THE UNIVERSITY *of* EDINBURGH

Analogue IC Design

Differential Amplifiers

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Dr Danial Chitnis

d.chitnis@ed.ac.uk

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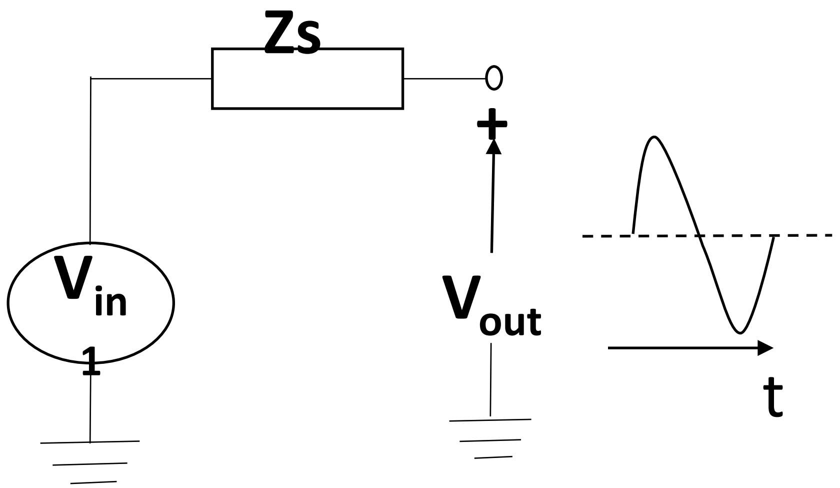
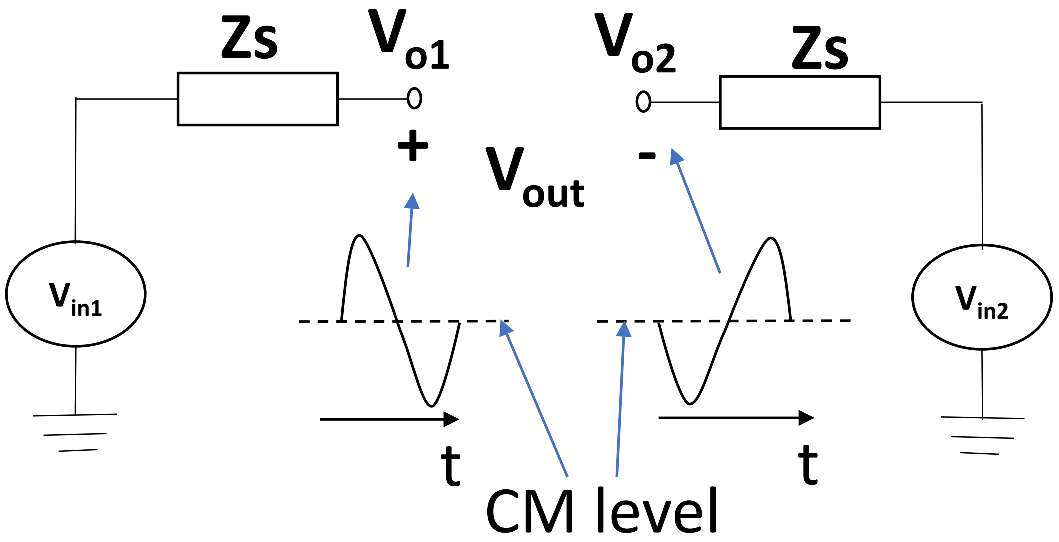
Differential Signalling



Single-ended signal – Measured with respect to a fixed potential (usually ground)

Differential Signalling

- A differential signal: Measured between two nodes that have equal and opposite signal excursions around a fixed potential.
- The ‘centre’ potential in differential signalling is called the ‘common-mode’’ (CM level).



$$V_{out} = (V_{o1} - V_{o2})$$

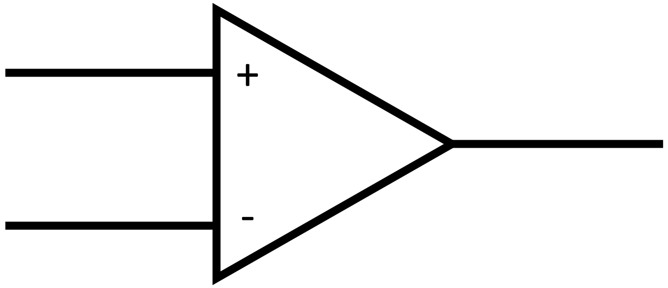
Differential Signalling

- A differential signal: Measured between two nodes that have equal and opposite signal excursions around a fixed potential.

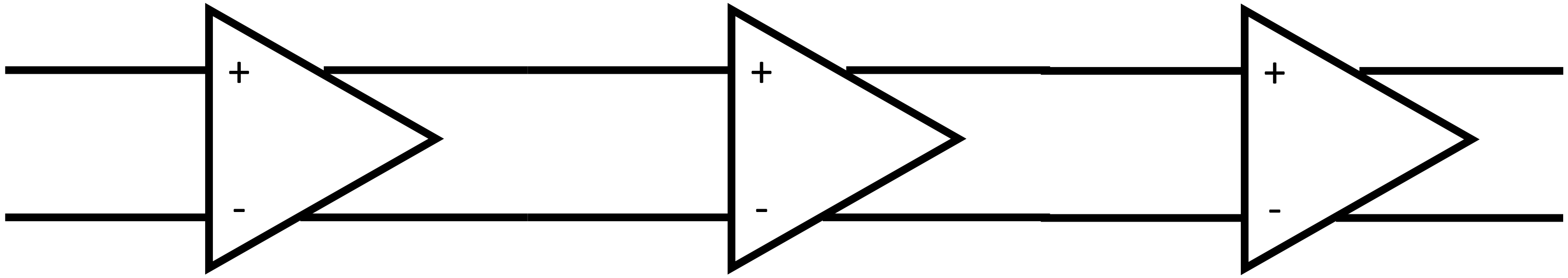
- The ‘centre’ potential in differential signalling is called the ‘common-mode’’ (CM level).

Differential Signalling

- Differential input, single ended output



- Differential input, differential output

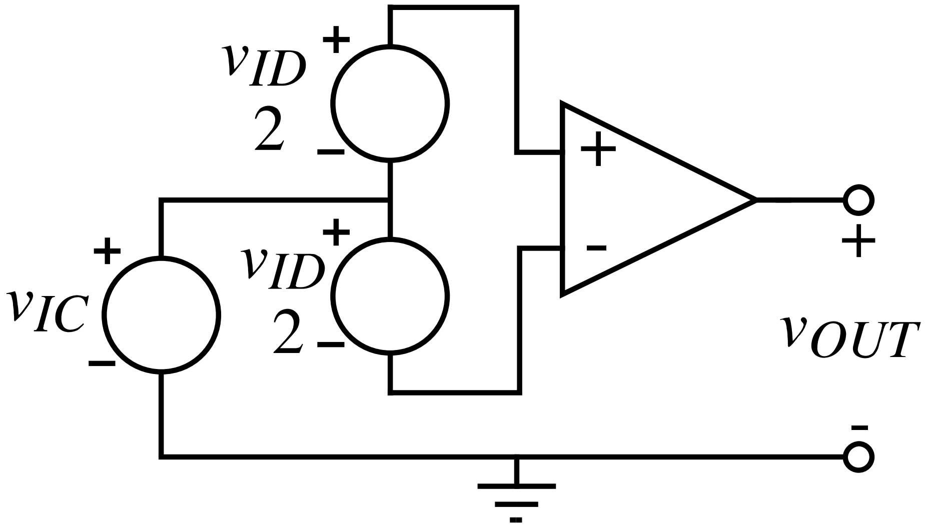


Differential Amps

- v_{ID} = v1 – v2
- v_{IC} = (v1 + v2) /2

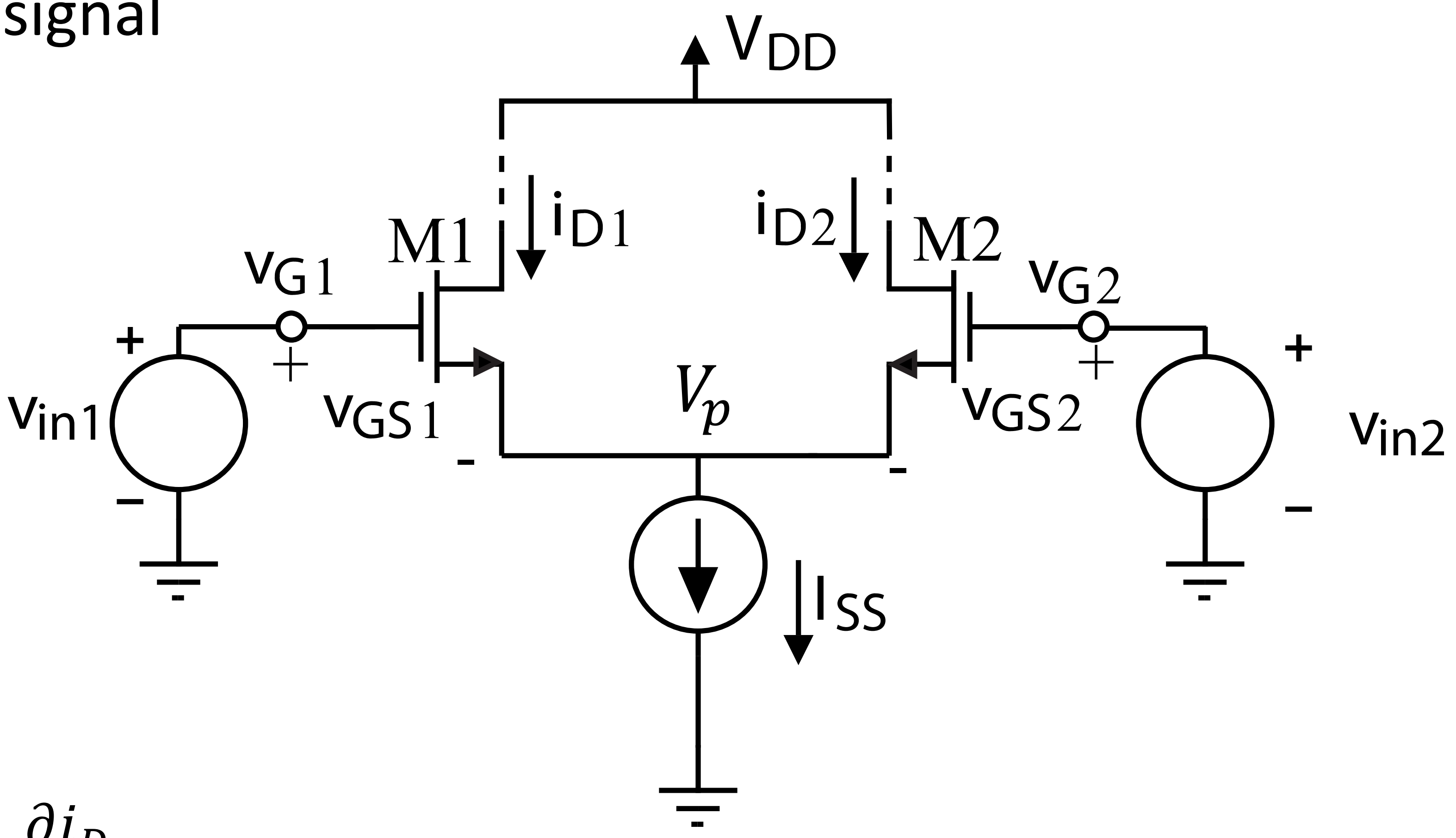
- AV_{ID} = differential-mode voltage gain
- AV_{CM} = common-mode voltage gain

