

Assignment 1

Due Wednesday 9/4/19

Reading Assignment:

- Required: Course Notes 1.1-1.3
- Recommended: PAF 1.1-2.6

Problems:

1. (PAF: 1.2.1) (1 pt each) Which of the following expressions are statements?

- (a) Today is a nice day.
- (b) Go to sleep.
- (c) Is it going to snow tomorrow?
- (d) The U.S. has 49 states.
- (e) I like to eat fruit, and you often think about traveling to Spain.
- (f) If we go out tonight, the babysitter will be unhappy.
- (g) Call me on Thursday if you are home.

Solution: Among these sentences, four are statements: Today is a nice day. The U.S. has 49 states. I like to eat fruit, and you often think about traveling to Spain. If we go out tonight, the babysitter will be unhappy.

2. (PAF: 1.2.5) (1 pt each) Let X = “Fred has red hair,” let Y = “Fred has a big nose” and R = “Fred likes to eat figs.” Translate the following statements into symbols.

- (a) Fred does not like to eat figs.

Solution: $\neg R$.

- (b) Fred has red hair, and does not have a big nose.

Solution: $X \wedge \neg Y$.

- (c) Fred has red hair or he likes to eat figs.

Solution: $X \vee R$.

- (d) Fred likes to eat figs, and he has red hair or he has a big nose.

Solution: $R \wedge (X \vee Y)$.

- (e) Fred likes to eat figs and he has red hair, or he has a big nose.

Solution: $(R \wedge X) \vee Y$.

- (f) It is not the case that Fred has a big nose or he has red hair.

Solution: $\neg(Y \vee X)$.

(g) It is not the case that Fred has a big nose, or he has red hair.

Solution: $\neg Y \vee X$.

(h) Fred has a big nose and red hair, or he has a big nose and likes to eat figs.

Solution: $(Y \wedge X) \vee (Y \wedge R)$.

3. (PAF: 1.2.6) (1 pt each) Let E = “The house is blue,” let F = “The house is 30 years old” and G = “The house is ugly.” Translate the following statements into symbols.

(a) If the house is 30 years old, then it is ugly. **Solution:** $F \rightarrow G$.

(b) If the house is blue, then it is ugly or it is 30 years old. **Solution:** $E \rightarrow (G \vee F)$.

(c) If the house is blue, then it is ugly, or it is 30 years old. **Solution:** $(E \rightarrow G) \vee F$.

(d) The house is not ugly if and only if it is 30 years old. **Solution:** $\neg G \leftrightarrow F$.

(e) The house is 30 years old if it is blue, and it is not ugly if it is 30 years old. **Solution:** $(E \rightarrow F) \wedge (F \rightarrow \neg G)$.

(f) For the house to be ugly, it is necessary and sufficient that it be ugly and 30 years old. **Solution:** $G \leftrightarrow (G \wedge F)$

4. (PAF: 1.5.13) (5 pt each) Which of the following statements is a tautology, which is a contradiction and which is neither?

(a) $P \vee (\neg P \wedge Q)$.

Solution: This is neither a tautology or a contradiction,

P	Q	$P \vee (\neg P \wedge Q)$				
T	T	T	T	F	F	T
T	F	T	T	F	F	F
F	T	F	T	T	T	T
F	F	F	F	T	F	F
		1	5	2	4	3

(b) $(X \vee Y) \leftrightarrow (\neg X \rightarrow Y)$.

Solution: This is a tautology,

X	Y	$(X \vee Y) \leftrightarrow (\neg X \rightarrow Y)$						
T	T	T	T	T	T	F	T	T
T	F	T	T	F	T	F	T	F
F	T	F	T	T	T	T	T	T
F	F	F	F	F	T	T	F	F
		1	5	2	7	3	6	4

(c) $(A \wedge \neg B) \wedge (\neg A \vee B)$.

Solution: This is a contradiction,

A	B	$(A \wedge \neg B) \wedge (\neg A \vee B)$							
T	T	T	F	F	F	F	T	T	
T	F	T	T	T	F	F	F	F	
F	T	F	F	F	F	T	T	T	
F	F	F	F	T	F	T	T	F	
		1	5	2	7	3	6	4	

(d) $(Z \vee (\neg Z \vee W)) \wedge \neg(W \wedge U)$.

