In [78]:

```
import numpy as np
from sklearn.linear_model import LogisticRegression
from sklearn.datasets import load_iris
from sklearn.model_selection import train_test_split
import pickle as cp
import matplotlib.pyplot as plt
```

In [79]:
```
class NBC:
    def __init__(self, feature_types, num_classes):
        self.feature_types = feature_types
        self.num_classes = num_classes

    def fit(self, X, y):
        classes, classCounts = np.unique(y, return_counts=True)

        pis = np.expand_dims(classCounts / y.shape[0], axis=1)

        allMus = []
        allSigmaSqrs = []

        for i in range(classes.shape[0]):
            # Get class examples
            classExampleIdxs = np.argwhere(y==classes[i])[:,0]

            # Calculate class parameters
            # Empirical mean / Bernoulli distribution parameter
            mus = np.average(X[classExampleIdxs,:], axis=0)
            # Emperical variance
            sigmaSqrs = np.var(X[classExampleIdxs,:], axis=0)

            allMus.append(mus)
            allSigmaSqrs.append(sigmaSqrs)

        allMus = np.array(allMus)
        allSigmaSqrs = np.array(allSigmaSqrs)

        self.pis = pis
        self.mus = allMus
        self.sigmaSqrs = allSigmaSqrs
        self.classes = classes
        self.classCounts = classCounts

    def calcRealProb(self, X, realIndices, clsIdx):
        mean = self.mus[clsIdx,realIndices]
        variances = self.sigmaSqrs[clsIdx,realIndices]
        realXs = X[:,realIndices]
        probs = np.exp(-np.square(realXs-mean)/(2*(variances+1e-6)))/np.sqrt(2
*np.pi*(variances+1e-6))
        return probs

    def calcCatProb(self, X, binIndices, clsIdx):
        params = self.mus[clsIdx,binIndices]
        binXs = X[:,binIndices]
        probs = np.zeros_like(binXs)
        for i in range(params.shape[0]):
            feature = binXs[:,i]
            featureProbs = np.zeros_like(feature)
            featureProbs[feature == 1] = params[i]
            featureProbs[feature == 0] = 1 - params[i]
            probs[:,i] = featureProbs
        return probs

    def calcClassProb(self, X, clsIdx):
        featureTypes = self.feature_types
        binIndices = [i for i, x in enumerate(featureTypes) if x == 'b']
        realIndices = [i for i, x in enumerate(featureTypes) if x == 'r']

        realProbs = self.calcRealProb(X, realIndices, clsIdx)
        catProbs = self.calcCatProb(X, binIndices, clsIdx)

        # Ensure no zeros
        realProbs[realProbs == 0] = 1e-6
        catProbs[catProbs == 0] = 1e-6

        realProbs = np.log(realProbs)
        catProbs = np.log(catProbs)
        realProbs = np.sum(realProbs,axis=1)
        catProbs = np.sum(catProbs, axis=1)

        return realProbs + catProbs + \
            (np.log(self.pis[clsIdx]))

    def predict(self, X):
        classProbs = []
        for i in range(self.classes.shape[0]):
            classProbs.append(self.calcClassProb(X, i))
        classProbs = np.array(classProbs)
        predictedClassIdx = np.argmax(classProbs,axis=0)
        return self.classes[predictedClassIdx]
```

In [80]:

```
nbc = NBC(feature_types=['b','r','b','r'], num_classes=2)
nbc.fit(np.array([[[1],[0.5],[1],[0.5]],[[1],[0.5],[0],[0.5]],[[1],[0.5],[0],[0.5]]]),np.array([[2],[0],[2]]))
```

