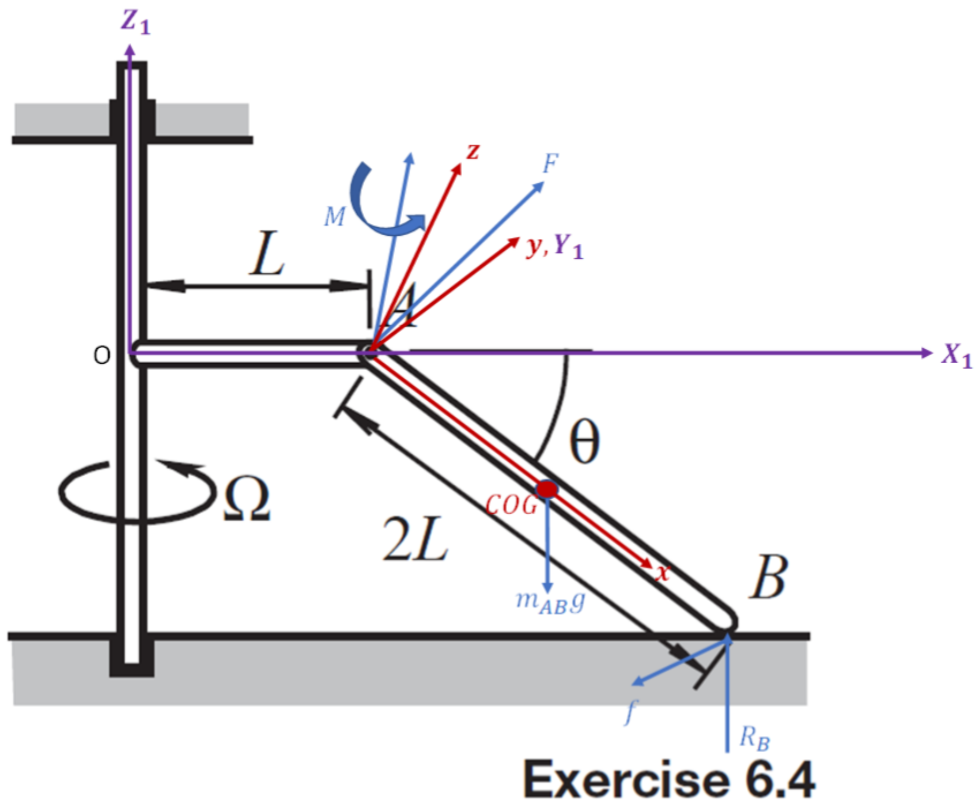
Question 1



XYZ is the moving reference frame that has an origin at point O.
xyz is the body fixed reference frame of bar AB
COG is the center of gravity, $m_{AB}\, g$
$X_1\, Y_1\, Z_1 \rightarrow R_Y(\theta) \rightarrow x\, y\, z$ (coordinate transformation is needed)
f is the friction caused by R_B (contact reaction force) is assumed that it acts perpendicular to the vertical plane depicted in the sketch which means the -y direction (given by the question.)
F_A is the reaction force at point A
M_A is the reaction moment at point A (does not have a y component

since it can move around y.

In[ ]:=
```
Quit[]
```

First I will define xyz as the Global Coordinate System. Then, I will compute XYZ with inverse transformation.

xyz → $R_Y^T(\theta) \rightarrow X_1\, Y_1\, Z_1$

In[ ]:=
```
i = {1, 0, 0}; j = {0, 1, 0}; k = {0, 0, 1}; (* define xyz as the GCS *)
```
$$\{I1, J1, K1\} = \begin{pmatrix} Cos[\theta] & 0 & -Sin[\theta] \\ 0 & 1 & 0 \\ Sin[\theta] & 0 & Cos[\theta] \end{pmatrix}^{T}.\{i, j, k\};$$
```
(* inverse rotational transformation to get I1J1K1 in unit vectors ijk *)

Print[MatrixForm[{"I1", "J1", "K1"}], " = ", MatrixForm[{I1, J1, K1}]]
```

$$\begin{pmatrix} I1 \\ J1 \\ K1 \end{pmatrix} = \begin{pmatrix} Cos[\theta] & 0 & Sin[\theta] \\ 0 & 1 & 0 \\ -Sin[\theta] & 0 & Cos[\theta] \end{pmatrix}$$

