Trigonometric Identities

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2} \qquad \sin(\alpha)\cos(\beta) = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2}$$

$$\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$\sin(\alpha) = \frac{1}{i2} \left[e^{j\alpha} - e^{-j\alpha} \right] \qquad \cos(\alpha) = \frac{1}{2} \left[e^{j\alpha} + e^{-j\alpha} \right]$$

Sum of a series

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a} \sum_{n=1}^{N} n = \frac{N(N+1)}{2} \sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\sum_{n=-N}^{N} \cos(2\alpha n) = \frac{\sin(\alpha(2N+1))}{\sin(\alpha)} \sum_{n=0}^{N} \sin(2\alpha n) = \frac{\sin(\alpha N)\sin(\alpha(N+1))}{\sin(\alpha)}$$

$$\sum_{n=-N}^{N} \cos(\alpha n) e^{-j\beta n} = \frac{\cos(\alpha(N+1))\cos(\beta N) - \cos(\alpha N)\cos(\beta(N+1))}{\cos(\alpha) - \cos(\beta)}$$

Fourier Transform Formulae

$$\begin{array}{lll} \textbf{Fourier series} & c_k = F_0 \int_{-1/2F_0}^{1/2F_0} x(t) e^{-j2\pi k F_0 t} dt & x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t} \\ \\ \textbf{Fourier} & X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi F t} dt & x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi F t} dF \\ \\ \textbf{Discrete-time} & Fourier & x(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega} & x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{jn\omega} d\omega \\ \\ \textbf{Discrete-time} & c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & Fourier & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & Fourier & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi k n/N} \\ \\ \textbf{Discrete} & x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} \\ \\ \textbf{Discrete} & x(n) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} & x(n) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N} \\ \\ \textbf{Discrete} & x(n) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N}$$

Correlation functions

$$r_{xy} = \sum_{-\infty}^{\infty} x(n)y(n-l)$$
 $r_{xy}(l) = \lim_{M \to \infty} \frac{1}{2M+1} \sum_{n=-M}^{M} x(n)y(n-l)$

Power density spectrum

$$\Gamma_{xx}(f) = \sum_{-\infty}^{\infty} \gamma_{xx}(m)e^{-j2\pi fm} \qquad \gamma_{xx}(m) = \int_{-1/2}^{1/2} \Gamma_{xx}(f)e^{j2\pi fm}df$$

$$w_n = 1 - \frac{|n - M|}{M + 1}$$

$$w_n = \frac{1}{2} \left(1 - \cos \left(\frac{2n\pi}{M - 1} \right) \right)$$

$$w_n = 0.54 - 0.46\cos\left(\frac{2n\pi}{M-1}\right)$$

Blackman window

$$w_n = 0.42 - 0.5 \cos\left(\frac{2n\pi}{M-1}\right) + 0.08 \cos\left(\frac{4n\pi}{M-1}\right)$$

$$w_n = \frac{1}{I_0(\beta)} I_0 \left(\beta \sqrt{1 - \left(\frac{2n}{M-1} - 1\right)^2} \right)$$

Window characteristics

Window	Transition Band	Stopband rejection (dB)		
Rectangular	$\frac{1.8\pi}{M}$	21		
Hanning	$\frac{6.2\pi}{M}$	44		
Hamming	$\frac{6.6\pi}{M}$	53		
Kaiser $\beta=6$	$\frac{8\pi}{M}$	63		
Blackman	$\frac{11\pi}{M}$	74		
Kaiser $\beta = 9$	$\frac{11.4\pi}{M}$	90		

