CS486 Assignment 3 CHEN YIFEI 20754459

# Part A:

# Q1:The construction of factor and the function needed are

## in the file 'factor.py':

```
from math import log10,floor
class Factor(object):
    # width during formatting
    FORMAT_WIDTH = 10
    # round up to roundTo decimal places
    ROUNDTO = 3
    # round up to n significant figures
    @staticmethod
    def significant(x):
         return round(x, -int(floor(log10(x))-(Factor.ROUNDTO-1)))
    def __init__(self, varList, varValues, mappedValues):
         self.factorTable = dict()
         self.mappedValues = mappedValues
         for i in xrange(0,len(varList)):
              self.factorTable[varList[i]] = varValues[i]
    # Restrict this factor on a variable with a particular value
    def restrict(self, variable, value):
         # search all the positions where value of indicated variable satisfy the parameter
'value'
         index = [i for i, val in enumerate(self.factorTable[variable]) if val == value]
```

```
del self.factorTable[variable]
         # then we do the restriction operation
         variables = self.factorTable.keys()
         restrictVars = map(lambda var: [val for i, val in enumerate(self.factorTable[var]) if i in
index], variables)
         self.mappedValues = [val for i, val in enumerate(self.mappedValues) if i in index]
         for i in xrange(0, len(variables)):
              self.factorTable[variables[i]] = restrictVars[i]
    # compute the multiplication of two factors
    @staticmethod
    def multiply(factorA, factorB):
         # get the variables list of each factor
         factorA_Vars = factorA.factorTable.keys()
         factorB_Vars = factorB.factorTable.keys()
         # figure out the variables which are common for these two factors and assign them
into newVars firstly
         newVars = [var for var in factorA_Vars if var in factorB_Vars]
         # select the rows in factorB that has to be multiplied with the rows in factorA
         multiply_A = [[] for i in xrange(len(factorA.mappedValues))]
         numMappedValues = 0
         for i in xrange(len(multiply_A)):
              for j in xrange(len(factorB.mappedValues)):
                   sameCommonValues = True
                   # we only multiply the tuple with the same values of common variables
                   for var in newVars:
                       if factorA.factorTable[var][i] != factorB.factorTable[var][j]:
                            sameCommonValues = False
                            break
                   # now the Common Variables should have the same values
```

# firstly we take out the 'column' from the factorTable to continue restrict operation

```
multiply_A[i].append(j)
                       numMappedValues+= 1
         # now the newVars should contain the union of the two factors
         newVars.extend([var for var in factorA_Vars if var not in newVars])
         newVars.extend([var for var in factorB_Vars if var not in newVars])
         newValues = [[] for i in xrange(len(newVars))]
         multiplyValues = []
         tuples = tuple(enumerate(newVars))
         # for each row in factorA
         for row in xrange(len(multiply_A)):
              # for each rows in factorB to be multiplied with a row in in factorA
              for row_in_factorB in multiply_A[row]:
                  for i, var in tuples:
                       # assign values of these variables row by row
                       newValues[i].append(factorA.factorTable[var][row]
                                                                              if
                                                                                    var
                                                                                           in
factorA_Vars \
                                               else factorB.factorTable[var][row_in_factorB])
                  multiplyValues.append(Factor.significant(factorA.mappedValues[row]
factorB.mappedValues[row_in_factorB]))
         return Factor(newVars, newValues, multiplyValues)
    # sum a variable in a factor
    def sumout(self, variable):
         # notice that we should do nothing for a factor with only one variable
         if len(self.factorTable.keys()) == 1:
              return
```