In[ ]:=
```
FCOG = -mab g K1; (* weight *)
F = Fx i + Fy j + Fz k; (* reaction force at point A*)
M = Mx i + Mz k; (* couple at point A*)
RB1 = RB K1; (* ground reaction force *)
f = -μ RB j; (*friction. -y direction is given by the question *)

Print[MatrixForm["FCOG"], " = ", MatrixForm[FCOG]]
Print[MatrixForm["F"], " = ", MatrixForm[F]]
Print[MatrixForm["RB1"], " = ", MatrixForm[RB1]]
Print[MatrixForm["f"], " = ", MatrixForm[f]]
```

$$FCOG = \begin{pmatrix} g\, mab\, Sin[\theta] \\ 0 \\ -g\, mab\, Cos[\theta] \end{pmatrix}$$

$$F = \begin{pmatrix} Fx \\ Fy \\ Fz \end{pmatrix}$$

$$RB1 = \begin{pmatrix} -RB\, Sin[\theta] \\ 0 \\ RB\, Cos[\theta] \end{pmatrix}$$

$$f = \begin{pmatrix} 0 \\ -RB\, \mu \\ 0 \end{pmatrix}$$

Angular velocity and accelerations is given as;

In[ ]:=
```
Ω' = 0; θ' = 0; θ'' = 0;
Ω1 = Ω K1;
Print[MatrixForm["Ω1"], " = ", MatrixForm[Ω1]]
```

$$\Omega 1 = \begin{pmatrix} -\Omega\, Sin[\theta] \\ 0 \\ \Omega\, Cos[\theta] \end{pmatrix}$$

Kinematics calculation

In[ ]:=

```
ω = Ω K1 + θ' j; (* overall ω *)
α = Ω' K1 + Ω1 × (Ω K1) + θ'' j + ω × (θ' j);

Print[MatrixForm["ω"], " = ", MatrixForm[ω]]
Print[MatrixForm["α"], " = ", MatrixForm[α]]

(*Velocity and Acceleration of  center of mass*)
```

$$\omega = \begin{pmatrix} -\Omega \operatorname{Sin}[\theta] \\ 0 \\ \Omega \operatorname{Cos}[\theta] \end{pmatrix}$$

$$\alpha = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

In[ ]:=

```
rOG = L I1 + L i;
VGrel = 0;
VO = O;
aO = 0;
vOG =  VO + VGrel +  ω × rOG;
aOG = aO + α × rOG + ω × (ω × rOG);

Print[MatrixForm["rOG"], " = ", MatrixForm[rOG]]
Print[MatrixForm["vOG"], " = ", MatrixForm[vOG]]
```

$$rOG = \begin{pmatrix} L + L \operatorname{Cos}[\theta] \\ 0 \\ L \operatorname{Sin}[\theta] \end{pmatrix}$$

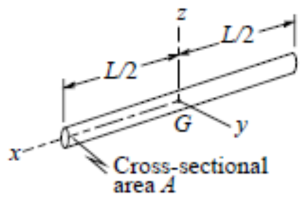
$$vOG = \begin{pmatrix} O \\ O + L\,\Omega \operatorname{Cos}[\theta] + L\,\Omega \operatorname{Cos}[\theta]^2 + L\,\Omega \operatorname{Sin}[\theta]^2 \\ O \end{pmatrix}$$

In[ ]:=

```
(* Inertia matrix of bar AB according to appendix (SLENDER BAR) *)
```

CENTROIDAL INERTIA PROPERTIES*

Slender bar



$$m = \rho A L,$$

$$I_{xx} \approx 0, \quad I_{yy} = I_{zz} = \frac{1}{12} m L^2.$$

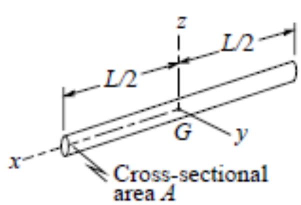
$$\text{IAB} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}\,\text{mab}\,(2\,L)^2 & 0 \\ 0 & 0 & \frac{1}{12}\,\text{mab}\,(2\,L)^2 \end{pmatrix};$$

```
(*xyz is attached to the body. Then, the moment equation can be given as;*)
HCOG = IAB.ω;
Print[MatrixForm["HCOG"], " = ", MatrixForm[HCOG]]
```

Out[ ]=

CENTROIDAL INERTIA PROPERTIES*

Slender bar



$$m = \rho A L,$$

$$I_{xx} \approx 0, \quad I_{yy} = I_{zz} = \frac{1}{12} m L^2.$$

$$\text{HCOG} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{3}\,L^2\,\text{mab}\,\Omega\,\text{Cos}[\theta] \end{pmatrix}$$

In[ ]:=
```
rACOG = -L i; rBCOG = L i;
Ftotal = F + FCOG + RB1 + f == mab aOG // Simplify

Mtotal = M + rACOG × F + rBCOG × (RB1 + f) == IAB.α + ω × HCOG // Simplify
```

