

IFT 4030/7030,
Machine Learning for Signal Processing
Week3: Signal Processing Primer

Cem Subakan



UNIVERSITÉ
Laval



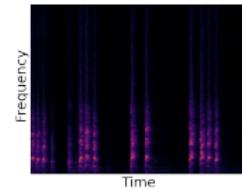
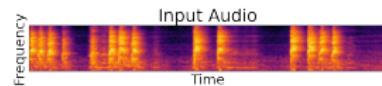
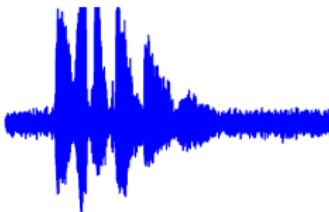
- Comment était le lab 1? Feedback?
 - ▶ How was the lab? Any feedback?
- On aura le deuxième lab en vendredi.
 - ▶ The next lab is on friday!
- Comment va la réflexion sur les projets? Les propositions de projets sont dûs la deuxième semaine de l'octobre!
 - ▶ How are the project proposals coming along? The proposals are due the second week of october.

Today

- Signals/Time series, what are they?
 - ▶ Qu'est-ce qu'on comprend quand on dit 'signaux/séries temporelles' ?

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 - ▶ Comment est-ce qu'on représente les signaux?
 - ▶ Time Domain / Frequency Domain / Time+Frequency Domain



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- The Fourier Transform
 - ▶ The Short-Time Fourier Transform

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 - ▶ The Short-Time Fourier Transform
- Filtrage
 - ▶ Convolution

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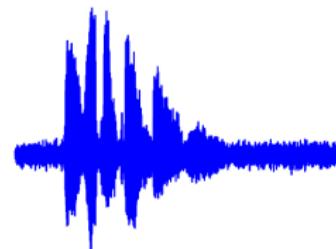


- The Fourier Transform
 - ▶ The Short-Time Fourier Transform
- Filtrage
 - ▶ Convolution
- Échantillonnage
 - ▶ Sampling

Signals

- A dry definition: A signal is an ordered collection of numbers
 - Une définition sec: Un signal est une collection des chiffres en ordre.

Forecast						
Sat 16 Sep	Sun 17 Sep	Mon 18 Sep	Tue 19 Sep	Wed 20 Sep	Thu 21 Sep	Fri 22 Sep
 22°C	 24°C	 24°C	 20°C	 20°C	 23°C	 24°C



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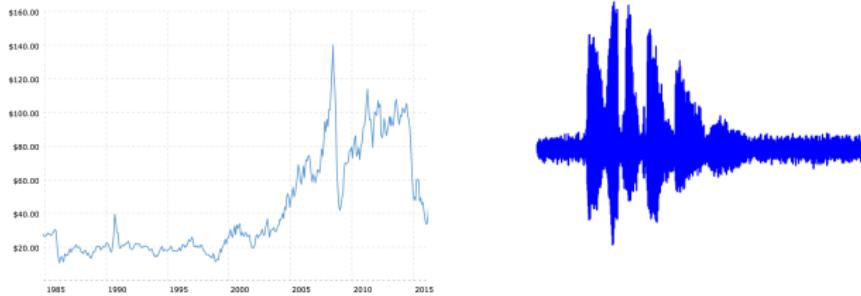


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Signal Representations

Fourier Series

DFT

Time-Frequency Representation

Analog to Digital Conversion

Quantization

Sampling

Convolution

Re-Sampling

Representing Signals/Time Series

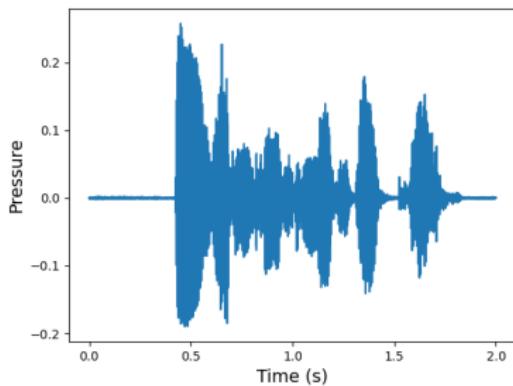
- There are different ways of representing time series/signals.
 - ▶ Il existe plusieurs façons de représenter les signaux.
- Each representation type has its pros / cons.
 - ▶ Chaque type de représentation a leurs avantages / désavantages.

Representing Signals/Time Series

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- Some typical options: Time Domain, Frequency Domain, Time+Frequency Domain

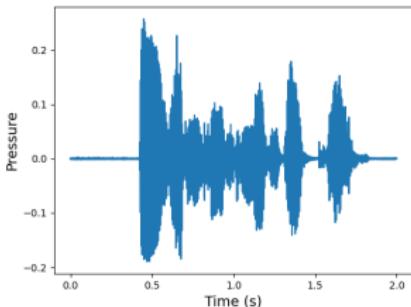
Sound as Signal

- We will start with sounds as an example, but any time series would do.
 - ▶ On commencera avec les sons comme un exemple, mais n'importe quel time series serait ok.
- Let's listen.



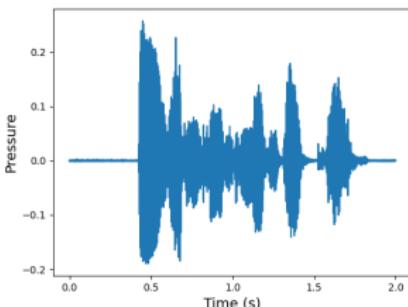
The time domain

- It's kinda difficult what to take out of this by looking at it.
 - ▶ Je sais pas quoi faire en regardant ça.



The time domain

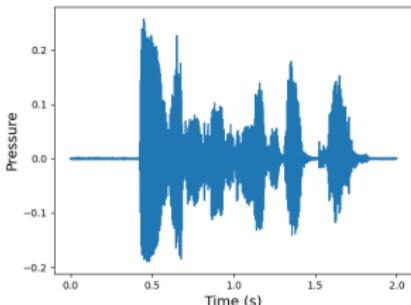
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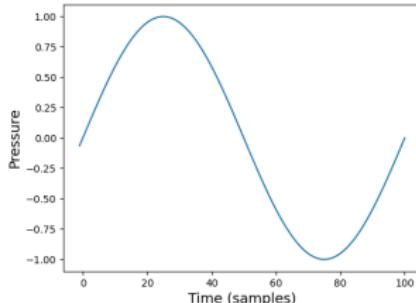
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 - ▶ C'est possible par contre de l'exprimer ce waveform en terme des waveform plus simples.

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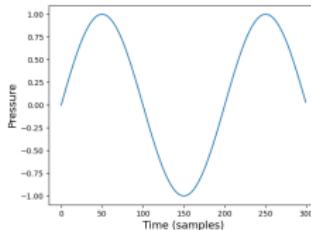


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 - ▶ C'est possible par contre de l'exprimer ce waveform en terme des waveform plus simples.
- Sinusoids!! (They are special)

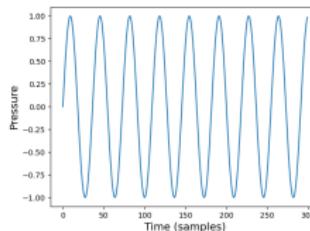


Sinusoids

- 80 Hz, listen.



- 440 Hz, listen. This is how we tune our instruments.
 - ▶ On tune nos instruments à La 440Hz.



- 880 Hz, listen
- 1760 Hz, listen
- 15000 Hz, are you able to hear this at all?
 - ▶ Pouvez-vous entendre ceci?

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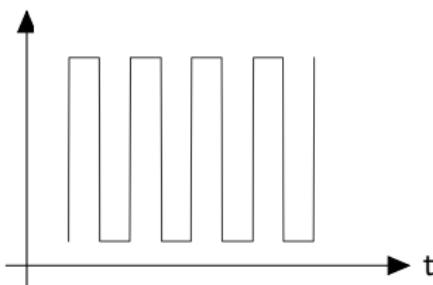
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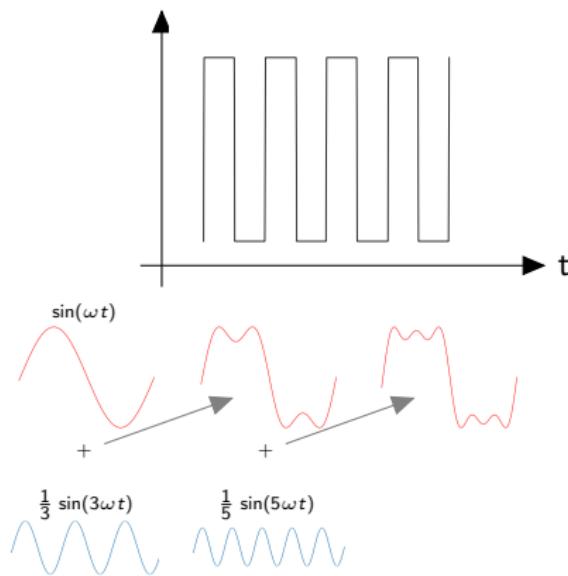
Decomposing a signal

- Let's decompose a square wave with sines.
 - Decomposons un square wave.