$$CMRR = \left|\frac{AV_{ID}}{AV_{CM}}\right|$$



Differential Amps

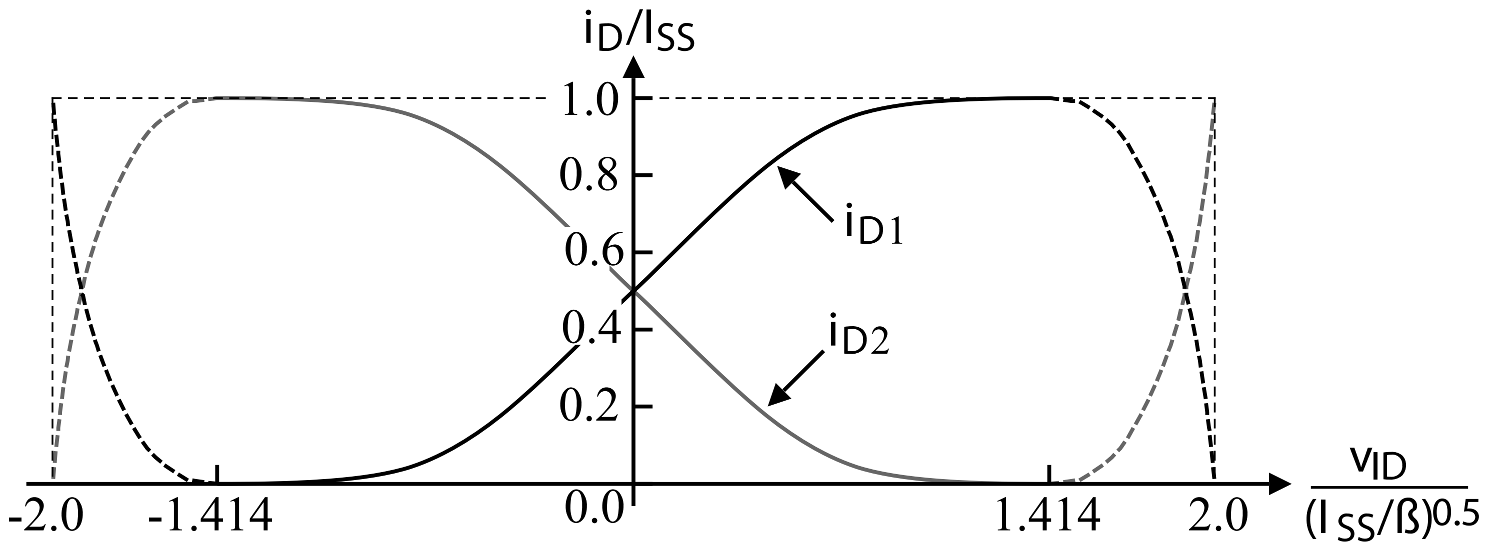
- Generic diff amp – large signal
- viD = vin1 – vin2
  =vgs1 – vgs2
- Is = iD1 + iD2



$$v_{gs} = V_T + \sqrt{\frac{2i_D}{\beta}}$$

$$g_{md} = \frac{\partial i_D}{\partial v_{iD}} \qquad g_{mc} = \frac{\partial i_D}{\partial v_{cm}}$$

Differential Amps



$$i_{D1} = \frac{I_{SS}}{2} + \frac{I_{SS}}{2}\left(\frac{\beta v_{ID}^2}{I_{SS}} - \frac{\beta^2 v_{ID}^4}{4I_{SS}^2}\right)^{1/2}$$

$$i_{D2} = \frac{I_{SS}}{2} + \frac{I_{SS}}{2}\left(\frac{\beta v_{ID}^2}{I_{SS}} - \frac{\beta^2 v_{ID}^4}{4I_{SS}^2}\right)^{1/2}$$

$$gm = \frac{di_{D1}}{dv_{ID}}(V_{ID} = 0) = \sqrt{\frac{\beta I_{SS}}{4}} = \sqrt{\frac{K'_1 I_{SS} W_1}{4L_1}}$$

Differential Amps

- From the large signal model we can conclude that:

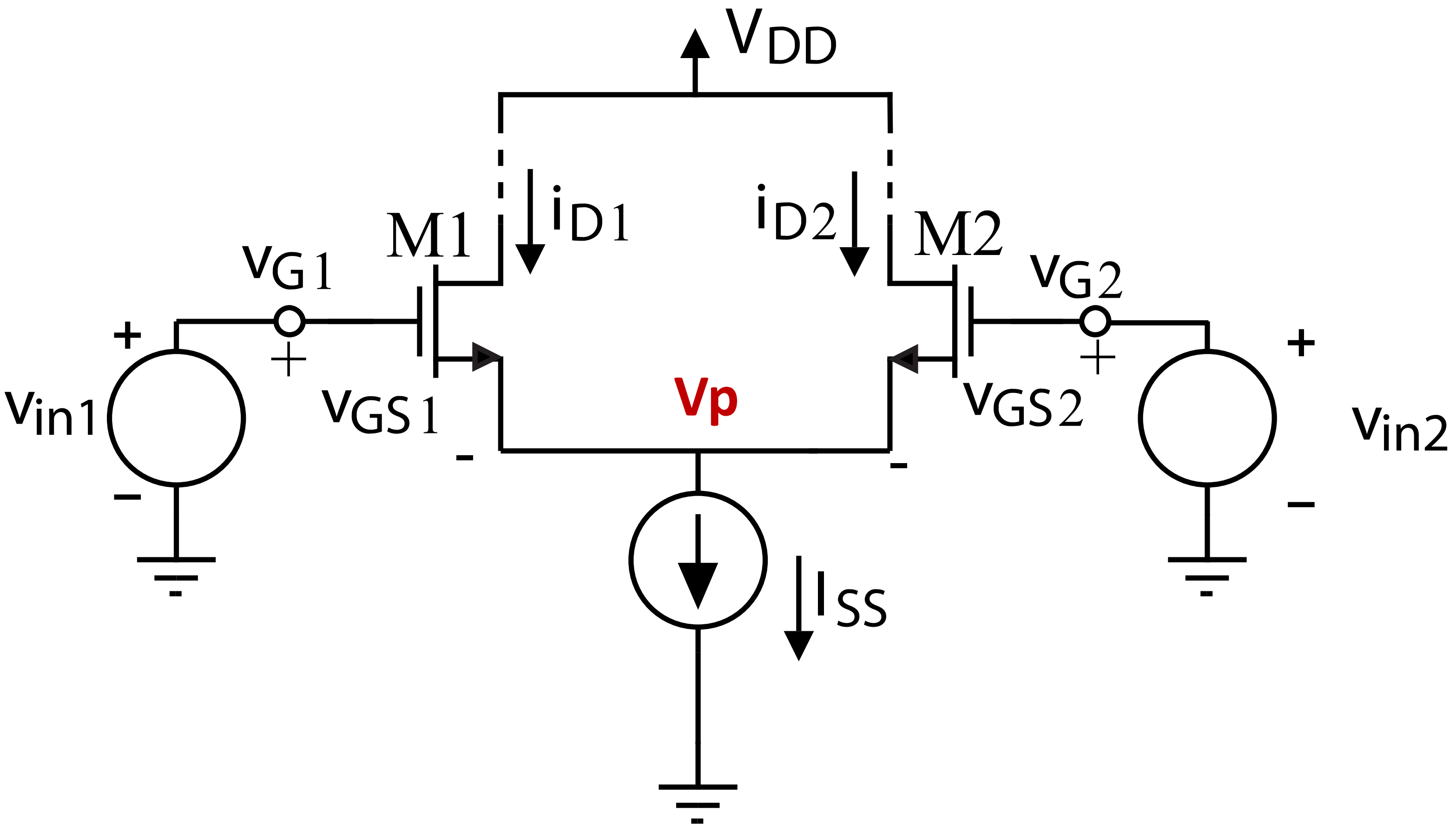
- The voltage swing is: $\Delta v_{swing} = 2\sqrt{2}\sqrt{\frac{I_{ss}}{\beta}}$

- The condition for large signal equations: $2\sqrt{\frac{I_{ss}}{\beta}} > v_{iD}$

- If $0 \ll v_{iD} \ll 2\sqrt{\frac{I_{ss}}{\beta}}$ Then $g_{md} = \frac{\partial i_D}{\partial v_{iD}} = \sqrt{\frac{I_{ss}\beta}{4}}$

Differential Amps

- What is Vp and is it changing with input voltages?
- Is = iD1 + iD2
- vgs1 = vin1 – Vp
- Vgs2 = vin2 – Vp



Differential Amps

- What is the common mode gain?

$$Vp = V_{CM} - V_T + \frac{1}{2}\sqrt{-v_{iD}^2 + \frac{4I_S}{\beta}}$$

$$0 < v_{iD} \ll \sqrt{\frac{4I_S}{\beta}} \quad \Rightarrow \quad Vp = V_{CM} - V_T + \frac{1}{2}\sqrt{\frac{4I_S}{\beta}}$$

$$\Rightarrow \quad g_{mc} = \frac{\partial i_D}{\partial v_{cm}} = 0$$

Differential Amps

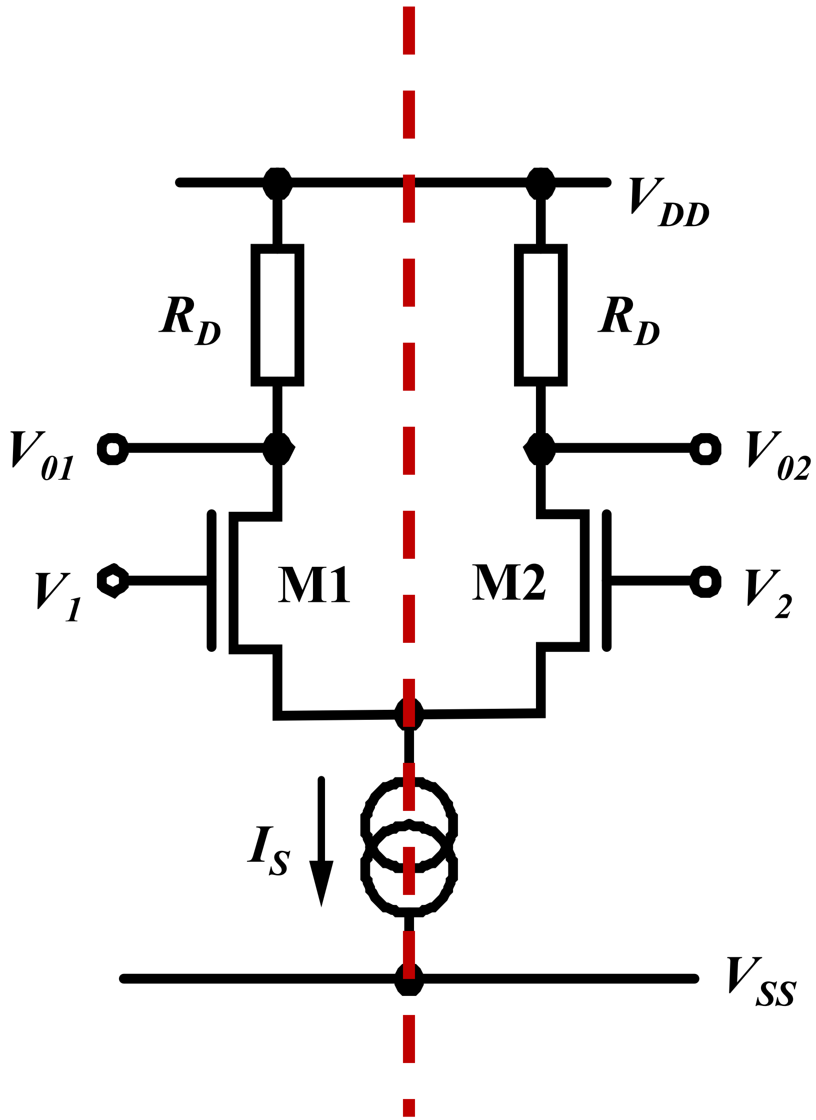
For small signal Analysis:

- Vp changes with Vcm
- Vp is constant with changing ViD --> Vp is virtual ground for diff analysis