Out[ ]= $\{Fx + 2\,L\,mab\,\Omega^2 \cos[\frac{\theta}{2}]^2 \cos[\theta] + (g\,mab - RB)\,\sin[\theta],\ Fy - RB\,\mu,\ Fz + (-g\,mab + RB)\cos[\theta] + L\,mab\,\Omega^2\,(1 + \cos[\theta])\,\sin[\theta]\} == \{0, 0, 0\}$

Out[ ]= $\{Mx,\ L\,(Fz - RB\cos[\theta]),\ -Fy\,L + Mz - L\,RB\,\mu\} == \{0,\ \frac{1}{3} L^2\,mab\,\Omega^2 \cos[\theta]\,\sin[\theta],\ 0\}$

In[ ]:= $\{Mx,\ L\,(Fz - RB\cos[\theta]),\ -Fy\,L + Mz - L\,RB\,\mu\} == \{0,\ \frac{1}{3} L^2\,mab\,\Omega^2 \cos[\theta]\,\sin[\theta],\ 0\}$

```
(* 6 equations 6 unknowns *)
```

Out[ ]= $\{Mx,\ L\,(Fz - RB\cos[\theta]),\ -Fy\,L + Mz - L\,RB\,\mu\} == \{0,\ \frac{1}{3} L^2\,mab\,\Omega^2 \cos[\theta]\,\sin[\theta],\ 0\}$

In[ ]:=
```
solvefor = Solve[{Ftotal, Mtotal}, {Fx, Fy, Fz, RB, Mx, Mz}] // FullSimplify // Flatten
```

Out[ ]= $\{Fx \to -\frac{1}{6} mab\,(3\,L\,\Omega^2 \cos[\theta] + L\,\Omega^2\,(5 + \cos[2\theta] + 3\sec[\theta]) + 3\,g\,\sin[\theta]),$

$Fy \to \frac{1}{6} mab\,\mu\,(3\,g - L\,\Omega^2\,(3 + 4\cos[\theta])\tan[\theta]),$

$Fz \to \frac{1}{6}(3\,g\,mab\cos[\theta] - L\,mab\,\Omega^2\,(3\sin[\theta] + \sin[2\theta])),$

$RB \to \frac{1}{6} mab\,(3\,g - L\,\Omega^2\,(3 + 4\cos[\theta])\tan[\theta]),\ Mx \to 0,$

$Mz \to -\frac{1}{3} L\,mab\,\mu\,(-3\,g + L\,\Omega^2\,(3 + 4\cos[\theta])\tan[\theta])\}$

In[ ]:=
```
Print["Force at pin A:\n F = ", MatrixForm[F] /. solvefor]
Print["Moment at pin A:\n M = ", MatrixForm[M] /. solvefor]
Print["Ground reaction force:\n R = ", RB /. solvefor]
```

Force at pin A:

$$F = \begin{pmatrix} -\frac{1}{6} mab\,(3\,L\,\Omega^2 \cos[\theta] + L\,\Omega^2\,(5 + \cos[2\theta] + 3\sec[\theta]) + 3\,g\sin[\theta]) \\ \frac{1}{6} mab\,\mu\,(3\,g - L\,\Omega^2\,(3 + 4\cos[\theta])\tan[\theta]) \\ \frac{1}{6}(3\,g\,mab\cos[\theta] - L\,mab\,\Omega^2\,(3\sin[\theta] + \sin[2\theta])) \end{pmatrix}$$

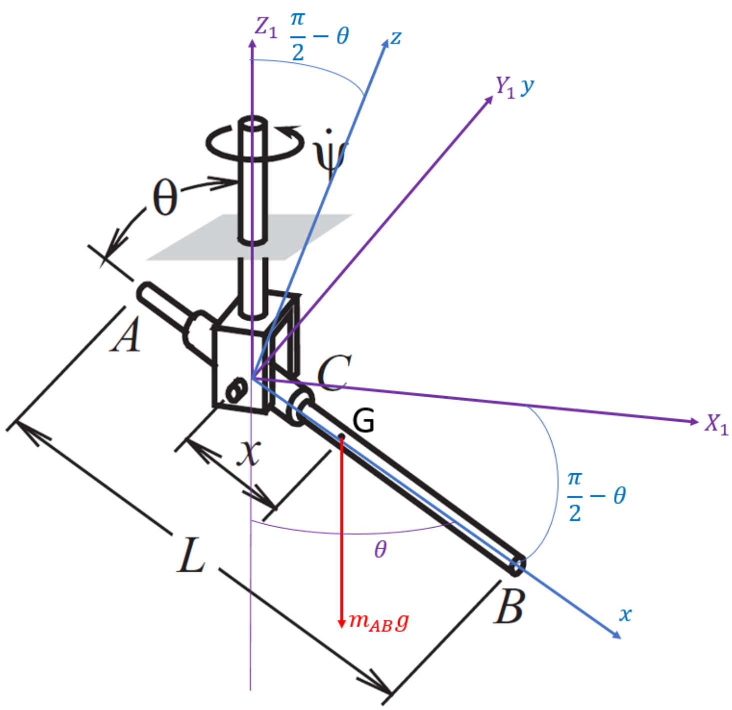
Moment at pin A:

$$M = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{3} L\,mab\,\mu\,(-3\,g + L\,\Omega^2\,(3 + 4\cos[\theta])\tan[\theta]) \end{pmatrix}$$