Decomposing a signal

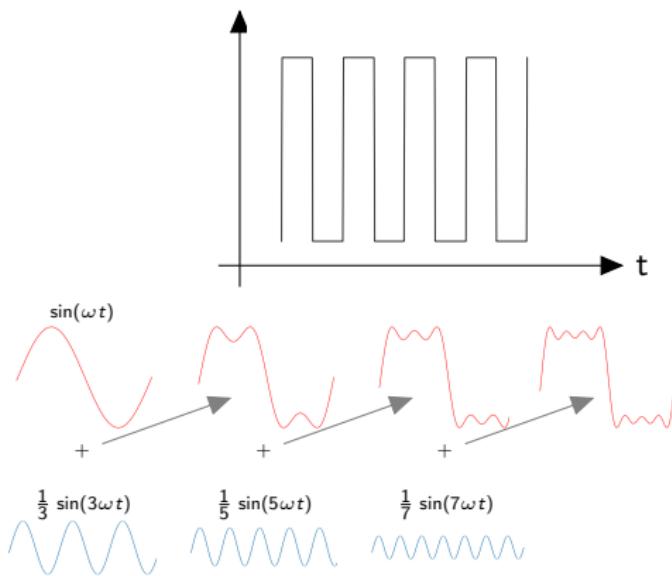
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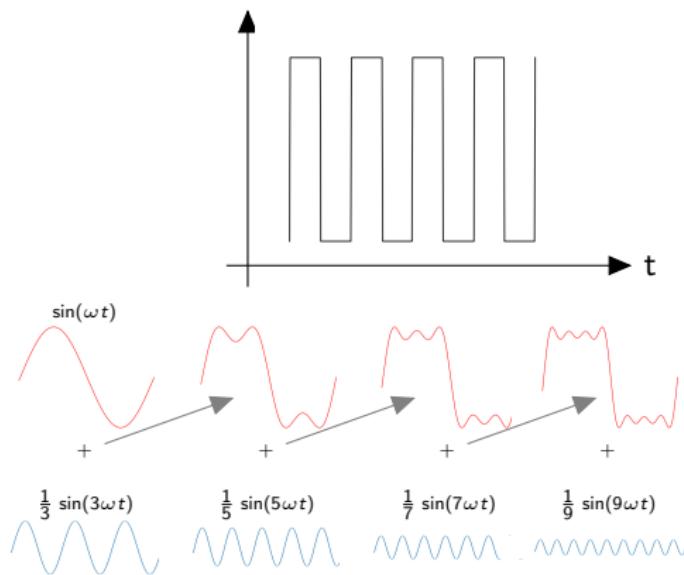
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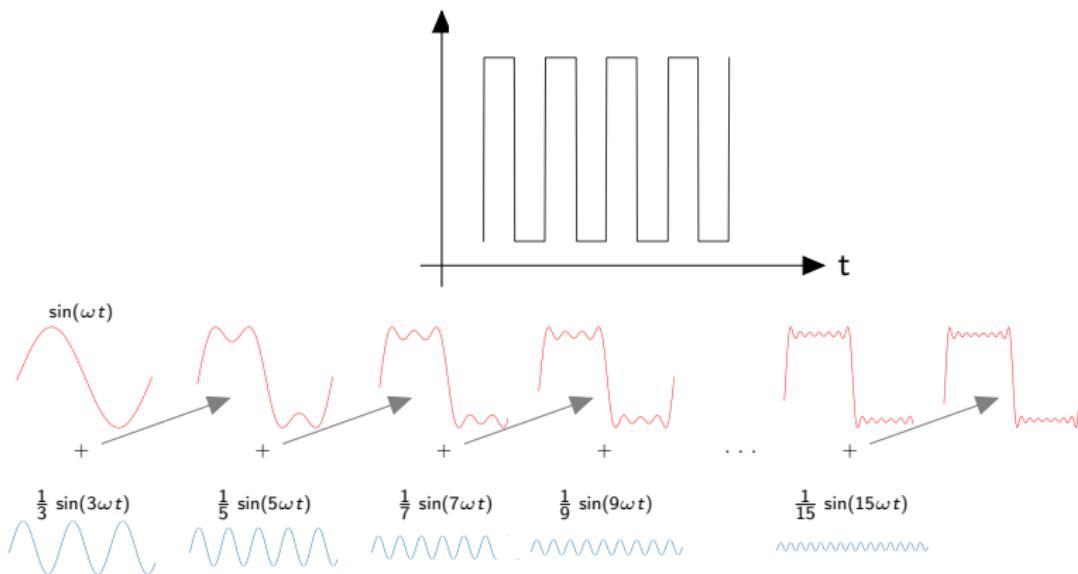
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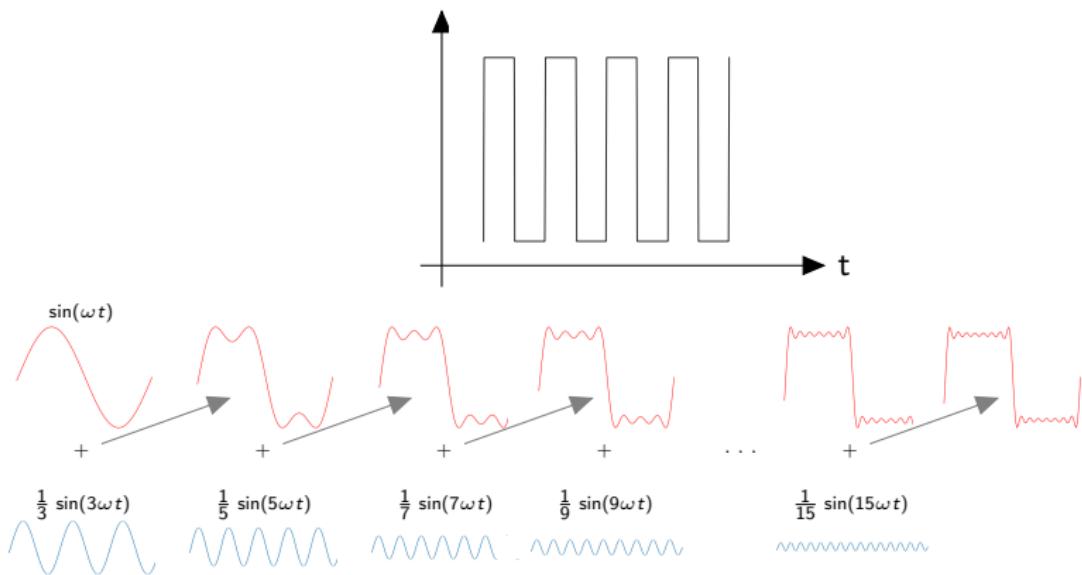
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Decomposing a signal

- Let's decompose a square wave with sines.

► Decomposons un square wave.



- So we can approximate a square wave with

► Alors on peut approximer un onde carré avec

$$SW(\omega t) \approx \sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{5} \sin(5\omega t) + \frac{1}{7} \sin(7\omega t) + \dots$$

Frequency Representation

- The goal is to find the contribution of each sinusoid (or 'frequency').
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- Do you remember how we calculate the similarity between two vectors?
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■ Inner Product!

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 - ▶ Dans le fond, FT est une projection sur des sinusoids.
- Let's do that for this signal.
 - ▶ Faisons ça pour ce signal.

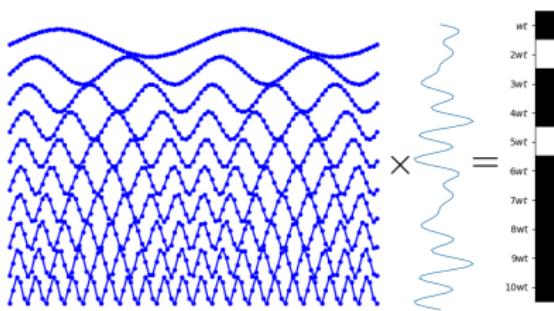


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- FT in principle:

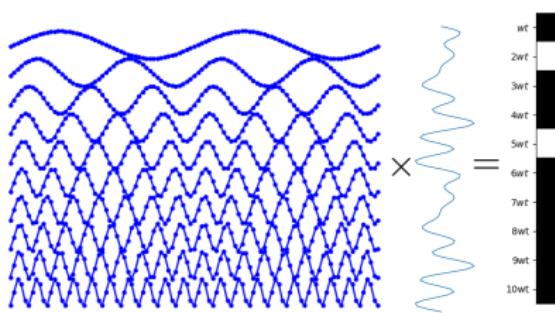


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- FT in principle:



- This is nice, but is this general enough?
 - ▶ Bon, mais est-ce assez générale?

No! You need cosines to span the space

- The signal we saw before was,
 - Le signal qu'on avait vu était:

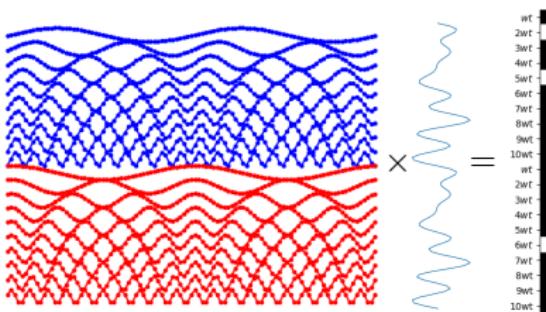
$$y(t) = \sin(2\omega t) + \sin(5\omega t) + \underbrace{\cos(6\omega t)}_{\text{We missed this guy!}}$$

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- Let's extend our bases,
 - Élargons notre ensemble de bases:

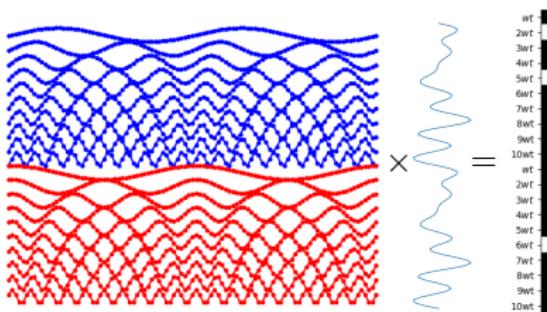


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 - Élargons notre ensemble de bases:



- But wait, I remember that Fourier Transform gave us complex numbers. What's that about?
 - Mais chuis confus-là, je me souviens que FT nous donnait des chiffres complexes?

Fourier Transform Formal Definition

- Remember Euler's formula:

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

- So it seems like, we can use complex exponentials to project onto cosines and sines at the same time.
 - ▶ On alors peut utiliser les exponentiels complexes pour projeter sur les cosines et sines en même temps.

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- Discrete Fourier Transform (DFT):

$$X_k = \sum_{n=0}^{N-1} x_n \exp\left(-j2\pi \frac{k}{N} n\right), \text{ where}$$
$$x = [x_0, x_1, \dots, x_{N-1}], k \in \{0, \dots, N-1\}.$$

- Note that k corresponds to frequencies, and we have the same number of frequencies as the signal length N .
 - ▶ Notez que l'indice k est la fréquence, et on a le même nombre de fréquences que la longueur N .

