16385-HW2

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Q2.1

1. There are 8 degrees of freedom in this case.
2. 4 pairs are required to solve h because the rank of the matrix is 8.
3. According to the problem, initially we have

$$\begin{bmatrix} x_1^i \\ y_1^i \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_2^i \\ y_2^i \\ 1 \end{bmatrix}$$

Multiplying the two matrices, we get

$$x_1^i = \alpha(h_1 x_2^i + h_2 y_2^i + h_3)(1)$$
$$y_1^i = \alpha(h_4 x_2^i + h_5 y_2^i + h_6)(2)$$
$$1 = \alpha(h_7 x_2^i + h_8 y_2^i + h_9)(3)$$

From (3), we know that (1) and (2) would still hold if we multiply $\alpha(h_7 x_2^i + h_8 y_2^i + h_9)$ to left side, after we devided out the scale factor, we will get

$$x_1^i(h_7 x_2^i + h_8 y_2^i + h_9) = h_1 x_2^i + h_2 y_2^i + h_3$$
$$y_1^i(h_7 x_2^i + h_8 y_2^i + h_9) = h_4 x_2^i + h_5 y_2^i + h_6$$

Rewrite the equation, we get

$$x_1^i h_7 x_2^i + x_1^i h_8 y_2^i + x_1^i h_9 - h_1 x_2^i - h_2 y_2^i - h_3 = 0$$
$$y_1^i h_7 x_2^i + y_1^i h_8 y_2^i + y_1^i h_9 - h_4 x_2^i - h_5 y_2^i - h_6 = 0$$

If we convert that into matrices, we get

$$\begin{bmatrix} -x_2^i & -y_2^i & -1 & 0 & 0 & 0 & x_1^i x_2^i & x_1^i y_2^i & x_1^i \\ 0 & 0 & 0 & -x_2^i & -y_2^i & -1 & y_1^i x_2^i & y_1^i y_2^i & y_1^i \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = 0$$

Which is $A_i h = 0$
Therefore,

$$A_i = \begin{bmatrix} -x_2^i & -y_2^i & -1 & 0 & 0 & 0 & x_1^i x_2^i & x_1^i y_2^i & x_1^i \\ 0 & 0 & 0 & -x_2^i & -y_2^i & -1 & y_1^i x_2^i & y_1^i y_2^i & y_1^i \end{bmatrix}$$

4. The trivial solution would be $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$. The rank would be 9.

Q3.1

Fast detector works by comparing pixels around a point in a circular way. Specifically, if the nearby pixels are significantly brighter or darker than the pixel at the center, the center would be identified as corner.

By comparison, Harris Corner Detector works by comparing the gradient of the intensity in the image. Specifically, the algorithm calculate the intensity gradients first, compute the eigenvalues through the covariance matrix, and then apply threshold to the response function generated by eigenvalues to identify corners.

In terms of complexity, Harris Corner Detector involves calculating eigenvalues, so it is computationally more expensive than Fast detector.
Harris corner detector have higher accuracy and provide more robust detection.

Q3.2

Filter banks works by using convolution to extract features such as edges and textures, while BRIEF compares the binary representation of pixel intensities. So they are different.

It is not common to use one of the filter banks as descriptor. This is because they are sensitive to the noises and orientations of the images. We often need additional steps to convert the continuous values produced by filter banks to discrete values in order for better feature matching.

Q3.3

The Hamming distance measures the similarity between two different binary strings of equal length by counting differing bits. In terms of application, nearest neighbor search is used to match interest points between two images, and the Hamming distance is employed to compare the similarities between interest points. Compared to the Euclidean distance, the Hamming distance works

better in BRIEF because BRIEF encodes data as binary strings, making it robust to noise and highly efficient for simple XOR operations. Additionally, it is easier to interpret the matching decisions.

Q3.4

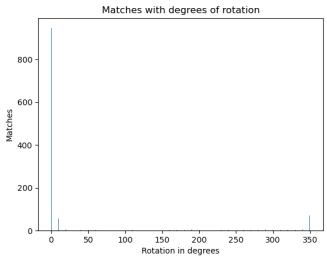

Figure 1: Matches and degrees of rotation

Q3.5

Above images are matches between book cover and the book cover with 10, 90 and 135 degrees of rotation. As we can see, BRIEF descriptor does not handle matches very well as rotation increases. I think it behaves this way because it matches by comparing binary representation of image intensities. When the image rotates, the intensities around the interest point would change, resulting in a worse matching.

3.9

The issue is our matrix "BestH2to1" only perform well if the picture has the same size as book cover and will not scale the image. Therefore, since the cover for Harry Potter cover does not have the same size as book cover, we have to resize it to the size of the cover of book and then apply homography.

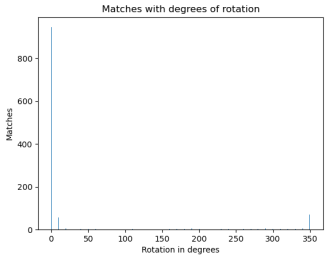

Figure 2: Matches and degrees of rotation

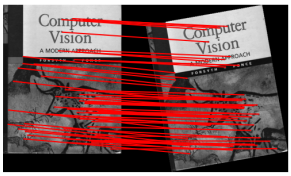

Figure 3: Rotate by 10 degrees

Figure 4: Rotate by 90 degrees

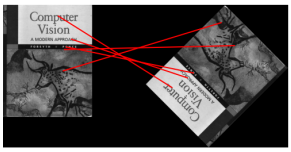

Figure 5: Rotate by 135 degrees

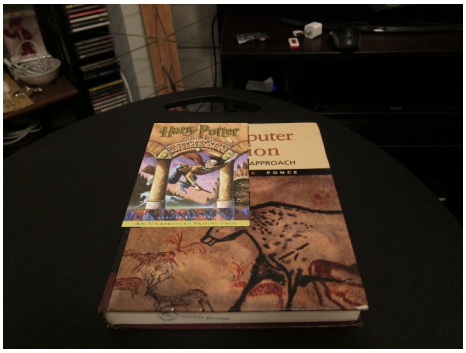

Figure 6: HarryPotter without resize

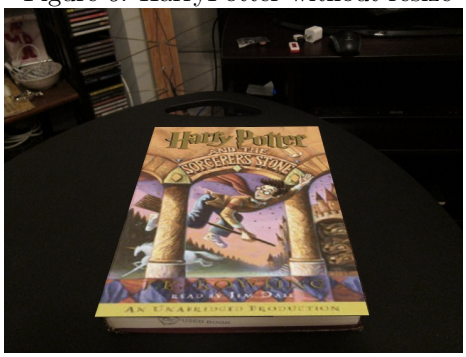

Figure 7: HarryPotter with resize