### Organisation of the Course

- First half 5 weeks Prof Rebecca Cheung
- Second half 5 weeks Prof Ian Underwood

- ELEE10003 exam only in December
- ELEE11049 exam 70%, course work 30% 15% 1st half, 15% 2nd half

#### Microelectronic Device Principles 4

- Quantum description of electronic materials
- Advanced electronic devices
- Electronic Information Displays
- Micro-displays

#### Recommended books

- Introduction to Quantum Mechanics by B. H. Bransden and C. J. Joachain
- Introduction to Solid State Physics by Charles Kittel
- GaAs Devices and Circuits by Michael Shur
- Lecture notes: on LEARN

## **Learning Outcomes**

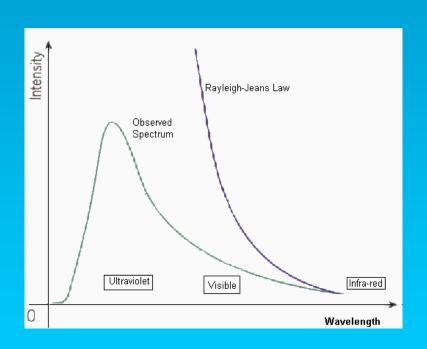
- Understand wave-particle duality
- Solve Schroedinger's equation for electron tunnelling
- Understand the origin of free electrons, periodic potentials and energy bands
- Know about electron transport and scattering mechanisms
- Derive the density of states in 3D, 2D, 1D
- Know the difference between ohmic and Schottky contacts; homo- and hetero- junctions
- Use bandgap engineering to design high electron mobility transistors, low dimensional structures
- Explain the impact of design and material properties on device performance
- Design simple microelectromechanical systems

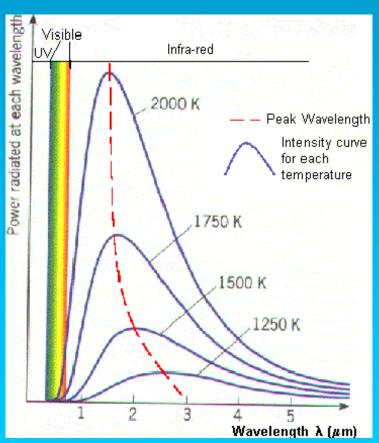
#### Course outline

- Bandgap engineering
- Wave-particle duality
- Schroedinger's equation and illustrative solutions
- Tunnelling
- Free electrons, periodic potentials and energy bands
- Electron transport and scattering mechanisms
- Ohmic and Schottky contacts
- Density of states 3D, 2D, 1D
- High electron mobility transistors
- Low dimensional structures
- Microelectromechanical systems

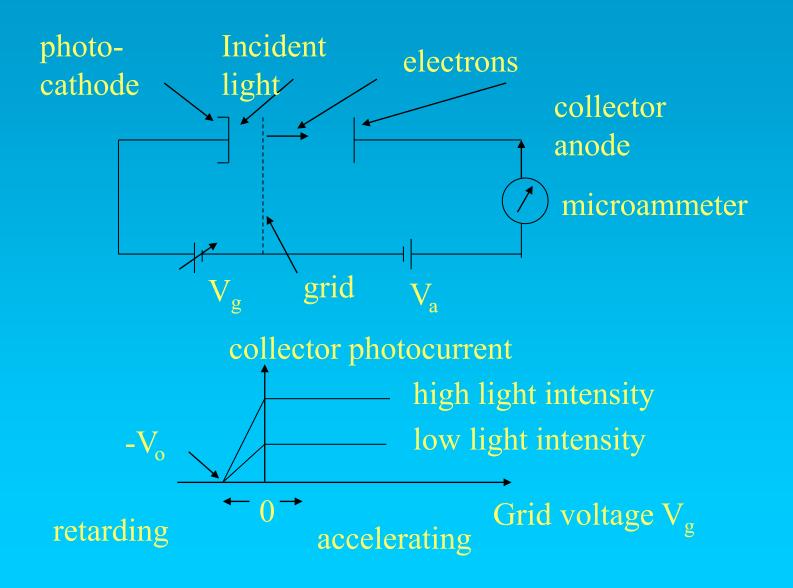
- electronic and optical properties of materials and the application of these properties to a physical understanding of the operation of electronic and optical devices
- classical mechanics → Newton
- Newton and Maxwell's electromagnetism theory → quantitative explanation before the beginning of the 20<sup>th</sup> century → for predicting dynamics of large scale systems
- to describe the dynamics of microscopic particles  $\rightarrow$  quantum or wave mechanics