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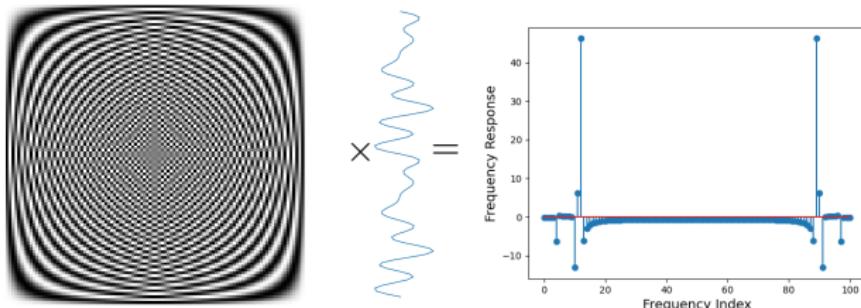
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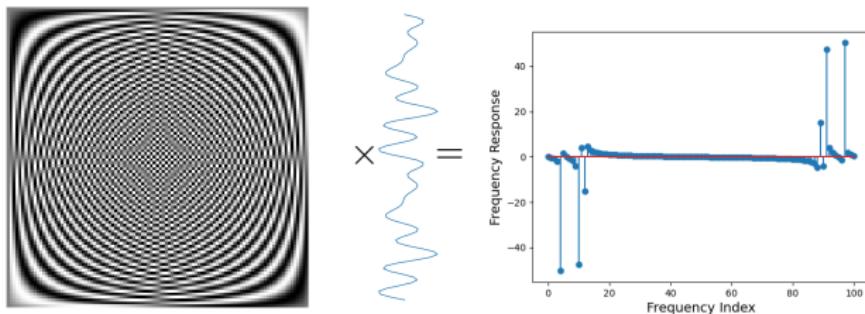
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 - ▶ Notez que l'indice k est la fréquence, et on a le même nombre de fréquences que la longueur N .
- By the way, do you see that this is a matrix? $\exp\left(-j2\pi \frac{k}{N} n\right)$
 - ▶ Vous voyez que c'est une matrice?

DFT in action

■ Real Part / La partie réel



■ Imaginary Part / La partie imaginaire



Remarks

- Notice that les results are symmetric. This is because of the construction of the DFT matrix.
 - ▶ Notez que les résultats sont symétriques. C'est à cause de la construction de la matrice DFT.

Remarks

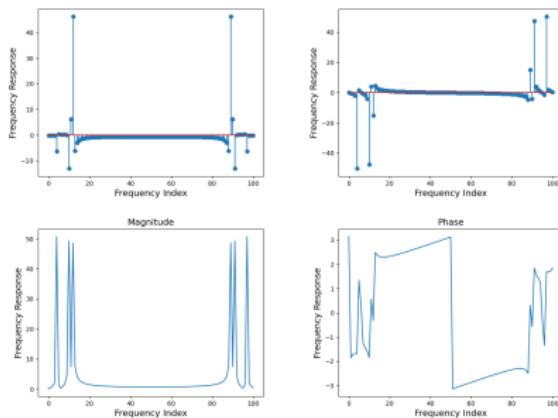
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 - ▶ Notez que les résultats sont symétriques. C'est à cause de la construction de la matrice DFT.
- Note that the DFT matrix repeats after the half. We will get back to this later.
 - ▶ Notez que les matrices DFT répètent après la moitié. On va parler de ça après.

Another way of interpreting DFT

- Note that we get complex numbers. We can calculate magnitude and phase.
 - ▶ Notez qu'on obtiens des nombres complex. Donc on peut calculer la magnitude et la phase.

$$|X_k| = \sqrt{\operatorname{Re}(X_k)^2 + \operatorname{Im}(X_k)^2}$$

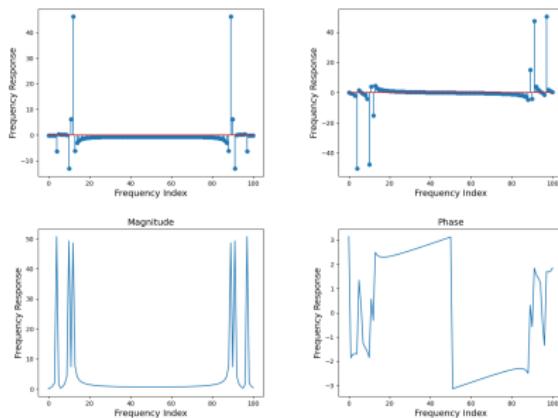
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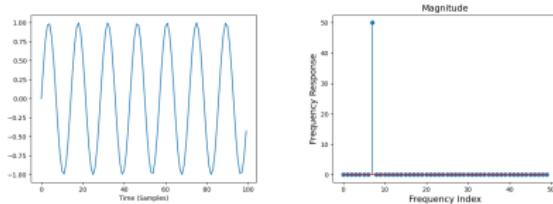
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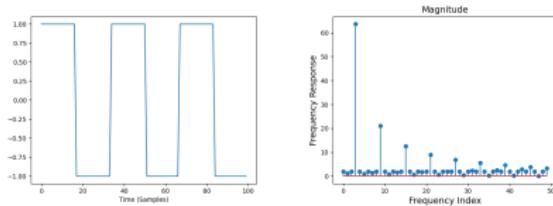
- Magnitude is easy to interpret. Phase is not as easy always.
 - ▶ Magnitude est facile est interpreter. La phase n'est pas toujours si facile à interpreter.

DFTs of different signals

■ A single sinusoid

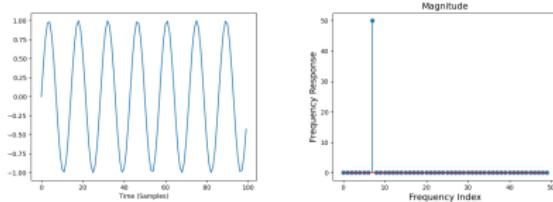


■ Square wave

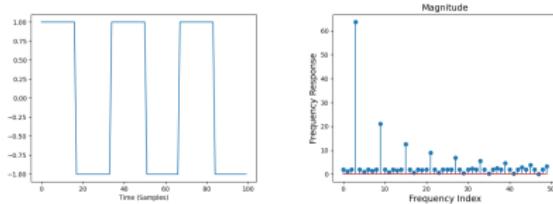


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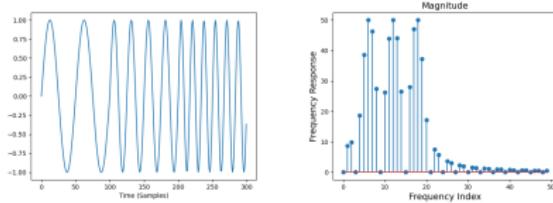
■ A single sinusoid



■ Square wave



■ Sinusoids with increasing frequencies



■ So, for time varying signals, DFT is not that great.

► Pour les signaux qui varie avec le temps DFT n'est pas idéal.

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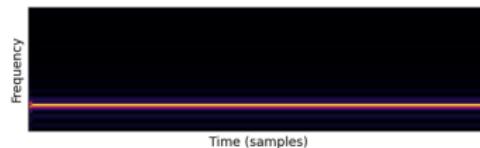
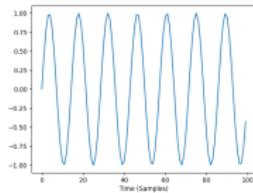
Sampling

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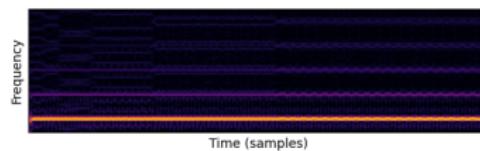
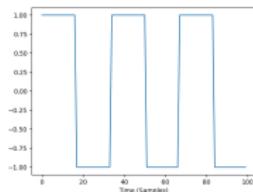
Re-Sampling

Time-frequency representation

- A single sinusoid

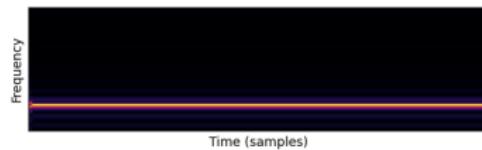
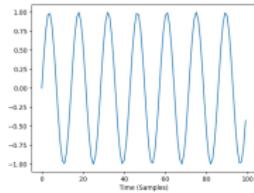


- Square wave

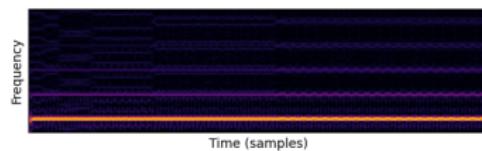
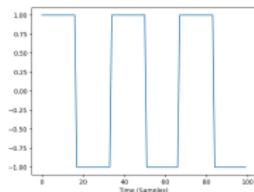


Time-frequency representation

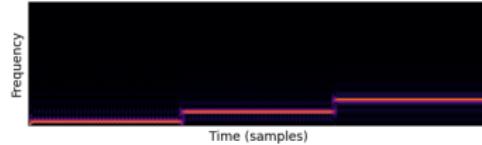
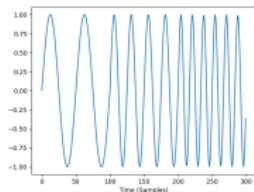
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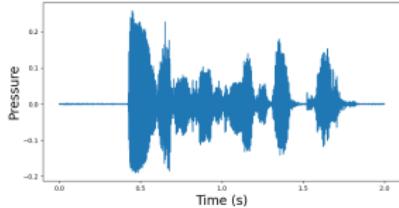


- Sinusoids with increasing frequencies



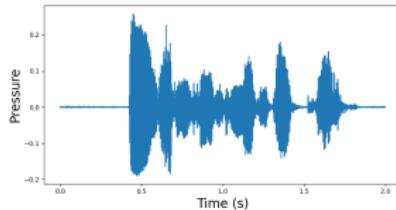
A real example for speech

Time Domain

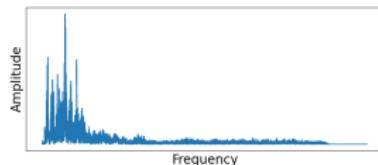


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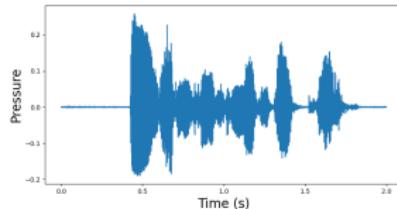


Frequency Domain – but
what is this really?
I see low freqs, but
where is time
information? – On
voit freq basses
mais pas d'info sur
temps.

