HASKELL
第六次课后作业

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1　第一题

Solution 1 *map* 和 *(++)* 定义如下

```
map :: (a -> b) -> [a] -> [b]
map _ [] = []                       (map.1)
map f (x:xs) = f x : map f xs       (map.2)
(++) :: [a] -> [a] -> [a]
[] ++ y = y                         (++.1)
(x:xs) ++ y = x : (xs ++ y)         (++.2)
```

下面证明 $\forall xs, ys, zs \in \{[a]\}, (xs++ys)++zs = xs++(ys++zs)$

1. 当 *xs=[]* 时

$$\begin{aligned}
map\ f\ (xs++ys) &= map\ f\ ([]++ys) \\
&= map\ f\ ys && (++.1) \\
&= []\ ++\ map\ f\ ys && (++.1) \\
&= map\ f\ []\ ++\ map\ f\ ys && (\text{map}.1) \\
&= map\ f\ xs\ ++\ map\ f\ ys
\end{aligned}$$

2. 当 *xs=(x:zs)* 时

$$
\begin{aligned}
map\ f\ (xs++ys) &= map\ f\ ((x:zs)++ys) \\
&= map\ f\ (x:zs++ys) && (++.2) \\
&= f\ x\ :\ map\ f\ (zs++ys) && (\text{map}.2) \\
&= f\ x\ :\ (map\ f\ zs\ ++\ map\ f\ ys) && (\text{归纳}) \\
&= (f\ x\ :\ map\ f\ zs)\ ++\ map\ f\ ys && (++.2) \\
&= map\ f\ (x:zs)\ ++\ map\ f\ ys && (\text{map}.2) \\
&= map\ f\ xs\ ++\ map\ f\ ys
\end{aligned}
$$

综上证得 $\forall xs, ys, zs \in \{[a]\}, (xs++ys)++zs = xs++(ys++zs)$。

2 第二题

fst, snd, zip, unzip 的定义如下

```haskell
fst :: (a, b) -> a
fst (x, _) = x                          (fst.1)
snd :: (a, b) -> b
snd (_, y) = y                          (snd.1)
zip :: [a] -> [b] -> [(a, b)]
zip [] _ = []                           (zip.1)
zip _ [] = []                           (zip.2)
zip (x:xs) (y:ys) = (x,y) : zip xs ys  (zip.3)
unzip :: [(a, b)] -> ([a], [b])
unzip [] = ([], [])                     (unzip.1)
unzip ((x, y) : ps) = (x : xs, y : ys) (unzip.2)
    where (xs, ys) = unzip ps
```

1.

Solution 2 下面证明对于所有有限列表 *ps*, *zip (fst (unzip ps)) (snd (unzip ps))* = *ps* 均成立

1. 当 *ps=[]* 时

$$
\begin{aligned}
& zip\ (fst\ (unzip\ ps))\ (snd\ (unzip\ ps)) \\
&= zip\ (fst\ ([],[]))\ (snd\ ([],[])) && \text{(unzip.1)} \\
&= zip\ []\ [] && \text{(fst.1, snd.1)} \\
&= [] && \text{(zip.1)} \\
&= ps && (1)
\end{aligned}
$$

2. 当 *ps=((x,y):zs)* 时

$$
\begin{aligned}
& zip\ (fst\ (unzip\ ps))\ (snd\ (unzip\ ps)) \\
&= zip\ (fst\ (x:xs,\ y:ys))\ (snd\ (x:xs,\ y:ys)) && \text{(unzip.2)} \\
&= zip\ (x:xs)\ (y:ys) && \text{(fst.1,snd.1)} \\
&= (x,y)\ :\ zip\ xs\ ys && \text{(zip.3)} \\
&= (x,y)\ :\ zip\ (fst\ (xs,\ ys))\ (snd\ (xs,\ ys)) && \text{(fst.1,snd.1)} \\
&= (x,y)\ :\ zip\ (fst\ unzip\ zs)\ (snd\ unzip\ zs) && \text{(unzip.2)} \\
&= (x,y)\ :\ zs && \text{(归纳)} \\
&= ps
\end{aligned}
$$

综上证得 *zip (fst (unzip ps)) (snd (unzip ps)) = ps*。

2.

Solution 3 当 *length xs = length ys* 时，*unzip (zip xs ys) = (xs, ys)*，下面给出证明

1. 当 *length xs = length ys = 0*，即 *xs=ys=[]*

$$
\begin{aligned}
unzip\ (zip\ xs\ ys) &= unzip\ (zip\ []\ []) \\
&= unzip[] && \text{(zip.1)} \\
&= ([],\ []) && \text{(unzip.1)} \\
&= (xs,\ ys) && (2)
\end{aligned}
$$

2. 当 *length xs = length ys = k > 0* 时，设 *xs=(x:xl),ys=(y,yl)*，则 *length xl = length yl = k-1*，由迭代知 *unzip (zip xl yl) = (xl, yl)*

$$
\begin{aligned}
unzip\ (zip\ xs\ ys) &= unzip\ (zip\ (x:xl)\ (y:yl)) \\
&= unzip((x,\ y):zip\ xl\ yl) && \text{(zip.3)} \\
&= (x:xl,\ y:yl) && \text{(unzip.2)} \\
&\qquad where\ (xl, yl) = unzip\ (zip\ xl\ yl) && \text{(归纳)} \\
&= (xs, ys)
\end{aligned}
$$

综上证得 *unzip (zip xs ys) = (xs, ys)*