Window	Resolution -3 dB Bandwidth	Equivalent Noise Bandwidth	Coherent gain (dB)	Peak sidelobe level (dB)	Asymptotic roll-off (dB/octave)	
Rectangular	$\frac{1.78\pi}{M}$	$\frac{2\pi}{M}$	0	-13	-6	
Triangular	$\frac{2.56\pi}{M}$	$\frac{2.66\pi}{M}$	-3	-27	-12	
Parzen	$\frac{3.64\pi}{M}$	$\frac{3.83\pi}{M}$	-4.3	-53	-24	
Hanning	$\frac{2.8\pi}{M}$	$\frac{3\pi}{M}$	$\frac{3\pi}{M}$ -3		-18	
Hamming	$\frac{2.6\pi}{M}$	$\frac{2.72\pi}{M}$	-2.7	-43	-6	
Bohman	$\frac{3.41\pi}{M}$	$\frac{3.58\pi}{M}$	-3.9	-46	-24	
Blackman	$\frac{3.04\pi}{M}$	$\frac{3.14\pi}{M}$	-3.8	-58	-6	
Dolph-Chebyshev	$\frac{2.88\pi}{M}$	$\frac{3.02\pi}{M}$	-3.2	-60	0	
Kaiser-Bessel	$\frac{3.14\pi}{M}$	$\frac{3.3\pi}{M}$	-3.6	-57	-6	

FIR filter design

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{RD}(\omega) e^{j\omega\left(n - \left(\frac{M-1}{2}\right)\right)} d\omega$$

$$\alpha = 0 \ \beta = 0 \quad h(n) = \frac{1}{M} \left\{ G(0) + 2 \sum_{k=1}^{U} G(k) \cos\left(\frac{2\pi k}{M} \left(n + \frac{1}{2}\right)\right) \right\}$$

$$\alpha = \frac{1}{2} \beta = 0 \quad h(n) = \frac{2}{M} \sum_{k=0}^{U} G(k + \alpha) \sin\left(\frac{2\pi (k + \alpha)}{M} \left(n + \frac{1}{2}\right)\right)$$

$$\alpha = 0 \ \beta = 1 \quad h(n) = \frac{1}{M} \left\{ (-1)^{n+1} G(M/2) - 2 \sum_{k=1}^{V} G(k) \sin\left(\frac{2\pi k}{M} \left(n + \frac{1}{2}\right)\right) \right\}$$

$$\alpha = \frac{1}{2} \beta = 1 \quad h(n) = \frac{2}{M} \sum_{k=0}^{V} G(k + \alpha) \cos\left(\frac{2\pi (k + \alpha)}{M} \left(n + \frac{1}{2}\right)\right)$$

$$G(k + \alpha) = (-1)^{k} H_{r} \left(\frac{2\pi (k + \alpha)}{M}\right)$$

$$U = \left\lfloor \frac{M-1}{2} \right\rfloor$$

$$V = \left\lceil \frac{M-1}{2} \right\rceil$$

Transition coefficients for 16 tap filter with $\alpha=0$

BW	Minimax (dB)	T1
1	-42.41981868	0.43306932
2	-41.38993328	0.41702849
3	-41.24929950	0.41001589
4	-41.66355813	0.40474097
5	-43.21753226	0.39666905
6	-49.33321372	0.37587659

BW	Minimax (dB)	T1	T2
1	-71.59789962	0.09225582	0.58621580
2	-70.66810992	0.09938976	0.58818920
3	-72.23796597	0.09528050	0.57815395
4	-76.63701678	0.08476728	0.55924120
5	-89.72938985	0.06534978	0.52474095

BW	Minimax (dB)	T1	T2	Т3
1	-104.19077428	0.01191092	0.17390102	0.66090319
2	-105.99219722	0.01294020	0.18866556	0.67271873
3	-115.06048549	0.01039931	0.17536927	0.65858025
4	-134.48576256	0.00625291	0.14601962	0.62676044

Analogue to digital converter

$$SQNR = 6.02b + 10.79 - 20 \log_{10} \frac{R}{\sqrt{P_x}} + 10(2L+1) \log_{10} O$$

Minimum variance spectral estimation
$$P_{xx}^{MV}(f) = \frac{1}{\mathbf{E}^H(f)\mathbf{R}_{xx}^{-1}\mathbf{E}(f)}$$

Estimate of mean

$$\widehat{m}_{x} = \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)$$

$$Var(\widehat{m}_{x}) = \frac{1}{2N+1} \sum_{l=-2N}^{2N} \left(1 - \frac{|l|}{2N+1}\right) c_{xx}(l)$$

$$c_{xx}(l) = E\left[(x(n) - E(X(n)))(x(n-l) - E(X(n)))\right]$$

Matrices

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \operatorname{adj}(\mathbf{A})$$

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow$$

$$\det(\mathbf{B}) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$\det(\mathbf{B}) = \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & - \begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$- \begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} - \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