Ground reaction force:

$$R = \frac{1}{6} mab\,(3\,g - L\,\Omega^2\,(3 + 4\cos[\theta])\tan[\theta])$$

Question 2



XYZ is the moving reference frame (moving with $\dot{\psi}$ with the vertical shaft)
Y is aligned with the pin.
To get xyz, we rotate XYZ about Y by angle $\frac{\pi}{2}$-θ
$X_1\, Y_1\, Z_1 \rightarrow\ R_Y(\theta) \rightarrow x\; y\; z$ (coordinate transformation is needed)
First I will define xyz as the Global Coordinate System. Then, I will compute XYZ with inverse transformation.
xyz $\rightarrow\ R_Y^T(\theta) \rightarrow X_1\, Y_1\, Z_1$
I did not show F and M reactions in the figure but of course, they will be calculated.

```
In[ ]:= Quit[]
```

In[ ]:=
```
i = {1, 0, 0}; j = {0, 1, 0}; k = {0, 0, 1};
```

$$\{I1, J1, K1\} = \begin{pmatrix} \mathrm{Cos}\left[\frac{\pi}{2}-\theta\right] & 0 & -\mathrm{Sin}\left[\frac{\pi}{2}-\theta\right] \\ 0 & 1 & 0 \\ \mathrm{Sin}\left[\frac{\pi}{2}-\theta\right] & 0 & \mathrm{Cos}\left[\frac{\pi}{2}-\theta\right] \end{pmatrix}^{T}.\{i, j, k\};$$

```
Print[MatrixForm[{"I1", "J1", "K1"}], " = ", MatrixForm[{I1, J1, K1}]]
```

$$\begin{pmatrix} I1 \\ J1 \\ K1 \end{pmatrix} = \begin{pmatrix} \mathrm{Sin}[\theta] & 0 & \mathrm{Cos}[\theta] \\ 0 & 1 & 0 \\ -\mathrm{Cos}[\theta] & 0 & \mathrm{Sin}[\theta] \end{pmatrix}$$

Question says bar AB can slide in the collar. So reaction force at pin C (F) does not have an X component.

Couple has no Y component since it is aligned with the axis of the pin.

I put an X component to couple since the question says “it may be assumed that bar does not spin about its own axis.”

In[ ]:=
```
WCOG = -mAB g K1;
F = Fy j + Fz k;
M = Mz k + Mx i;
ψ'' = 0;
Ω = ψ' K1;
ω = ψ' K1 - θ' j;
α = ψ'' k1 + Ω × (ψ' K1) - θ'' j - ω × (θ' j);
(*  Position, velocity and acceleration of
 center of gravity (G) relative to xyz coordinate system *)
RGCrel = x i;
VGCrel = x' i;
AGCrel = x'' i;
AC = 0;
VC = 0;
(* Point C does not move. aC=0 and vC=0. *)
VG = VC + VGCrel + ω × (RGCrel);
AG = AC + AGCrel + α × RGCrel + 2 ω × VGCrel + ω × (ω × RGCrel);

Print[MatrixForm["Ω"], " = ", MatrixForm[Ω]]
Print[MatrixForm["ω"], " = ", MatrixForm[ω]]
Print[MatrixForm["α"], " = ", MatrixForm[α]]
Print[MatrixForm["VG"], " = ", MatrixForm[VG]]
Print[MatrixForm["AG"], " = ", MatrixForm[AG]]
```

$$\Omega = \begin{pmatrix} -\text{Cos}[\theta]\ \psi' \\ 0 \\ \text{Sin}[\theta]\ \psi' \end{pmatrix}$$

$$\omega = \begin{pmatrix} -\text{Cos}[\theta]\ \psi' \\ -\theta' \\ \text{Sin}[\theta]\ \psi' \end{pmatrix}$$

$$\alpha = \begin{pmatrix} \text{Sin}[\theta]\ \theta'\ \psi' \\ -\theta'' \\ \text{Cos}[\theta]\ \theta'\ \psi' \end{pmatrix}$$