Differential Amps

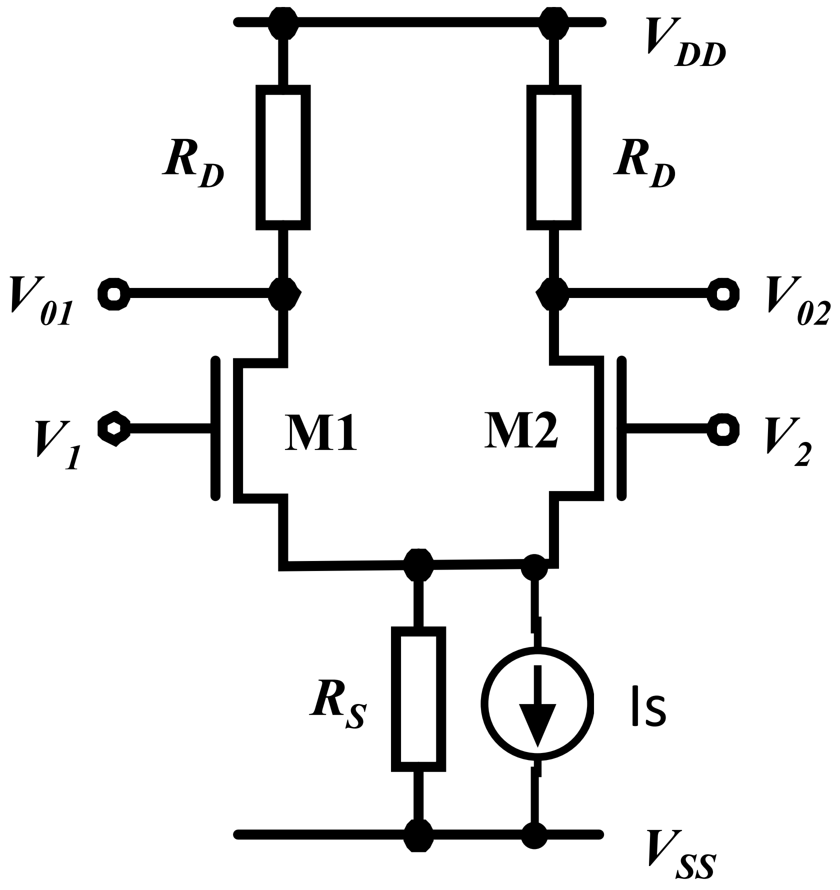
- Small signal analysis

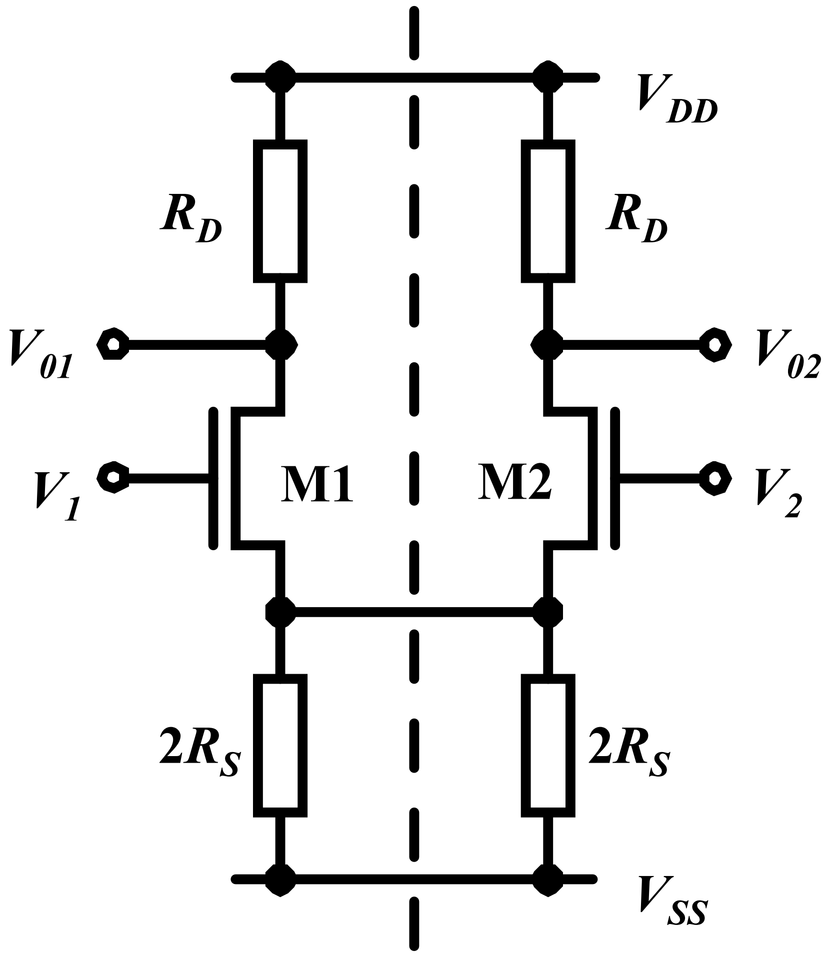
- Half circuit due to symmetry



Differential Amps

- Small-signal common mode

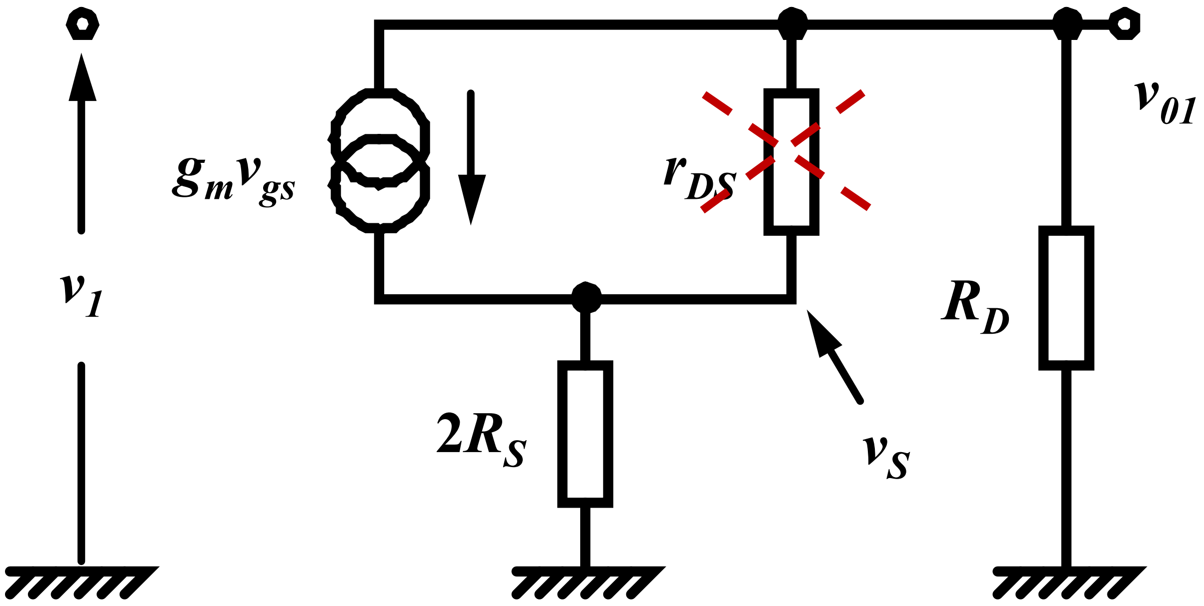




Differential Amps

- DC current source → high impedance
- DC voltage source → low impedance

$$AV_{CM} = \frac{v_{out}}{v_{cm}} = \frac{-R_D}{\frac{1}{g_m} + 2R_S}$$



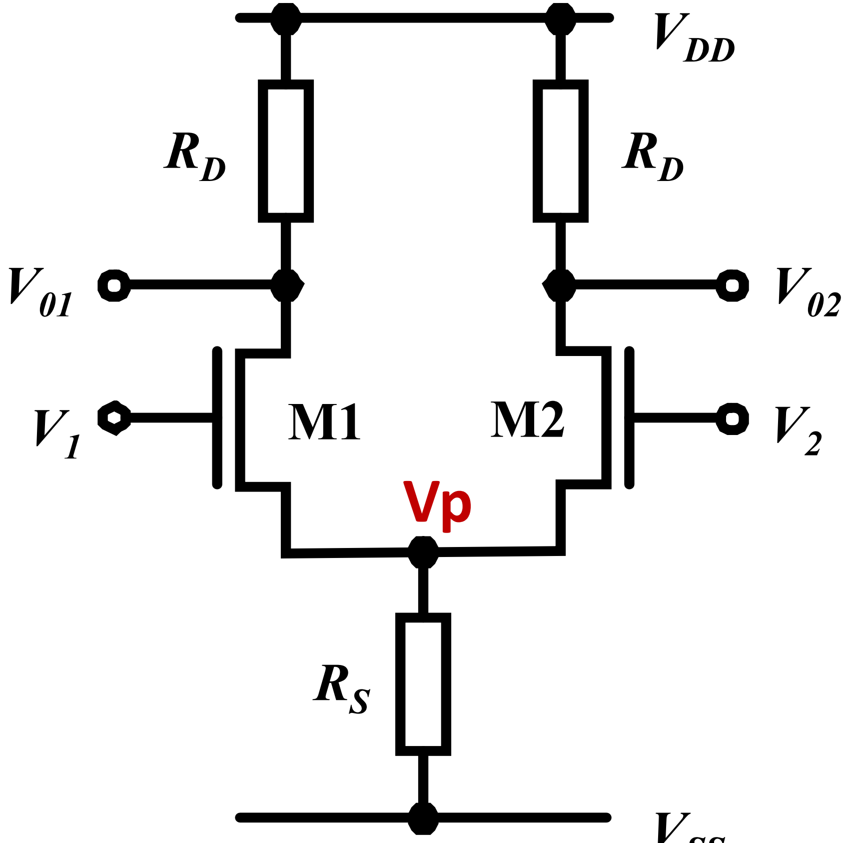
Differential Amps

- Small signal differential gain

- Vin1,2 = Vcm ± (vid /2)
- Vp = const for diff

$$\frac{v_{o1} - v_{o2}}{v_{iD}} = -g_m R_D$$

