Homework #3

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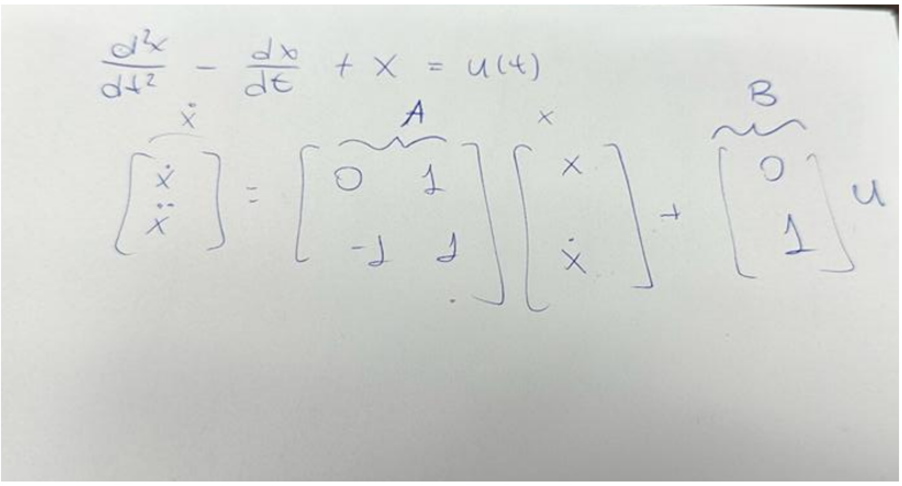
In[ ]:= `Quit[]`

Problem #3.15



$$\frac{d^2x}{dt^2} - \frac{dx}{dt} + x = u(t)$$

$$\overbrace{\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}}^{\dot{x}} = \overbrace{\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} x \\ \dot{x} \end{bmatrix}}^{x} + \overbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}^{B} u$$

In[ ]:= $A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix};$

$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix};$

In[ ]:= `Eigensystem[A]`

Out[ ]= $\left\{\left\{\frac{1}{2}\left(1 + i\sqrt{3}\right), \frac{1}{2}\left(1 - i\sqrt{3}\right)\right\}, \left\{\left\{-\frac{1}{2} i \left(i + \sqrt{3}\right), 1\right\}, \left\{\frac{1}{2} i \left(-i + \sqrt{3}\right), 1\right\}\right\}\right\}$

Approximate Poles (Open-Loop)

In[ ]:= `eig = Eigenvalues[A] // N`

Out[ ]= $\{0.5 + 0.866025\, i, 0.5 - 0.866025\, i\}$

Closed - loop poles (Optimal)

Since r -> ∞ , optimal closed loop poles are the reflection of open loop poles.
Because of that,

In[ ]:= `clpoles = -eig`

Out[ ]= $\{-0.5 - 0.866025\, i, -0.5 + 0.866025\, i\}$

Characteristic polynomial (Optimal):

In[ ]:= `α = (s - clpoles[[1]]) (s - clpoles[[2]])`

Out[ ]= $((0.5 - 0.866025\, i) + s)\,((0.5 + 0.866025\, i) + s)$

C-L GAINS

In[ ]:= `KK = ( k1  k2 );`

In[ ]:= `KK`

Out[ ]= `{{k1, k2}}`

In[ ]:=
```
II = (1 0
      0 1);
```

In[ ]:=
```
δ = Det[s II - (A - B.KK)]
```

Out[ ]= $1 + k1 - s + k2\, s + s^2$

In[ ]:=
```
ksolved = SolveAlways[δ == α, {s}] // Chop
```

Out[ ]= {{k1 → 0, k2 → 2.}}

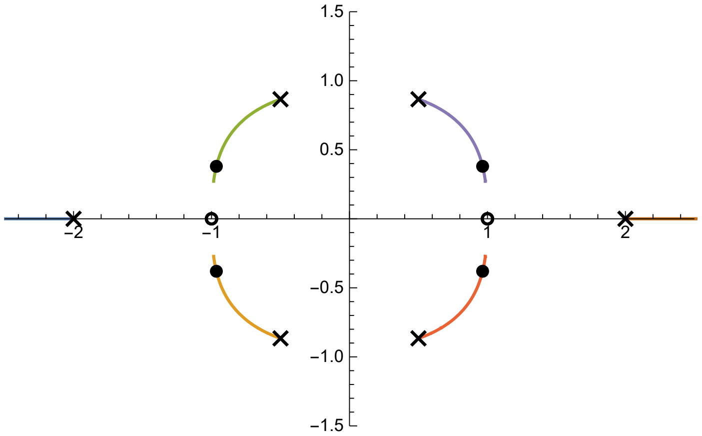
In[ ]:=
```
Koptimal = KK /. ksolved // Flatten
```

Out[ ]= {0, 2.}

Problem #3.17

In[54]:=
```
Tf[s_] := ((s + 1) (s - 1)) / ((s^2 - s + 1) (s + 2))
H[s_] := 1 / r ((Numerator[Tf[s]] × Numerator[Tf[-s]]) /
     (Denominator[Tf[s]] × Denominator[Tf[-s]]))
RootLocusPlot[H[s] /. r^-1 → k, {k, 0, 30}, PlotRange → {{-2.5, 2.5}, {-1.5, 1.5}}]
```

Out[56]=




OPEN-LOOP

In[11]:=
```
zerosOL = s /. Solve[Numerator[Tf[s]] == 0, s]
```

Out[11]= {-1, 1}

In[14]:=
```
polesOL = s /. Solve[Denominator[Tf[s]] == 0, s] // N
```

Out[14]= {-2., 0.5 + 0.866025 i, 0.5 - 0.866025 i}

b) CLOSED-LOOP (r->0)

Reflection: -1,-1

3rd pole --> -(infinity)

For non-zero (small) r value:

$b_0 = 1$

In[15]:=
```
Exponent[Denominator[Tf[s]], s] - Exponent[Numerator[Tf[s]], s]
```

n-m

Out[15]= 1

$$\left(b_0^2 r^{-1}\right)^{1/2(n-m)} = \left(1^2 \cdot \frac{1}{r}\right)^{\frac{1}{2.1}} = \sqrt{\frac{1}{r}}$$

c) CLOSED-LOOP (r→∞)

Reflection:

```
In[18]:= polesCL1 = -polesOL[[2]]
Out[18]= -0.5 - 0.866025 i

In[19]:= polesCL2 = -polesOL[[3]]
Out[19]= -0.5 + 0.866025 i

In[20]:= polesCL3 = polesOL[[1]]
Out[20]= -2.
```