990.

4. Ans:

There is a real number $\epsilon > 0$, for every positive integer k such that there exists a positive integer n, it is the case that $|a_n - k^2| \geq \epsilon$.

5. Ans:

(a) Let n be an integer. The contrapositive of the statement is: If n is not even, then n^2 is not even.

Direct Proof: n is not even, so it can be expressed as $n = 2k+1$ where k is an integer. $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ which is also not even. So if n is not even, then n^2 is not even. It follows that if n^2 is even, then n is even.

(b) Let x and y be real number. The contrapositive is: If x is rational and y is rational, then xy is rational.

Direct Proof: x and y are rational, so $x = \frac{a}{b}$, $y = \frac{c}{d}$ with $a, b, c, d \in \mathbb{Z}$, $b, d \neq 0$. Then $xy = \frac{ac}{bd}$ where $ac, bd \in \mathbb{Z}$, $bd \neq 0$. So xy is rational. The contrapositive has been proved. It follows that if xy is irrational, then x is irrational and y is irrational.

6. Ans:

(a) A relation is called an equivalence relation if reflexive, symmetric and transive.

(1) Reflexive: Because $f: X \to Y$ is a function, $\forall x \in X$ we can get $f(x) = f(x)$. So $x R x$ ($x \sim x$).

(2) Symmetric: If $x_1 \sim x_2$ then $f(x_1) = f(x_2)$, then $f(x_2) = f(x_1)$ then $x_2 \sim x_1$ for $\forall x_1, x_2 \in X$.

(3) Transitive: If $x_1 \sim x_2$ and $x_2 \sim x_3$, then $f(x_1) = f(x_2)$ and $f(x_2) = f(x_3)$, then $f(x_1) = f(x_3)$, the $x_1 \sim x_3$ for $\forall x_1, x_2, x_3 \in X$.

Therefore, the relation $x_1 \sim x_2$ iff $f(x_1) = f(x_2)$ is an equivalence relation.

(b) the set of equivalence classes:

$X / \sim = \{[x] \mid x \in X\}$ can be partitioned into the equivalence class $[\cancel{x}] = \{x \in R \mid f(x) = \sin(x)\}$

(c) $\tilde{f}$ is not unique but one-to-one

Because there is other sets can satisfy, so not unique

Elements in $[x]$ satisfy $\forall x, x' \in X$, if $f(x) = f(x')$ then $x = x'$, so one-to-one.