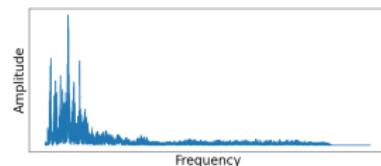


A real example for speech

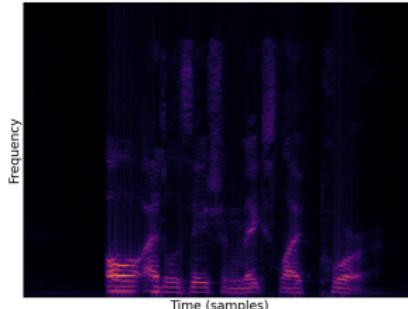
Time Domain



Frequency Domain – but what is this really?
I see low freqs, but where is time information? – On voit freq basses mais pas d'info sur temps.

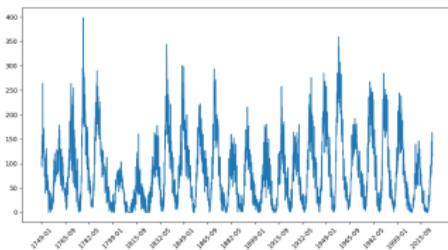


Time-Frequency Domain – We 'see' the signal. – on voit le signal.



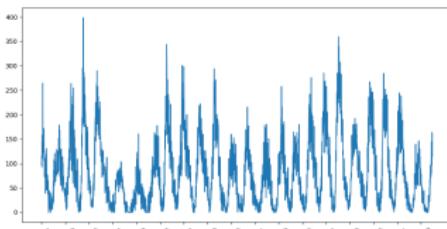
An example from another domain

Time Domain
Counts of
sunspots wrt.
time

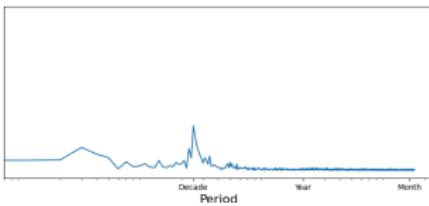


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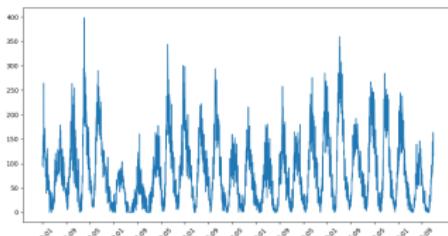


Frequency Domain

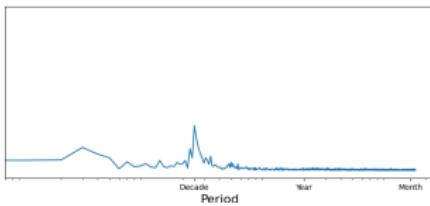


An example from another domain

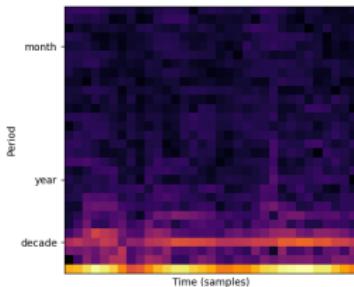
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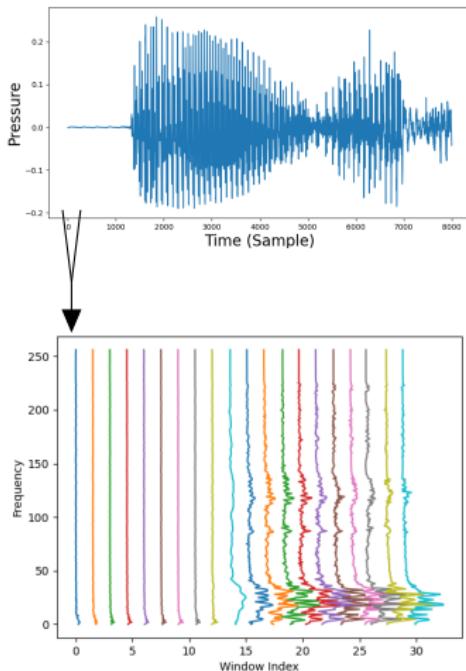
Frequency Domain



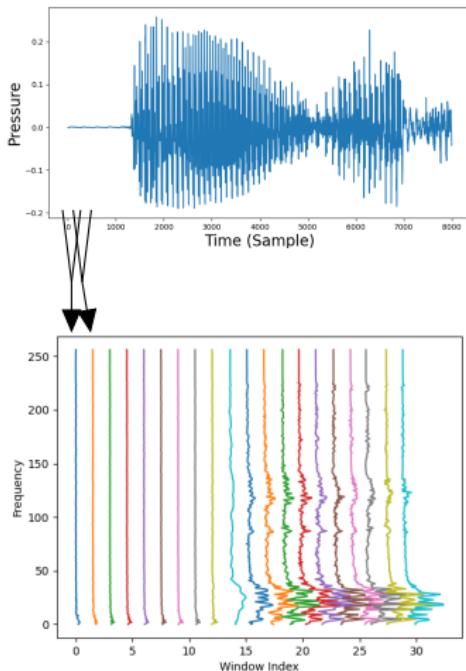
Time-Frequency Domain



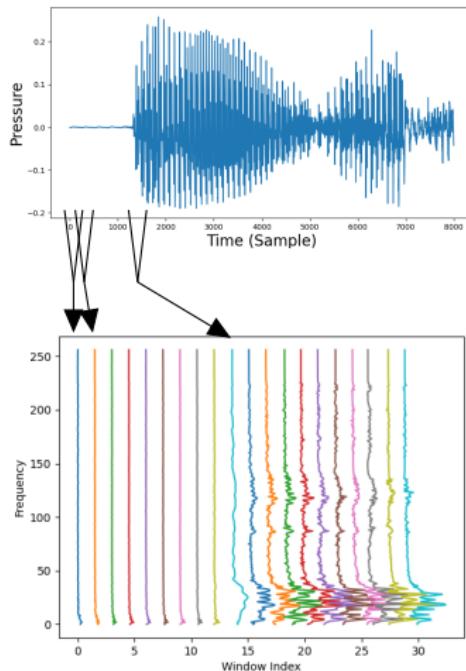
How to Calculate the Spectrogram (STFT)



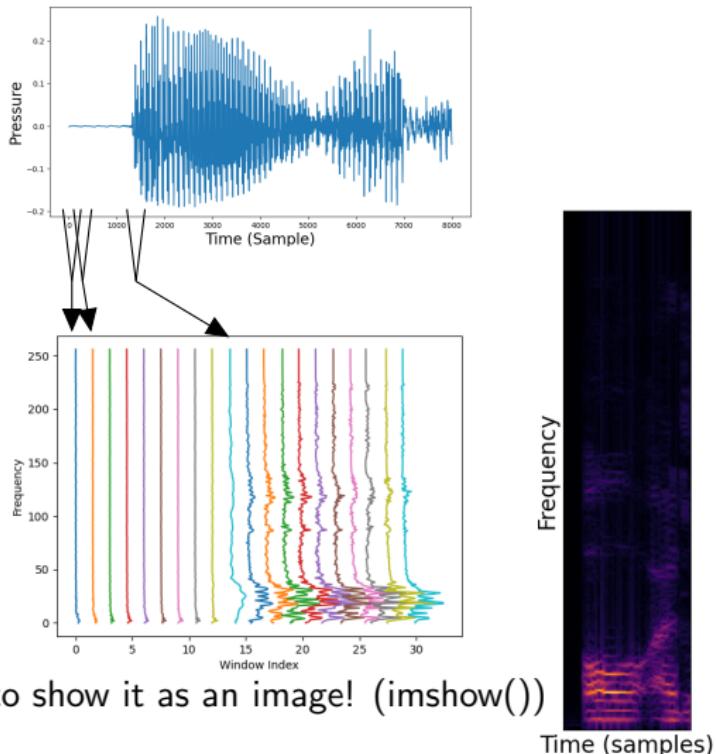
How to Calculate the Spectrogram (STFT)



How to Calculate the Spectrogram (STFT)



How to Calculate the Spectrogram (STFT)

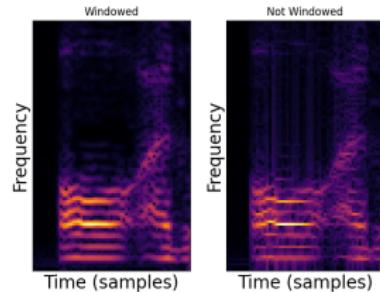
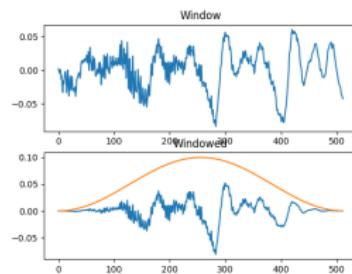


It's nice to show it as an image! (`imshow()`)

Time (samples)

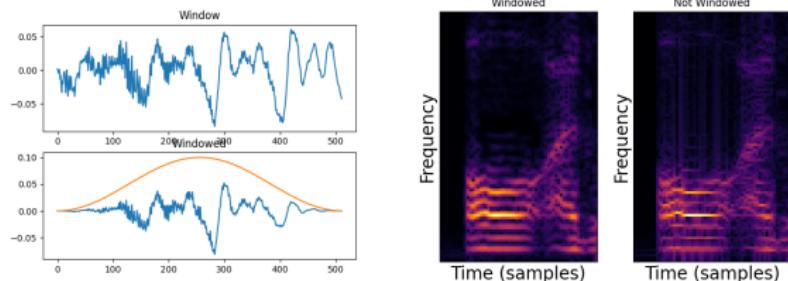
STFT considerations

- Windowing (Utilisation de fenêtres)
 - ▶ We apply a window, before calculating the DFT.
 - ▶ On applique une fenêtre avant de calculer le DFT.