$$g_{md} = \frac{\partial i_D}{\partial v_{iD}} = \sqrt{\frac{\beta I_s}{4}}$$

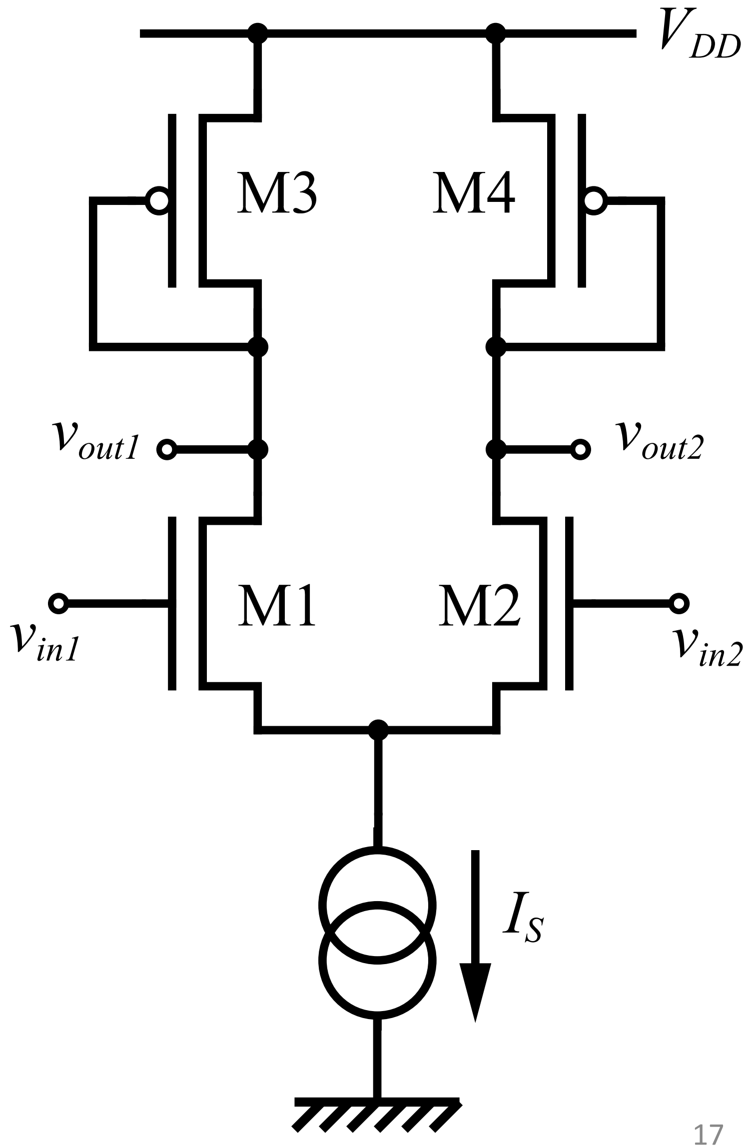


Diode connected load

Diode connected load

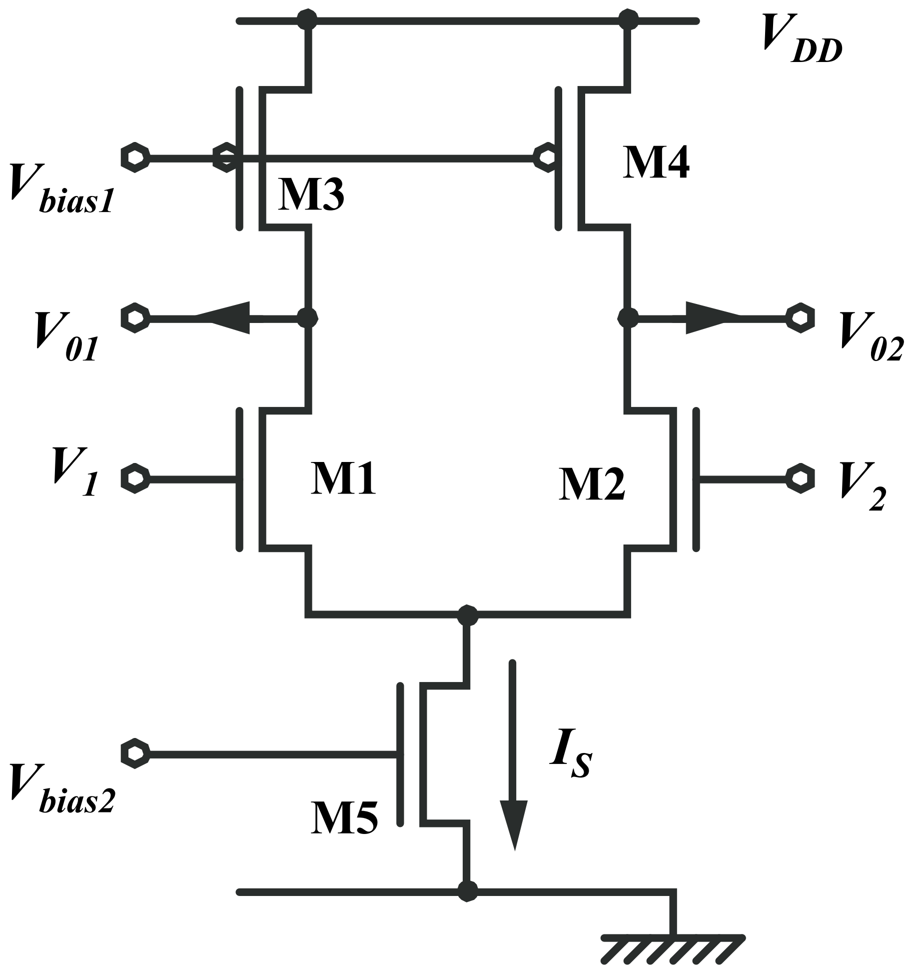
$$Av_d = \frac{v_{od}}{v_{id}} \approx -\frac{g_{m2}}{g_{m3}} = -\sqrt{\frac{\mu_N (W/L)_N}{\mu_P (W/L)_P}}$$

$$Av_{cm} = \frac{v_{ocm}}{v_{icm}} \approx -\frac{r_{o4}}{r_s}$$



Current Source load

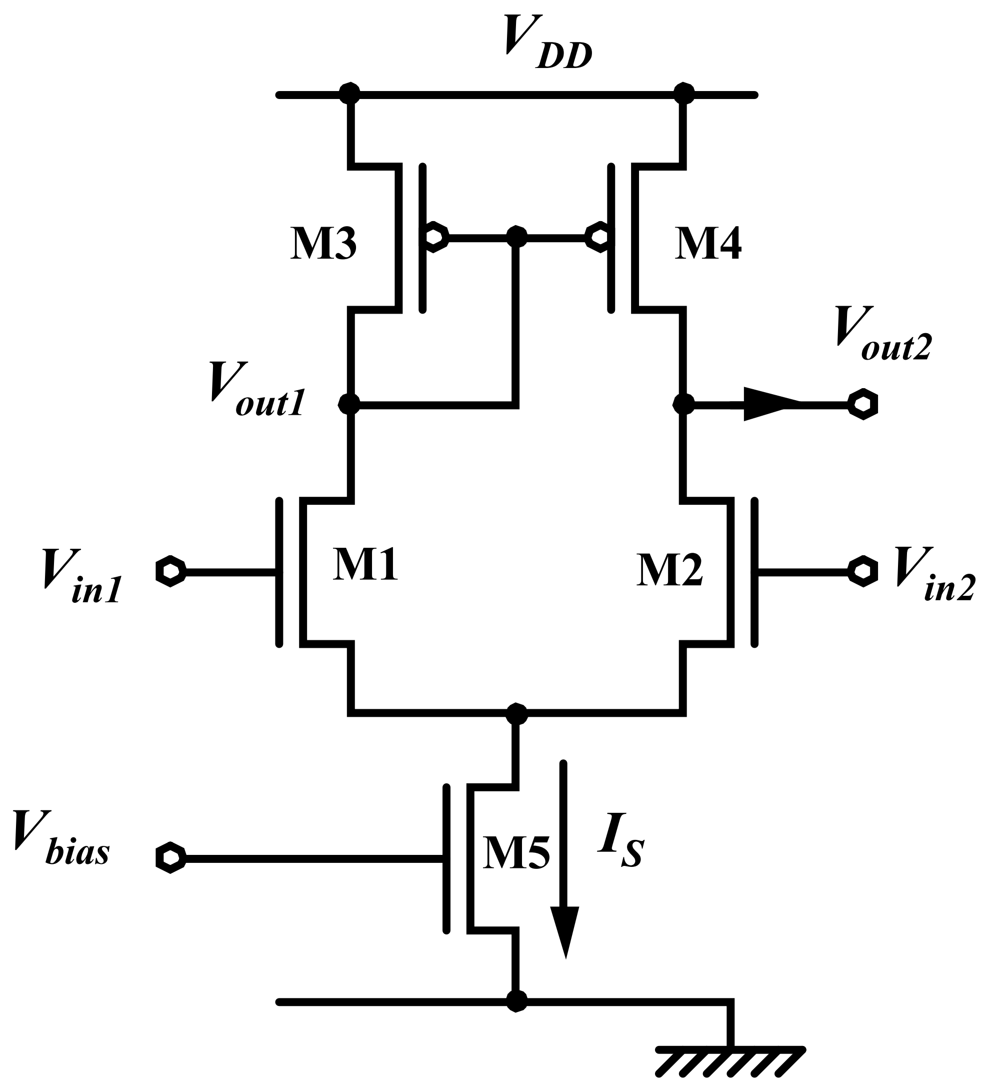
- Current source load
- Higher gain than diode connected
- This would be near impossible to bias correctly
- Is = I3 + I4
- Requires excellent matching



