

# HASKELL

## 第六次课后作业

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### 1 第一题

**Solution 1** *map* 和 *(++)* 定义如下

```
map :: (a -> b) -> [a] -> [b]
map _ [] = []                                (map.1)
map f (x:xs) = f x : map f xs                (map.2)
(++) :: [a] -> [a] -> [a]
[] ++ y = y                                    (++.1)
(x:xs) ++ y = x : (xs ++ y)                  (++.2)
```

下面证明  $\forall xs, ys, zs \in \{[a]\}, (xs ++ ys) ++ zs = xs ++ (ys ++ zs)$

1. 当  $xs=[]$  时

$$\begin{aligned} \text{map } f (xs ++ ys) &= \text{map } f ([] ++ ys) \\ &= \text{map } f ys && (++.1) \\ &= [] ++ \text{map } f ys && (++.1) \\ &= \text{map } f [] ++ \text{map } f ys && (\text{map}.1) \\ &= \text{map } f xs ++ \text{map } f ys \end{aligned}$$

2. 当  $xs=(x:zs)$  时

$$\begin{aligned}
\text{map } f \ (xs ++ ys) &= \text{map } f \ ((x : zs) ++ ys) \\
&= \text{map } f \ (x : zs ++ ys) && (++) . 2 \\
&= f \ x : \text{map } f \ (zs ++ ys) && (\text{map}.2) \\
&= f \ x : (\text{map } f \ zs ++ \text{map } f \ ys) && (\text{迭代}) \\
&= (f \ x : \text{map } f \ zs) ++ \text{map } f \ ys && (++) . 2 \\
&= \text{map } f \ (x : zs) ++ \text{map } f \ ys && (\text{map}.2) \\
&= \text{map } f \ xs ++ \text{map } f \ ys
\end{aligned}$$

综上证得  $\forall xs, ys, zs \in \{[a]\}, (xs ++ ys) ++ zs = xs ++ (ys ++ zs)$ 。

## 2 第二题

fst, snd, zip, unzip 的定义如下

```

fst :: (a, b) -> a
fst (x, _) = x                                (fst.1)
snd :: (a, b) -> b
snd (_, y) = y                                (snd.1)
zip :: [a] -> [b] -> [(a, b)]
zip [] _ = []                                  (zip.1)
zip _ [] = []                                  (zip.2)
zip (x:xs) (y:ys) = (x,y) : zip xs ys         (zip.3)
unzip :: [(a, b)] -> ([a], [b])
unzip [] = ([], [])                            (unzip.1)
unzip ((x, y) : ps) = (x : xs, y : ys)         (unzip.2)
    where (xs, ys) = unzip ps

```

1.

**Solution 2** 下面证明对于所有有限列表  $ps$ ,  $\text{zip } (\text{fst } (\text{unzip } ps)) (\text{snd } (\text{unzip } ps)) = ps$  均成立

1. 当  $ps=[]$  时

$$\begin{aligned}
& zip (fst (unzip ps)) (snd (unzip ps)) \\
&= zip (fst ([])) (snd ([])) && (\text{unzip.1}) \\
&= zip [] [] && (\text{fst.1, snd.1}) \\
&= [] && (\text{zip.1}) \\
&= ps && (1)
\end{aligned}$$

2. 当  $ps=((x,y):zs)$  时

$$\begin{aligned}
& zip (fst (unzip ps)) (snd (unzip ps)) \\
&= zip (fst (x : xs, y : ys)) (snd (x : xs, y : ys)) && (\text{unzip.2}) \\
&= zip (x : xs) (y : ys) && (\text{fst.1, snd.1}) \\
&= (x, y) : zip xs ys && (\text{zip.3}) \\
&= (x, y) : zip (fst (xs, ys)) (snd (xs, ys)) && (\text{fst.1, snd.1}) \\
&= (x, y) : zip (fst unzip zs) (snd unzip zs) && (\text{unzip.2}) \\
&= (x, y) : zs && (\text{迭代}) \\
&= ps
\end{aligned}$$

综上证得  $zip (fst (unzip ps)) (snd (unzip ps)) = ps$ 。

2.

**Solution 3** 当  $length\ xs = length\ ys$  时,  $unzip (zip\ xs\ ys) = (xs, ys)$ , 下面给出证明

1. 当  $length\ xs = length\ ys = 0$ , 即  $xs=ys=[]$

$$\begin{aligned}
unzip (zip\ xs\ ys) &= unzip (zip [] []) \\
&= unzip [] && (\text{zip.1}) \\
&= ([], []) && (\text{unzip.1}) \\
&= (xs, ys) && (2)
\end{aligned}$$

2. 当  $\text{length } xs = \text{length } ys = k > 0$  时, 设  $xs=(x:xl), ys=(y,yl)$ , 则  $\text{length } xl = \text{length } yl = k-1$ , 由迭代知  $\text{unzip } (\text{zip } xl \ yl) = (xl, yl)$

$$\begin{aligned}
 \text{unzip } (\text{zip } xs \ ys) &= \text{unzip } (\text{zip } (x : xl) \ (y : yl)) \\
 &= \text{unzip}((x, y) : \text{zip } xl \ yl) && (\text{zip.3}) \\
 &= (x : xl, y : yl) && (\text{unzip.2}) \\
 &\quad \text{where } (xl, yl) = \text{unzip } (\text{zip } xl \ yl) \quad (\text{迭代}) \\
 &= (xs, ys)
 \end{aligned}$$

综上证得  $\text{unzip } (\text{zip } xs \ ys) = (xs, ys)$