STFT considerations

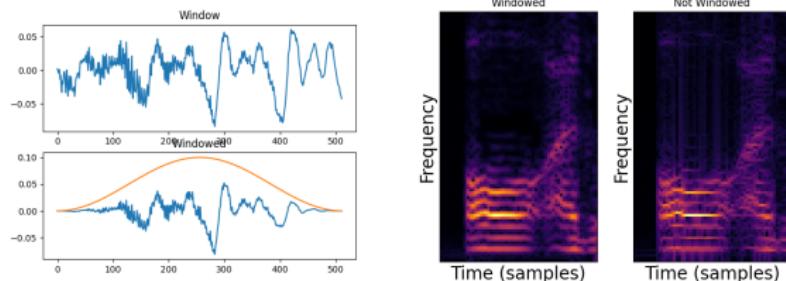
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- Notice that not windowed STFT contains more high frequency artifacts
 - ▶ Notez que STFT qui n'est pas windowé contient plus des artefacts de haute fréquence.

STFT considerations

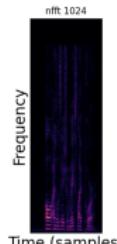
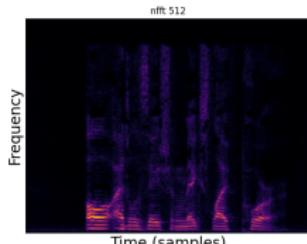
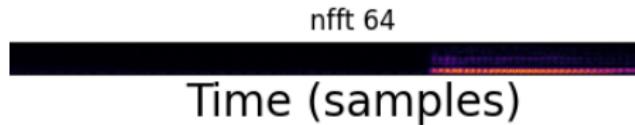
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- Notice that not windowed STFT contains more high frequency artifacts
 - ▶ Notez que STFT qui n'est pas windowé contient plus des artefacts de haute fréquence.
- We add an overlap to make up for using tapering windows. / On ajoute un overlap aussi pour compenser pour les fenêtres qui réduisent en amplitude.

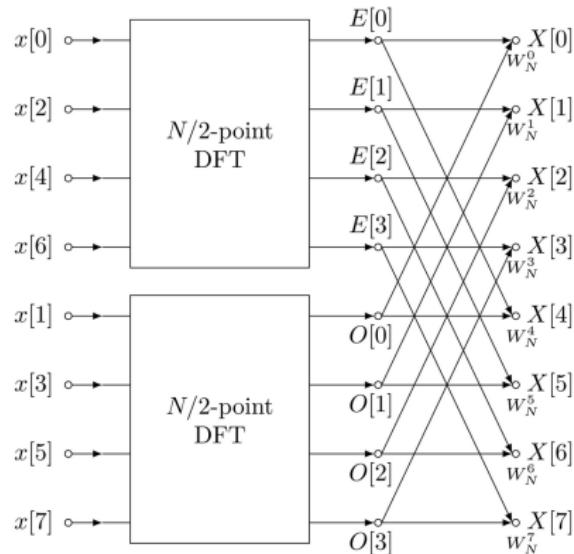
Time/Frequency Tradeoff

- Heisenberg's uncertainty principle - We can not know the frequency and time location of a wave.
 - ▶ On ne peut pas déterminer la fréquence et la localisation temporelle d'une onde.
- In the context of spectrograms: Big DFT sacrifice temporal resolution, Small DFTs have bad frequency resolution
 - ▶ Grand DFT a une mauvaise résolution temporelle, Petit DFTs ont une mauvaise résolution fréquentielle



Fast Fourier Transform (FFT)

- DFT matrix has symmetries.
- DFT can be decomposed into DFTs of half the size.
 - La DFT peut être décomposée aux 2 DFTs avec la moitié de la taille originale.
- We reduce the complexity from $\mathcal{O}(N^2)$ (*why?*) to $\mathcal{O}(N \log N)$.



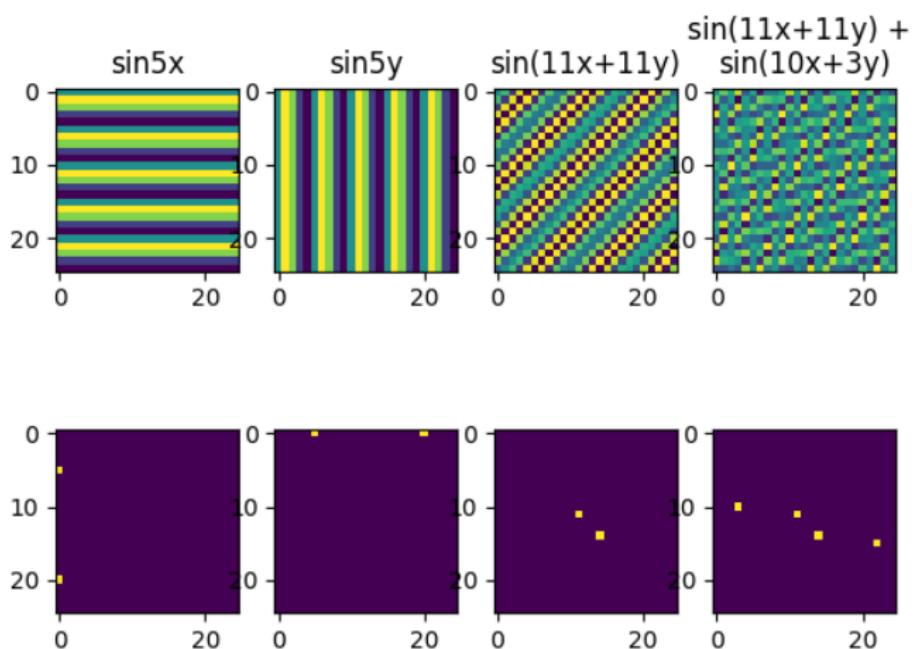
- Whenever you can use FFTs, use em!

Image DFTs

- We can généralize to images as well
 - ▶ On peut généraliser aux images aussi.
- The DFT bases in 2d
 - ▶ Les bases DFT en 2d



Some example DFTs



How to calculate 2d DFTs?

- For images $Y = FXF$
- For Tensors $F_{il_1} X_{ijk} F_{jl_2} F_{kl_3} \rightarrow Y_{l_1, l_2, l_3}$

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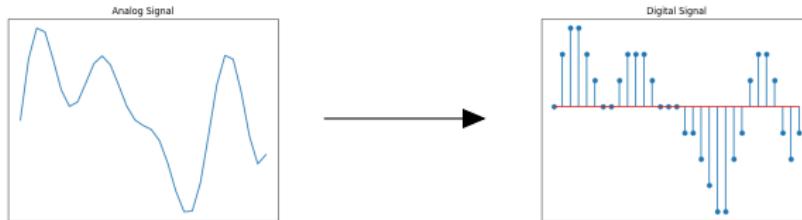
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Convolution

Re-Sampling

Analog to Digital

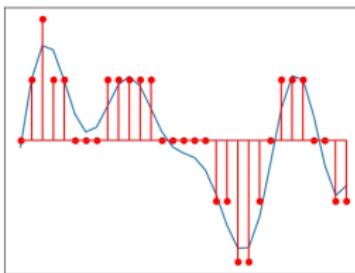
- How do we store / acquire signals?
 - ▶ Comment est-ce qu'on mets les signaux dans les ordinateurs?
- We convert the analog signals to digital.
 - ▶ On convertit signaux analogues au digital.



- It's not straightforward how to do this conversion.
 - ▶ C'est pas trivial comment faire cette conversion.

Signal Representation – Quantization

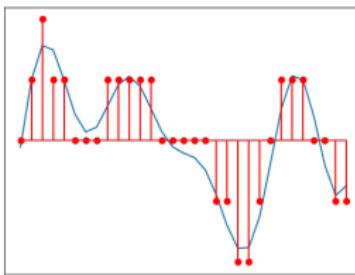
- We have to quantize a signal in order to digitize. (Quantize the y-axis)
 - ▶ On est obligé de quantiser pour digitiser. (l'axe y)



- We measure the precision in terms of bits. More bits we use, more signal-to-noise ratio we have.
 - ▶ On mesure la précision de la quantification en terme de bits. Plus de bits qu'on utilise, ça donne plus de SNR.

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 - ▶ On mesure la précision de la quantification en terme de bits. Plus de bits qu'on utilise, ça donne plus de SNR.
- Example average numbers:
 - ▶ 8bit - 48dB poor
 - ▶ 12 bits - 72dB okayish
 - ▶ 16 bits - 96 dB good
 - ▶ 24 bits - 144 dB overkill

Quantization in practice



8, 7, 5 bits



4, 3, 2 bits, 1 bit

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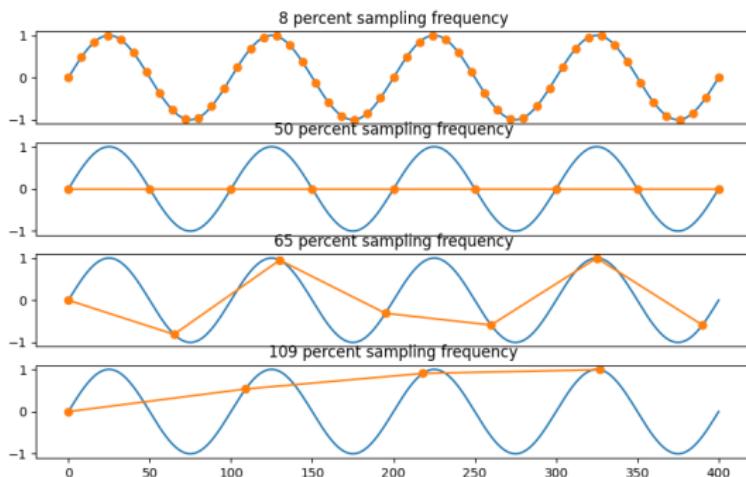
Re-Sampling

Sampling / Échantillonnage

- How often we sample is also important!
 - ▶ C'est aussi important la fréquence avec laquelle on échantillon. (l'axe de temps)
 - ▶ We use Hz to measure sampling freq. / On utilise Hz pour mesurer la fréquence d'échantillonnage.
- Important: Nyquist Rate: We must sample x2 above the highest frequency we want to represent.
 - ▶ Important: Le taux de Nyquist: On doit échantillonner 2 fois au-delà de la fréquence qu'on veut représenter.
- Perceptual Limits:
 - ▶ Our ears: We hear up to 20kHz (declines with age) So above 40kHz sampling is required.
 - ▶ Seeing: We perceive only up to 60Hz, so 120 Hz or up is required.
- Limites perceptuels:
 - ▶ Nos oreilles: La limite est 20kHz. Donc on a besoin d'une fréquence d'échantillonnage de 40Hz.
 - ▶ Nos yeux: La limite est 60Hz. On a besoin donc 120Hz.
- Common sampling rates:
 - ▶ Speech: 8kHz, 16kHz, Music: 32kHz, 44.1kHz, Pro-Audio 96kHz
 - ▶ Movies: 24fps (recently 48fps) HDTV: 60fps, ...