Solution: This is neither a tautology or a contradiction,

U	W	Z	$(Z \vee (\neg Z \vee W)) \wedge \neg (W \wedge U)$									
T	T	T	T	T	F	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T	F	F	T	T	T
T	F	T	T	T	F	F	F	T	T	F	F	T
T	F	F	F	T	T	T	F	T	T	F	F	T
F	T	T	T	T	F	T	T	T	T	T	F	F
F	T	F	F	T	T	T	T	T	T	T	F	F
F	F	T	T	T	F	F	F	T	T	F	F	F
F	F	F	F	T	T	T	F	T	T	F	F	F
			1	5	2	4	3	10	9	6	8	7

(e) $(L \rightarrow (M \rightarrow N)) \rightarrow (M \rightarrow (L \rightarrow N))$.

Solution: This is a tautology,

L	M	N	$(L \rightarrow (M \rightarrow N)) \rightarrow (M \rightarrow (L \rightarrow N))$										
T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F	T	T	F	T	F	F
T	F	T	T	T	F	T	T	T	F	T	T	T	T
T	F	F	T	T	F	T	F	T	F	T	T	F	F
F	T	T	F	T	T	T	T	T	T	T	F	T	T
F	T	F	F	T	T	F	F	T	T	T	F	T	F
F	F	T	F	T	F	T	T	T	F	T	F	T	T
F	F	F	F	T	F	T	F	T	F	T	F	T	F
			1	5	2	4	3	11	6	10	7	9	8

(f) $((X \leftrightarrow Z) \wedge (X \leftrightarrow Y)) \wedge X$.

Solution: This is neither a tautology or a contradiction,

X	Y	Z	$((X \leftrightarrow Z) \wedge (X \leftrightarrow Y)) \wedge X$								
T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T	T	T	F	T
T	F	T	T	T	T	F	T	F	F	F	T
T	F	F	T	F	F	F	T	F	F	F	T
F	T	T	F	F	T	F	F	F	T	F	F
F	T	F	F	T	F	F	F	F	T	F	F
F	F	T	F	F	T	F	F	T	F	F	F
F	F	F	F	T	F	T	F	T	F	F	F
			1	6	2	8	3	7	4	9	5

(g) $((P \leftrightarrow \neg Q) \wedge P) \wedge Q$.

Solution: This is a contradiction,

P	Q	$((P \leftrightarrow \neg Q) \wedge P) \wedge Q$							
T	T	T	F	F	F	T	F	T	
T	F	T	T	T	T	T	F	F	
F	T	F	T	F	F	F	F	T	
F	F	F	F	T	F	F	F	F	
		1	5	2	6	3	7	4	

5. (PAF: 1.3.12) (5 pt each) Simplify the following statements (making use of any equivalences of statements given so far in the text or exercises).

(a) $\neg(P \rightarrow \neg Q)$ **Solution:** $\Leftrightarrow P \wedge Q$.

(b) $A \rightarrow (A \wedge B)$ **Solution:** $\Leftrightarrow \neg A \vee (A \wedge B) \Leftrightarrow (\neg A \vee A) \wedge (\neg A \vee B) \Leftrightarrow \neg A \vee B$.

(c) $(X \wedge Y) \rightarrow X$ **Solution:** $\Leftrightarrow \neg(X \wedge Y) \vee X \Leftrightarrow (\neg X \vee \neg Y) \vee X \Leftrightarrow (\neg X \vee X)$.

(d) $\neg(M \vee L) \wedge L$ **Solution:** $\Leftrightarrow (\neg M \wedge \neg L) \wedge L \Leftrightarrow (L \wedge \neg L)$.

(e) $(P \rightarrow Q) \vee Q$ **Solution:** $\Leftrightarrow (\neg P \vee Q) \vee Q \Leftrightarrow \neg P \vee Q$.

(f) $\neg(X \rightarrow Y) \vee Y$ **Solution:** $\Leftrightarrow (X \wedge \neg Y) \vee Y \Leftrightarrow (X \vee Y) \wedge (\neg Y \vee Y) \Leftrightarrow X \vee Y$.

6. (PAF: 1.5.1) (3 pt each) Suppose that the possible values of x are all people. Let $Y(x)$ = “ x has green hair,” let $Z(x)$ = “ x likes pickles” and $W(x)$ = “ x has a pet frog.” Translate the following statements into words.

(a) $(\forall x)Y(x)$. **Solution:**

All people have green hair.

