Jber. d. Dt. Math.-Verein. cy. 1997 B. G. Teubner Stuttgart
99 (1997) 9(1-100

Joachim A. Nitsche (192W1996)

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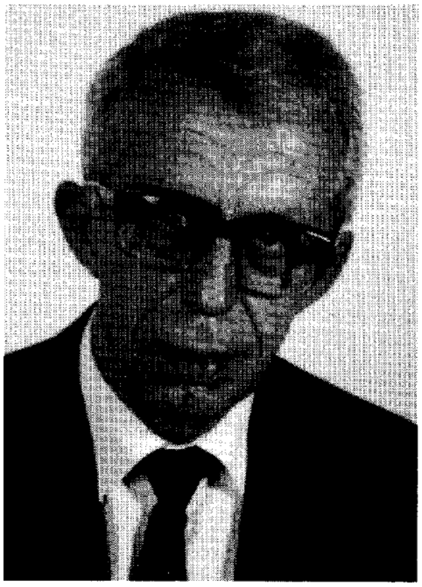

After a serious illness that confined him to a wheelchair for the last five years of his life, Joachim A. Nitsche died on January 12, 1996 at his home in Freiburg.

1 Curriculum vitae

Joachim Nitsche was bom on September 2, 1926 in Nossen, Saxony, where his parents worked as high school teachers for mathematics and physics. He was conscripted into the Wehrmacht at the age of seventeen and then spent the end of the war in captivity. In 1946, he graduated from high school in Bischofswerda and enrolled at the University of Göttingen in the summer semester of 1947 to study mathematics. After only six semesters, he obtained his diploma. For the

He was awarded a distinction for his diploma thesis written under the supervision of F. Rellich.

In Göttingen, he received his first introduction to differential geometry through a lecture by W. Haak, which he gave as part of a visiting professorship. Attracted by this field, he followed Haak to the Technische Hochschule Berlin-Charlottenburg, which later became the TU Berlin. He was awarded his doctorate in 1951 for his thesis "Integral relations for systems of quasilinear differential equations of 1st order of the elliptic type in two variables with an application to the embedding of surfaces with a given line element of positive curvature", for which he was again awarded a distinction.

Embedding and rigidity theorems, as well as related problems in the field of partial differential equations, defined J. Nitsche's main field of work over the next six to seven years. After completing his doctorate, he moved to an assistant position at the Free University of Berlin. There he was awarded the Venia Legendi in 1953 for his habilitation thesis "Boundary value problems for the embedding and bending of positively curved bounded surfaces". This work was written under the influence of W. Blaschke, whom he regarded as his "unofficial" teacher and supervisor, while his "formal" supervisor was F.W. Levi.

In 1952, Joachim Nitsche married Gisela Lange. The marriage produced three children.

From 1955 to 1957, Nitsche held a lectureship at the Free University, which he then gave up in favor of a job at IBM in Böblingen. There he became familiar with questions of applied mathematics, in particular with problems arising from the use of modern computing systems for the practical solution of concrete tasks. This led to a far-reaching reorientation of his scientific interests. From then on, his main field of research was theoretical investigations into numerical mathematics. Here he combined his old love of the theory of partial differential equations with the new aspect of the effective numerical determination of solutions to such equations. With his work on the finite element method, which emerged in the following years, he gained a place in the first rank of applied mathematicians.

From Böblingen, J. Nitsche was appointed to an associate professorship at the University of Freiburg by H. Görtler in 1958. There he was appointed to the chair in 1962

"Modern Computing Technology" at the Vienna University of Technology. However, the University of Freiburg managed to retain him by appointing him to a chair in applied mathematics in the same year. Nitsche remained loyal to Freiburg until his retirement in 1991, even when the state of North Rhine-Westphalia offered him a position as head of the Institute of Mathematics at the Society for Mathematics and Data Processing in conjunction with a chair at the University of Bonn in 1975.

As already mentioned, J. Nitsche's main field of work during his time in Freiburg was numerical mathematics. In addition to approximation statements in the area of spline functions, he devoted himself to

He studied the theory of finite elements for solving elliptic and parabolic differential equations in detail. He was one of the world's leading researchers in this field. He was in close contact with numerous specialist colleagues and was a frequent invited speaker at relevant congresses. The two conferences on finite elements that he organized at the Mathematisches Forschungsinstitut Oberwolfach in 1977 and 1980 attracted a great deal of international attention.