if sameCommonValues:

```
# get the domain of the variable
domainOfVar = set(self.factorTable[variable])
# delete the column of this variable from table entirely
del self.factorTable[variable]
row_to_sum = []
allOfVars = self.factorTable.keys()
for i in xrange(len(self.mappedValues)):
    if i in row_to_sum:
         # if i already found should be sum, continue to next iteration
         continue
    limitOfMatch = len(self.mappedValues) / len(domainOfVar)
     for j in xrange(i+1, len(self.mappedValues)):
         # check if the remaining values of vars of two row are the same
         sameValues = True
         for var in allOfVars:
              if self.factorTable[var][i] != self.factorTable[var][j]:
                   sameValues = False
         if sameValues and j not in row_to_sum:
              row_to_sum.append(j)
              limitOfMatch-= 1
         # if we already found pair of matches up to limitOfMatch
         if limitOfMatch == 0:
              break
newValues = [[] for i in xrange(len(allOfVars))]
# create a new list of mapped values with length
sumoutValues = []
```

```
lists = list(enumerate(allOfVars))
         sumRowIndex = 0
         for i in xrange(len(self.mappedValues)):
             if i in row_to_sum:
                  continue
             for w, var in lists:
                  newValues[w].append(self.factorTable[var][i])
             # add the rows as a sum of a kind of variable values
             sum = self.mappedValues[i]
             for j in xrange(0, len(domainOfVar)-1):
                  sum+= self.mappedValues[row_to_sum[sumRowIndex]]
                  sumRowIndex+= 1
             sumoutValues.append(sum)
         for j, var in lists:
             # clear former values
             del self.factorTable[var][:]
             self.factorTable[var].extend(newValues[j])
         # clear old mappedValues
         del self.mappedValues[:]
         self.mappedValues.extend(sumoutValues)
    def normalize(self):
         totalMappedValues = sum(self.mappedValues)
         self.mappedValues[:] = map(lambda x: Factor.significant(x/totalMappedValues),
self.mappedValues)
```

```
@staticmethod
    def inference(factorList, queryVariables, orderedListOfHiddenVariables, evidenceList):
         # figure out the variables which are in the query and evidence from
orderedListOfHiddenVariables
         step = 0
         if len(evidenceList) > 0:
             evidenceOfVars = zip(*evidenceList)[0]
         else:
              evidenceOfVars = []
         orderedListOfHiddenVariables = filter(lambda x: x not in queryVariables \
                                                           and x not in evidenceOfVars,
orderedListOfHiddenVariables)
         for evidence in evidenceList:
              for factor in factorList:
                  # if the evidence variable is in this factor, we should restrict them
                  if evidence[0] in factor.factorTable.keys():
                       factor.restrict(evidence[0], evidence[1])
         evidences = reduce(lambda x,y: '{!s}, {!s}'.format(x,y), evidenceList) if
len(evidenceList) > 0 else 'nothing'
         print ("step ",step)
         step += 1
         Factor.textFormattedPrint('Restricting factors on evidence {!s}'.format(evidences), 50,
factorList)
         for hiddenVariable in orderedListOfHiddenVariables:
              factorsWithHiddenVar = []
              for factor in factorList:
```

```
factorsWithHiddenVar.append(factor)
              # take out all the factors with a hidden variable from the list
              factorList = filter(lambda x: x not in factorsWithHiddenVar, factorList)
              # compute product of hidden variables
              if len(factorsWithHiddenVar) > 0:
                  print ("step ",step)
                  step += 1
                  Factor.textFormattedPrint('Summing over {!s}'.format(hiddenVariable), 30,
[]
                  multiplyFactor = reduce(Factor.multiply, factorsWithHiddenVar)
              else:
                  continue
              print ("step ",step)
              step += 1
              Factor.textFormattedPrint('Intermediate
                                                             multiply
                                                                             factor
                                                                                          on
{!s}'.format(hiddenVariable),30,[multiplyFactor])
              # do the sumout operation among the hidden variable on the product factor
              print ("step ",step)
              step += 1
              multiplyFactor.sumout(hiddenVariable)
              Factor.textFormattedPrint('Summing
                                                                                     variable
                                                                  out
{!s}'.format(hiddenVariable),30,[multiplyFactor])
              # insert the result back to the list
              factorList.append(multiplyFactor)
```

# now the only issue we have is about factors with only the query variable, we

if hiddenVariable in factor.factorTable.keys():