(b) $(\exists x)Z(x)$. **Solution:**

There exists a person who likes pickles.

(c) $(\forall x)[W(x) \wedge Z(x)]$. **Solution:**

Every person has a pet frog and likes pickles.

(d) $(\exists x)[Y(x) \rightarrow W(x)]$. **Solution:**

There is a person such that if he has green hair then he has a frog.

(e) $(\forall x)[W(x) \leftrightarrow \neg Z(x)]$. **Solution:**

Every person has a pet frog if and only if he does not like pickles.

Practice Problems (do not hand in):

1. (PAF: 1.2.2) Which of the following expressions are statements? (Assume $w, x, y, z, a, b, c \in \mathbb{R}$)

(a) $4 < 3$.

(b) If $x \geq 2$ then $x^3 \geq 1$.

(c) $y < 7$.

(d) $x + y = z$.

(e) $(a + b)^2 = a^2 + 2ab + b^2$.

(f) $a^2 + b^2 = c^2$.

(g) If $w = 3$ then $z^w \neq 0$.

Solution: The first two expressions are statements (e.g., the first is false and the second is true). All the remaining expressions have free variables and, unfortunately, this means that the answer requires a very precise definition of a statement. A strict reading would say that these expressions require quantifiers to be statements (e.g., $y < 7$ is not true or false without knowledge of y). A loose reading would say that these are all statements that depend on free variables.

The answer is even murkier for $(a + b)^2 = a^2 + 2ab + b^2$, which is true for all $a, b \in \mathbb{R}$. No matter what quantifiers are chosen, the quantified expression is a true statement. But, without knowing this, how should one interpret the statement? Again, the best answer is that this is a statement that depends on free variables.

2. (PAF: 1.2.4) Let $X =$ “I am happy,” let $Y =$ “I am watching a movie” and $Z =$ “I am eating spaghetti.” Translate the following statements into words.

(a) $Z \rightarrow X$.

Solution: If I am eating spaghetti then I am happy.

(b) $X \leftrightarrow Y$.

Solution: I am happy if and only if I am watching a movie.

(c) $(Y \vee Z) \rightarrow X$.

Solution: If I am watching a movie or eating spaghetti then I am happy.

(d) $Y \vee (Z \rightarrow X)$.

Solution: I am watching a movie or if I am eating spaghetti then I am happy.

(e) $(Y \rightarrow \neg X) \wedge (Z \rightarrow \neg X)$.

Solution: If I am watching a movie then I am not happy, and if I am eating spaghetti then I am not happy.

(f) $(X \wedge \neg Y) \leftrightarrow (Y \vee Z)$.

Solution: I am happy and I am not watching a movie if and only if I am watching a movie or eating spaghetti.

3. (PAF: 1.2.11) Make a truth table for each of the following statements.

(a) $P \wedge \neg Q$. **Solution:**

P	Q	P	\wedge	$\neg P$
T	T	T	F	F
T	F	T	T	T
F	T	F	F	F
F	F	F	F	T
		1	3	2

(b) $(R \vee S) \wedge \neg R$. **Solution:**

R	S	$(R$	\vee	$S)$	\wedge	$\neg R$
T	T	T	T	T	F	F
T	F	T	T	F	F	F
F	T	F	T	T	T	T
F	F	F	F	F	F	T
		1	3	2	5	4

(c) $X \vee (\neg Y \vee Z)$. **Solution:**

X	Y	Z	X	\vee	$(\neg Y$	\wedge	$Z)$
T	T	T	T	T	F	T	T
T	T	F	T	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	F
F	T	T	F	T	F	T	T
F	T	F	F	F	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	F
			1	5	2	4	3

(d) $(A \vee B) \wedge (A \vee C)$. **Solution:**

A	B	C	$(A \vee B) \wedge (A \vee C)$		
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	F	T
F	F	F	F	F	F
			1	5	2

(e) $(P \wedge R) \vee \neg(Q \wedge S)$. **Solution:**

P	Q	R	S	$(P \wedge R) \vee \neg(Q \wedge S)$		
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	T	F	T	T	F
T	F	F	T	T	F	T
T	F	F	F	T	F	F
F	T	T	T	F	F	T
F	T	T	F	F	F	F
F	T	F	T	F	F	T
F	T	F	F	F	F	F
F	F	T	T	F	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	T
F	F	F	F	F	T	F
				1	5	2