In [81]:

```
# Hand-in 1:
# The value of C: 0.5 * 10.0 = 5.0
```

In [82]:

```
# Iris Dataset
iris = load_iris()
XIris, yIris = iris['data'], iris['target']
XTrainIris, XTestIris, yTrainIris, yTestIris =\
    train_test_split(XIris, yIris, test_size=0.25)
XTrainIrisExpand = np.expand_dims(XTrainIris,axis=2)
XTestIrisExpand = np.expand_dims(XTestIris,axis=2)
yTrainIrisExpand = np.expand_dims(yTrainIris,axis=1)
yTestIrisExpand = np.expand_dims(yTestIris,axis=1)

nbc = NBC(feature_types=['r','r','r','r'], num_classes=3)
nbc.fit(XTrainIrisExpand, yTrainIrisExpand)
nbcYTrainPredict = nbc.predict(XTrainIrisExpand)
nbcYTestPredict = nbc.predict(XTestIrisExpand)
nbcTrainAccuracy = np.mean(nbcYTrainPredict == yTrainIrisExpand)
nbcTestAccuracy = np.mean(nbcYTestPredict == yTestIrisExpand)

logReg = LogisticRegression(\
    solver='lbfgs', multi_class='multinomial', max_iter=1000)
logReg.fit(np.squeeze(XTrainIris), np.squeeze(yTrainIris))
logRegYTrainPredict = logReg.predict(np.squeeze(XTrainIris))
logRegYTestPredict = logReg.predict(np.squeeze(XTestIris))
logRegTrainAccuracy = np.mean(logRegYTrainPredict == yTrainIris)
logRegTestAccuracy = np.mean(logRegYTestPredict == yTestIris)

print("Naive Bayes Training Accuracy", nbcTrainAccuracy)
print("Naive Bayes Test Accuracy", nbcTestAccuracy)
print("Logistic Regression Training Accuracy", logRegTrainAccuracy)
print("Logistic Regression Test Accuracy", logRegTestAccuracy)
```

```
Naive Bayes Training Accuracy 0.9642857142857143
Naive Bayes Test Accuracy 0.9473684210526315
Logistic Regression Training Accuracy 0.9732142857142857
Logistic Regression Test Accuracy 0.9736842105263158
```

In [83]:

```
# Congressional Voting Records
XVote, yVote = cp.load(open('voting.pickle', 'rb'))
featureTypes = ['b' for i in range(XVote.shape[1])]
XTrainVote, XTestVote, yTrainVote, yTestVote = \
    train_test_split(XVote, yVote, test_size=0.25)
XTrainVoteExpand = np.expand_dims(XTrainVote,axis=2)
XTestVoteExpand = np.expand_dims(XTestVote,axis=2)
yTrainVoteExpand = np.expand_dims(yTrainVote,axis=1)
yTestVoteExpand = np.expand_dims(yTestVote,axis=1)

nbc = NBC(feature_types=featureTypes, num_classes=2)
nbc.fit(XTrainVoteExpand, yTrainVoteExpand)
nbcYTrainPredict = nbc.predict(XTrainVoteExpand)
nbcYTestPredict = nbc.predict(XTestVoteExpand)
nbcTrainAccuracy = np.mean(nbcYTrainPredict == yTrainVoteExpand)
nbcTestAccuracy = np.mean(nbcYTestPredict == yTestVoteExpand)

logReg = LogisticRegression(\
    solver='lbfgs', max_iter=10000)
logReg.fit(XTrainVote, yTrainVote)
logRegYTrainPredict = logReg.predict(XTrainVote)
logRegYTestPredict = logReg.predict(XTestVote)
logRegTrainAccuracy = np.mean(logRegYTrainPredict == yTrainVote)
logRegTestAccuracy = np.mean(logRegYTestPredict == yTestVote)

print("Naive Bayes Training Accuracy", nbcTrainAccuracy)
print("Naive Bayes Test Accuracy", nbcTestAccuracy)
print("Logistic Regression Training Accuracy", logRegTrainAccuracy)
print("Logistic Regression Test Accuracy", logRegTestAccuracy)
```

```
Naive Bayes Training Accuracy 0.9482758620689655
Naive Bayes Test Accuracy 0.9310344827586207
Logistic Regression Training Accuracy 0.9770114942528736
Logistic Regression Test Accuracy 0.9137931034482759
```

In [84]:

```
# Split data
def splitData(X,y):
    XTrain, XTest, yTrain, yTest = \
    train_test_split(X, y, test_size=0.25)
    XTrain = np.expand_dims(XTrain,axis=2)
    XTest = np.expand_dims(XTest,axis=2)
    yTrain = np.expand_dims(yTrain,axis=1)
    yTest = np.expand_dims(yTest,axis=1)
    return XTrain, XTest, yTrain, yTest
```