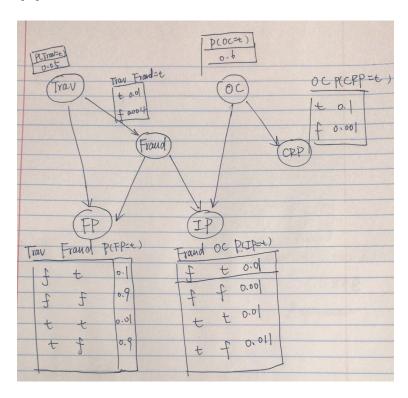
```
product them together
         inferredFactor = reduce(Factor.multiply, factorList)
         print ("step ",step)
         step += 1
         Factor.textFormattedPrint('multiplying factors with query variables: {!s}'.\
                                                format(queryVariables),45,[inferredFactor])
         # normalize stage
         inferredFactor.normalize()
         print ("step ",step)
         step += 1
         Factor.textFormattedPrint('Finally normalize the result',30,[inferredFactor])
    def copy(self):
         valuesOfKey = self.factorTable.items()
         return Factor(map(lambda x: x[0], valuesOfKey), map(lambda x: x[1], valuesOfKey),
self.mappedValues)
    # delete opertation which take out the 'column' in matrix
    def __del__(self):
         del self.mappedValues
         self.factorTable.clear()
         del self.factorTable
    # used to get the string form of this factor
    def __str__(self):
         formatting = "
         variables = self.factorTable.keys()
         # create a format string to use
         for i in xrange(0, len(variables)+1):
```

```
formatting+= '{!s:{width}}'
             if i == len(variables):
                  formatting+= '\r\n'
         # create the formatted header
         returnValue
                                       formatting.format('MappedVal', *variables,
width=Factor.FORMAT_WIDTH)
         zippedValues = zip(self.mappedValues, *map(lambda x: self.factorTable[x],
variables))
         # for each value create the formatted row
         for values in zippedValues:
             returnValue+= formatting.format(*values, width=Factor.FORMAT_WIDTH)
         return return Value
    @staticmethod
    def textFormattedPrint(statement, Num, factors):
         print statement, '\r'
         print '+' * Num, '\r'
         for factor in factors:
             print factor, '\r'
         print '+' * Num, '\r'
```

## Q2:The construction of factor and the function needed are

# in the file 'factor.py':

(a)



# (b) the code:

```
from factor import Factor

def main():
    hiddenVariables = ['Trav','FP','Fraud','IP','OC','CRP']
    factorTrav = Factor(['Trav'], [['t','f']], [0.05,0.95])
    factorFraudTrav = Factor(['Fraud','Trav'], [['t','t','f','f'], ['t','t','f','f'], ['t','f'], ['t'], ['t','f'], ['t','f
```

### now we can see the steps of two inference from the terminal:

```
[v1022-rn-76-14:a3partA chenyifei$ python q2b.py
('step ', 0)
Restricting factors on evidence nothing
MappedVal Fraud Trav
    t
t
f
0.004
          f
t
0.99
0.996 f
MappedVal Trav
0.05
0.95
Summing over Trav
('step ', 2)
Intermediate multiply factor on Trav
MappedVal Fraud Trav 0.0005 t t t 0.0038 t f
0.0495 f t
0.946 f f
0.946
    f
           f
('step', 3)
Summing out variable Trav
MappedVal Fraud
0.0043 t
0.9955
('step ', 4)
multiplying factors with query variables: ['Fraud']
MappedVal Fraud
0.0043 t
0.9955
('step', 5)
Finally normalize the result
MappedVal Fraud
0.0043 t
0.996 f
0.996
```

```
('step ', 0)
Restricting factors on evidence ['FP', 't'], ['CRP', 't'], ['IP', 'f']
MappedVal Trav
0.05
   t
0.95
     f
MappedVal Fraud
           Trav
0.01
     t
            t
0.004
      t
            f
0.99
0.996
     f
           f
MappedVal Fraud
           Trav
   t
0.9
            t
0.1
            f
      +
0.9
      f
           t
0.01
      f
MappedVal Fraud
           OC
0.98
      t
            t
0.989
      t
           f
0.99
      f
           t
0.999
      f
MappedVal OC
0.6
0.4
MappedVal OC
0.1 t
0.001
('step ', 1)
Summing over Trav
('step ', 2)
Intermediate multiply factor on Trav
MappedVal Fraud Trav 0.00045 t t
0.00045 t
           t
f
0.0446
     f
0.00038 t
0.00946 f
('step', 3)
Summing out variable Trav
MappedVal Fraud
0.00083 t
0.05406
('step ', 4)
Summing over OC
('step ', 5)
Intermediate multiply factor on OC
MappedVal Fraud OC
0.0588 t
           t
0.000396 t
0.0594 f
           t
f
     f
0.0004
```

```
('step ', 6)
Summing out variable OC
MappedVal Fraud
0.059196 t
0.0598
('step ', 7)
multiplying factors with query variables: ['Fraud']
MappedVal Fraud
4.91e-05 t
0.00323
('step', 8)
Finally normalize the result
MappedVal Fraud
0.015
    t
0.985
```