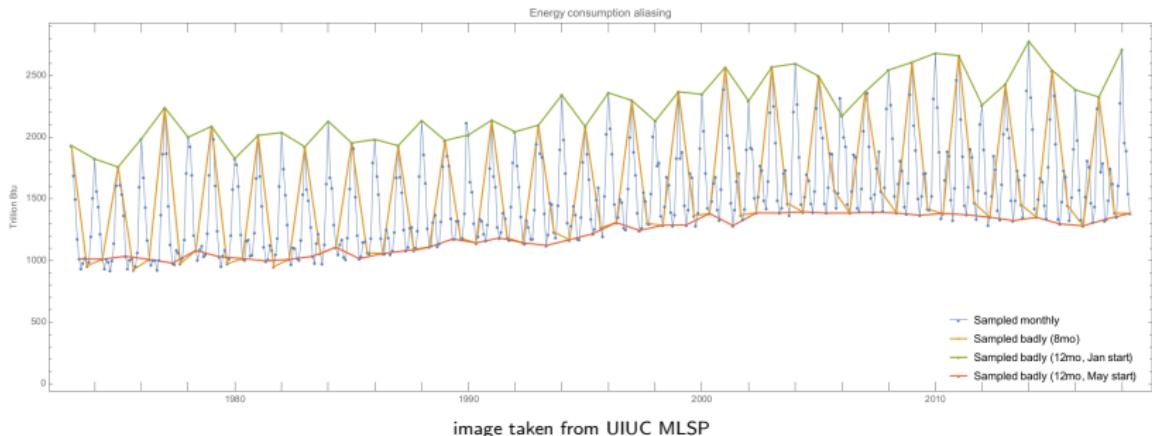
Aliasing

- Frequencies above 50% of the sampling rate are mis-represented.
 - ▶ Les fréquences sur 50% du rate d'échantillonnage sont mal représentés.



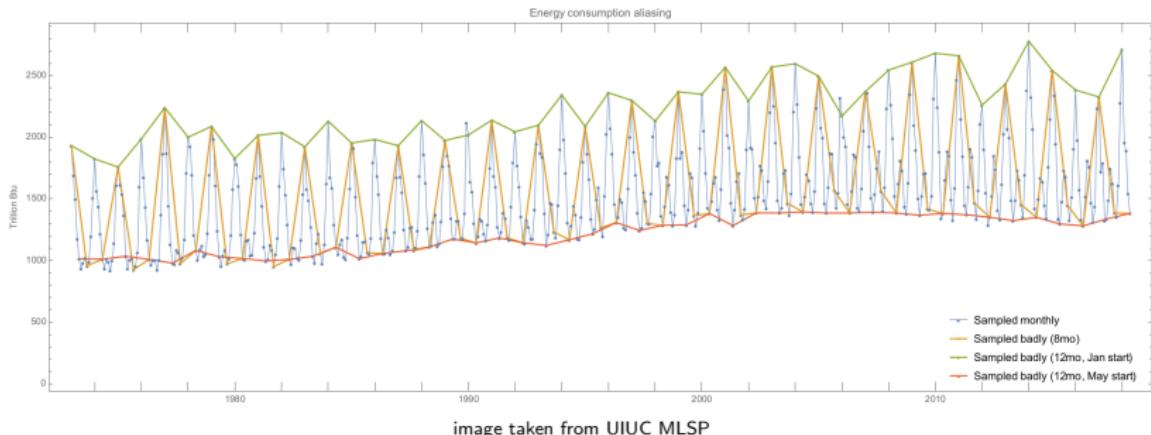
In real-life

- Electricity consumption sampled poorly.
 - ▶ Une mauvaise échantillonnage de la consomptions de l'électricité.

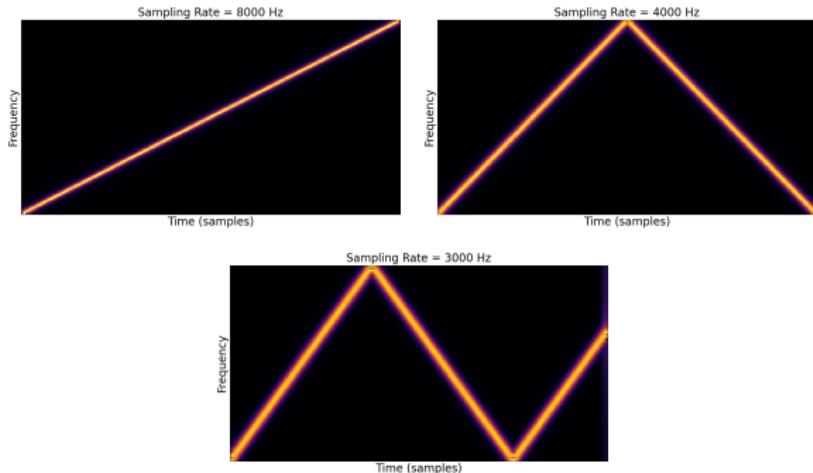


In real-life

- Electricity consumption sampled poorly.
 - ▶ Une mauvaise échantillonnage de la consomptions de l'électricité.
- Different sampling leads to different conclusions.
 - ▶ Différent l'échantillonnage mène à des conclusions différentes.



Aliasing in Sinusoid Sweeping

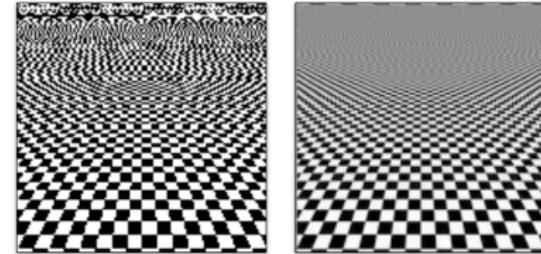


We have frequency sweeps from 0-4kHz, for different sampling rates.

Notice the aliasing when sampling rate is lower!

Sweep 1 Sweep 2 Sweep 3

Aliasing in Images/Video



<https://www.youtube.com/watch?v=R-IVw8OKjvQ>

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- This is an extremely important operation that comes up all the time.
(e.g. filtering)
 - ▶ C'est une opération extremement importante qu'on va voir souvent.

$$\begin{aligned}x(t) * w(t) &:= \sum_{i=0}^{M-1} x(i)w(t-i) \\&= x(0)w(t) + x(1)w(t-1) + x(1)w(t-2) + \dots \\&\quad + x(M)w(t-M)\end{aligned}$$

Convolution

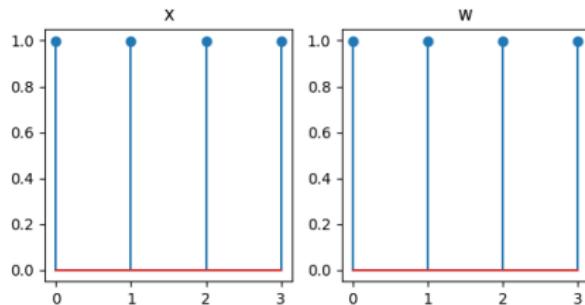
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- If x is of length M and w is of length N , the result is of length $M + N - 1$.
 - ▶ Si x est de longeur M et w est du longeur N , le résultat est de longeur $M + N - 1$.

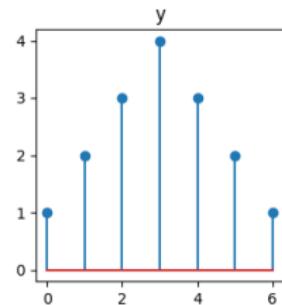
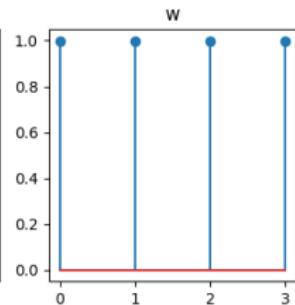
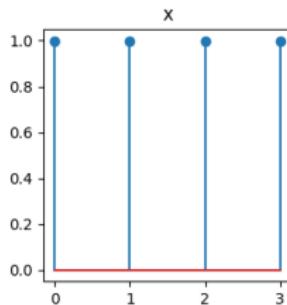
Convolution