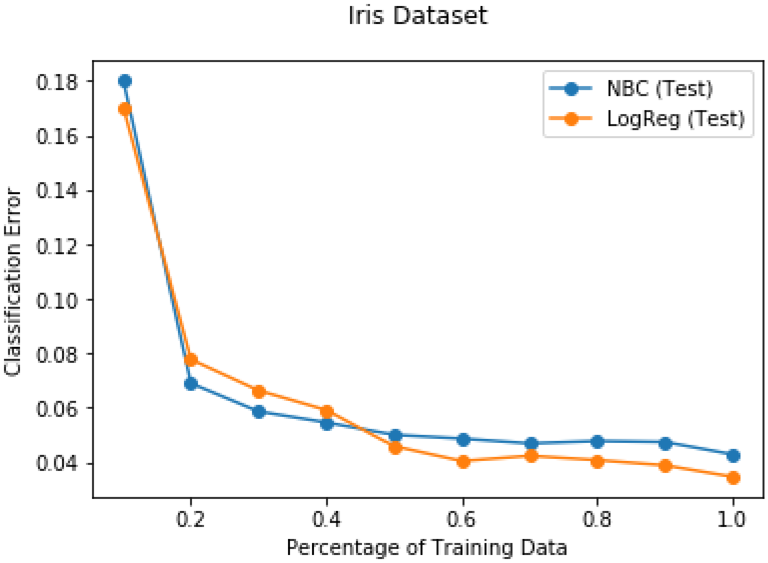
In [85]:

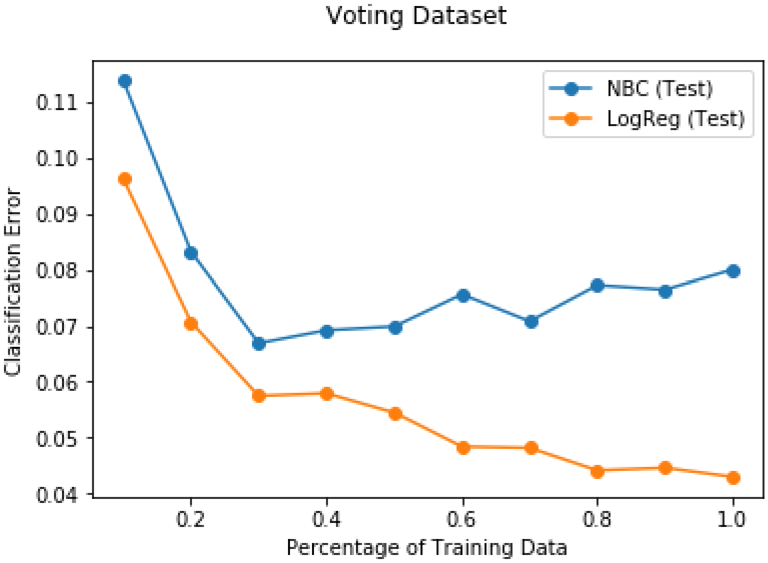
```
# Learning Curves of NBC and LogReg
def getLearningCurveNBC(X, y, estimator, numPerm):
    dataPerc = np.linspace(0.1,1.0,10)
    trainAccuracies = []
    testAccuracies = []
    for i in range(len(dataPerc)):
        totTrainAccuracy = 0
        totTestAccuracy = 0
        for j in range(numPerm):
            XTrain, XTest, yTrain, yTest = splitData(X,y)
            numTrain = XTrain.shape[0]
            numTrainSub = int(dataPerc[i] * numTrain)
            XTrainSub = XTrain[:numTrainSub]
            yTrainSub = yTrain[:numTrainSub]
            estimator.fit(XTrainSub, yTrainSub)
            yTrainPredict = estimator.predict(XTrainSub)
            yTestPredict = estimator.predict(XTest)
            totTrainAccuracy += \
                np.mean(yTrainPredict \
                == yTrainSub)
            totTestAccuracy += \
                np.mean(yTestPredict == yTest)
        trainAccuracies.append(totTrainAccuracy / numPerm)
        testAccuracies.append(totTestAccuracy / numPerm)
    return trainAccuracies, testAccuracies

def getLearningCurveLogReg(X, y, estimator, numPerm):
    dataPerc = np.linspace(0.1,1.0,10)
    trainAccuracies = []
    testAccuracies = []
    for i in range(len(dataPerc)):
        totTrainAccuracy = 0
        totTestAccuracy = 0
        for j in range(numPerm):
            XTrain, XTest, yTrain, yTest = train_test_split(X, y, test_size=0.
25)
            numTrain = XTrain.shape[0]
            numTrainSub = int(dataPerc[i] * numTrain)
            XTrainSub = XTrain[:numTrainSub]
            yTrainSub = yTrain[:numTrainSub]
            estimator.fit(XTrainSub, yTrainSub)
            yTrainPredict = estimator.predict(XTrainSub)
            yTestPredict = estimator.predict(XTest)
            totTrainAccuracy += \
                np.mean(yTrainPredict \
                == yTrainSub)
            totTestAccuracy += \
                np.mean(yTestPredict == yTest)
        trainAccuracies.append(totTrainAccuracy / numPerm)
        testAccuracies.append(totTestAccuracy / numPerm)
    return trainAccuracies, testAccuracies
```