## Blackbody radiation





#### Photoelectric effect



#### Observations

- unless the frequency of the incident light is greater than some critical values f<sub>o</sub>, (which is dependent on the material of the cathode), no emission is observed, no matter how intense the light
- For constant light frequency, and provided f
   f<sub>o</sub>, the photocurrent can be measured as a
   function of grid voltage V<sub>g</sub> and light
   intensity, keeping the anode voltage
   constant, see above figure

#### Observations

• no matter how intense the light is, there exists constant retarding voltage, at  $-V_o$ , when emission is entirely prohibited. This implies that the *maximum kinetic energy* of emitted electrons is constant and independent of the intensity of the incident light. Thus, the number of emitted photoelectrons is a function of the intensity of the light but their maximum energy is constant

#### Conclusion

- light energy is quantised, transported in discrete packets or photons
- The minimum energy required for an electron to be just emitted from a metal surface is called the *work function* of the particular metal and is usually designated eφ where φ is in volts.
- Therefore, kinetic energy of emitted electron = photon energy work function

$$\frac{1}{2} * mv^2 = hf - e\varphi$$

#### Conclusion

• Limiting case when an electron is just emitted with no kinetic energy. Then

$$f = f_o = e\phi/h$$

• At frequencies less than this critical value, the photon energy, hf, is not even sufficient to overcome the work function and no emission occurs

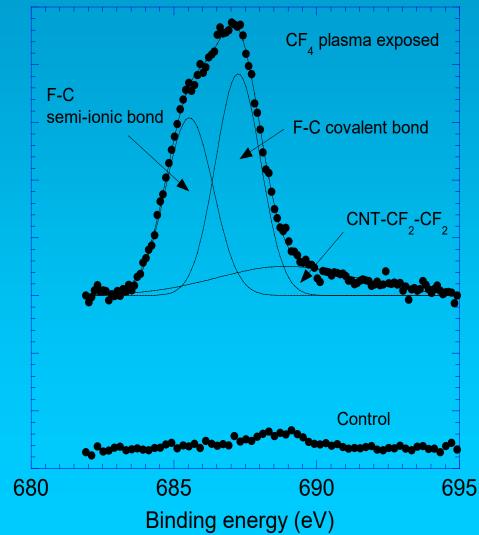
# Experimental example

# X-ray Photoelectron Spectroscopy of fluorinated Carbon Nanotubes (F1s spectra)

Normalised intensity (arb. units)

F is present in the CF<sub>4</sub> plasma exposed tubes with the existence of F-C covalent and semi-ionic bonding

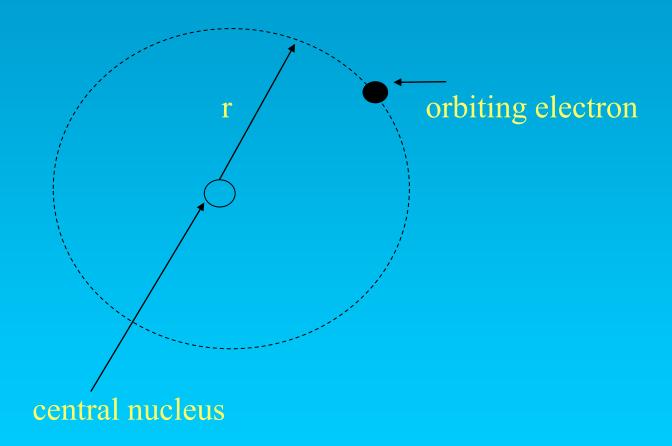
• APL vol. 83 pp 2426-2428 (2003)