Integration by parts

$$\int u \, dv = uv - \int v \, du$$

Indefinite Integrals

Indefinite Integrals
$$\int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \tan^{-1} \left(\frac{bx}{a} \right) \qquad \int_0^\infty \frac{\sin^2(ax)}{x^2} \, dx = |a| \frac{\pi}{2}$$

$$\int \frac{a \, dx}{\sqrt{b^2 - x^2}} = a \sin^{-1} \left(\frac{x}{b} \right) \qquad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \exp\left(-\frac{t^2}{2} \right) \, dt = 1$$

$$\int \frac{-a \, dx}{\sqrt{b^2 - x^2}} = a \cos^{-1} \left(\frac{x}{b} \right) \qquad \frac{1}{2\pi} \int_{-\pi}^\pi \frac{\left(1 - a^2 \right) e^{j\omega m}}{1 + a^2 - 2a \cos(\omega)} d\omega = a^{|m|} \; ; |a| < 1$$

$$\int \frac{a \, dx}{\sqrt{x^2 - b^2}} = a \ln\left(x + \sqrt{x^2 - b^2} \right)$$

$$\int \sin^{m}(ax+b) \, dx = -\frac{\sin^{m-1}(ax+b)\cos(ax+b)}{ma} + \frac{m-1}{m} \int \sin^{m-2}(ax+b) \, dx$$

Modified Bessel function of the first kind

$$I_0(z) = \sum_{k=0}^{\infty} \frac{\left(\frac{z^2}{4}\right)^k}{\left(k!\right)^2}$$

Trigonometric transform Identity
$$\cos(n\omega_0) \xleftarrow{F}_{F^{-1}} \pi \left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

Gaussian process

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$
 $E[X] = \mu$ $Var(X) = \sigma^2$