Current Mirror Load

- Current mirror load

- Guarantees that I3 = I4

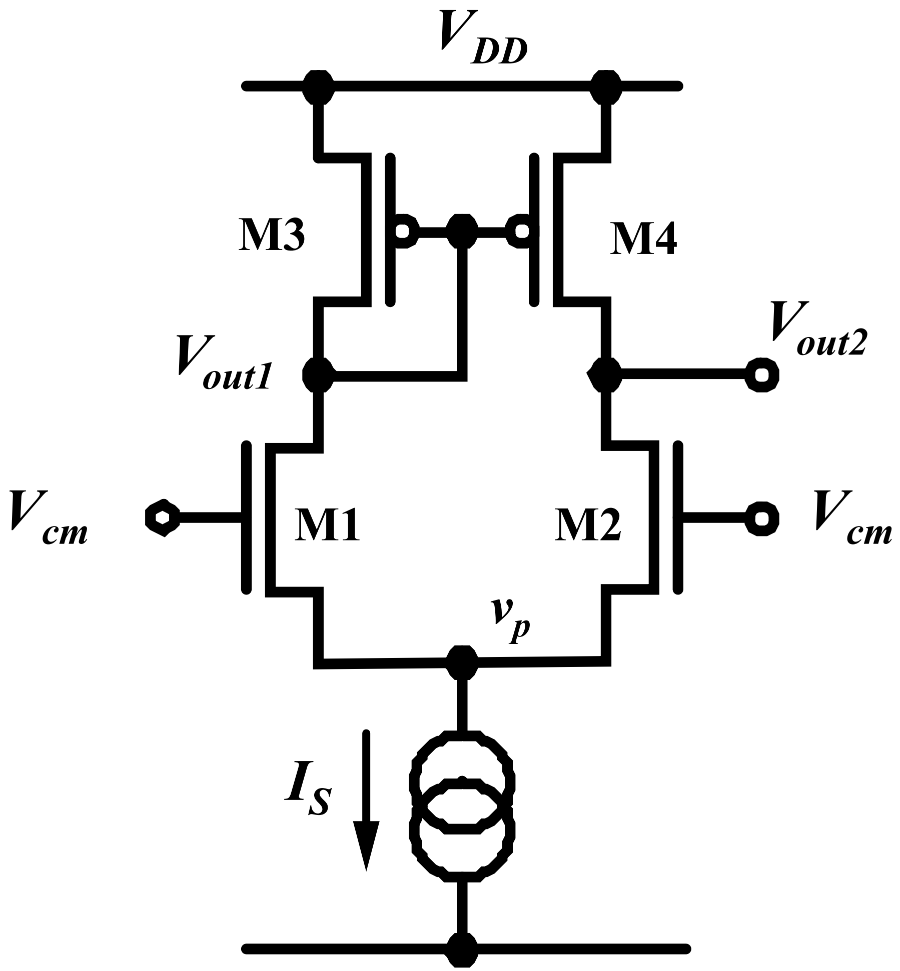


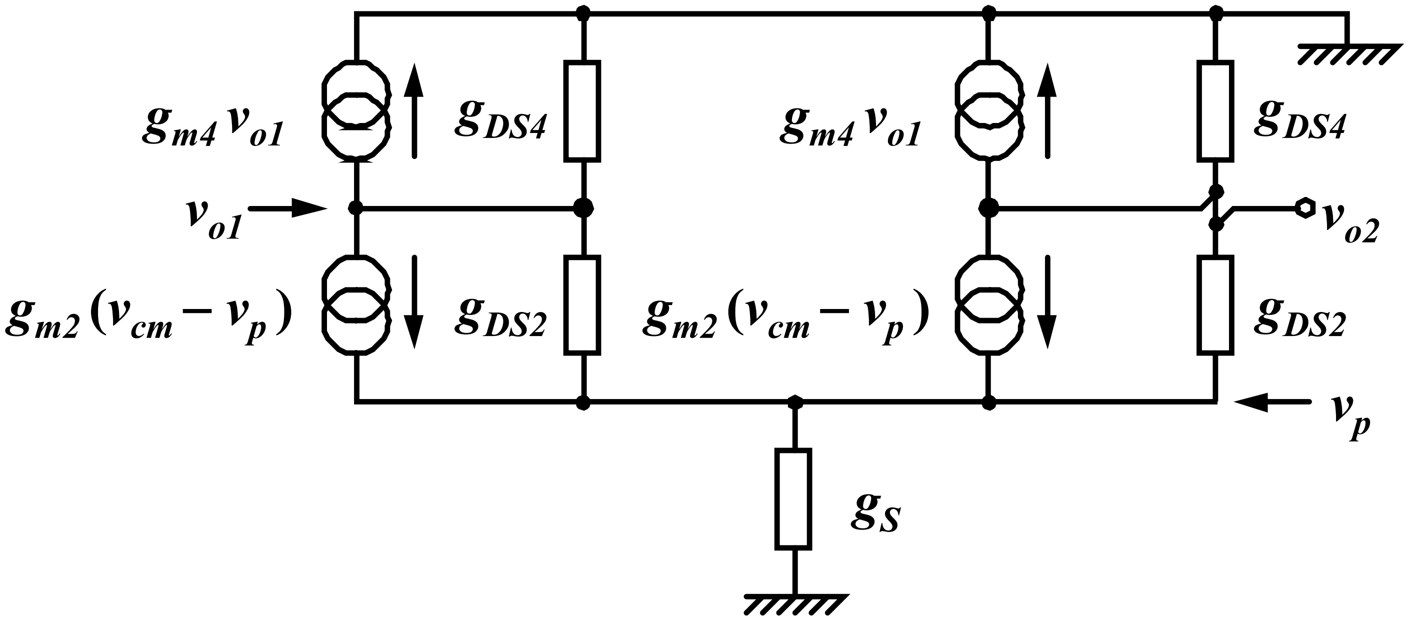
Current Mirror Load

Initial assumptions for simplification:

- $g_{m1} = g_{m2}$
- $g_{m3} = g_{m4}$
- $r_{o1} = r_{o2}$
- $r_{o3} = r_{o4}$

Current Mirror Load





Current Mirror Load

Node vp

gm2 (vcm - vp) + gDS2 (vo1 - vp) + gm2 (vcm - vp) + gDS2 (vo2 - vp) = gs vp

Node vo1

-gm2 (vcm - vp) + gDS2 (vp - vo1) - gm4 vo1 + (0 - vo1) gDS4 = 0

Node vo2

-gm2 (vcm - vp) + gDS2 (vp - vo2) - gm4 vo1 + (0 - vo2) gDS4 = 0

Current Mirror Load

In[15]:=
```
Eliminate[{gm2 (vcm - vp) + gDS2 (vo1 - vp) + gm2 (vcm - vp) + gDS2 (vo2 - vp) == gs vp
, -gm2 (vcm - vp) + gDS2 (vp - vo1) - gm4 vo1 + (0 - vo1) gDS4 == 0,
   -gm2 (vcm - vp) + gDS2 (vp - vo2) - gm4 vo1 + (0 - vo2) gDS4 == 0
},
 {vo1,
  vp}]
```

Out[15]=

$$gs\left(gDS2\, gm2\, vcm + gDS4\, gm2\, vcm + gDS2^2\, vo2 + 2\, gDS2\, gDS4\, vo2 + gDS4^2\, vo2 + gDS2\, gm4\, vo2 + gDS4\, gm4\, vo2\right) == \left(-2\, gDS2^2\, gDS4 - 2\, gDS2\, gDS4^2 - 2\, gDS2\, gDS4\, gm2 - 2\, gDS4^2\, gm2 - 2\, gDS2^2\, gm4 - 2\, gDS2\, gDS4\, gm4 - 2\, gDS2\, gm2\, gm4 - 2\, gDS4\, gm2\, gm4\right)\, vo2$$

In[16]:=
```
Solve[%, vo2]
```

Out[16]=

$$\left\{\left\{vo2 \to -\frac{gm2\, gs\, vcm}{2\, gDS2\, gDS4 + 2\, gDS4\, gm2 + 2\, gDS2\, gm4 + 2\, gm2\, gm4 + gDS2\, gs + gDS4\, gs + gm4\, gs}\right\}\right\}$$

Current Mirror Load

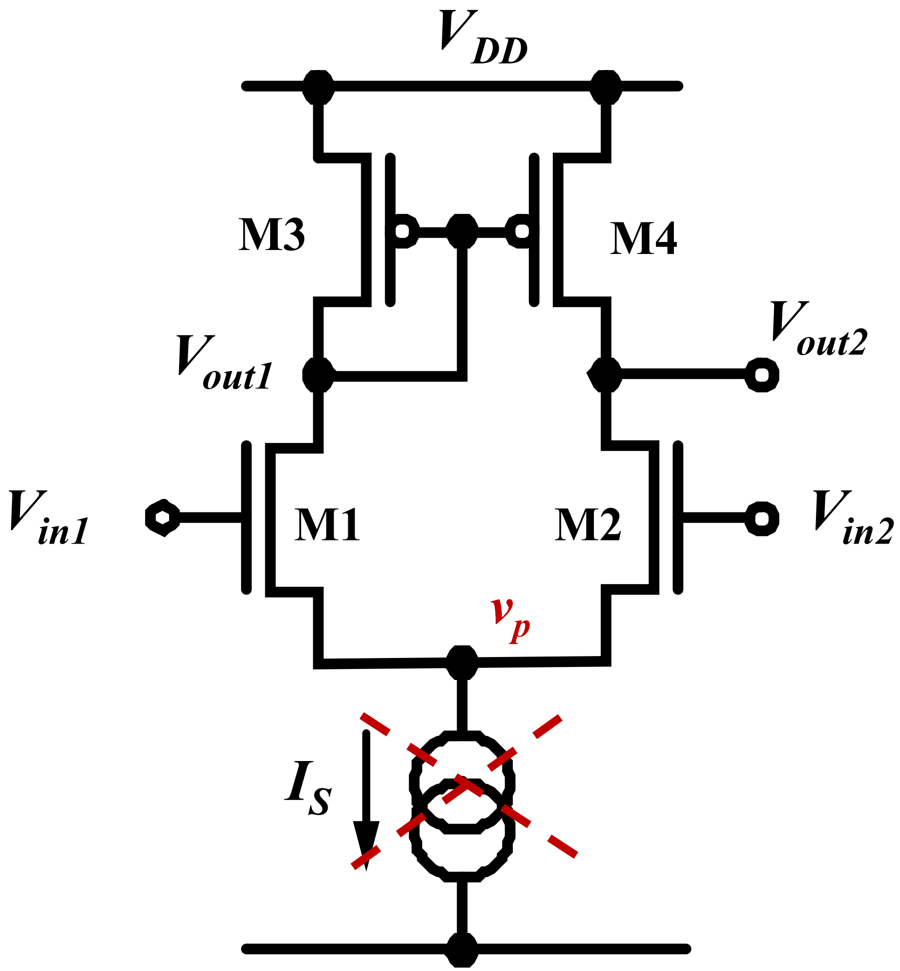
$$v_{o2} = \frac{-g_{m2}\, g_s\, v_{cm}}{2\, g_{DS2} g_{DS4} \ + \ 2 g_{DS4} g_{m2} + \ 2 g_{DS2} g_{m4} \ + \ 2 g_{m2} g_{m4} + g_{DS2} g_s + g_{DS4} gs \ + g_{m4} g_s}$$

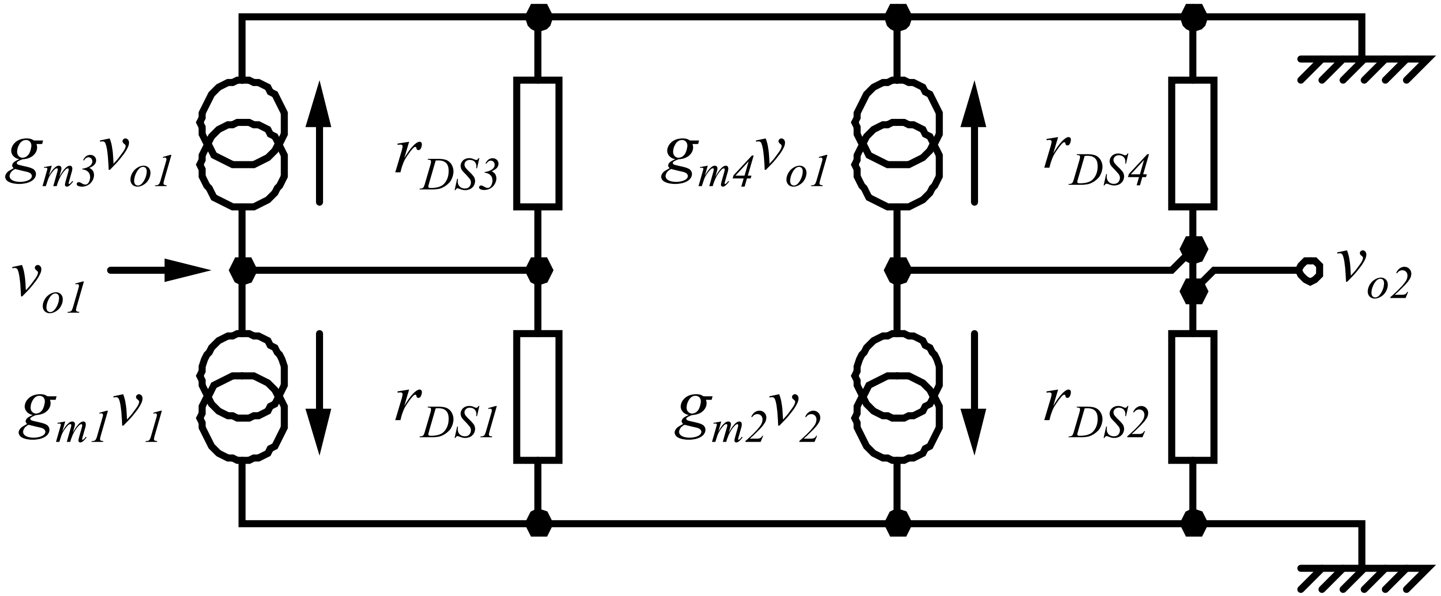
$$g_{DS} \ll g_m$$

$$v_{o2} \approx -\frac{g_{m2}\, g_s\, v_{cm}}{2 g_{m2} g_{m4}}$$

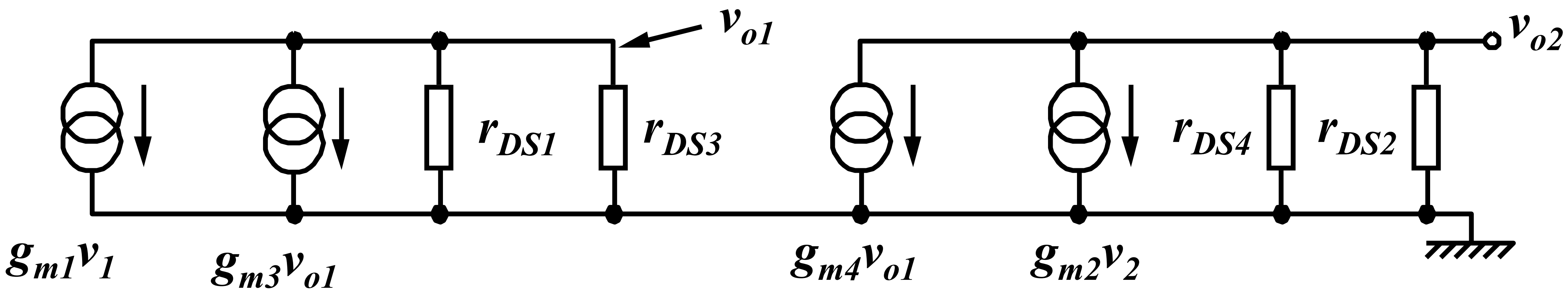
$$\frac{v_{o2}}{v_{cm}} \approx -\frac{g_s}{2 g_{m4}} = -\frac{1}{2 g_{m4} r_s}$$