In [86]:

```
# Hand-in 2
dataPerc = np.linspace(0.1,1.0,10)
nbc = NBC(feature_types=featureTypes, num_classes=2)
nbcVoteTrainCurve, nbcVoteTestCurve = \
    getLearningCurveNBC(XVote, yVote, nbc, 150)
nbcVoteTestCurve = np.array(nbcVoteTestCurve)
nbcVoteTestCurve = 1 - nbcVoteTestCurve

nbc = NBC(feature_types=['r','r','r','r'], num_classes=3)
nbcIrisTrainCurve, nbcIrisTestCurve = \
    getLearningCurveNBC(XIris, yIris, nbc, 150)
nbcIrisTestCurve = np.array(nbcIrisTestCurve)
nbcIrisTestCurve = 1 - nbcIrisTestCurve

logReg = LogisticRegression(max_iter=1000, solver='lbfgs')
logRegVoteTrainCurve, logRegVoteTestCurve = \
    getLearningCurveLogReg(XVote, yVote, logReg, 150)
logRegVoteTestCurve = np.array(logRegVoteTestCurve)
logRegVoteTestCurve = 1 - logRegVoteTestCurve

logReg = LogisticRegression(max_iter=1000, solver='lbfgs', multi_class='multinomial')
logRegIrisTrainCurve, logRegIrisTestCurve = \
    getLearningCurveLogReg(XIris, yIris, logReg, 150)
logRegIrisTestCurve = np.array(logRegIrisTestCurve)
logRegIrisTestCurve = 1 - logRegIrisTestCurve

plt.figure()
#plt.plot(dataPerc, nbcIrisTrainCurve, '-o', label='NBC (Train)')
plt.plot(dataPerc, nbcIrisTestCurve, '-o', label='NBC (Test)')
#plt.plot(dataPerc, logRegIrisTrainCurve, '-o', label='LogReg (Train)')
plt.plot(dataPerc, logRegIrisTestCurve, '-o', label='LogReg (Test)')
plt.xlabel("Percentage of Training Data")
plt.ylabel("Classification Error")
plt.suptitle("Iris Dataset")
plt.legend()

plt.figure()
#plt.plot(dataPerc, nbcVoteTrainCurve, '-o', label='NBC (Train)')
plt.plot(dataPerc, nbcVoteTestCurve, '-o', label='NBC (Test)')
#plt.plot(dataPerc, logRegVoteTrainCurve, '-o', label='LogReg (Train)')
plt.plot(dataPerc, logRegVoteTestCurve, '-o', label='LogReg (Test)')
plt.xlabel("Percentage of Training Data")
plt.ylabel("Classification Error")
plt.suptitle("Voting Dataset")
plt.legend()
```

```
Out[86]: <matplotlib.legend.Legend at 0x2195f5eca48>
```


Iris Dataset
Classification Error
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.2
0.4
0.6
0.8
1.0
Percentage of Training Data
NBC (Test)
LogReg (Test)


Voting Dataset
Classification Error
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.2
0.4
0.6
0.8
1.0
Percentage of Training Data
NBC (Test)
LogReg (Test)