### (c) The code:

### steps:

```
[v1022-rn-76-14:a3partA chenyifei$ python q2c.py
('step ', 0)
Restricting factors on evidence ['FP', 't'], ['CRP', 't'], ['IP', 'f'], ['Trav',
't']
MappedVal
0.05
MappedVal Fraud
0.01 t
0.99 f
MappedVal Fraud
0.9 t
0.9 f
MappedVal Fraud
0.989
0.999
MappedVal OC 0.6 t f
MappedVal OC
0.1
0.001
+++++
('step ', 1)
Summing over OC
('step ', 3)
Summing out variable OC
MappedVal Fraud
0.059196 t
0.0598 f
2.66e-05 t
0.00267 f
```

## (d) code:

### steps:

```
Summing over FP
[v1022-rn-76-14:a3partA chenyifei$ python q2d.py
('step', 0)
Restricting factors on evidence ['IP', 't']
                                     ('step ', 5)
                                     Intermediate multiply factor on FP
MappedVal Trav
0.05 t
0.95 f
                                     0.00083
MappedVal Fraud
              Trav
                                     0.00347
                                     0.05406
0.94195
0.996
                                     ('step ', 6)
Summing out variable FP
MappedVal FP
              Fraud
0.9
                                     0.1
                                     0.0043 t
0.99601 f
0.01
0.1
0.9
0.1
0.99
                                     ('step ', 7)
Summing over OC
MappedVal Fraud
              OC
                                     0.02
0.011
                                     ('step', 8)
0.01
0.001
                                     Intermediate multiply factor on OC
                                     MappedVal OC
                                     MappedVal CRP
                                                    Fraud
0.6
0.4
                                     0.0012
                                     0.0108
                                     4.4e-06
MappedVal CRP
              OC
                                     0.0044
0.1
0.001
                                     0.0006
                                     0.0054
0.999
                                     4e-07
                                     0.0004
('step', 2)
Intermediate multiply factor on Trav
HappedVal FP Fraud Trav
4.000065 t t t
                                     ('step ', 9)
                                     Summing out variable OC
                                     MappedVal CRP
                                     0.0012044 t
                                     0.0152 f
0.0006004 t
0.00045
5e-05
                                     0.0058
0.0446
0.00495
0.00038
                                     0.00342
                                     Summing over CRP
0.00946
0.937
                                     ('step ', 11)
('step ', 3)
                                     Intermediate multiply factor on CRP
Fraud
                                     MappedVal CRP
0.0012044 t
0.00083
                                     0.0152
0.0006004 t
0.00347
0.05406
0.94195
('step ', 12)
Summing out variable CRP
MappedVal Fraud
0.0164044 t
0.0064004 f
('step', 13)
multiplying factors with query variables: ['Fraud']
MappedVal Fraud
7.05e-05 t
0.00637
('step ', 14)
Finally normalize the result
+++++++++++++++++++++++++++++
MappedVal Fraud
0.0109
         t
f
0.989
```