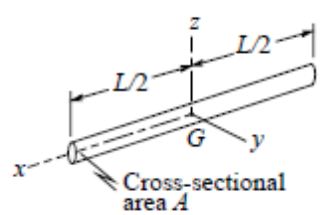
$$\text{VG} = \begin{pmatrix} x' \\ x\,\text{Sin}[\theta]\ \psi' \\ x\,\theta' \end{pmatrix}$$

$$\text{AG} = \begin{pmatrix} -x\ (\theta')^2 - x\,\text{Sin}[\theta]^2\ (\psi')^2 + x'' \\ 2\,\text{Sin}[\theta]\ x'\ \psi' + 2\,x\,\text{Cos}[\theta]\ \theta'\ \psi' \\ 2\,x'\ \theta' - x\,\text{Cos}[\theta]\ \text{Sin}[\theta]\ (\psi')^2 + x\,\theta'' \end{pmatrix}$$

```
(* Inertia Properties from Appendix *)
```

CENTROIDAL INERTIA PROPERTIES*

Slender bar



$$m = \rho A L,$$

$$I_{xx} \approx 0, \quad I_{yy} = I_{zz} = \frac{1}{12} m L^2.$$

In[ ]:=

$$\textbf{IAB} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}\,\text{mAB}\,L^2 & 0 \\ 0 & 0 & \frac{1}{12}\,\text{mAB}\,L^2 \end{pmatrix};$$

In[ ]:=

```
(*xyz is attached to the body. Then, the moment equation can be simplified as; *)
HG = IAB.ω;
Print[MatrixForm["HG"], " = ", MatrixForm[HG]]
```

$$\text{HG} = \begin{pmatrix} 0 \\ -\frac{1}{12}\,L^2\,\text{mAB}\,\theta' \\ \frac{1}{12}\,L^2\,\text{mAB}\,\text{Sin}[\theta]\ \psi' \end{pmatrix}$$

In[•]:=
```
rCG = -x i;
Ftotal = F + WCOG == mAB AG;
Mtotal = M + rCG × F == IAB.α + ω × HG;

Print[MatrixForm["Ftotal"], " = ", MatrixForm[Ftotal]]
Print[MatrixForm["Mtotal"], " = ", MatrixForm[Mtotal]]
```

Ftotal = $\{g\,mAB\,\mathrm{Cos}[\theta],\ Fy,\ Fz - g\,mAB\,\mathrm{Sin}[\theta]\} == \{mAB\,(-x\,(\theta')^2 - x\,\mathrm{Sin}[\theta]^2\,(\psi')^2 + x''),$
$mAB\,(2\,\mathrm{Sin}[\theta]\,x'\,\psi' + 2\,x\,\mathrm{Cos}[\theta]\,\theta'\,\psi'),\ mAB\,(2\,x'\,\theta' - x\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,(\psi')^2 + x\,\theta'')\}$

Mtotal = $\{Mx,\ Fz\,x,\ Mz - Fy\,x\} == \left\{0,\ \frac{1}{12}L^2\,mAB\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,(\psi')^2 - \frac{1}{12}L^2\,mAB\,\theta'',\ \frac{1}{6}L^2\,mAB\,\mathrm{Cos}[\theta]\,\theta'\,\psi'\right\}$

```
(* 6 equations 6 unknowns *)
solvefor2 = Solve[{Ftotal, Mtotal}, {Fy, Fz, Mx, Mz, x'', θ''}] // Flatten // FullSimplify
```

Out[•]=
$\left\{Fy \to 2\,mAB\,(\mathrm{Sin}[\theta]\,x' + x\,\mathrm{Cos}[\theta]\,\theta')\,\psi',\ Fz \to \frac{L^2\,mAB\,(g\,\mathrm{Sin}[\theta] + 2\,x'\,\theta')}{L^2 + 12\,x^2},\right.$
$Mx \to 0,\ Mz \to \frac{1}{6}\,mAB\,(12\,x\,\mathrm{Sin}[\theta]\,x' + (L^2 + 12\,x^2)\,\mathrm{Cos}[\theta]\,\theta')\,\psi',$
$\left.x'' \to g\,\mathrm{Cos}[\theta] + x\,(\theta')^2 + x\,\mathrm{Sin}[\theta]^2\,(\psi')^2,\ \theta'' \to -\frac{12\,x\,(g\,\mathrm{Sin}[\theta] + 2\,x'\,\theta')}{L^2 + 12\,x^2} + \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,(\psi')^2\right\}$

In[•]:=
```
Print["x'' and θ'' :\n
 x'' = ", x'' /. solvefor2, "\n θ'' = ", θ'' /. solvefor2]
Print["Force Reaction at pin C: F = ", MatrixForm[F /. solvefor2]]
Print["Couple Reaction at pin C: M = ", MatrixForm[M /. solvefor2]]
```

x'' and θ'' :