In [87]:

```
class NBC2:
    def __init__(self, feature_types, num_classes):
        self.feature_types = feature_types
        self.num_classes = num_classes

    def fit(self, X, y):
        classes, classCounts = np.unique(y, return_counts=True)

        pis = np.expand_dims(classCounts / y.shape[0], axis=1)

        allParams = []

        for i in range(classes.shape[0]):
            # Get class examples
            classExampleIdxs = np.argwhere(y==classes[i])[:,0]

            # Calculate class parameters
            # Bernoulli distribution parameter
            numOnes = np.count_nonzero(X == 1, axis=0)
            numZeros = np.count_nonzero(X == 0, axis=0)
            allParams.append(np.divide(numOnes, numOnes + numZeros))

        allParams = np.array(allParams)

        self.pis = pis
        self.params = allParams
        self.classes = classes
        self.classCounts = classCounts

    def calcCatProb(self, X, binIndices, clsIdx):
        params = self.params[clsIdx,binIndices]
        binXs = X[:,binIndices]
        probs = np.zeros_like(binXs)
        for i in range(params.shape[0]):
            feature = binXs[:,i]
            featureProbs = np.zeros_like(feature)
            featureProbs[feature == 1] = params[i]
            featureProbs[feature == 0] = 1 - params[i]
            probs[:,i] = featureProbs
        return probs

    def calcClassProb(self, X, clsIdx):
        featureTypes = self.feature_types
        binIndices = [i for i, x in enumerate(featureTypes) if x == 'b']
        realIndices = [i for i, x in enumerate(featureTypes) if x == 'r']

        catProbs = self.calcCatProb(X, binIndices, clsIdx)
        # Ensure no zeros
        catProbs[catProbs == 0] = 1e-6

        # Catch empty entries
        catProbs[catProbs != 2] = np.log(catProbs[catProbs != 2])
        catProbs[catProbs == 2] = 0

        # Calculate probability
        catProbs = np.sum(catProbs, axis=1)
        return catProbs + \
            (self.classCounts * np.log(self.pis[clsIdx]))

    def predict(self, X):
        classProbs = []
        for i in range(self.classes.shape[0]):
            classProbs.append(self.calcClassProb(X, i))
        classProbs = np.array(classProbs)
        predictedClassIdx = np.argmax(classProbs,axis=0)
        return self.classes[predictedClassIdx]
```

In [88]:

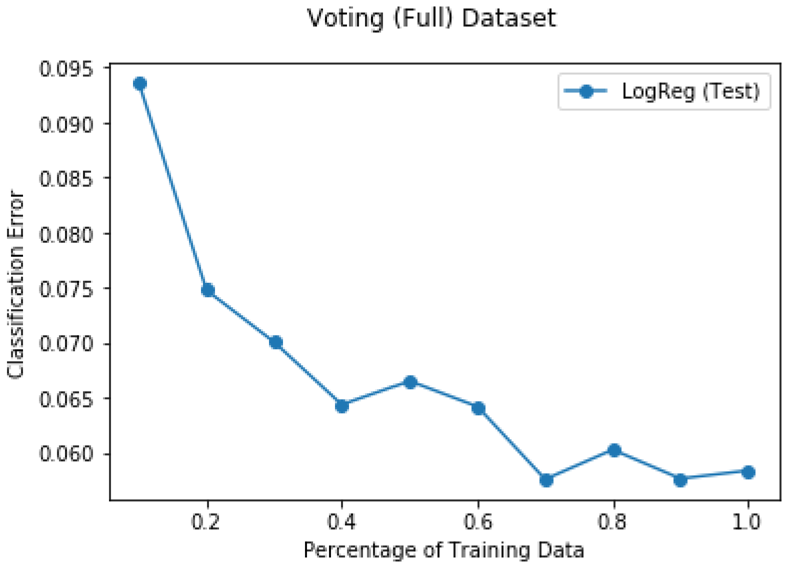
```
# Congressional Voting Records (Full)
XVoteFull, yVoteFull = cp.load(open('voting-full.pickle', 'rb'))
featureTypes = ['b' for i in range(XVote.shape[1])]
nbc2 = NBC2(feature_types=featureTypes, num_classes=2)
nbcVoteFullTrainCurve, nbcVoteFullTestCurve = \
    getLearningCurveNBC(XVoteFull, yVoteFull, nbc2, 150)
nbcVoteFullTestCurve = np.array(nbcVoteFullTestCurve)
nbcVoteFullTestCurve = 1 - nbcVoteFullTestCurve

logReg = LogisticRegression(max_iter=1000, solver='lbfgs')
logRegVoteFullTrainCurve, logRegVoteFullTestCurve = \
    getLearningCurveLogReg(XVoteFull, yVoteFull, logReg, 150)
logRegVoteFullTestCurve = np.array(logRegVoteFullTestCurve)
logRegVoteFullTestCurve = 1 - logRegVoteFullTestCurve
```