PartB Code:

```
preprocess.py
from sets import Set
                     \mbox{\#} the function to produce the matrix from the content of the files \mbox{\#} which has the same functionality with load.m script in matlab
                     def setMatrix(data, label, numFeatures):
                                          Matrix = []
                                         \label{eq:matrix} \begin{tabular}{ll} $\mathsf{Matrix} = \mathsf{I}_1$ & $\mathsf{Ma
                                                            docId = 0
for line in f:
    Matrix.append([0] * (numFeatures + 1))
    Matrix[docId][-1] = int(line)
                                                                                docId+= 1
                                         #read the data file and record the words existence of each article in the matrix
                                         with open(data, 'r') as f:
    for line in f:
        lineSplit = line.split()
                                                                               fileID = int(lineSplit[0])
featureNum = int(lineSplit[1])
Matrix[fileID-1][featureNum-1] = 1
     24
                                         return Matrix
                      # obtain the arrays of numfeature and exactly words in correspond order
     28
                     def readWords(haveDone):
                                         numFeatures = 0
features = []
                                          words = []
                                         if haveDone:
    featuresSet = Set(featuresSet)
                                  with open('words.txt', 'r') as f:
   35
                                                              for line in f:
if haveDone:
                                                                                                  features.add(numFeatures)
                                                                              else:
features.append(numFeatures)
                                                                               numFeatures = numFeatures +
                                                                                wordsSet.append(line.rstrip())
                                         return (numFeatures, features, words)
   45
```

```
naiveBayes.py
🔡 < > 📝 naiveBayes.py > 🛐 _init_
       from math import log, fabs
from __future__ import division
import heapq
       class NaiveBayes(object):
    # labbel
    ALTATHEISM = 1
    COMPGRAPHICS = 2
              def __init__(self, featuresSet, matrix):|
    # get groups of Atheism and graphics separately
    Atheism = filter(lambda doc: doc[-1] == NaiveBayesLearner.ALTATHEISM, matrix)
    Graphics = filter(lambda doc: doc[-1] == NaiveBayesLearner.COMPGRAPHICS, matrix)
10
                     # compute the prior probability
self.priorAtheism = len(Atheism)/len(matrix)
self.priorGraphics = len(Graphics)/len(matrix)
                     # for each feature calculate the likelihood
                     def predict(self, doc):
    posteriorAtheism = log(self.priorAtheism)
                     posteriorGraphics = log(self.priorGraphics)
                     for i in xrange(0,len(self.likelihoods)):
    if doc[i]:
                                  posteriorAtheism += log(self.likelihoods[i][0])
                                  posteriorAtheism -= log(1-self.likelihoods[i][0])
                           in uuc(1]:
    posteriorGraphics += log(self.likelihoods[i][1])
else:
                                  posteriorGraphics -= log(1-self.likelihoods[i][1])
                     # now we return the result with higher ML
return NaiveBayesLearner.ALTATHEISM if posteriorAtheism >= posteriorGraphics else NaiveBayesLearner.COMPGRAPHICS
               # for question 2
               def DiscriminativeWords(self, wordsSet):
                    DiscriminativeWords(self, wordsSet):
array = []
# compute the absolute value of difference of likelihoods
for i in xrange(0,len(self.likelihoods)):
    difference = fabs(log(self.likelihoods[i][0], 2) - log(self.likelihoods[i][1], 2))
array.append((i,difference))
# sort the result
array.sort(lambda x,y: cmp(x[1],y[1]), None, True)
for i in xrange(0, 10):
    print ("word: %s discrimination: %d" %(wordsSet[res[i][0]], res[i][1]))
```

```
main.py
1 | from __future__ import division
2 | from naiveBayes import NaiveBayes
  3 from preprocess import setMatrix, readWords
  5 # implement the prediction
     # and check the correction rate
  7 def getAccuracy(learner, matrix):
         numCorrectPredictions = 0
         for rows in matrix:
              if learner.predict(rows) == rows[-1]:
                  numCorrectPredictions+= 1
 13
         return numCorrectPredictions/len(matrix)
 14
 15
 16 def main():
         numFeatures, featuresSet, wordsSet = readWords(False)
         trainMatrix = setMatrix('trainData.txt', 'trainLabel.txt', numFeatures)
testMatrix = setMatrix('testData.txt', 'testLabel.txt', numFeatures)
 19
 20
 21
        naiveLearner = NaiveBayes (featuresSet, trainMatrix)
 22
         # get the accuracy for train and text file
 23
         trainAccuracy = getAccuracy(naiveLearner, trainMatrix)
        testAccuracy = getAccuracy(naiveLearner, testMatrix)
         print ("trainAccuracy: %s testAccuracy: %d" %(trainAccuracy, testAccuracy))
         naiveLearner.DiscriminativeWords(wordsSet)
    main()
```