Current Mirror Load





Current Mirror Load



r24 = r2 || r4

vo1 = -(gm1 v1 + gm3 vo1) r24

vo2 = -(gm1 v2 + gm3 vo1) r24

Current Mirror Load

```
In[22]:= Eliminate[{vo1 == - (gm1v1 + gm3 vo1) r13, vo2 == - (gm2 v2 + gm4 vo1) r24}, {vo1}]

Out[22]= (1 + gm3 r13) vo2 == r24 (gm1v1 gm4 r13 - gm2 v2 - gm2 gm3 r13 v2)

In[23]:= Solve[%, vo2]
```

Out[23]= $\left\{\left\{vo2 \to \frac{r24\,(gm1v1\;gm4\;r13 - gm2\;v2 - gm2\;gm3\;r13\;v2)}{1 + gm3\;r13}\right\}\right\}$

Current Mirror Load

In[25]:= $\mathbf{Solve}\left[\left\{\mathbf{vo2} == \dfrac{\mathbf{r24\ (gm1v1\ gm4\ r13 - gm2\ gm3\ r13\ v2)}}{\mathbf{gm3\ r13}}\right\}\right]$

Out[25]= $\left\{\left\{\text{vo2} \to -\dfrac{\text{r24 }(-\text{gm1v1 gm4} + \text{gm2 gm3 v2})}{\text{gm3}}\right\}\right\}$

Current Mirror Load

$$v_{o2} = \frac{r24\,(-gm1v1\;gm4\; + \;gm2\;gm3\;v2)}{gm3}$$

$$v_{o2} = -\frac{r24\,(gm1gm4v1 - \;gm2\;gm3\;v2)}{gm3}$$

$$v_{o2} \approx -\frac{gm2gm4}{gm3}r24(v1 - \;v2)$$

$$v_{o2} \approx -g_{m2}(r_{o2} \parallel ro_4)v_{id}$$

Current Mirror Load

- What about vo1?

From previous slides: vo1 = -(gm1 v1 + gm3 vo1) r24

$$v_{o1} = -\frac{g_{m1} r_{24} v_1}{1 + g_{m3} r_{24}} \approx -\frac{g_{m1}}{g_{m3}} v_1$$

Current Mirror Load

- What vo1 and vo2 are not the same!

$$v_{o1} \approx -\frac{g_{m1}}{g_{m3}} v_1$$

$$v_{o2} \approx -g_{m2}(r_{o2} \parallel r_{o4}) v_{id}$$

Current Mirror Load

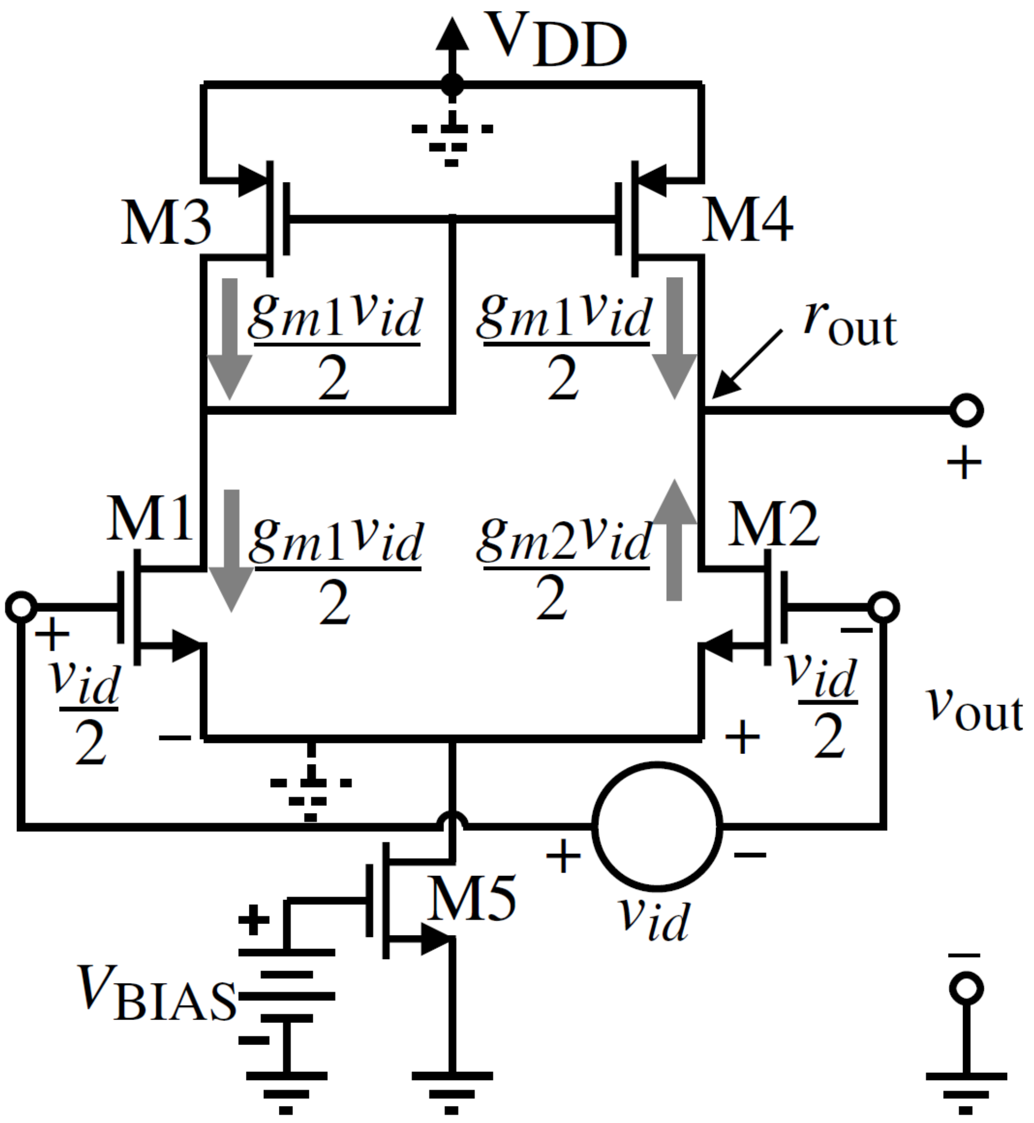
Simple intuitive analysis

- The extra current at *vo2* node must flow into rout

$$r_{out} = r_{o2} \parallel r_{o4}$$

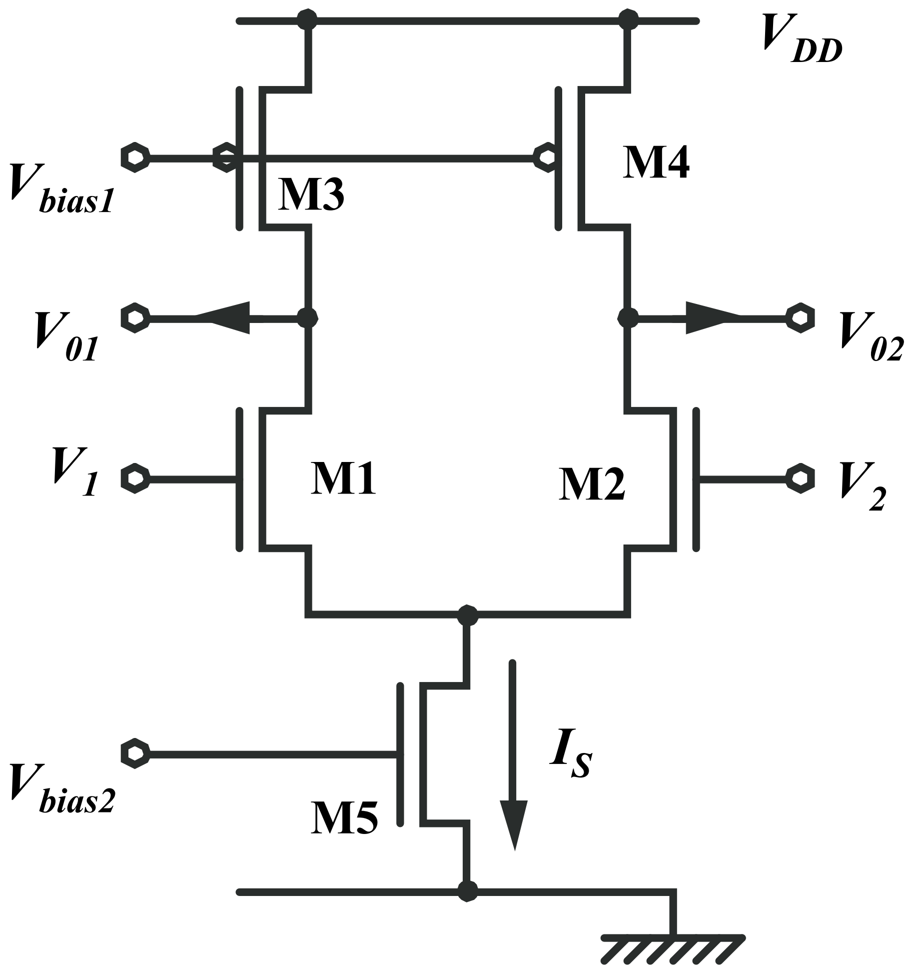
$$v_{o2} = (-g_{m1}\frac{v_{id}}{2} - g_{m2}\frac{v_{id}}{2})(r_{o2} \parallel r_{o4})$$