Joachim Nitsche was a dedicated teacher. In addition to beginners' lectures on analysis and linear algebra, as well as the standard lectures on numerical mathematics, he repeatedly offered special lectures and seminars on current research topics, which encouraged his students to carry out their own scientific work. In the early years in Freiburg, his lecture repertoire also included courses on game theory and business research.

In addition to the usual activities in academic self-administration
-In 1971 and 1972, he held the office of Dean of the Faculty of Mathematics and also took on tasks in the area of scientific self-administration. He was co-editor of the journals "Numerical Functional Analysis and Optimization", "Japan Journal of Applied Mathematics" and "RAIRO - Analyse Numérique", as well as the series "Pitman Research Notes in Mathematics" and the book series "Monographs and Studies in Mathematics" published by the same publishing house.

2 The scientific work

As already stated above, Joachim Nitsche's publications can essentially be divided into two categories: works of a purely analytical nature and theoretical studies of questions relating to numerical mathematics. In addition, there are isolated contributions on game theory ([22]) and optimization issues ([14, 21]).

2.a The analytical work

The majority of the analytical works were written during the Berlin period and deal with questions of surface bending and related problems from the theory of partial differential equations ([1-13, 16, 19]), although some of the later works from this period were written jointly with his brother Johannes C.C. Nitsche.

Edged surface pieces with a given metric allow isometric de- formations. Nitsche examines the question of which properties differentiate the various bendings and how the possible embeddings can be realized. The determination of bending surfaces is equivalent to finding solutions to the Gauss-Codazzi equations, which he transforms into a quasilinear system of two partial differential equations in two variables by suitable elimi- nation. This system is elliptic, parabolic or hyperbolic, depending on whether the Gaussian curvature of the surface is positive, zero or negative. Nitsche mainly studies the case of positive curvature. He derives integral relations, which the solutions to the problem must satisfy, and the geometric quantities, such as the curvature of the boundary curve. He then shows that simply connected surface pieces with a given metric are uniquely determined by the curvature of the boundary curve. By skillful transformations he can reduce the associated quasilinear elliptic system to a semilinear one in normal form, which is then solved by iteration. In this way, he also obtains existence theorems under suitable smallness conditions.

In [8] he shows that uniqueness is no longer guaranteed in the case of doubly connected surfaces. In the case of isometric bending of a spherical zone, he can prove by means of bifurcation methods that the integro-differential equation associated with the problem has exactly two solutions in the neighborhood of the unbent spherical zone under certain geometric conditions.

During his years in Freiburg, Joachim Nitsche published two more purely analytical papers. In [66] he gives a simple and elegant proof of the second Korn inequality. This is a coercivity inequality for the strain tensor of linear elasticity theory, where no boundary conditions are imposed. Using an elementary mirror method, he reduces the general situation to the case of Dirichletscher zero boundary conditions.

In his most recent analytical publication [73], he is also concerned with elementary evidence for known regularity statements.

2.b Work on numerics

Joachim Nitsche's first works [15, 20] on numerics are probably to be seen in connection with his work at IBM in Böblingen and deal with error estimates for eigenvalue problems with matrices.

At about the same time, he wrote an article [17] with Johannes C.C. Nitsche on convergence estimates for difference methods for partial differential equations of the elliptic type in two space dimensions. The difference solution to the mesh size h is continued by suitable trigonometric interpolation to a subspace of dimension n - - 0(/i^2). With the help of embedding theorems for Sobolev spaces with fractional index, convergence in the maximum norm can be shown under very weak conditions on the coefficient functions. Only square integrability is assumed for the right-hand side f . This means that the function values must be replaced by integral averages for discretization. The numerical calculation of the integrals is discussed in [18].