## print out of code:

#### preprocess:

```
from sets import Set
# the function to produce the matrix from the content of the files
# which has the same functionality with load.m script in matlab
def setMatrix(data, label, numFeatures):
    Matrix = []
    #construct the matrix with the size [numArticle][numFeatures + 1]
    #and assign the label value of each article is in the last column
    with open(label, 'r') as f:
         docld = 0
         for line in f
              Matrix.append([0] * (numFeatures + 1))
              Matrix[docld][-1] = int(line)
              docld+= 1
```

#read the data file and record the words existence of each article in the matrix

```
with open(data, 'r') as f:
```

```
for line in f:
              lineSplit = line.split()
              fileID = int(lineSplit[0])
              featureNum = int(lineSplit[1])
              Matrix[fileID-1][featureNum-1] = 1
    return Matrix
# obtain the arrays of numfeature and exactly words in correspond order
def readWords(haveDone):
    numFeatures = 0
    features = []
    words = []
    if haveDone:
         featuresSet = Set(featuresSet)
     with open('words.txt', 'r') as f:
         for line in f:
             if haveDone:
                  features.add(numFeatures)
              else:
                  features.append(numFeatures)
              numFeatures = numFeatures + 1
              wordsSet.append(line.rstrip())
    return (numFeatures, features, words)
```

### Naïvebayes

```
from math import log, fabs
from _future_ import division
import heapq
class NaiveBayes(object):
    # labbel
    ALTATHEISM = 1
    COMPGRAPHICS = 2
    def __init__(self, featuresSet, matrix):
        # get groups of Atheism and graphics separately
        Atheism = filter(lambda doc: doc[-1] == NaiveBayesLearner.ALTATHEISM, matrix)
        Graphics = filter(lambda doc: doc[-1] == NaiveBayesLearner.COMPGRAPHICS, matrix)
        # compute the prior probability
        self.priorAtheism = len(Atheism)/len(matrix)
        self.priorGraphics = len(Graphics)/len(matrix)
        # for each feature calculate the likelihood
        (reduce(lambda x,y: (1 if y[feature] else 0) + x, Graphics, 0) + 1)/(len(Graphics) + 2))\
                                  for feature in featuresSet]
    def predict(self, doc):
        posteriorAtheism = log(self.priorAtheism)
        posteriorGraphics = log(self.priorGraphics)
        for i in xrange(0,len(self.likelihoods)):
            if doc[i]:
                 posteriorAtheism += log(self.likelihoods[i][0])
            else:
                 posteriorAtheism -= log(1-self.likelihoods[i][0])
            if doc[i]:
                 posteriorGraphics += log(self.likelihoods[i][1])
```

```
else:
                   posteriorGraphics -= log(1-self.likelihoods[i][1])
          # now we return the result with higher ML
          return
                      NaiveBayesLearner.ALTATHEISM
                                                               if
                                                                       posteriorAtheism
                                                                                                          posteriorGraphics
                                                                                                                                    else
{\tt Naive Bayes Learner. COMPGRAPHICS}
     # for question 2
     def DiscriminativeWords(self, wordsSet):
          array = []
          # compute the absolute value of difference of likelihoods
          for i in xrange(0,len(self.likelihoods)):
              difference = fabs(log(self.likelihoods[i][0], 2) - log(self.likelihoods[i][1], 2))
              array.append((i,difference))
          # sort the result
          array.sort(lambda x,y: cmp(x[1],y[1]), None, True)
          for i in xrange(0, 10):
              print ("word: %s discrimination: %d" %(wordsSet[res[i][0]], res[i][1]))
main:
from _future_ import division
from naiveBayes import NaiveBayes
from preprocess import setMatrix, readWords
# implement the prediction
# and check the correction rate
def getAccuracy(learner, matrix):
    numCorrectPredictions = 0
    for rows in matrix:
        if learner.predict(rows) == rows[-1]:
             numCorrectPredictions+= 1
    return numCorrectPredictions/len(matrix)
```

```
def main():

numFeatures, featuresSet, wordsSet = readWords(False)

trainMatrix = setMatrix('trainData.txt', 'trainLabel.txt', numFeatures)

testMatrix = setMatrix('testData.txt', 'testLabel.txt', numFeatures)

naiveLearner = NaiveBayes (featuresSet, trainMatrix)

# get the accuracy for train and text file

trainAccuracy = getAccuracy(naiveLearner, trainMatrix)

testAccuracy = getAccuracy(naiveLearner, testMatrix)

print ("trainAccuracy: %s testAccuracy: %d" %(trainAccuracy,testAccuracy))
```

