Trigonometric Identities

$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$
$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$
$$\cos(\alpha)\cos(\beta)=\frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2} \qquad \sin(\alpha)\cos(\beta)=\frac{\sin(\alpha-\beta)+\sin(\alpha+\beta)}{2}$$
$$\sin(\alpha)\sin(\beta)=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$$
$$\sin(\alpha)=\frac{1}{j2}\left[e^{j\alpha}-e^{-j\alpha}\right] \qquad \cos(\alpha)=\frac{1}{2}\left[e^{j\alpha}+e^{-j\alpha}\right]$$

Sum of a series

$$\sum_{n=0}^{N-1}a^n=\frac{1-a^N}{1-a} \quad \sum_{n=1}^{N}n=\frac{N(N+1)}{2} \quad \sum_{n=1}^{N}n^2=\frac{N(N+1)(2N+1)}{6}$$
$$\sum_{n=-N}^{N}\cos(2\alpha n)=\frac{\sin(\alpha(2N+1))}{\sin(\alpha)} \qquad \sum_{n=0}^{N}\sin(2\alpha n)=\frac{\sin(\alpha N)\sin(\alpha(N+1))}{\sin(\alpha)}$$
$$\sum_{n=-N}^{N}\cos(\alpha n)e^{-j\beta n}=\frac{\cos(\alpha(N+1))\cos(\beta N)-\cos(\alpha N)\cos(\beta(N+1))}{\cos(\alpha)-\cos(\beta)}$$

Fourier Transform Formulae

Fourier series	$c_k=F_0\int_{-1/2F_0}^{1/2F_0}x(t)e^{-j2\pi kF_0t}dt$	$x(t)=\sum_{k=-\infty}^{\infty}c_ke^{j2\pi kF_0t}$
Fourier transform	$X(F)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi Ft}dt$	$x(t)=\int_{-\infty}^{\infty}X(F)e^{j2\pi Ft}dF$
Discrete-time Fourier transform	$X(\omega)=\sum_{n=-\infty}^{\infty}x(n)e^{-jn\omega}$	$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\omega)e^{jn\omega}d\omega$
Discrete-time Fourier series	$c_k=\frac{1}{N}\sum_{n=0}^{N-1}x(n)e^{-j2\pi kn/N}$	$x(n)=\sum_{k=0}^{N-1}c_ke^{j2\pi kn/N}$
Discrete Fourier transform	$X(k)=\sum_{n=0}^{N-1}x(n)e^{-j2\pi kn/N}$	$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{j2\pi kn/N}$

Correlation functions

$$r_{xy}=\sum_{-\infty}^{\infty}x(n)y(n-l) \qquad r_{xy}(l)=\lim_{M\to\infty}\frac{1}{2M+1}\sum_{n=-M}^{M}x(n)y(n-l)$$

Power density spectrum

$$\Gamma_{xx}(f)=\sum_{-\infty}^{\infty}\gamma_{xx}(m)e^{-j2\pi fm} \qquad \gamma_{xx}(m)=\int_{-1/2}^{1/2}\Gamma_{xx}(f)e^{j2\pi fm}df$$

Bartlett window
$$w_n = 1 - \frac{|n - M|}{M + 1}$$

Hanning window
$$w_n = \frac{1}{2}\left(1 - \cos\left(\frac{2n\pi}{M-1}\right)\right)$$

Hamming window
$$w_n = 0.54 - 0.46\cos\left(\frac{2n\pi}{M-1}\right)$$

Blackman window
$$w_n = 0.42 - 0.5\cos\left(\frac{2n\pi}{M-1}\right) + 0.08\cos\left(\frac{4n\pi}{M-1}\right)$$

Kaiser window
$$w_n = \frac{1}{I_0(\beta)} I_0\left(\beta\sqrt{1 - \left(\frac{2n}{M-1} - 1\right)^2}\right)$$

Window characteristics

Window	Transition Band	Stopband rejection (dB)
Rectangular	$\frac{1.8\pi}{M}$	21
Hanning	$\frac{6.2\pi}{M}$	44
Hamming	$\frac{6.6\pi}{M}$	53
Kaiser $\beta = 6$	$\frac{8\pi}{M}$	63
Blackman	$\frac{11\pi}{M}$	74
Kaiser $\beta = 9$	$\frac{11.4\pi}{M}$	90