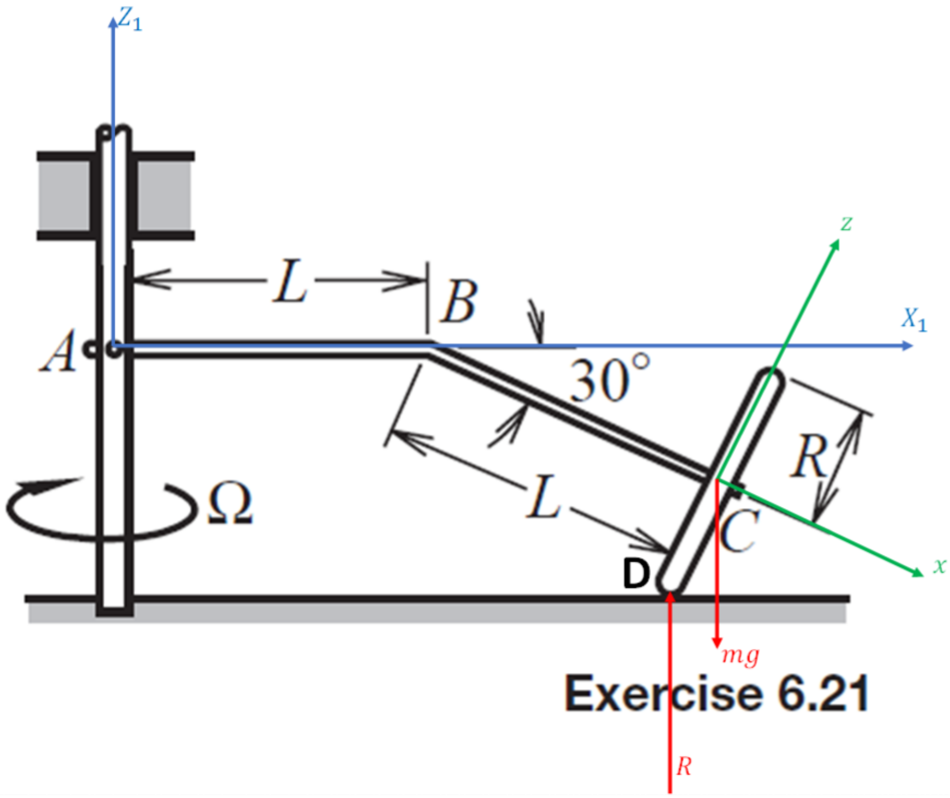
$x'' = g\,\mathrm{Cos}[\theta] + x\,(\theta')^2 + x\,\mathrm{Sin}[\theta]^2\,(\psi')^2$

$\theta'' = -\frac{12\,x\,(g\,\mathrm{Sin}[\theta] + 2\,x'\,\theta')}{L^2 + 12\,x^2} + \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,(\psi')^2$

Force Reaction at pin C: F = $\begin{pmatrix} 0 \\ 2\,mAB\,(\mathrm{Sin}[\theta]\,x' + x\,\mathrm{Cos}[\theta]\,\theta')\,\psi' \\ \frac{L^2\,mAB\,(g\,\mathrm{Sin}[\theta]+2\,x'\,\theta')}{L^2+12\,x^2} \end{pmatrix}$

Couple Reaction at pin C: M = $\begin{pmatrix} 0 \\ 0 \\ \frac{1}{6}\,mAB\,(12\,x\,\mathrm{Sin}[\theta]\,x' + (L^2 + 12\,x^2)\,\mathrm{Cos}[\theta]\,\theta')\,\psi' \end{pmatrix}$

Question 3



Exercise 6.21

$X_1\,Y_1\,Z_1$ is moving reference frame has origin at Point A and xyz is the body fixed coordinate system. $X_1\,Y_1\,Z_1 \rightarrow R_Y(\theta) \rightarrow x\,y\,z$

xyz will be selected as GCS and xyz $\rightarrow R_Y^T(\theta) \rightarrow X_1\,Y_1\,Z_1$

θ=30°

X_1 axis is aligned with horizontal axis.

I did not show F and M reactions in the figure but of course, they will be calculated.

In[ ]:= `Quit[]`

```
i = {1, 0, 0}; j = {0, 1, 0}; k = {0, 0, 1}; (* define xyz as the GCS *)
{I1, J1, K1} = ( Cos[θ]  0  -Sin[θ] )ᵀ
               (   0     1     0    )  .{i, j, k};
               ( Sin[θ]  0   Cos[θ] )
(*inverse rotation to get I1J1K1 in terms if GCS *)

Print[MatrixForm[{"I1", "J1", "K1"}], " = ", MatrixForm[{I1, J1, K1}]]
```

$$\begin{pmatrix} I1 \\ J1 \\ K1 \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}[\theta] & 0 & \mathrm{Sin}[\theta] \\ 0 & 1 & 0 \\ -\mathrm{Sin}[\theta] & 0 & \mathrm{Cos}[\theta] \end{pmatrix}$$

Force and couple reactions at pin A should be written.

