Assignment 1

Due Wednesday 9/4/19

Reading Assignment:

- Required: Course Notes 1.1-1.3
- Recommended: PAF 1.1-2.6

Problems:

1. (PAF: 1.2.1) (1 pt each) Which of the following expressions are statements?
   (a) Today is a nice day.
   (b) Go to sleep.
   (c) Is it going to snow tomorrow?
   (d) The U.S. has 49 states.
   (e) I like to eat fruit, and you often think about traveling to Spain.
   (f) If we go out tonight, the babysitter will be unhappy.
   (g) Call me on Thursday if you are home.

2. (PAF: 1.2.5) (1 pt each) Let X = "Fred has red hair," let Y = "Fred has a big nose" and R = "Fred likes to eat figs." Translate the following statements into symbols.
   (a) Fred does not like to eat figs.
   (b) Fred has red hair, and does not have a big nose.
   (c) Fred has red hair or he likes to eat figs.
   (d) Fred likes to eat figs, and he has red hair or he has a big nose.
   (e) Fred likes to eat figs and he has red hair, or he has a big nose.
   (f) It is not the case that Fred has a big nose or he has red hair.
   (g) It is not the case that Fred has a big nose, or he has red hair.
   (h) Fred has a big nose and red hair, or he has a big nose and likes to eat figs.

3. (PAF: 1.2.6) (1 pt each) Let E = "The house is blue," let F = "The house is 30 years old" and G = "The house is ugly." Translate the following statements into symbols.
   (a) If the house is 30 years old, then it is ugly.
   (b) If the house is blue, then it is ugly or it is 30 years old.
   (c) If the house is blue, then it is ugly, or it is 30 years old.
   (d) The house is not ugly if and only if it is 30 years old.

(e) The house is 30 years old if it is blue, and it is not ugly if it is 30 years old.

(f) For the house to be ugly, it is necessary and sufficient that it be ugly and 30 years old.

4. (PAF: 1.5.13) (5 pt each) Which of the following statements is a tautology, which is a contradiction and which is neither?

   (a) $P \vee (\neg P \wedge Q)$.

   (b) $(X \vee Y) \leftrightarrow (\neg X \rightarrow Y)$.

   (c) $(A \wedge \neg B) \wedge (\neg A \vee B)$.

   (d) $(Z \vee (\neg Z \vee W)) \wedge \neg(W \wedge U)$.

   (e) $(L \rightarrow (M \rightarrow N)) \rightarrow (M \rightarrow (L \rightarrow N))$.

   (f) $((X \leftrightarrow Z) \wedge (X \leftrightarrow Y)) \wedge X$.

   (g) $((P \leftrightarrow \neg Q) \wedge P) \wedge Q$.

5. (PAF: 1.3.12) (5 pt each) Simplify the following statements (making use of any equivalences of statements given so far in the text or exercies).

   (a) $\neg(P \rightarrow \neg Q)$

   (b) $A \rightarrow (A \wedge B)$

   (c) $(X \wedge Y) \rightarrow X$

   (d) $\neg(M \vee L) \wedge L$

   (e) $(P \rightarrow Q) \vee Q$

   (f) $\neg(X \rightarrow Y) \vee Y$

6. (PAF: 1.5.1) (3 pt each) Suppose that the possible values of x are all people. Let $Y(x) =$ "x has green hair," let $Z(x) =$ "x likes pickles" and $W(x) =$ "x has a pet frog." Translate the following statements into words.

   (a) $(\forall x)Y(x)$.

   (b) $(\exists x)Z(x)$.

   (c) $(\forall x)\,[W(x) \wedge Z(x)]$.

   (d) $(\exists x)\,[Y(x) \rightarrow W(x)]$.

   (e) $(\forall x)\,[W(x) \leftrightarrow \neg Z(x)]$.

Optional Problems:

1. (PAF: 1.2.2) Which of the following expressions are statements? (Assume $w, x, y, z, a, b, c \in \mathbb{R}$)

   (a) $4 < 3$.

   (b) If $x \geq 2$ then $x^3 \geq 1$.

(c) $y < 7$.

(d) $x + y = z$.

(e) $(a + b)^2 = a^2 + 2ab + b^2$.

(f) $a^2 + b^2 = c^2$.

(g) If $w = 3$ then $z^w \neq 0$.

2. (PAF: 1.2.4) Let $X =$ "I am happy," let $Y =$ "I am watching a movie" and $Z =$ "I am eating spaghetti." Translate the following statements into words.

(a) $Z \to X$.

(b) $X \leftrightarrow Y$.

(c) $(Y \vee Z) \to X$.

(d) $Y \vee (Z \to X)$.

(e) $(Y \to \neg X) \wedge (Z \to \neg X)$.

(f) $(X \wedge \neg Y) \leftrightarrow (Y \vee Z)$.

3. (PAF: 1.2.11) Make a truth table for each of the following statements.

(a) $P \wedge \neg Q$.

(b) $(R \vee S) \wedge \neg R$.

(c) $X \vee (\neg Y \vee Z)$.

(d) $(A \vee B) \wedge (A \vee C)$.

(e) $(P \wedge R) \vee \neg(Q \wedge S)$.