HASKELL
第九次课后作业

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1 Maybe

Proof 1

```
fmap id Nothing === Nothing
                === id Nothing
fmap id (Just a) === Just (id a)
                 === Just a
                 === id (Just a)
fmap f . fmap g Nothing === fmap f (fmap g Nothing)
                        === fmap f Nothing
                        === Nothing
                        === fmap (f.g) Nothing
fmap f . fmap g (Just a) === fmap f (fmap g (Just a))
                         === fmap f (Just g a)
                         === Just f (g a)
                         === Just (f.g) a
                         === fmap (f.g) (Just a)
```

综上证得对于 *Maybe*

```
fmap id === id
```

$$fmap\ (f.g) === fmap\ f\ .\ fmap\ g$$

。

2 []

Proof 2

$$\begin{aligned}
fmap\ id\ [] &=== [] \\
&=== id\ [] \\
fmap\ id\ (a:as) &=== id\ a\ :\ fmap\ id\ as \\
&=== a\ :\ id\ as && \text{(归纳)} \\
&=== a:as \\
&=== id\ (a:as) \\
fmap\ f\ .\ fmap\ g\ [] &=== fmap\ f\ (fmap\ g\ []) \\
&=== fmap\ f\ [] \\
&=== [] \\
&=== fmap\ (f.g)\ [] \\
fmap\ f\ .\ fmap\ g\ (a:as) &=== fmap\ f\ (fmap\ g\ (a:as)) \\
&=== fmap\ f\ (g\ a\ :\ fmap\ g\ as) \\
&=== f\ (g\ a)\ :\ fmap\ f\ (fmap\ g\ as) \\
&=== f.g\ a\ :\ (fmap\ f\ .\ fmap\ .g)\ as \\
&=== f.g\ a\ :\ fmap\ (f.g)\ as && \text{(归纳)} \\
&=== fmap\ (f.g)\ (a:as)
\end{aligned}$$

综上证得对于 []

$$fmap\ id === id$$

$$fmap\ (f.g) === fmap\ f\ .\ fmap\ g$$

。

3 Either

Proof 3

$$
\begin{aligned}
\mathit{fmap}\ \mathit{id}\ (\mathit{Left}\ c) &=== \mathit{Left}\ c \\
&=== \mathit{id}\ (\mathit{Left}\ c) \\
\mathit{fmap}\ \mathit{id}\ (\mathit{Right}\ a) &=== \mathit{Right}\ (\mathit{id}\ a) \\
&=== \mathit{Right}\ a \\
&=== \mathit{id}\ (\mathit{Right}\ a) \\
\mathit{fmap}\ f\ .\ \mathit{fmap}\ g\ (\mathit{Left}\ c) &=== \mathit{fmap}\ f\ (\mathit{fmap}\ g\ (\mathit{Left}\ c)) \\
&=== \mathit{fmap}\ f\ (\mathit{Left}\ c) \\
&=== \mathit{Left}\ c \\
&=== \mathit{fmap}\ (f.g)\ (\mathit{Left}\ c) \\
\mathit{fmap}\ f\ .\ \mathit{fmap}\ g\ (\mathit{Right}\ a) &=== \mathit{fmap}\ f\ (\mathit{fmap}\ g\ (\mathit{Right}\ a)) \\
&=== \mathit{fmap}\ f\ (\mathit{Right}\ (g\ a)) \\
&=== \mathit{Right}\ (f\ (g\ a)) \\
&=== \mathit{Right}\ (f.g\ a) \\
&=== \mathit{fmap}\ (f.g)\ (\mathit{Right}\ a)
\end{aligned}
$$

综上证得对于 *Either*

$$\mathit{fmap}\ \mathit{id} === \mathit{id}$$

$$\mathit{fmap}\ (f.g) === \mathit{fmap}\ f\ .\ \mathit{fmap}\ g$$

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