A few years later, Joachim Nitsche returned to these questions from the new perspective of seeing the order of convergence in the context of regularity and approximability of the solutions of differential equations. In particular, the question of quasi-optimal error estimates is one of the central topics **in his** work over the next few years. Error estimates for a sequence u of numerical solutions in subspaces of dimension n generally have the form ||o, - u|| < cz f | for linear equations Au -- f. The n-dimensional diameter d of the set {o | | Au < 1} is a lower bound for the error factors c,. A method is called *quasi-optimal* if c, = $O(d\ )$ applies.

Joachim Nitsche [25, 26, 32] can improve the earlier result [17] for the error order of O(/i '25) to O(h^2) and show that this estimate is quasi-optimal with respect to the L2 norm if onlyf *C* L2 is assumed via the right-hand side.

After the difference methods, Joachim Nitsche goes on in [24, 28, 30, 31] to treat the problem of quasi-optimality for projective methods (Ritz, Galerkin and error square methods) first for the abstract case of a self-adjoint operator *A* in Hilbert space. In addition to optimality, questions of stability, defect convergence and the comparison of the different methods are addressed. It is in this context that the famous and short paper [27] is written, in which it is shown in an abstract way that the quasi-optimality of the Ritz method in the energy norm also implies that in the original norm. This proof technique is later referred to as *Nitsche's trick*. For finite elements, it follows that convergence in the L2 norm is better by a power of h than in the N^l norm. J. Nitsche can show in [40] that quasi-optimality in the L2 norm also implies it in stronger norms if the subspaces satisfy suitable inverse estimates as known from classical approximation theory for trigonometric functions.

This period also saw several studies on the question of how the approximation quality of spline functions [33, 35, 36, 37] is related to the regularity of the functions to be approximated, as well as a paper [38] on the convergence of the Galerkin method for nonlinear equations.

Further work on spline functions [29, 34] led J. Nitsche to his main field of work of the next twenty years, namely his investigations of convergence properties of finite elements, for which he became internationally renowned. In [39] he proves for the first time that for linear elliptic differential equations of second order the error is of the order 0(h^2) in the *L* -norm and *O(h)* in the *Lg* -norm. This Lg estimate is derived from the L2 estimate using inverse estimates. All estimates prove to be quasi-optimal under the assumption that the right-hand side lies in 2.

Subsequent work is devoted to the question of how boundary conditions are to be taken into account in projective methods if the initial spaces do not satisfy these conditions. By including the boundary conditions in the variational approach and a clever choice of weighted boundary integrals, J. Nitsche can show in [41] that the convergence order **0(/2^2**) is preserved for linear splines. Extensions of these investigations can be found in [42, 44, 47] and in a paper published with J.H. Bramble's work on the error square method [42]. In a later paper [57] Nitsche takes up the treatment of boundary conditions again, this time for the plate problem.

Together with A.H. Schatz, and later with J.H. Bramble, he then deals in [43, 45, 46 with the question of whether locally better regularity properties of the solution also require locally better convergence properties. It turns out that the situation with finite elements is generally more favorable than in the classical case of Fourier approximation, where a singularity has an influence on the global convergence behavior. Locally higher regularity in the case of elliptic and parabolic differentials causes tial equations, as J. Nitsche [64] later shows, have a locally better approximation behavior.

The situation is different for differential equations with singularities in the coefficient functions or in regions with corners. In [48, 55] he can show that in such cases the global convergence behavior is also deteriorated. For nonlinear problems such investigations are carried out in [71]. Somewhat later, Joachim Nitsche turns to Lg-estimates for parabolic equations [58,70]. Time-dependent problems for free edges, as they occur in the Stefan problem, take up a broader space. Error estimates for the one- and two-dimensional case [60, 61, 62, 63, 65, 67, 69, 72] are reduced to the case of fixed boundaries by transformation, after which a nonlinear parabolic differential equation is to be treated. Singularities also occur here, the effects of which on the convergence behavior are carefully analyzed.

2.c Lg estimates

One focus of Joachim Nitsche's research in the field of finite element methods is convergence investigations in *Lg* standards.

Projection methods, such as Ritz's method for elliptic boundary value problems or the Galerkin method for parabolic initial boundary value problems, are formulated in a natural way using 2-norms within the framework of the theory of partial differential equations. For this reason, convergence statements arise first in the corresponding +2-norms, while the convergence behavior in Lp-norms, especially in the Lg-norm, must be investigated separately.