MDP IV R.Cheung SMC G 1.06

- demonstration of quantum (theory) of light, in preference to classical wave theory – contrast with light diffraction.
- If a light wave can behave as a particle, can a particle, say, an electron, behave as a wave?

• Consider a simple model of a hydrogen atom consisting of a central nucleus with an electron travelling in a circular orbit around it, at some distance r:



- The coulomb force on the electron due to the efield of the positive nucleus is just sufficient to provide inward acceleration for circular motion at a constant radius r.
- Now, the electron in the circular orbit is constantly being accelerated and it can be shown by e-m theory that such an accelerated charge radiates e-m energy, with a corresponding loss of energy.

- But, conservation of energy indicates that as the electron radiates energy, its total energy must decrease, and the radius of the orbit must decrease => continual loss of energy and spiralling of the electron towards the nucleus.
- This clearly does not happen.

- Bohr postulated that the electron could only exist in discrete energy levels, corresponding in certain allowed stable orbits, without radiating any energy. The electron behaves as a wave interfering with itself thus setting up standing waves.
- Radiation from atom occurs only when the electron makes a transition from one allowed energy level to another, when the energy lost by the atom is converted into the energy of a single photon.

$$E_1 - E_2 = hf_{12}$$
  
where  $h = Planck's constant = 6.626e-34 J-s$ 

- only a discrete set of energy levels is postulated, only a discrete set of characteristic frequency is present in the output spectrum
- $E_n = -13.6/n^2 \text{ eV}$
- Where  $E_n$  are energy states for hydrogen atom, with ground states  $E_1 = -13.6 \text{eV}$ ,  $E_2 = -3.4 \text{eV}$ ,  $E_3 = -1.51 \text{eV}$

Velocity of light  $c = f \lambda$ Crude argument to derive momentum of photons.

Photons are subjected to an external force F which acts over some distance dx.

The change in photon energy dE = Fdx

If the photon momentum is p,

By Newton's law,

F = dp/dt

Therefore.

dE = dpdx/dt = (dx/dt).dp = c.dp

Integrating, we get

p = E/c

The momentum of the photons p = E/c, where energy E is transported with velocity c. Therefore  $p=hf/c = hf/f \lambda = h/\lambda = p$ 

De Broglie wavelength of a particle. In 1924, de Broglie argued that if light photons with wavelength  $\lambda$  have momentum p=h/ $\lambda$ , it might be possible for particles with momentum p to have some associated wavelength  $\lambda$  also, and behave in a wavelike manner, under some circumstances.

It was found that electrons can be diffracted, just like light. Electron diffraction has become an important tool to study interatomic spacing and the structure of molecules.

Using the above hypothesis, we calculate the wavelength of a classical Newtonian particle – an apple!

$$m = 1/5 \text{ kg}$$
  $v = 10 \text{m/s}$ 

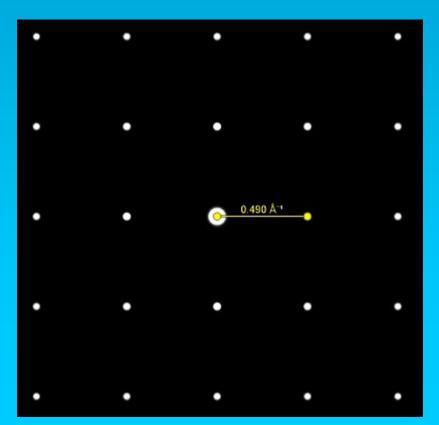
$$p=10/5 = 2kg.m/s$$

And its associated wavelength is

$$\lambda = h/p = (6.6 \times 10^{-34}) / 2 \approx 10^{-34} \text{ m}$$

Very small wavelength indeed!!!