$$v_{o2} = -g_{m1}\,(r_{o2} \parallel r_{o4})v_{id}$$

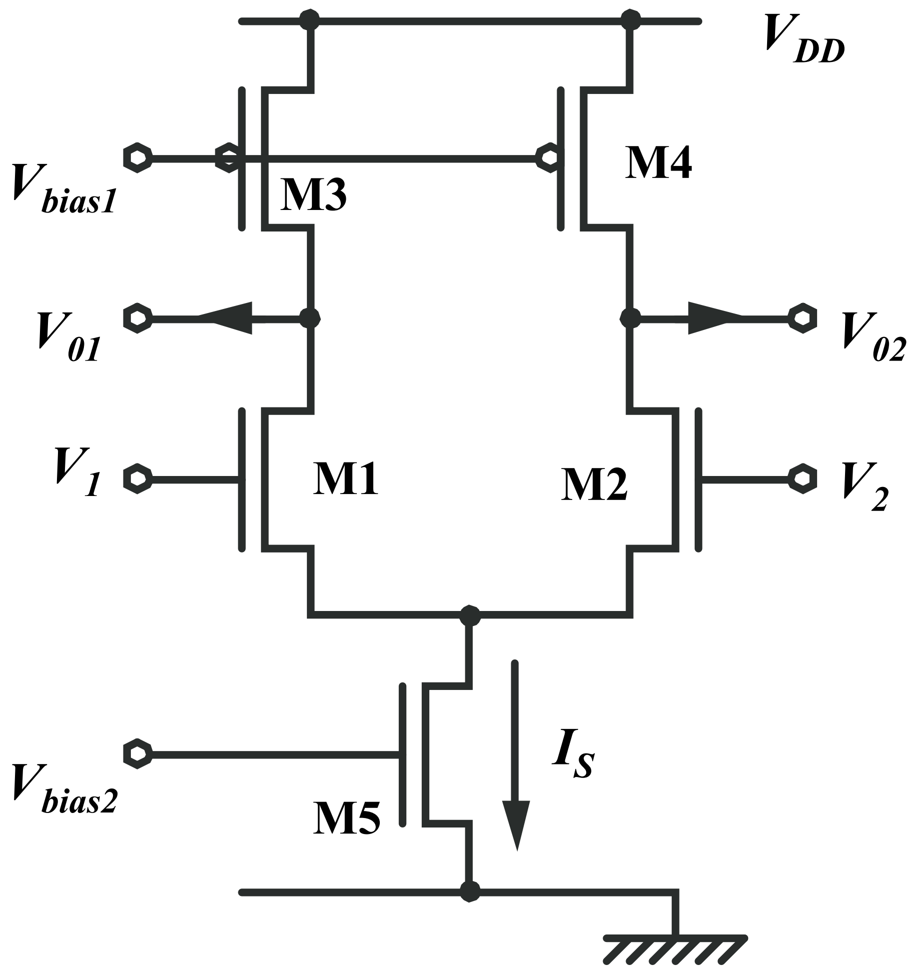


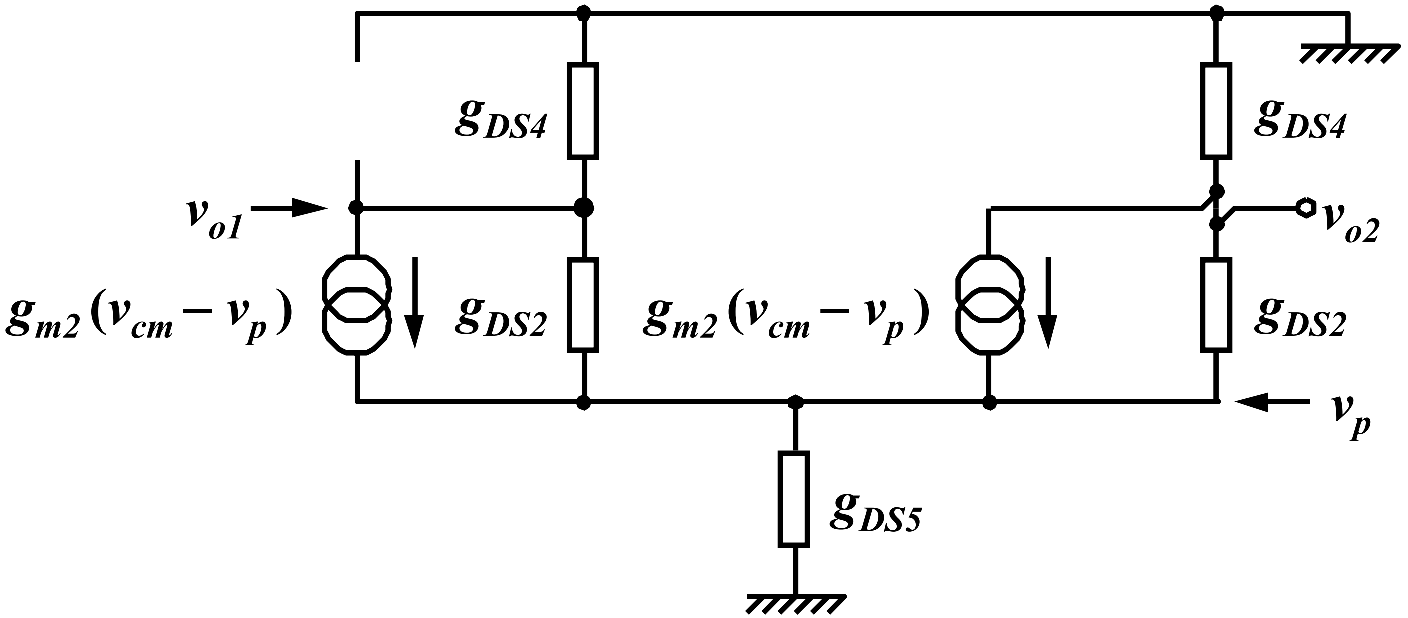
Current Source Load

- Let's calculate current source load



Current Source Load





Current Source Load

```
In[38]:= Eliminate[{gm1 (vcm - vp) + gDS1 (vo1 - vp) + gm2 (vcm - vp) + gDS2 (vo2 - vp) == gDS5 vp
  , -gm1 (vcm - vp) + gDS1 (vp - vo1) + (0 - vo1) gDS3 == 0,
    -gm2 (vcm - vp) + gDS2 (vp - vo2) + (0 - vo2) gDS4 == 0
  },
   {vo1,
    vp}]

Out[38]= gDS2 gDS3 gm1 vcm - gDS1 gDS3 gm2 vcm - gDS1 gDS5 gm2 vcm - gDS3 gDS5 gm2 vcm -
    gDS1 gDS2 gDS3 vo2 - gDS1 gDS2 gDS5 vo2 - gDS2 gDS3 gDS5 vo2 - gDS2 gDS3 gm1 vo2 == gDS4
    (gDS1 gDS2 + gDS1 gDS3 + gDS2 gDS3 + gDS1 gDS5 + gDS3 gDS5 + gDS3 gm1 + gDS1 gm2 + gDS3 gm2) vo2

In[39]:= Solve[%, vo2]

Out[39]= {{vo2 → (gDS2 gDS3 gm1 vcm - gDS1 gDS3 gm2 vcm - gDS1 gDS5 gm2 vcm - gDS3 gDS5 gm2 vcm) /
      (gDS1 gDS2 gDS3 + gDS1 gDS2 gDS4 + gDS1 gDS3 gDS4 + gDS2 gDS3 gDS4 +
        gDS1 gDS2 gDS5 + gDS2 gDS3 gDS5 + gDS1 gDS4 gDS5 + gDS3 gDS4 gDS5 +
        gDS2 gDS3 gm1 + gDS3 gDS4 gm1 + gDS1 gDS4 gm2 + gDS3 gDS4 gm2)}}
```

Current Source Load

```
In[40]:= Simplify[{vo2 == (gDS2 gDS3 gm1 vcm - gDS1 gDS3 gm2 vcm - gDS1 gDS5 gm2 vcm - gDS3 gDS5 gm2 vcm) /
            (gDS1 gDS2 gDS3 + gDS1 gDS2 gDS4 + gDS1 gDS3 gDS4 + gDS2 gDS3 gDS4 +
              gDS1 gDS2 gDS5 + gDS2 gDS3 gDS5 + gDS1 gDS4 gDS5 + gDS3 gDS4 gDS5 +
              gDS2 gDS3 gm1 + gDS3 gDS4 gm1 + gDS1 gDS4 gm2 + gDS3 gDS4 gm2)}]

Out[40]= {vo2 == ((gDS2 gDS3 gm1 - (gDS3 gDS5 + gDS1 (gDS3 + gDS5)) gm2) vcm) /
          (gDS1 gDS2 (gDS3 + gDS4 + gDS5) + gDS2 gDS3 (gDS4 + gDS5 + gm1) +
            gDS1 gDS4 (gDS3 + gDS5 + gm2) + gDS3 gDS4 (gDS5 + gm1 + gm2))}

In[41]:= Simplify[vo2 == ((gDS2 gDS3 gm1 - (gDS3 gDS5 + gDS1 (gDS3 + gDS5)) gm2) vcm) /
          (0 + gDS2 gDS3 (gDS4 + gDS5 + gm1) +
            gDS1 gDS4 (gDS3 + gDS5 + gm2) + gDS3 gDS4 (gDS5 + gm1 + gm2))]
```

$$\text{Out[41]= } vo2 == \frac{(gDS2\ gDS3\ gm1 - (gDS3\ gDS5 + gDS1\ (gDS3 + gDS5))\ gm2)\ vcm}{gDS2\ gDS3\ \boxed{(gDS4 + gDS5 + gm1)} + gDS1\ gDS4\ \boxed{(gDS3 + gDS5 + gm2)} + gDS3\ gDS4\ \boxed{(gDS5 + gm1 + gm2)}}$$

$$\text{In[42]:= Simplify}\left[\left\{vo2 == \frac{(gDS2\ gDS3\ gm1 - (gDS3\ gDS5 + gDS1\ (gDS3 + gDS5))\ gm2)\ vcm}{gDS2\ gDS3\ (0 + gm1) + gDS1\ gDS4\ (0 + gm2) + gDS3\ gDS4\ (0 + gm1 + gm2)}\right\}\right]$$

$$\text{Out[42]= } \left\{vo2 == \frac{(gDS2\ gDS3\ gm1 - (gDS3\ gDS5 + gDS1\ (gDS3 + gDS5))\ gm2)\ vcm}{gDS2\ gDS3\ gm1 + gDS1\ gDS4\ gm2 + gDS3\ gDS4\ (gm1 + gm2)}\right\}$$

Current Source Load

```
In[45]:= Eliminate[{vo2 == ((gDS2 gDS3 gm1 - (gDS3 gDS5 + gDS1 (gDS3 + gDS5)) gm2) vcm) / (gDS2 gDS3 gm1 + gDS1 gDS4 gm2 + gDS3 gDS4 (gm1 + gm2)),
    gDS1 == gDS2, gDS3 == gDS4, gm1 == gm2, gm3 == gm4}, {gDS1, gDS3, gm2, gm3}]
Out[45]= -2 gDS4 vo2 == gDS5 vcm
```

$$v_{o2} = -\frac{g_{DS5}}{2g_{DS4}} v_{cm} = -\frac{r_{o4}}{2r_{o5}} v_{cm}$$

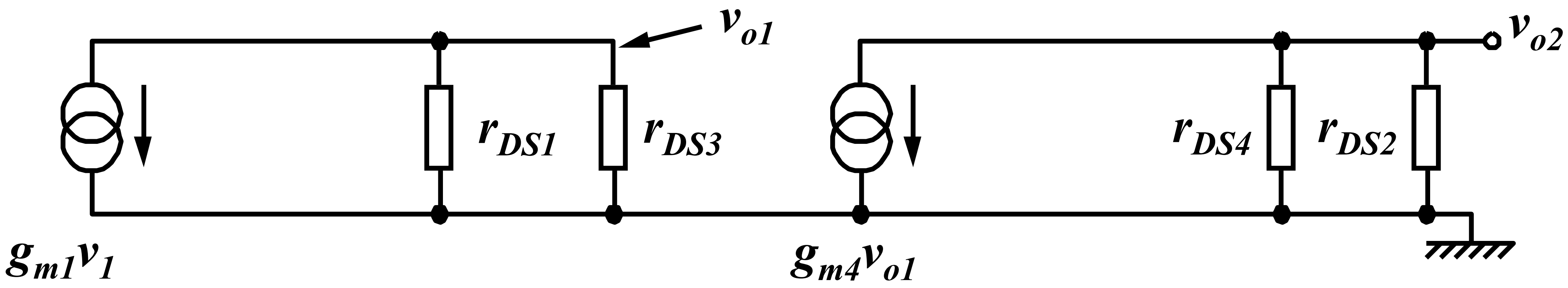
Current Source Load

$$v_{o1,2} = -\frac{g_{DS5}}{2g_{DS4}} v_{cm} = -\frac{r_{o4}}{2r_{o5}} v_{cm}$$

$$v_{ocm} = \frac{v_{o1} + vo_2}{2}$$

$$v_{ocm} = -\frac{g_{DS5}}{2g_{DS4}} v_{icm} = -\frac{r_{o4}}{2r_{o5}} v_{icm}$$

