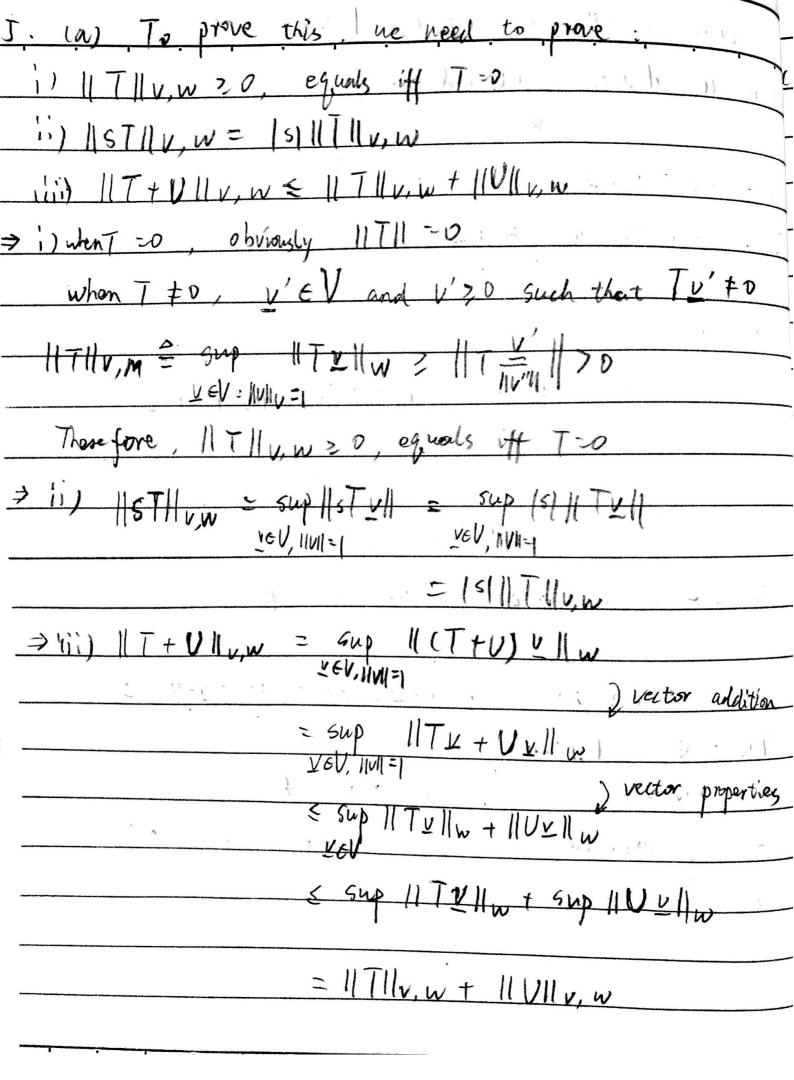


Date
Also, d(x17/2) = /(x1-X1) = /(x2-X1) = d(x,x1)
The induced distance meet all the three properties, so it's a metric
b) when $ X_1 < X_2 $
$ x_1 - x_2 = x_2 - x_1 = x_2 + x_1 - x_1 - x_1 \le x_2 - x_1 + x_1 - x_1 $
$= \ x_{\nu} - x_{1}\ $
when 1/22/21/21/
$ x_1 - x_2 = x_1 - x_2 = x_1 - x_2 + x_1 - x_1 \le x_1 - x_2 + x_4 - x_4 $
= X1-X2
= 11- X2 - X111
Therefore 11x1+11x11 < /1x1-x11
10) f= 11.11 : X -> R f(7) = 11 x 11
For 4 € 70, if d(x1, x2) = 11x1-1211< €
11×111- xx \le x1-Xx = d(x1, xx) < & from quegtion (b)
3870, re assume 8= E
So d(x111, xx11) = d(f(x1), f(x1)) < E = 8
Therefore, the norm : : X -> R is a continuous function.



b) From question (a) ne get 11711v, w = sup 11711w 2 11711v1
Then ITYIIW < ITTIV, WILVIN
Replace w by v, then 1/TXIIv = 1/TIIv, v 1/4/1V
Since U: V > V, then U v EV, he can replace V by UV
Then take induced operator
Sup 17 TUUIL & Sup 11 TILV, V 11 UVILV VOV, 1VII=1 VOV, 11M1=1
« Sup 11 T/1 v. v /1 V/1 v, v /1 v /1 v / =)
= Sup (TT/v, v - sup (1 U/l v, v - veV, 11 vi)=1
Therefore, the induced operator norm 11:11 v, v is submuttiplicative
b. Ans: Since 11.1143 a matrix norm satisfying the submittidia
11 TU \(\int 1) T
comtropositive: it I-F is singular, then 1/ F/1/21
It I-F is singular, there is a vector to such that (I-F)x=0
Then $\hat{I}X = \underline{x} = f\underline{x}$
11×11 = 11 F × 11 € 11 F 11 11 1
11F1121
Therefore, if 11.11 satisfys submultiplicative property and 11.51/11.
then I-F is non-singular