Normal distribution

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt$$

z 0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.0 0.00000 0.01128 0.02256 0.03384 0.04511 0.05637 0.06762 0.07886 0.09008 0.10128 0.1 0.11246 0.12362 0.13476 0.14587 0.15695 0.16800 0.17901 0.18999 0.2004 0.21184 0.2 0.22270 0.23352 0.24430 0.25502 0.26570 0.27633 0.28690 0.29742 0.30788 0.31828 0.3 0.32863 0.33931 0.34913 0.35928 0.36936 0.37938 0.38933 0.39921 0.40901 0.14184 0.5 0.52050 0.52924 0.53790 0.54646 0.55494 0.56332 0.57162 0.57982 0.58792 0.59594 0.6 0.60386 0.61168 0.61941 0.62705 0.63459 0.64203 0.64938 0.65633 0.66378 0.67844 0.7 0.748			T	1			T	1			
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0.7 0.67780 0.68467 0.69143 0.69810 0.70468 0.71116 0.71754 0.72382 0.73001 0.73610 0.8 0.74210 0.74800 0.75381 0.75952 0.76514 0.77067 0.77610 0.78144 0.78669 0.79184 0.9 0.79691 0.804881 0.85074 0.81156 0.81627 0.82089 0.82542 0.82987 0.83423 0.83851 1.0 0.84270 0.84681 0.85084 0.85478 0.85865 0.86244 0.86614 0.86977 0.87333 0.87680 1.1 0.88021 0.89383 0.89612 0.89910 0.90200 0.90484 0.90761 1.2 0.91031 0.91296 0.91553 0.91805 0.92251 0.92290 0.92524 0.92751 0.92973 0.93190 1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94376 0.94556 0.94731 0.94902 0.95661 1.5 0.96611 0.96728	0.5	0.52050	0.52924	0.53790	0.54646	0.55494	0.56332	0.57162	0.57982	0.58792	0.59594
0.8 0.74210 0.74800 0.75381 0.75952 0.76514 0.77067 0.77610 0.78144 0.78669 0.79184 0.9 0.79691 0.80188 0.80677 0.81156 0.81627 0.82089 0.82542 0.82987 0.83423 0.83851 1.0 0.84270 0.84681 0.85084 0.85478 0.85865 0.86244 0.86614 0.86977 0.87333 0.87680 1.1 0.88021 0.88353 0.88679 0.89308 0.89612 0.89910 0.90200 0.90484 0.90761 1.2 0.91031 0.91296 0.91553 0.91805 0.92051 0.92290 0.92524 0.92751 0.92973 0.93190 1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94556 0.94731 0.94902 0.95686 0.95830 0.95970 0.96105 0.96237 0.96365 0.96410 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263	0.6	0.60386	0.61168	0.61941	0.62705	0.63459	0.64203	0.64938	0.65663	0.66378	0.67084
0.9 0.79691 0.80188 0.80677 0.81156 0.81627 0.82089 0.82542 0.82987 0.83423 0.83851 1.0 0.84270 0.84681 0.85084 0.85478 0.85865 0.86244 0.86614 0.86977 0.87333 0.87680 1.1 0.88021 0.88353 0.88679 0.88997 0.89308 0.89612 0.89910 0.90200 0.90484 0.90761 1.2 0.91031 0.91296 0.91553 0.91805 0.92051 0.92290 0.92524 0.92751 0.92973 0.93190 1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94376 0.94556 0.94731 0.94902 0.95067 1.4 0.95229 0.95385 0.95686 0.95830 0.95970 0.96105 0.96365 0.96490 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97360 0.97455 0.97451 1.7 0.98379 0.98441 <	0.7	0.67780	0.68467	0.69143	0.69810	0.70468	0.71116	0.71754	0.72382	0.73001	0.73610
1.0 0.84270 0.84681 0.85084 0.85478 0.85865 0.86244 0.86614 0.86977 0.87333 0.87680 1.1 0.88021 0.88353 0.88679 0.88997 0.89308 0.89612 0.89910 0.90200 0.90484 0.90761 1.2 0.91031 0.91296 0.91553 0.91805 0.92051 0.92290 0.92524 0.92751 0.92973 0.93190 1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94376 0.94556 0.94731 0.94002 0.95836 0.95836 0.95830 0.95970 0.96105 0.96237 0.96365 0.96490 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263 0.97360 0.97455 0.97546 1.6 0.97635 0.97721 0.97804 0.97884 0.97962 0.98038 0.98110 0.98181 0.98249 0.98315 1.7 0.988099 0.98952 0.989944 0.99035 <td>8.0</td> <td>0.74210</td> <td>0.74800</td> <td>0.75381</td> <td>0.75952</td> <td>0.76514</td> <td>0.77067</td> <td>0.77610</td> <td>0.78144</td> <td>0.78669</td> <td>0.79184</td>	8.0	0.74210	0.74800	0.75381	0.75952	0.76514	0.77067	0.77610	0.78144	0.78669	0.79184