Couple has no J1 component since it is aligned with the pin axis.

Ground reaction force R should only have a K1 component (perpendicular to the ground)

Mass of the shafts are assumed to be zero.

```
WC = -m g K1;
F = Fx I1 + Fy J1 + Fz K1;
M = Mx I1 + Mz K1;
R1 = R K1;

Ω' = 0;
```

Angular velocity of disk (ωc) is not known. We should apply no slip condition. VD=0 VC1=VC2

```
In[]:= Ω1 = -Ω K1;
ω = Ω1 + ωc i;
ωc' = 0;
α = -Ω' K1 - Ω1 × (Ω K1) + ωc' i + ω × (ωc i);

rCA = L I1 + L i;
rAC = -rCA;
(* Point A is not moving *)
vC1 = Ω1 × rCA // Simplify;
aC = Ω1 × (Ω1 × rCA) // Simplify;

Print[MatrixForm["ω"], " = ", MatrixForm[ω]]
Print[MatrixForm["α"], " = ", MatrixForm[α]]
Print[MatrixForm["vC1"], " = ", MatrixForm[vC1]]
Print[MatrixForm["aC"], " = ", MatrixForm[aC]]
```

$$\omega = \begin{pmatrix} \omega c + \Omega \,\mathrm{Sin}[\theta] \\ 0 \\ -\Omega \,\mathrm{Cos}[\theta] \end{pmatrix}$$

$$\alpha = \begin{pmatrix} 0 \\ -\Omega\, \omega c \,\mathrm{Cos}[\theta] \\ 0 \end{pmatrix}$$

$$\mathrm{vC1} = \begin{pmatrix} 0 \\ -L\, \Omega\, (1 + \mathrm{Cos}[\theta]) \\ 0 \end{pmatrix}$$

$$\mathrm{aC} = \begin{pmatrix} -2\, L\, \Omega^2 \,\mathrm{Cos}\left[\frac{\theta}{2}\right]^2 \mathrm{Cos}[\theta] \\ 0 \\ -L\, \Omega^2\, (1 + \mathrm{Cos}[\theta])\, \mathrm{Sin}[\theta] \end{pmatrix}$$

In[∘]:=
```
vD = 0;
rCD = r k (* r is the radius of the disk *)
rDC = -rCD;
vC2 = vD + ω × rCD;
Print[MatrixForm["vC2"], " = ", MatrixForm[vC2]]
```

Out[∘]= {0, 0, r}

$$\mathrm{vC2} = \begin{pmatrix} 0 \\ -r\, \omega c - r\, \Omega \,\mathrm{Sin}[\theta] \\ 0 \end{pmatrix}$$

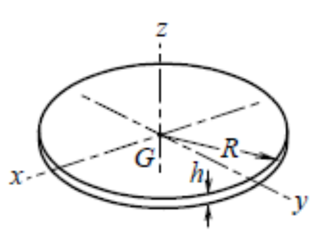
In[∘]:=
```
solvefor = Solve[vC1 == vC2, ωc]

Print["ωdisk = ",
 ωdisk = ω /. solvefor // FullSimplify // Flatten ]
Print["αdisk = ",
 αdisk = α /. solvefor // FullSimplify // Flatten ]
```

Out[∘]= $\left\{\left\{\omega c \to \frac{L\, \Omega + L\, \Omega \,\mathrm{Cos}[\theta] - r\, \Omega \,\mathrm{Sin}[\theta]}{r}\right\}\right\}$

$$\omega\mathrm{disk} = \left\{\frac{L\, \Omega\, (1 + \mathrm{Cos}[\theta])}{r},\ 0,\ -\Omega \,\mathrm{Cos}[\theta]\right\}$$

$$\alpha\mathrm{disk} = \left\{0,\ -\frac{\Omega^2 \,\mathrm{Cos}[\theta]\, \left(L + L\, \mathrm{Cos}[\theta] - r\, \mathrm{Sin}[\theta]\right)}{r},\ 0\right\}$$

Inertia Properties can be written from Appendix



$$m = \pi \rho R^2 h,$$

$$I_{xx} = I_{yy} = \frac{1}{4} m R^2, \quad I_{zz} = \frac{1}{2} m R^2.$$

z axis is x axis for our selected unit vectors. R=r. Thus;