$$y = X * W$$



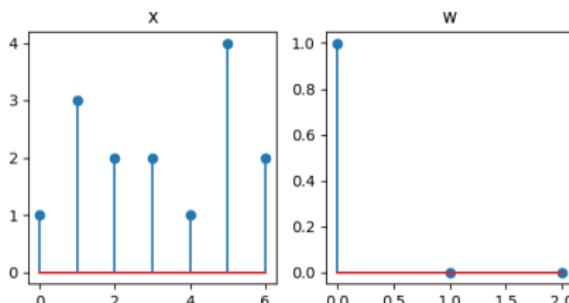
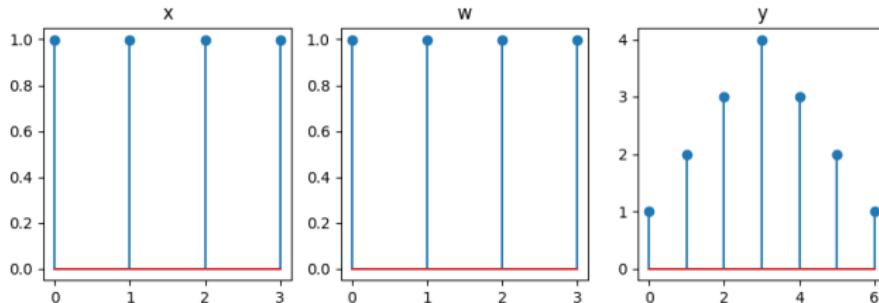
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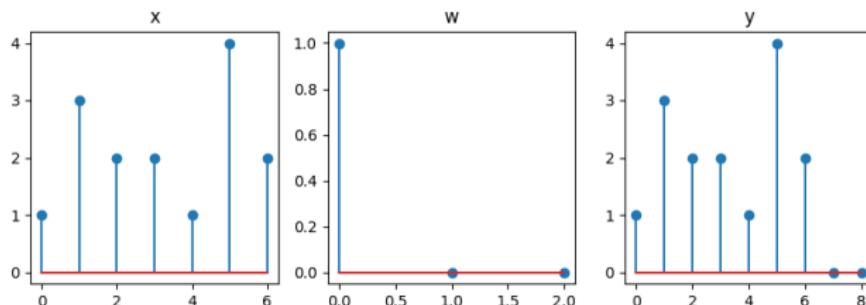
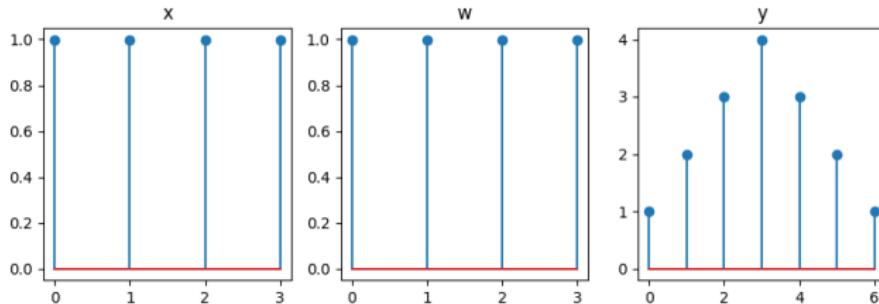
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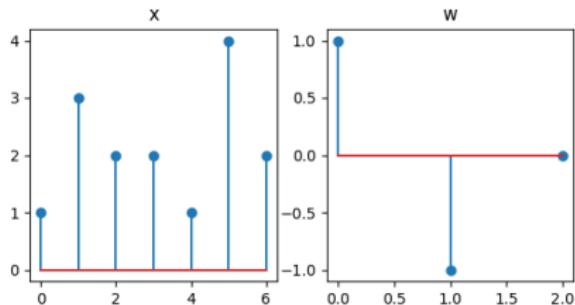
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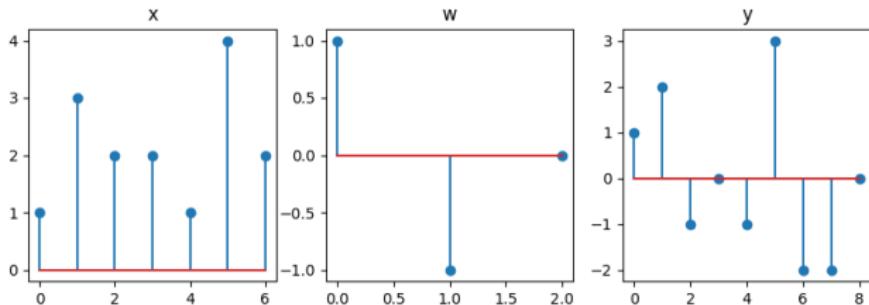
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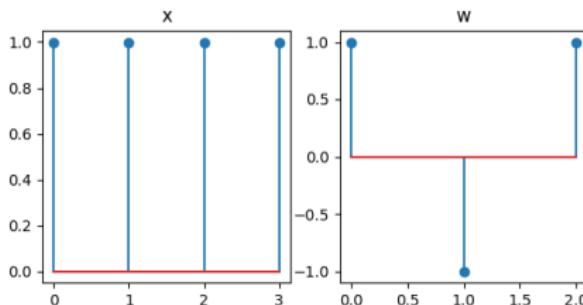
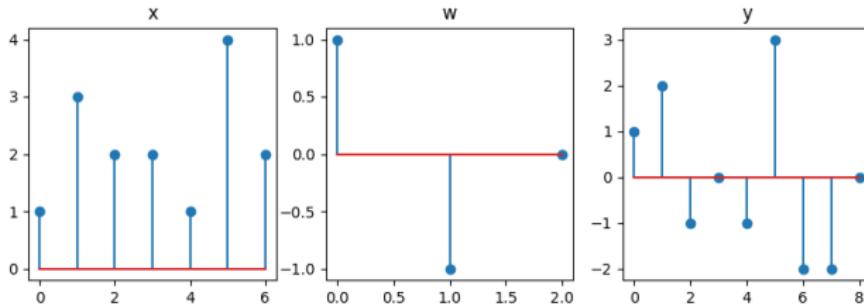
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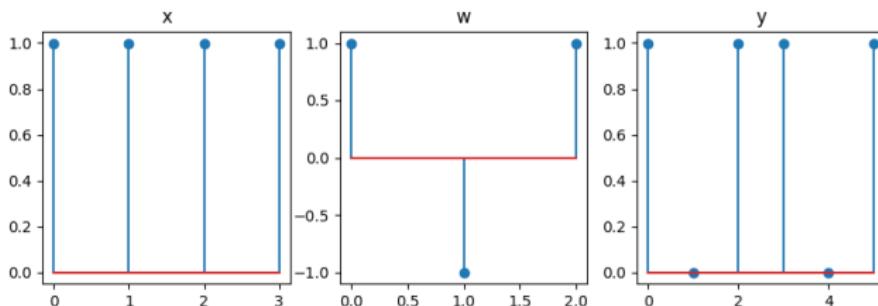
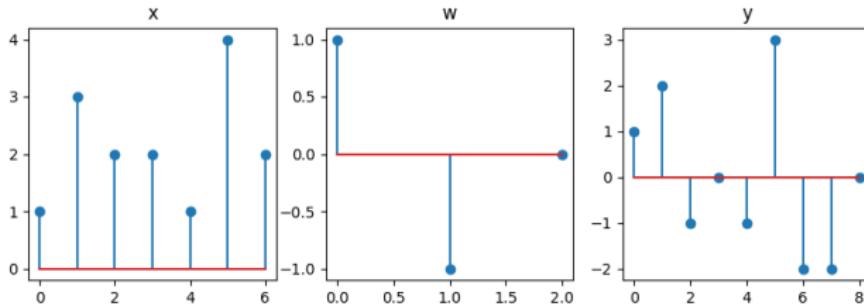


Image Convolution

$$y = x * w$$

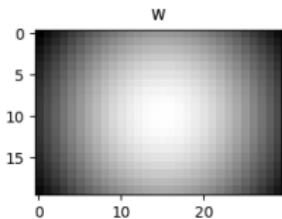
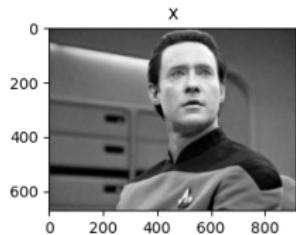


Image Convolution

$$y = x * w$$

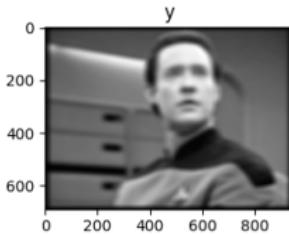
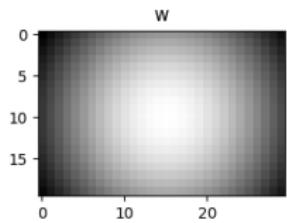
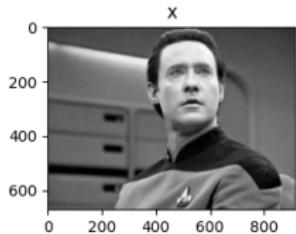


Image Convolution

$$y = x * w$$

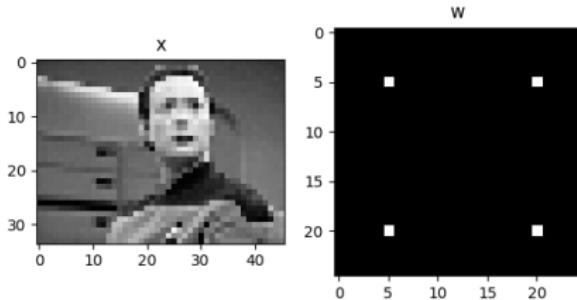
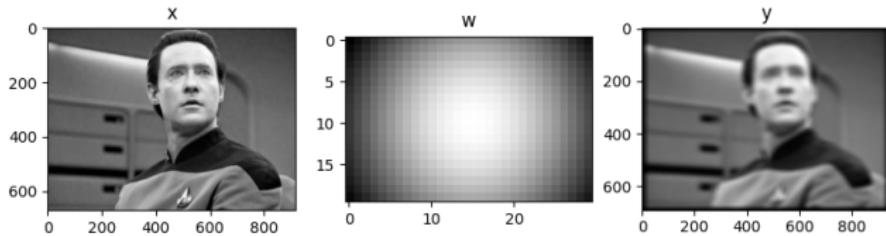
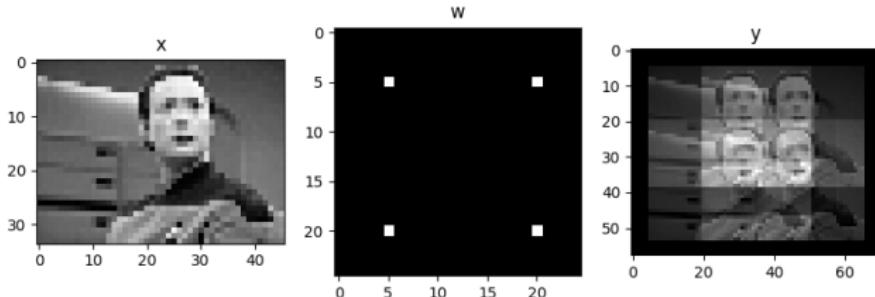
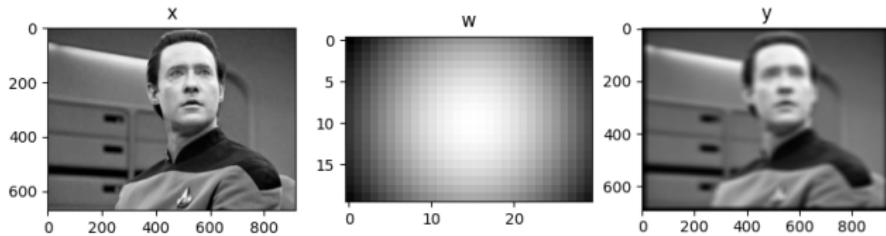


