Assignment 2

Due Wednesday 9/11/19

Reading:

- Required: Course Notes 1.4-1.5
- Recommended: PAF 3.1-5.3

Problems:

1. (PAF: 1.5.2) (1 pt each) Suppose that the possible values of x and y are all cars. Let $L(x, y)$ = "x is as fast as y," let $M(x, y)$ = "x is as expensive as y" and $N(x, y)$ = "x is as old as y." Translate the following statements into words.

   (a) $(\exists x)(\forall y)L(x, y)$.

   (b) $(\forall x)(\exists y)M(x, y)$.

   (c) $(\exists y)(\forall x)[L(x, y) \vee N(x, y)]$.

   (d) $(\forall y)(\exists x)[\neg M(x, y) \rightarrow L(x, y)]$.

2. (PAF: 1.5.4) (1 pt each) Suppose that the possible values of p and q are all fruit. Let $A(p, q)$ = "p tastes better than q," let $B(p, q)$ = "p is riper than q" and $C(p, q)$ = "p is the same species as q." Translate the following into symbols.

   (a) There is a fruit such that all fruit taste better than it.

   (b) For every fruit, there is a fruit that is riper than it.

   (c) There is a fruit such that all fruit taste better than it and are not riper than it.

   (d) For every fruit, there is a fruit of the same species that does not taste better than it.

3. (PAF: 1.5.6) (1 pt each) Write a negation of each statement. Do not write the word "not" applied to any of the objects being quantified (for example, do not write "Not all boys are good" for the first part).

   (a) All boys are good.

   (b) There are bats that weigh 50 lbs. or more.

   (c) The equation $x^2 - 2x > 0$ holds for all real numbers x.

   (d) Every parent has to change diapers.

   (e) Every flying saucer is aiming to conquer some galaxy.

   (f) There is an integer n such that n^2 is a perfect number.

   (g) There is a house in Kansas such that every one who enters the house goes blind.

   (h) Every house has a door that is white.

(i) At least one person in New York City owns every book published in 1990.

4. (PAF: 1.5.8) (5 pts) Negate the following statement: For every real number $\epsilon > 0$ there exists a positive integer k such that for all positive integers n, it is the case that $|a_n - k^2| < \epsilon$.

5. (PAF: 2.3.2*)(5 pts each) Prove the following by giving direct proofs for their contrapositives.

   (a) Let n be an integer. If n^2 is even, then n is even.

   (b) Let x and y be real numbers. If xy is irrational, then x is irrational or y is irrational.

6. (EF: 1.4.1) (5 pts each) Let X, Y be sets and $f : X \to Y$ be a function.

   (a) Show that the relation $x_1 \sim x_2$ iff $f(x_1) = f(x_2)$ is an equivalence relation.

   (b) Describe the set of equivalence classes $X\backslash\sim$ for $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(x)$.

   (c) Consider any $\widetilde{f} : X\backslash\sim \to Y$ satisfying $\widetilde{f}([x]) = f(x)$ for $x \in X$. Is $\widetilde{f}$ unique? one-to-one?

Practice Problems (do not hand in):

1. (PAF: 1.5.3) Suppose that the possible values of x are all cows. Let $P(x)$ = "x is brown," let $Q(x)$ = "x is four years old" and $R(x)$ = "x has white spots." Translate the following statements into symbols.

   (a) There is a brown cow.

   (b) All cows are four years old.

   (c) There is a brown cow with white spots.

   (d) All four year old cows have white spots.

   (e) There exists a cow such that if it is four years old, then it has no white spots.

   (f) All cows are brown if and only if they are not four years old.

   (g) There are no brown cows.

2. (TOP: 1.2.1) The image of a function applied to a set-valued argument is defined by $f(A) \triangleq \{f(x) | x \in A\}$ and $f^{-1}(B) \triangleq \{x \in X | f(x) \in B\}$. Let $f : X \to Y$, $A \subseteq X$, and $B \subseteq Y$.

   (a) Show that $f^{-1}(f(A)) \supseteq A$ and that equality holds if f is injective.

   (b) Show that $f(f^{-1}(B)) \subseteq B$ and that equality holds if f is surjective.

3. (PAF: 2.2.4) Let n be an integer. Find a direct proof that shows: if n is even then n^2 is even, and if n is odd then n^2 is odd.

4. (PAF: 2.2.5) Let n and m be integers. Assume that n and m are divisible by 3.

   (a) Find a direct proof that shows $n + m$ is divisible by 3

   (b) Find a direct proof that shows nm is divisible by 3

5. (PAF: 2.2.6) Let a, b, c, m and n be integers. Find a direct proof to show that if $a|b$ and $a|c$, then $a|(bm + cn)$.