# Experimental observation Electron diffraction pattern of single crystal gold



Face centre cubic with a lattice parameter of ~ 4 angstroms

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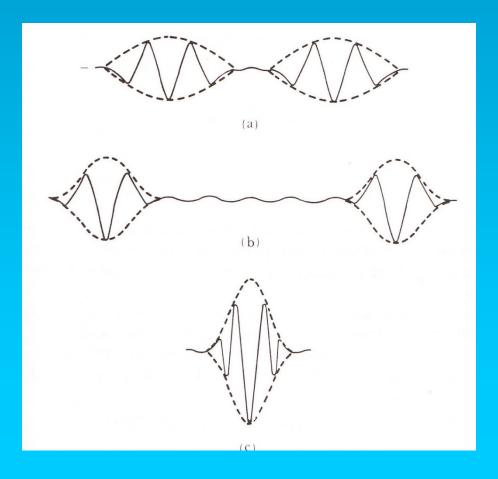
# Wave-packets - group and phase velocity

- To consider a geometric representation of how an object may simultaneously possess both wave and particle properties
- by studying the addition of waves of differing wavelength which has particle properties

# Wave-packets - group and phase velocity

- Two waves of slightly different wavelength
- Three waves of slightly different  $\lambda \rightarrow$  interference maxima larger and spreaded at wider intervals
- $\rightarrow$  infinite number of waves  $\rightarrow$  only one region of constructive interference exists  $\rightarrow$  wave packet
- The wavepacket geometrically represents an object with wave and particle properties.

# Formation of a 2D wavepacket



# Wave-packets - group and phase velocity

What is the velocity of the wavepacket?

A wave travelling in the x- direction with an amplitude  $A_o$  and  $\omega$ , angular frequency and  $\beta$  the phase constant:

$$A_o \cos (\omega t - \beta x)$$
  
where  $\beta = 2\pi/\lambda$ 

or  $A_o$  Re exp  $\{j(\omega t-\beta x)\}$ 

# Wave-packets - group and phase velocity

Phase velocity  $\rightarrow$  defined as the velocity of planes of constant phase along the propagation direction of the wave

 $(\omega t - \beta x) = constant$ 

Differentiating w.r.t. time gives

 $\omega - \beta dx/dt = 0$ 

 $v_{ph} = \omega/\beta \rightarrow$  the velocity at which some arbitrary phase propagates

Nothing material propagates at this velocity, it is possible for  $v_{ph}$  to be > speed of light

# Wave-packets - group and phase velocity

Consider mathematically what happens when two waves of equal amplitude but with slightly different λ propagates simultaneously in the x-direction:

$$A_o \cos (\omega t - \beta x) + A_o \cos [(\omega + \delta \omega)t - (\beta + \delta \beta)x]$$

$$= 2 A_o \cos \frac{1}{2} [(2\omega + \delta \omega)t - (2\beta + \delta \beta)x] \cos \frac{1}{2} [\delta \omega t - \delta \beta x]$$

Since 
$$\delta \omega \ll 2\omega$$
 and  $\delta \beta \ll 2\beta$ 

 $\approx 2A_0 \cos \frac{1}{2} (\delta \omega t - \delta \beta x) \cdot \cos (\omega t - \beta x)$ 

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# Wave-packets - group and phase velocity

• group velocity  $\rightarrow$  defined as the velocity of propagation of a plane of constant phase on the envelope  $\rightarrow$  the velocity of a group. Or a packet of waves along the direction of propagation

• A plane of constant phase on the envelope is  $\delta \omega t - \delta \beta x = const.$ 

 $v_g = \partial \omega / \partial \beta \rightarrow$  is the velocity at which energy is transmitted along the direction of propagation

# Wave-packets - group and phase velocity