Current Source Load



r24 = r2 || r4

vo1 = -(gm1 v1) r24

vo2 = -(gm1 v2) r24

Current Source Load

$$v_{o1} = -g_m(r_2 \parallel r_4)v_1$$

$$v_{o2} = -g_m(r_2 \parallel r_4)v_2$$

$$v_{o1} - v_{o2} = -g_m(r_2 \parallel r_4)(v_1 - v_2)$$

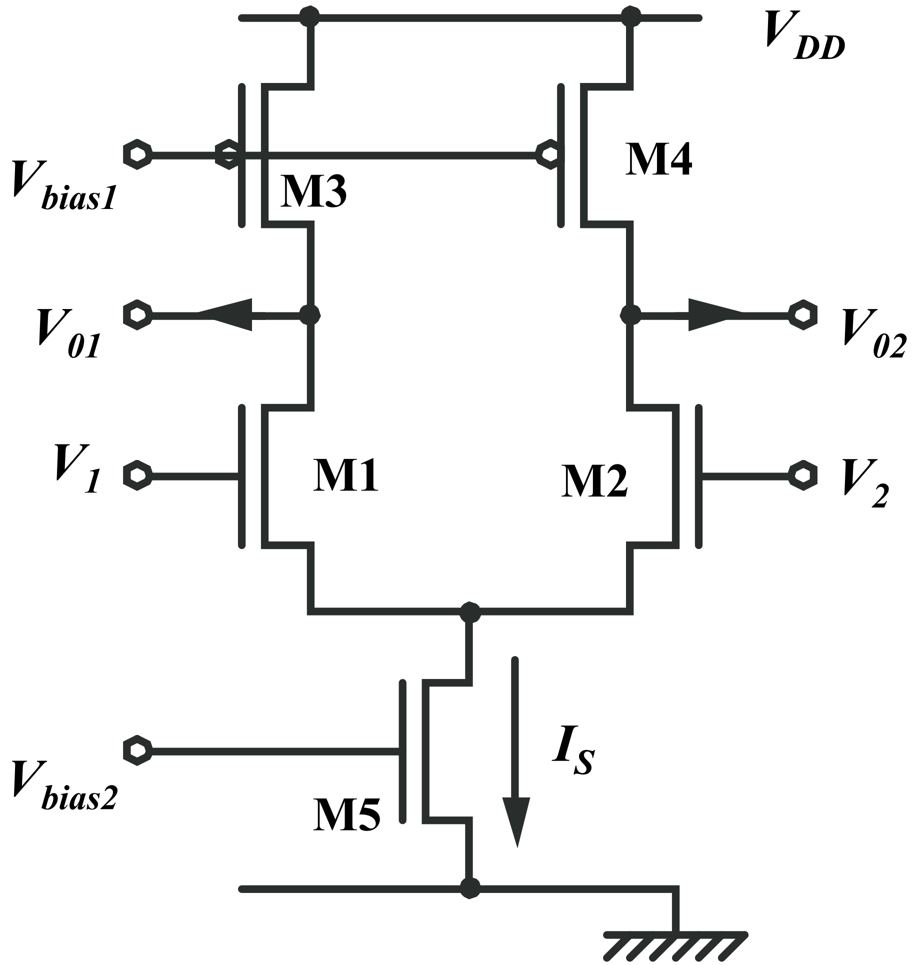
$$\frac{v_{od}}{v_{id}} = -g_m(r_2 \parallel r_4)$$

Current Source Load

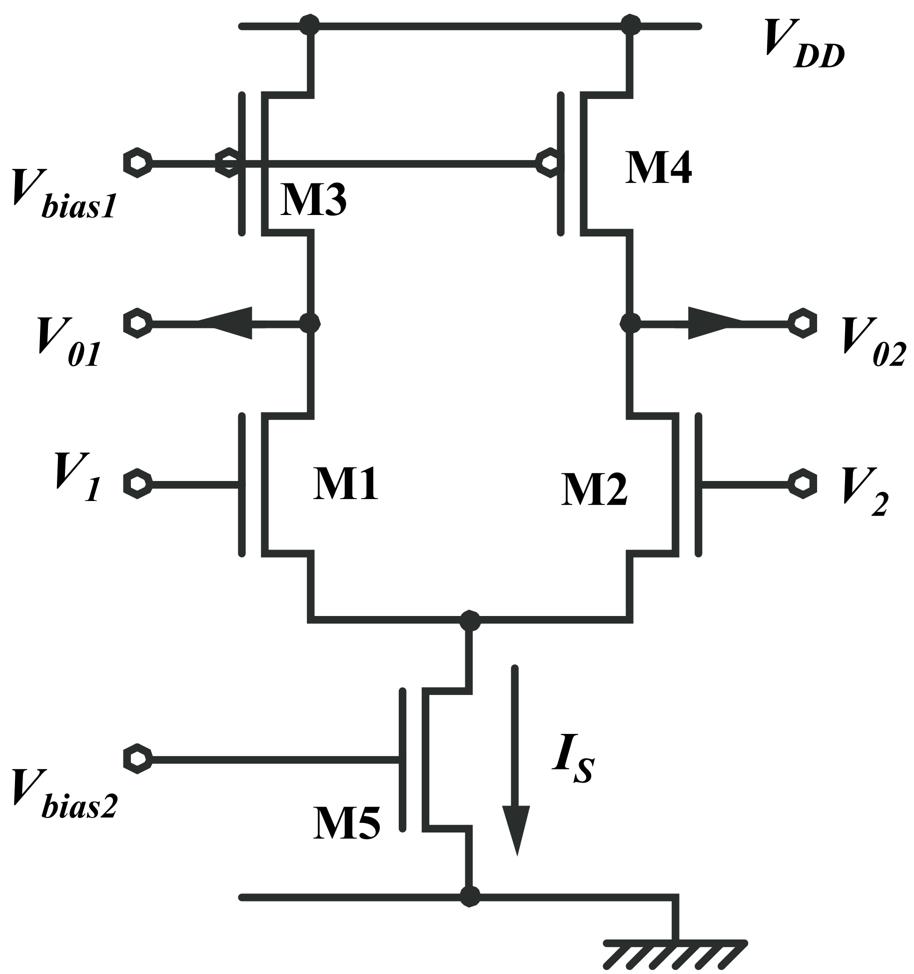
- Fully differential!!
- But needs feedback to work

$$\frac{v_{od}}{v_{id}} = -g_m(r_{o2} \parallel r_{o4})$$

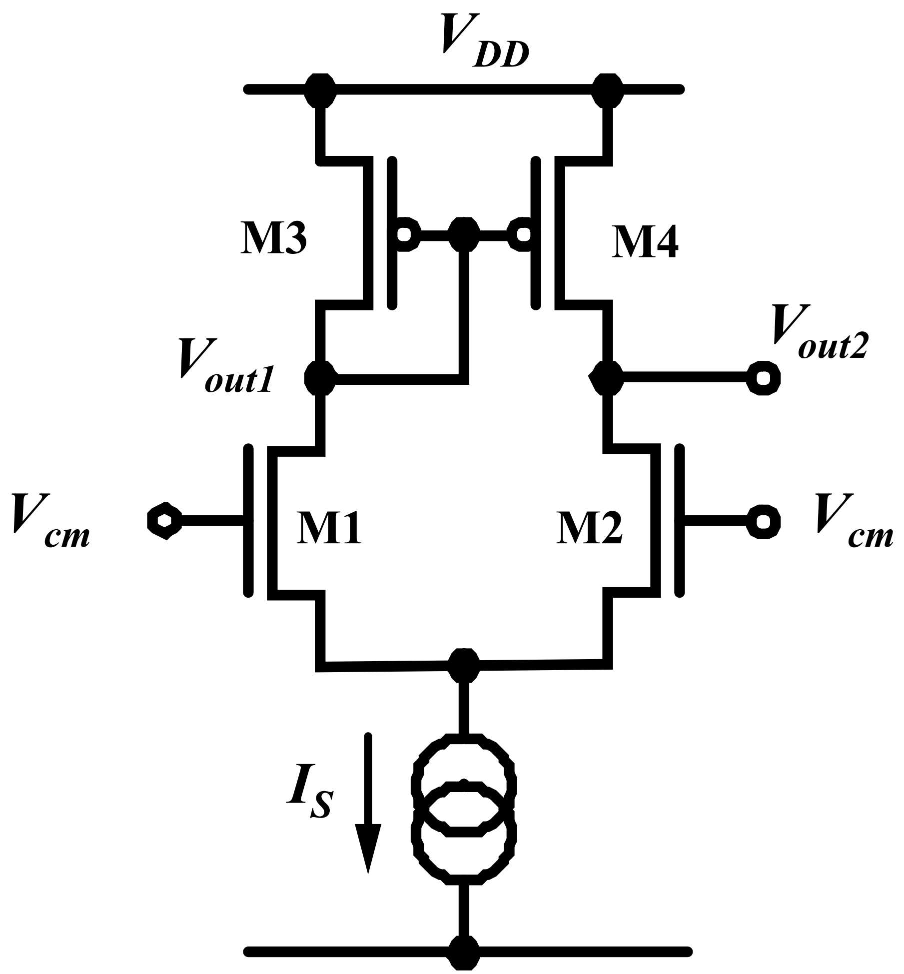
$$\frac{v_{ocm}}{v_{icm}} = -\frac{r_{o4}}{2r_{o5}}$$



Comparison

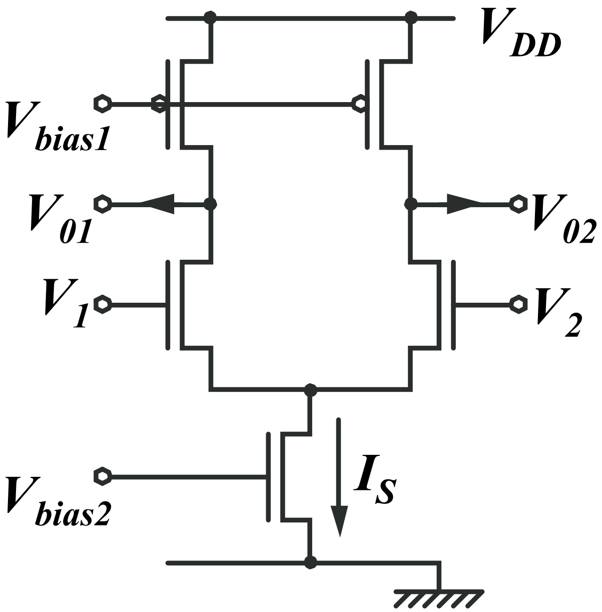


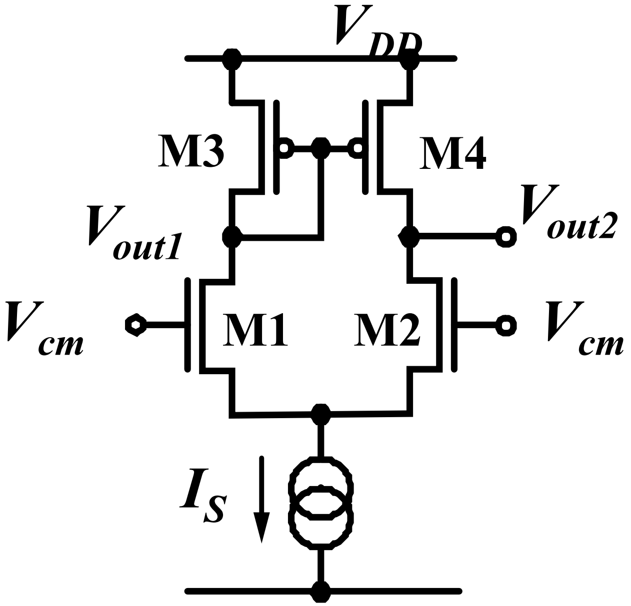
V_{DD}
V_{bias1}
M3
M4
V_{01}
V_{02}
V_1
M1
M2
V_2
I_S
V_{bias2}
M5



V_{DD}
M3
M4
V_{out2}
V_{out1}
V_{cm}
M1
M2
V_{cm}
I_S

Comparison





$$\frac{v_{od}}{v_{id}} = -g_{m2}(r_{o2} \parallel r_{o4})$$

$$\frac{v_{ocm}}{v_{icm}} = -\frac{r_{o4}}{2r_s}$$

$$v_{o1} \approx -\frac{g_{m1}}{g_{m3}} v_1$$

$$v_{o2} \approx -g_{m2}(r_{o2} \parallel r_{o4}) v_{id}$$

$$\frac{v_{o1,2}}{v_{cm}} \approx -\frac{1}{2g_{m4}r_s}$$

Long road to OpAmps...