1.1 0.88021 0.88353 0.88679 0.88997 0.89308 0.89612 0.89910 0.90200 0.90484 0.90761 1.2 0.91031 0.91296 0.91553 0.91805 0.92051 0.92290 0.92524 0.92751 0.92973 0.93190 1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94376 0.94556 0.94731 0.94902 0.95067 1.4 0.95229 0.95385 0.95538 0.95686 0.95830 0.95970 0.96105 0.96237 0.96365 0.96490 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263 0.97360 0.97455 0.97546 1.6 0.97635 0.97721 0.97804 0.97884 0.97962 0.98038 0.98110 0.98181 0.98249 0.98315 1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.99182 0.99216 0.99248 1.9 0.99529 0.999520 0.99952 0.99953 0.99954 0.99	0.9	0.79691	0.80188	0.80677	0.81156	0.81627	0.82089	0.82542	0.82987	0.83423	0.83851
1.2 0.91031 0.91296 0.91553 0.91805 0.92051 0.92290 0.92524 0.92751 0.92973 0.93190 1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94376 0.94556 0.94731 0.94902 0.95067 1.4 0.95229 0.95385 0.95538 0.95686 0.95830 0.95970 0.96105 0.96237 0.96365 0.96490 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263 0.97360 0.97455 0.97546 1.6 0.97635 0.97721 0.97804 0.97884 0.97962 0.98038 0.98110 0.98181 0.98249 0.98315 1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.98769 0.98817 0.98864 1.8 0.98909 0.98952 0.98994 0.99035 0.99074 0.99111 0.99147 0.99182 0.99216 0.99248 1.9 0.99953 0.999572 0.99591 0.99960 0.99626 0.99	1.0	0.84270	0.84681	0.85084	0.85478	0.85865	0.86244	0.86614	0.86977	0.87333	0.87680
1.3 0.93401 0.93606 0.93807 0.94002 0.94191 0.94376 0.94556 0.94731 0.94902 0.95067 1.4 0.95229 0.95385 0.95538 0.95686 0.95830 0.95970 0.96105 0.96237 0.96365 0.96490 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263 0.97360 0.97455 0.97546 1.6 0.97635 0.97721 0.97804 0.97844 0.97962 0.98038 0.98110 0.98110 0.98249 0.98315 1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.98769 0.98817 0.98864 1.8 0.98909 0.99309 0.99338 0.99366 0.99392 0.99418 0.99469 0.99466 0.99489 0.99511 2.0 0.99532 0.99572 0.99571 0.99753 0.99764 0.99775 0.99785 0.99795 0.99868 2.1 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.999	1.1	0.88021	0.88353	0.88679	0.88997	0.89308	0.89612	0.89910	0.90200	0.90484	0.90761
1.4 0.95229 0.95385 0.95538 0.95686 0.95830 0.95970 0.96105 0.96237 0.96365 0.96490 1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263 0.97360 0.97455 0.97546 1.6 0.97635 0.97721 0.97804 0.97884 0.97962 0.98038 0.98110 0.98181 0.98249 0.98315 1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.98769 0.98817 0.98864 1.8 0.98909 0.98952 0.98994 0.99035 0.99074 0.99111 0.99147 0.99182 0.99216 0.99248 1.9 0.99279 0.99309 0.99338 0.99366 0.99392 0.99418 0.99443 0.99466 0.99489 0.99511 2.0 0.99532 0.99552 0.99572 0.99591 0.99609 0.99660 0.99642 0.99658 0.9975 0.99865 2.1 0.99702 0.99715 0.99971 0.99831 0.99846 0.9985	1.2	0.91031	0.91296	0.91553	0.91805	0.92051	0.92290	0.92524	0.92751	0.92973	0.93190
1.5 0.96611 0.96728 0.96841 0.96952 0.97059 0.97162 0.97263 0.97360 0.97455 0.97546 1.6 0.97635 0.97721 0.97804 0.97884 0.97962 0.98038 0.98110 0.98181 0.98249 0.98315 1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.98769 0.98817 0.98864 1.8 0.98909 0.98952 0.98994 0.99035 0.99074 0.99111 0.99147 0.99182 0.99216 0.99248 1.9 0.99279 0.99309 0.99338 0.99366 0.99392 0.99418 0.99443 0.99466 0.99489 0.99511 2.0 0.99532 0.99572 0.99591 0.99609 0.99626 0.99642 0.99658 0.99673 0.99688 2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99764 0.99755 0.99785 0.99805 2.2 0.99814 0.99822 0.99831 0.99839 0.99946 0.99954 0.99955 0.999	1.3	0.93401	0.93606	0.93807	0.94002	0.94191	0.94376	0.94556	0.94731	0.94902	0.95067