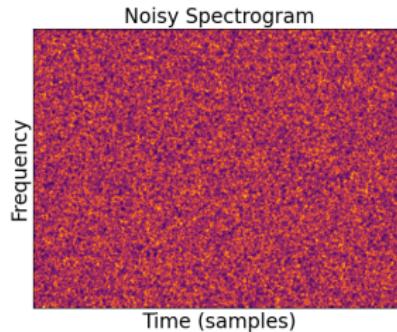
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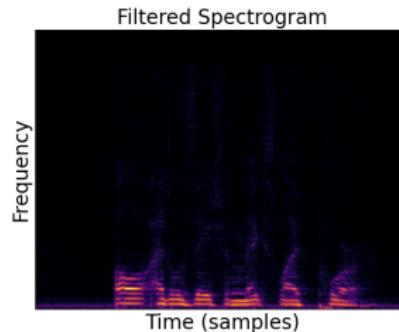
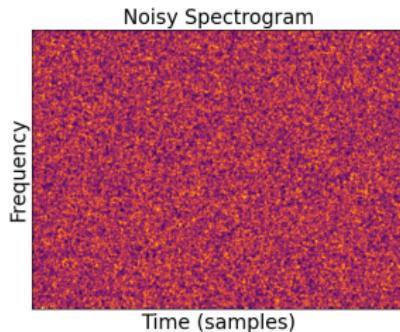
Filtering Audio with Convolution

- Consider this noisy audio. Listen.
- The filtered version with an averaging kernel. Listen.
 - ▶ La version filtrée avec un noyau de moyenne.



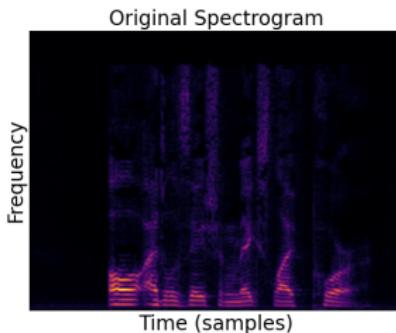
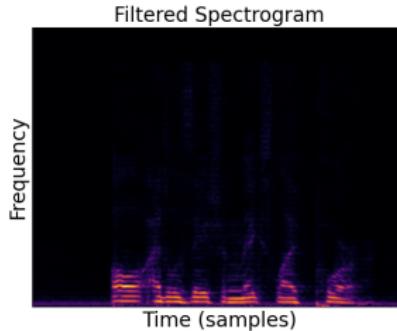
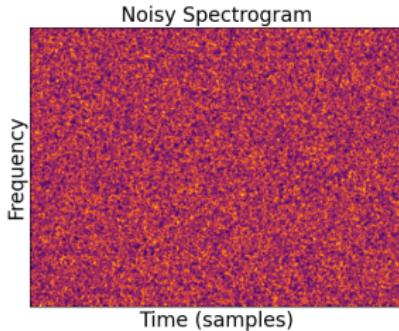
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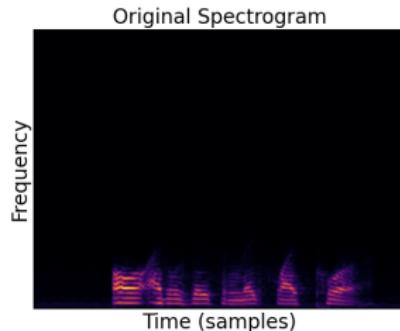
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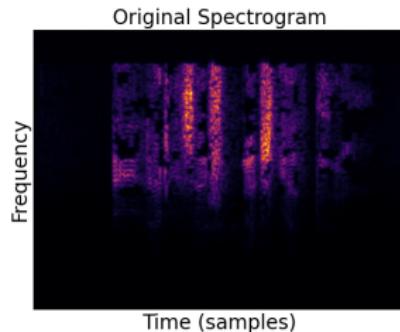


More on filtering

- Low-pass filtering (cut-off at 1kHz) Listen.

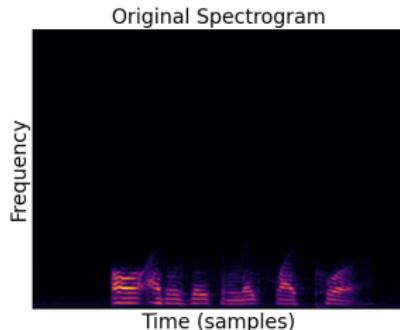


- High-pass filtering (cut-off at 4kHz) Listen.

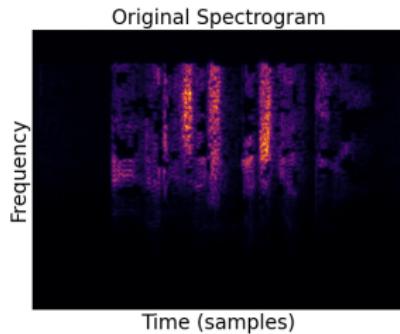


More on filtering

- Low-pass filtering (cut-off at 1kHz) Listen.



- High-pass filtering (cut-off at 4kHz) Listen.



- There are also band-stop, band-pass filtering..
 - ▶ Il existe aussi band-stop, band-pass filtering.

Fast Convolution

- Convolution is an $\mathcal{O}(N^2)$ operation.
- However we can use FFT to get it down to $\mathcal{O}(N \log N)$.
 - ▶ On peut prendre avantage de le FFT pour avoir une complexité de $\mathcal{O}(N \log N)$.
- Convolution in time domain, is multiplication in the Fourier Domain.
 - ▶ Convolution dans le domaine de temps est multiplication dans le domaine de Fourier.

$$\begin{aligned} F(x * w) &= Fx \odot Fw \\ \rightarrow x * w &= F^{-1}(Fx \odot Fw) \end{aligned}$$

- Use FFT whenever you can!

Convolution as a Matrix Multiplication

- Convolution can be expressed as a matrix multiplication.
 - ▶ Convolution peut-être exprimée comme une multiplication de matrices.

$$w * x = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} w_1 & 0 & 0 & 0 \\ w_2 & w_1 & 0 & 0 \\ 0 & w_2 & w_1 & 0 \\ 0 & 0 & w_2 & w_1 \\ 0 & 0 & 0 & w_2 \end{bmatrix}}_{:=C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

- C is a Circulant / Toeplitz matrix.

Table of Contents

Signal Representations

- Fourier Series

- DFT

- Time-Frequency Representation

Analog to Digital Conversion

- Quantization

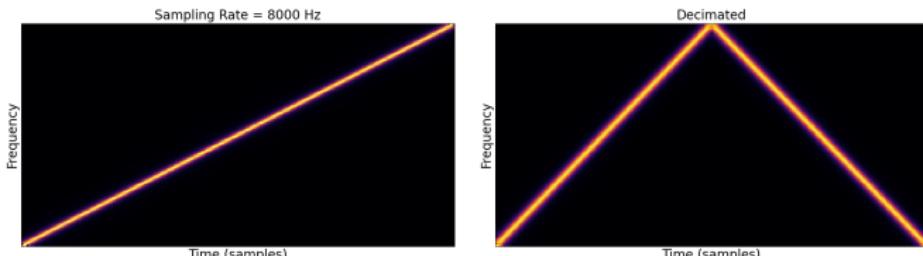
- Sampling

Convolution

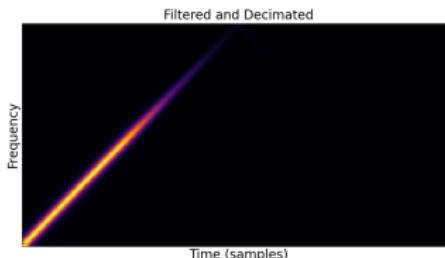
Re-Sampling

Downsampling

- Let's say we want to downsample. Simple decimation introduces aliasing.
- Si on sous-échantillonne, simple décimation introduit de l'aliasing.



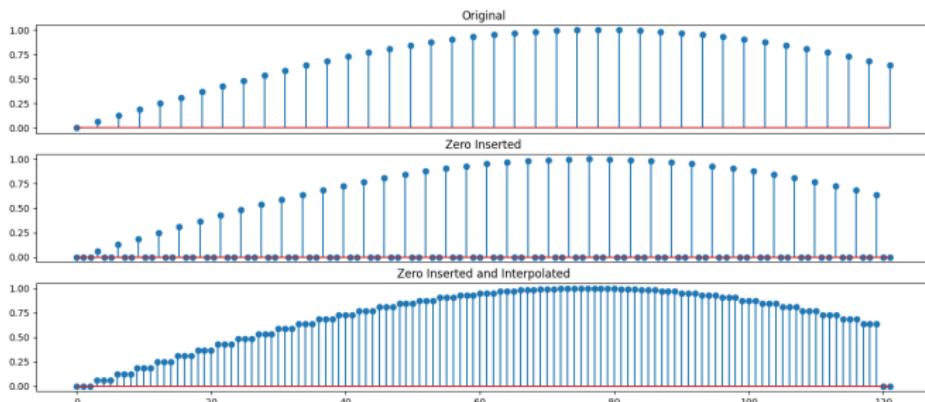
- The solution is to first filter and then downsample.
 - La solution est de filtrer et puis downsample



- We lose the high-freqs, but we at least avoid aliasing.
 - On perd les hautes fréquences mais on moins pas d'aliasing.

Upsampling

- To upsample by a factor of L , the procedure is to first insert $L - 1$ zeros, and then to interpolate.
 - Pour upsampler de la facteur L , le procedure est de d'abord inserer $L - 1$ zéros, et puis faire de l'interpolation.



Recap

- Signal Representations
 - ▶ Time, Frequency, Time-Frequency
- Discrete Fourier Transform
- Short-Time Fourier Transform
- Sampling, Resampling
- Convolution

Further Reading

- <http://www.dspsguide.com/pdfbook.htm> – nice free book, check it out.
- <https://dspguru.com/dsp/howtos/>

Next week

- We get started with machine learning (sometimes for signal processing)