Window	Resolution -3 dB Bandwidth	Equivalent Noise Bandwidth	Coherent gain (dB)	Peak sidelobe level (dB)	Asymptotic roll-off (dB/octave)
Rectangular	$\frac{1.78\pi}{M}$	$\frac{2\pi}{M}$	0	-13	-6
Triangular	$\frac{2.56\pi}{M}$	$\frac{2.66\pi}{M}$	-3	-27	-12
Parzen	$\frac{3.64\pi}{M}$	$\frac{3.83\pi}{M}$	-4.3	-53	-24
Hanning	$\frac{2.8\pi}{M}$	$\frac{3\pi}{M}$	-3	-31	-18
Hamming	$\frac{2.6\pi}{M}$	$\frac{2.72\pi}{M}$	-2.7	-43	-6
Bohman	$\frac{3.41\pi}{M}$	$\frac{3.58\pi}{M}$	-3.9	-46	-24
Blackman	$\frac{3.04\pi}{M}$	$\frac{3.14\pi}{M}$	-3.8	-58	-6
Dolph-Chebyshev	$\frac{2.88\pi}{M}$	$\frac{3.02\pi}{M}$	-3.2	-60	0
Kaiser-Bessel	$\frac{3.14\pi}{M}$	$\frac{3.3\pi}{M}$	-3.6	-57	-6

FIR filter design

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{RD}(\omega)e^{j\omega\left(n-\left(\frac{M-1}{2}\right)\right)}d\omega$$

$\alpha = 0\ \beta = 0$	$h(n) = \frac{1}{M}\left\{G(0) + 2\sum_{k=1}^{U} G(k)\cos\left(\frac{2\pi k}{M}\left(n+\frac{1}{2}\right)\right)\right\}$
$\alpha = \frac{1}{2}\ \beta = 0$	$h(n) = \frac{2}{M}\sum_{k=0}^{U} G(k+\alpha)\sin\left(\frac{2\pi(k+\alpha)}{M}\left(n+\frac{1}{2}\right)\right)$
$\alpha = 0\ \beta = 1$	$h(n) = \frac{1}{M}\left\{(-1)^{n+1}G(M/2) - 2\sum_{k=1}^{V} G(k)\sin\left(\frac{2\pi k}{M}\left(n+\frac{1}{2}\right)\right)\right\}$
$\alpha = \frac{1}{2}\ \beta = 1$	$h(n) = \frac{2}{M}\sum_{k=0}^{V} G(k+\alpha)\cos\left(\frac{2\pi(k+\alpha)}{M}\left(n+\frac{1}{2}\right)\right)$
$G(k+\alpha) = (-1)^k H_r\left(\frac{2\pi(k+\alpha)}{M}\right)$ $U = \left\lfloor\frac{M-1}{2}\right\rfloor$ $V = \left\lceil\frac{M-1}{2}\right\rceil$	

Transition coefficients for 16 tap filter with $\alpha = 0$

BW	Minimax (dB)	T1
1	-42.41981868	0.43306932
2	-41.38993328	0.41702849
3	-41.24929950	0.41001589
4	-41.66355813	0.40474097
5	-43.21753226	0.39666905
6	-49.33321372	0.37587659

BW	Minimax (dB)	T1	T2
1	-71.59789962	0.09225582	0.58621580
2	-70.66810992	0.09938976	0.58818920
3	-72.23796597	0.09528050	0.57815395
4	-76.63701678	0.08476728	0.55924120
5	-89.72938985	0.06534978	0.52474095

BW	Minimax (dB)	T1	T2	T3
1	-104.19077428	0.01191092	0.17390102	0.66090319
2	-105.99219722	0.01294020	0.18866556	0.67271873
3	-115.06048549	0.01039931	0.17536927	0.65858025
4	-134.48576256	0.00625291	0.14601962	0.62676044

Analogue to digital converter

$$\mathrm{SQNR} = 6.02b + 10.79 - 20\log_{10}\frac{R}{\sqrt{P_x}} + 10(2L+1)\log_{10} O$$

Minimum variance spectral estimation

$$P_{xx}^{MV}(f) = \frac{1}{\mathbf{E}^H(f)\mathbf{R}_{xx}^{-1}\mathbf{E}(f)}$$

Estimate of mean

$$\widehat{m_x} = \frac{1}{2N+1}\sum_{n=-N}^{N} x(n)$$

$$\mathrm{Var}(\widehat{m_x}) = \frac{1}{2N+1}\sum_{l=-2N}^{2N}\left(1-\frac{|l|}{2N+1}\right)c_{xx}(l)$$

$$c_{xx}(l) = E\left[\left(x(n)-E\left(X(n)\right)\right)\left(x(n-l)-E\left(X(n)\right)\right)\right]$$

Matrices

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\,\mathrm{adj}(\mathbf{A})$$

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow$$

$$\det(\mathbf{B}) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei-fh) - b(di-fg) + c(dh-eg)$$

$$\mathrm{adj}(\mathbf{B}) = \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ e & f \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}^T$$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Indefinite Integrals

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{a\,dx}{\sqrt{b^2-x^2}} = a\sin^{-1}\left(\frac{x}{b}\right)$$

$$\int \frac{-a\,dx}{\sqrt{b^2-x^2}} = a\cos^{-1}\left(\frac{x}{b}\right)$$

$$\int \frac{a\,dx}{\sqrt{x^2-b^2}} = a\ln\left(x+\sqrt{x^2-b^2}\right)$$

Definite Integrals

$$\int_0^\infty \frac{\sin^2(ax)}{x^2}\,dx = |a|\frac{\pi}{2}$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left(-\frac{t^2}{2}\right)dt = 1$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\left(1-a^2\right)e^{j\omega m}}{1+a^2-2a\cos(\omega)}d\omega = a^{|m|}\;;|a|<1$$

$$\int \sin^m(ax+b)\,dx = -\frac{\sin^{m-1}(ax+b)\cos(ax+b)}{ma} + \frac{m-1}{m}\int \sin^{m-2}(ax+b)\,dx$$