1.6 0.97635 0.97721 0.97804 0.97884 0.97962 0.98038 0.98110 0.98181 0.98249 0.98315 1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.98769 0.98817 0.98864 1.8 0.98909 0.98952 0.98994 0.99035 0.99074 0.99111 0.99147 0.99182 0.99216 0.99248 1.9 0.99279 0.99309 0.99338 0.99366 0.99392 0.99418 0.99443 0.99466 0.99489 0.99511 2.0 0.99532 0.99572 0.99591 0.99609 0.99626 0.99642 0.99658 0.99673 0.99688 2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99861 0.99867 0.99875 0.99880 2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99920 0.99924 0.99928 2.4 <	1.4	0.95229	0.95385	0.95538	0.95686	0.95830	0.95970	0.96105	0.96237	0.96365	0.96490
1.7 0.98379 0.98441 0.98500 0.98558 0.98613 0.98667 0.98719 0.98769 0.98817 0.98864 1.8 0.98909 0.98952 0.98994 0.99035 0.99074 0.99111 0.99147 0.99182 0.99216 0.99248 1.9 0.99279 0.99309 0.99338 0.99366 0.99392 0.99418 0.99443 0.99466 0.99489 0.99511 2.0 0.99532 0.99552 0.99572 0.99591 0.99609 0.99626 0.99642 0.99658 0.99673 0.99688 2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99764 0.99775 0.99785 0.99795 0.99805 2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99874 0.99988 2.3 0.99886 0.99891 0.99931 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99959 0.99963 0.99963 0.99965 0.99967 0.999	1.5	0.96611	0.96728	0.96841	0.96952	0.97059	0.97162	0.97263	0.97360	0.97455	0.97546
1.8 0.98909 0.98952 0.98994 0.99035 0.99074 0.99111 0.99147 0.99182 0.99216 0.99248 1.9 0.99279 0.99309 0.99338 0.99366 0.99392 0.99418 0.99443 0.99466 0.99489 0.99511 2.0 0.99532 0.99552 0.99572 0.99591 0.99609 0.99626 0.99642 0.99658 0.99673 0.99688 2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99764 0.99775 0.99785 0.99795 0.99805 2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99874 0.99880 2.3 0.99886 0.99891 0.99897 0.99902 0.99906 0.99911 0.99915 0.99920 0.99924 0.99928 2.4 0.99931 0.99933 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99988 0.99989 0.99989 0.99999 0.999	1.6	0.97635	0.97721	0.97804	0.97884	0.97962	0.98038	0.98110	0.98181	0.98249	0.98315
1.9 0.99279 0.99309 0.99338 0.99366 0.99392 0.99418 0.99443 0.99466 0.99489 0.99511 2.0 0.99532 0.99552 0.99572 0.99591 0.99609 0.99626 0.99642 0.99658 0.99673 0.99688 2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99764 0.99775 0.99785 0.99795 0.99805 2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99874 0.99880 2.3 0.99886 0.99891 0.99897 0.99902 0.99906 0.99911 0.99915 0.99920 0.99924 0.99928 2.4 0.99931 0.99935 0.99938 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99976 0.99978 0.99978 0.99986 0.99989 0.99989 0.99989 0.99989 0.99989 0.99999 0.99991 0.99991 0.99995 0.99995 0.99995 0.99995 0.99996 <	1.7	0.98379	0.98441	0.98500	0.98558	0.98613	0.98667	0.98719	0.98769	0.98817	0.98864
2.0 0.99532 0.99552 0.99572 0.99591 0.99609 0.99626 0.99642 0.99658 0.99673 0.99688 2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99764 0.99775 0.99785 0.99795 0.99805 2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99874 0.99880 2.3 0.99886 0.99891 0.99897 0.99902 0.99906 0.99911 0.99915 0.99920 0.99924 0.99928 2.4 0.99931 0.99935 0.99938 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99959 0.99961 0.99963 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99983 0.99983 0.99984 0.99984 0.999991 0.99991 0.99991 0.99991 0.99992 0.99992 0.99993 0.99997	1.8	0.98909	0.98952	0.98994	0.99035	0.99074	0.99111	0.99147	0.99182	0.99216	0.99248
2.1 0.99702 0.99715 0.99728 0.99741 0.99753 0.99764 0.99775 0.99785 0.99795 0.99805 2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99874 0.99880 2.3 0.99886 0.99891 0.99897 0.99902 0.99906 0.99911 0.99915 0.99920 0.99924 0.99928 2.4 0.99931 0.99935 0.99938 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99959 0.99961 0.99963 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99982 0.99983 0.99984 0.99985 0.99986 2.7 0.99987 0.99998 0.99998 0.99999 0.99999 0.99991 0.99991 0.99995 0.99995 0.99995 2.8 0.99996 0.99996 0.99997 0.99997 0.99997 0.999	1.9	0.99279	0.99309	0.99338	0.99366	0.99392	0.99418	0.99443	0.99466	0.99489	0.99511