In[∘]:=
$$\text{II} = \begin{pmatrix} \frac{1}{2} m r^2 & 0 & 0 \\ 0 & \frac{1}{4} m r^2 & 0 \\ 0 & 0 & \frac{1}{4} m r^2 \end{pmatrix};$$

```
HC = II.ωdisk;
Print[MatrixForm["HC"], " = ", MatrixForm[HC]]

(*HC=II.ωd; since xyz is attached to the body. *)
```

$$\text{HC} = \begin{pmatrix} \frac{1}{2} L\, m\, r\, \Omega\, (1 + \text{Cos}[\theta]) \\ 0 \\ -\frac{1}{4} m\, r^2\, \Omega\, \text{Cos}[\theta] \end{pmatrix}$$

In[∘]:=
```
Print ["Force Equation is: "]
MatrixForm[ Ftotal = F + WC + R1 == m aC // FullSimplify]

Print["Moment Equation is:"]
MatrixForm[ Mtotal = M + rDC × R1 + rAC × F  == II.αdisk + ωdisk × HC // FullSimplify]
```

Force Equation is:

Out[∘]//MatrixForm=

$$\{\text{Cos}[\theta]\, (Fx + L\, m\, \Omega^2 + L\, m\, \Omega^2\, \text{Cos}[\theta]) - (Fz - g\, m + R)\, \text{Sin}[\theta],\ Fy,\ (Fz - g\, m + R)\, \text{Cos}[\theta] + Fx\, \text{Sin}[\theta] + L\, m\, \Omega^2\, (1 + \text{Cos}[\theta])\, \text{Sin}[\theta]\} == \{0, 0, 0\}$$

Moment Equation is:

$$\{Mx\, \text{Cos}[\theta] + (Fy\, L - Mz)\, \text{Sin}[\theta],\ Fz\, L\, (1 + \text{Cos}[\theta]) + (Fx\, L + r\, R)\, \text{Sin}[\theta],\ -Fy\, L + (-Fy\, L + Mz)\, \text{Cos}[\theta] + Mx\, \text{Sin}[\theta]\} == \left\{0,\ \frac{1}{4} m\, r\, \Omega^2\, \text{Cos}[\theta]\, (-2 L\, (1 + \text{Cos}[\theta]) + r\, \text{Sin}[\theta]),\ 0\right\}$$

```
(* 6 equations 6 unknowns *)
```

In[∘]:=
```
solvefor2 = Solve[{Ftotal, Mtotal}, {Fx, Fy, Fz, R, Mx, Mz}] // Flatten // FullSimplify
```

Out[∘]=

$$\left\{Fx \to -L\, m\, \Omega^2\, (1 + \text{Cos}[\theta]),\ Fy \to 0,\ Fz \to \frac{-2 L\, m\, r\, \Omega^2\, \text{Cos}[\theta]\, (1 + \text{Cos}[\theta]) + m\, (-4 g\, r + 4 L^2\, \Omega^2 + (4 L^2 + r^2)\, \Omega^2\, \text{Cos}[\theta])\, \text{Sin}[\theta]}{4\, (L + L\, \text{Cos}[\theta] - r\, \text{Sin}[\theta])},\ R \to \frac{2 L\, m\, (1 + \text{Cos}[\theta])\, (2 g + r\, \Omega^2\, \text{Cos}[\theta]) - m\, \Omega^2\, (4 L^2 + (4 L^2 + r^2)\, \text{Cos}[\theta])\, \text{Sin}[\theta]}{4\, (L + L\, \text{Cos}[\theta] - r\, \text{Sin}[\theta])},\ Mx \to 0,\ Mz \to 0\right\}$$

```
Print["Ground reaction force:R = ", R /. solvefor2 // FullSimplify]
(* θ was givben as equal to π/6:*)
Print["Solve for θ = π/6: R1 =", R /. solvefor2 /. θ → π / 6 // FullSimplify]
```

Ground reaction force:R $= \dfrac{2\,L\,m\,(1+\mathrm{Cos}[\theta])\,\left(2\,g+r\,\Omega^2\,\mathrm{Cos}[\theta]\right)-m\,\Omega^2\,\left(4\,L^2+\left(4\,L^2+r^2\right)\,\mathrm{Cos}[\theta]\right)\,\mathrm{Sin}[\theta]}{4\,\left(L+L\,\mathrm{Cos}[\theta]-r\,\mathrm{Sin}[\theta]\right)}$

Solve for $\theta = \dfrac{\pi}{6}$: R1 $= \dfrac{m\,\left(8\,\left(2+\sqrt{3}\right)\,g\,L-\left(4\,\left(2+\sqrt{3}\right)\,L^2-2\,\left(3+2\,\sqrt{3}\right)\,L\,r+\sqrt{3}\,r^2\right)\,\Omega^2\right)}{8\,\left(2+\sqrt{3}\right)\,L-8\,r}$