Modified Bessel function of the first kind

$$I_0(z) = \sum_{k=0}^{\infty} \frac{\left(\frac{z^2}{4}\right)^k}{(k!)^2}$$

Trigonometric transform Identity

$$\cos(n\omega_0) \overset{F}{\underset{F^{-1}}{\longleftrightarrow}} \pi\left(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right)$$

Gaussian process

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad E[X]=\mu \qquad \mathrm{Var}(X)=\sigma^2$$

Normal distribution $$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.01128	0.02256	0.03384	0.04511	0.05637	0.06762	0.07886	0.09008	0.10128
0.1	0.11246	0.12362	0.13476	0.14587	0.15695	0.16800	0.17901	0.18999	0.20094	0.21184
0.2	0.22270	0.23352	0.24430	0.25502	0.26570	0.27633	0.28690	0.29742	0.30788	0.31828
0.3	0.32863	0.33891	0.34913	0.35928	0.36936	0.37938	0.38933	0.39921	0.40901	0.41874
0.4	0.42839	0.43797	0.44747	0.45689	0.46623	0.47548	0.48466	0.49375	0.50275	0.51167
0.5	0.52050	0.52924	0.53790	0.54646	0.55494	0.56332	0.57162	0.57982	0.58792	0.59594
0.6	0.60386	0.61168	0.61941	0.62705	0.63459	0.64203	0.64938	0.65663	0.66378	0.67084
0.7	0.67780	0.68467	0.69143	0.69810	0.70468	0.71116	0.71754	0.72382	0.73001	0.73610
0.8	0.74210	0.74800	0.75381	0.75952	0.76514	0.77067	0.77610	0.78144	0.78669	0.79184
0.9	0.79691	0.80188	0.80677	0.81156	0.81627	0.82089	0.82542	0.82987	0.83423	0.83851
1.0	0.84270	0.84681	0.85084	0.85478	0.85865	0.86244	0.86614	0.86977	0.87333	0.87680
1.1	0.88021	0.88353	0.88679	0.88997	0.89308	0.89612	0.89910	0.90200	0.90484	0.90761
1.2	0.91031	0.91296	0.91553	0.91805	0.92051	0.92290	0.92524	0.92751	0.92973	0.93190
1.3	0.93401	0.93606	0.93807	0.94002	0.94191	0.94376	0.94556	0.94731	0.94902	0.95067
1.4	0.95229	0.95385	0.95538	0.95686	0.95830	0.95970	0.96105	0.96237	0.96365	0.96490
1.5	0.96611	0.96728	0.96841	0.96952	0.97059	0.97162	0.97263	0.97360	0.97455	0.97546
1.6	0.97635	0.97721	0.97804	0.97884	0.97962	0.98038	0.98110	0.98181	0.98249	0.98315
1.7	0.98379	0.98441	0.98500	0.98558	0.98613	0.98667	0.98719	0.98769	0.98817	0.98864
1.8	0.98909	0.98952	0.98994	0.99035	0.99074	0.99111	0.99147	0.99182	0.99216	0.99248
1.9	0.99279	0.99309	0.99338	0.99366	0.99392	0.99418	0.99443	0.99466	0.99489	0.99511
2.0	0.99532	0.99552	0.99572	0.99591	0.99609	0.99626	0.99642	0.99658	0.99673	0.99688
2.1	0.99702	0.99715	0.99728	0.99741	0.99753	0.99764	0.99775	0.99785	0.99795	0.99805
2.2	0.99814	0.99822	0.99831	0.99839	0.99846	0.99854	0.99861	0.99867	0.99874	0.99880
2.3	0.99886	0.99891	0.99897	0.99902	0.99906	0.99911	0.99915	0.99920	0.99924	0.99928
2.4	0.99931	0.99935	0.99938	0.99941	0.99944	0.99947	0.99950	0.99952	0.99955	0.99957
2.5	0.99959	0.99961	0.99963	0.99965	0.99967	0.99969	0.99971	0.99972	0.99974	0.99975
2.6	0.99976	0.99978	0.99979	0.99980	0.99981	0.99982	0.99983	0.99984	0.99985	0.99986
2.7	0.99987	0.99987	0.99988	0.99989	0.99989	0.99990	0.99991	0.99991	0.99992	0.99992
2.8	0.99992	0.99993	0.99993	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	0.99996
2.9	0.99996	0.99996	0.99996	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998
3.0	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999	0.99999

Table entries correspond to z = row heading + column heading