2.2 0.99814 0.99822 0.99831 0.99839 0.99846 0.99854 0.99861 0.99867 0.99874 0.99880 2.3 0.99886 0.99891 0.99897 0.99902 0.99906 0.99911 0.99915 0.99920 0.99924 0.99928 2.4 0.99931 0.99935 0.99938 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99959 0.99961 0.99963 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99982 0.99983 0.99984 0.99985 0.99986 2.7 0.99987 0.99988 0.99989 0.99989 0.99999 0.99991 0.99991 0.99992 0.99992 0.99995 0.99995 0.99995 0.99996 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997	2.0	0.99532	0.99552	0.99572	0.99591	0.99609	0.99626	0.99642	0.99658	0.99673	0.99688
2.3 0.99886 0.99891 0.99897 0.99902 0.99906 0.99911 0.99915 0.99920 0.99924 0.99928 2.4 0.99931 0.99935 0.99938 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99959 0.99961 0.99963 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99982 0.99983 0.99984 0.99985 0.99986 2.7 0.99987 0.99988 0.99989 0.99989 0.99990 0.99991 0.99991 0.99992 0.99992 0.99995 0.99995 0.99996 0.99996 0.99996 0.99997 0.999997 0.999997 0.999997 0.99997 0.99997 0	2.1	0.99702	0.99715	0.99728	0.99741	0.99753	0.99764	0.99775	0.99785	0.99795	0.99805
2.4 0.99931 0.99935 0.99938 0.99941 0.99944 0.99947 0.99950 0.99952 0.99955 0.99957 2.5 0.99959 0.99961 0.99963 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99982 0.99983 0.99984 0.99985 0.99986 2.7 0.99987 0.99988 0.99989 0.99989 0.99990 0.99991 0.99991 0.99992 0.99992 2.8 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.999997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.9	2.2	0.99814	0.99822	0.99831	0.99839	0.99846	0.99854	0.99861	0.99867	0.99874	0.99880
2.5 0.99959 0.99961 0.99963 0.99965 0.99967 0.99969 0.99971 0.99972 0.99974 0.99975 2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99982 0.99983 0.99984 0.99985 0.99986 2.7 0.99987 0.99988 0.99989 0.99989 0.99990 0.99991 0.99991 0.99992 0.99992 2.8 0.99992 0.99993 0.99993 0.99994 0.99994 0.99994 0.99995 0.99995 0.99995 0.99995 0.99997 0.99998 2.9 0.99996 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99999 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99998 0.99998 0.99998 0.99999 0.99999 0.99999 0.99999 0.99999 0.999997 0.999997 0.99999 0.99999 0.	2.3	0.99886	0.99891	0.99897	0.99902	0.99906	0.99911	0.99915	0.99920	0.99924	0.99928
2.6 0.99976 0.99978 0.99979 0.99980 0.99981 0.99982 0.99983 0.99984 0.99985 0.99986 2.7 0.99987 0.99988 0.99989 0.99989 0.99990 0.99991 0.99991 0.99992 0.99992 2.8 0.99992 0.99993 0.99994 0.99994 0.99994 0.99995 0.99995 0.99995 0.99996 2.9 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99998	2.4	0.99931	0.99935	0.99938	0.99941	0.99944	0.99947	0.99950	0.99952	0.99955	0.99957
2.7 0.99987 0.99988 0.99988 0.99989 0.99989 0.99990 0.99991 0.99991 0.99992 0.99992 0.99992 2.8 0.99992 0.99993 0.99994 0.99994 0.99994 0.99994 0.99995 0.99995 0.99995 0.99995 0.99996 2.9 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99998	2.5	0.99959	0.99961	0.99963	0.99965	0.99967	0.99969	0.99971	0.99972	0.99974	0.99975
2.8 0.99992 0.99993 0.99993 0.99994 0.99994 0.99994 0.99995 0.99995 0.99995 0.99995 0.99996 2.9 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99997 0.99998	2.6	0.99976	0.99978	0.99979	0.99980	0.99981	0.99982	0.99983	0.99984	0.99985	0.99986
2.9 0.99996 0.99996 0.99996 0.99997 0.99997 0.99997 0.99997 0.99997 0.99998	2.7	0.99987	0.99987	0.99988	0.99989	0.99989	0.99990	0.99991	0.99991	0.99992	0.99992
	2.8	0.99992	0.99993	0.99993	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	0.99996
3.0 0.99998 0.99998 0.99998 0.99998 0.99998 0.99999 0.99999 0.99999 0.99999	2.9	0.99996	0.99996	0.99996	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998
	3.0	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999	0.99999

Table entries correspond to z = row heading + column heading