naiveLearner.DiscriminativeWords(wordsSet)

main

#### Question b

```
trainAccuracy: 0.92837 testAccuracy: 0.88967 word: graphics discrimination: 6.38374874578 word: atheism discrimination: 5.73389435561 word: religion discrimination: 5.66678015975 word: moral discrimination: 5.55986495583 word: keith discrimination: 5.55986495583 word: evidence discrimination: 5.55986495583 word: atheists discrimination: 5.52239025041 word: god discrimination: 5.45768456103 word: bible discrimination: 5.40374575392 word: christian discrimination: 5.36192557822
```

#### Question c

This assumption might be reasonable in some situations where the variables are not correlated firmly, and calculating the accurate result can be hard, therefore the choice of naïve Bayesian is reasonable.

However, we do production directly for conditional probabilities, which is not accurate. we saw that in this part of assignment the algorithm is not that good for estimation when faced with test data.

#### Question d

Suppose we have a dataset  $\mathbf{e}$ , where the  $i^{th}$  data assign a values  $\mathbf{x}_i$  to observed variables  $\mathbf{X}$ , leaving  $\mathbf{Z}$  latent variables (with values  $\mathbf{z}_j$ ) unassigned, then

**Recall**: Bayes Net Maximum Likelihood (Complete data -  $Z = \{\}$ )

$$\theta_{V=true,pa(V)=\mathbf{v}} = \frac{\text{number in } \mathbf{e} \text{ with } (V=true \land pa(V)=\mathbf{v})}{\text{number in } \mathbf{e} \text{ with } pa(V)=\mathbf{v}}$$

**Now**: Bayes Net Expectation Maximization (incomplete data) Start with some guess for  $\theta$ ,

E Step: Compute weights for each data  $\mathbf{x}_i$  and latent variable(s) value(s)  $\mathbf{z}_i$  (using e.g. variable elimination)

$$w_{ij} = P(\mathbf{z}_j | \theta, \mathbf{x}_i)$$

M Step: Update parameters:

$$\theta_{V=true,pa(V)=\mathbf{v}} = \frac{\sum_{ij} w_{ij} | V = true \land pa(V) = \mathbf{v} \text{ in } \{\mathbf{x}_i,\mathbf{z}_j\}}{\sum_{ij} w_{ij} | pa(V) = \mathbf{v} \text{ in } \{\mathbf{x}_i,\mathbf{z}_j\}}$$