Until the mid-seventies, the convergence of finite elements in several spatial dimensions was mainly investigated in the +z- and ff^k norms. Apart from the comparatively simple one-dimensional case, fig-estimates are obtained by applying embedding theorems and inverse estimations for the initial spaces. As already mentioned, these methods provide quasi-optimal estimates for the case where the right-hand side is in Lzband the solution is in ff^2 . The paper [50] written with J.H. Bramble and A.H. Schatz on "inner" Lg estimates provided the first approaches for dealing with the case that the solution lies in the space W2. The question of optimal estimates in the *Lg* norm under this condition was considered very difficult and spurred on many mathematicians at that time.

In order to prove quasi-optimal convergence orders, two methods have emerged at about the same time, at least for second-order elliptic boundary value problems: the regularization of the Green's function and the weighted 2-norm method. The latter method is expanded, refined and transferred to other problems by Nitsche.

In a series of papers, convergence statements for the Ritz approximation of the solution of the Laplace equation are first proved. In [49] it is shown that for finite elements of order r > 3 the asymptotic convergence of the Ritz approximation is quasi-optimal. The case of linear finite elements is more difficult to treat.

In [52] Nitsche proves that for r = 2, except for a logarithmic factor, quasi-optimality also holds, and derives a corresponding estimate for variational inequalities. In [56], the result for the Laplace equation is improved by replacing the Lp norms of the second derivatives of the solution by the *Lz* -non:n of the right-hand side of the differential equation in the error estimation.

These estimates were so technically complex that Ciarlet asked Nitsche for a simplified version for his book on finite elements, which is never written there.

In [51] error estimates for the elasticity equations are proved, and in [54] the results for the linear second order elliptic problem are transferred to nonlinear elliptic boundary value problems in divergence form. Finally, Nitsche shows in [68] that it follows from his results that the Ritz operator is bounded (except for a logarithmic term in the case of linear finite elements) in the *Lp* - norm and in Hölder or Lipschitz norms, a result which had been partially proved before by other methods.

In [58, 59] and in the paper [70] written jointly with M.F. Wheeler, Nitsche addresses the question of asymptotic error estimates in the *Lg(Lq)* norm (i.e. in the supremum norm with respect to time and space) for the Galerkin approximations of the solutions of linear parabolic initial boundary value problems. Here the methods of the weighted +z-norms are suitably transferred. Finally, in several papers he treats the Stefan problem in one space dimension and also proves ńp-estimates for the Galerkin approximation of the solution of this problem in [60], among others.

In [74] he discusses the application of finite element methods to singular integral operators as they occur in the theory of conformal mappings. Such equations do not fit into the usual theory of finite elements, since neither Gårding-type inequalities nor the usual shift theorems for partial differential equations apply. Nitsche shows how these equations can be treated with the abstract theory of finite elements. Using the method of weighted norms, he proves that the Galerkin operator in the Lg-norm is bounded, and transfers this result using methods from Arbeit [40] on Hölder norms.

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2. Hans-Peter Helfrich, Optimal linear approximation of bounded sets in normierten Räumen, Freiburg, SS 1969.
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4. Reinhard Scholz, Approximation with spline functions in weighted Lebesgue-Spaces above the interval (0, on), Freiburg, WS 1971/72.
5. Reinhardt Link, Asymptotic error representations in approximation methods with spline functions, Freiburg, WS 1974/75.
6. Jürgen Schormann, Vergleich der Normen von Galerkin- und Ritz-Operatoren, Freiburg, SS 1975.

7. Fritz Grenacher, Über die Konvergenz bei Regularisierungsverfahren für nicht sachgemäß gestellte Anfangswertprobleme, Freiburg, WS 1976/77.
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9. Klaus Braun, Lokale Konvergenz der Ritz-Projektion auf Finite Elemente für Pseudo- Differentialgleichungen negativer Ordnung, Freiburg, SS 1985.
10. Sigrid Lang, *Lg* -Convergence of the finite element method for the capillary surface problem, Freiburg, WS 1991/92 (completed under G. Dziuk).

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