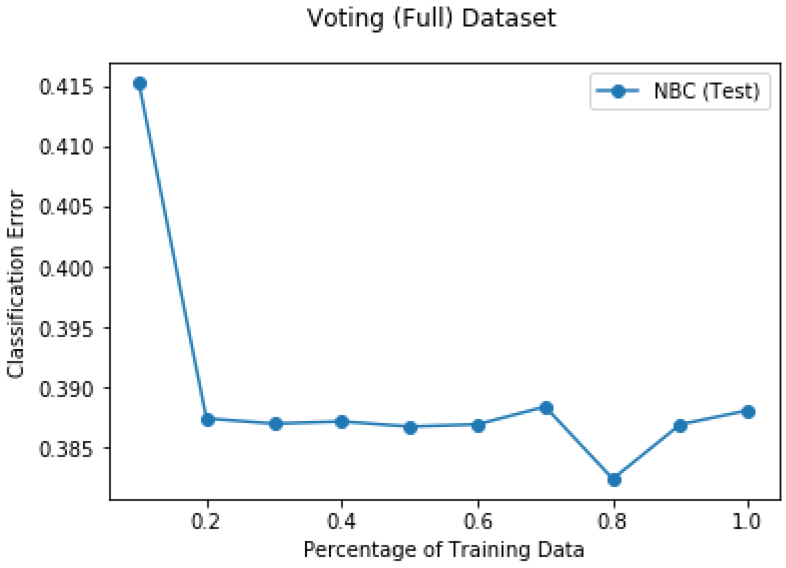
In [89]:

```
dataPerc = np.linspace(0.1,1.0,10)
plt.figure()
plt.plot(dataPerc, logRegVoteFullTestCurve, '-o', label='LogReg (Test)')
plt.xlabel("Percentage of Training Data")
plt.ylabel("Classification Error")
plt.suptitle("Voting (Full) Dataset")
plt.legend()
plt.figure()
plt.plot(dataPerc, nbcVoteFullTestCurve, '-o', label='NBC (Test)')
plt.xlabel("Percentage of Training Data")
plt.ylabel("Classification Error")
plt.suptitle("Voting (Full) Dataset")
plt.legend()
```

Out[89]:

```
<matplotlib.legend.Legend at 0x2195f69a188>
```



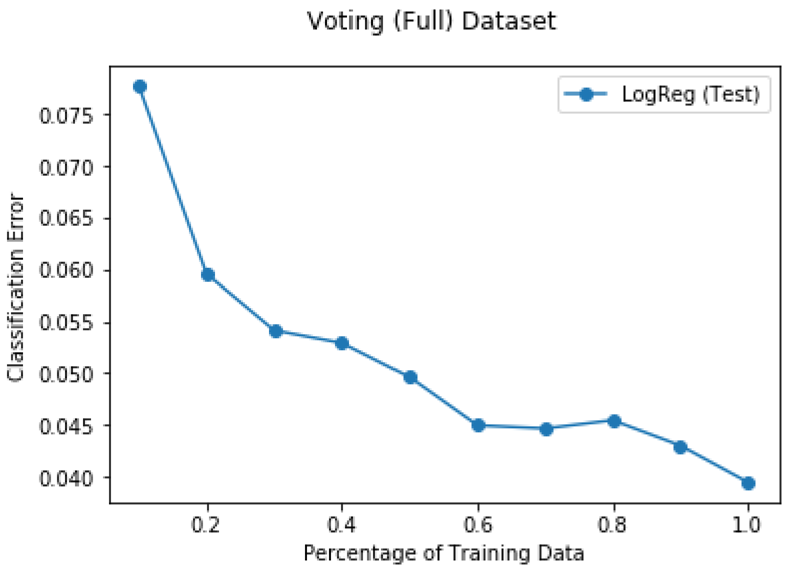


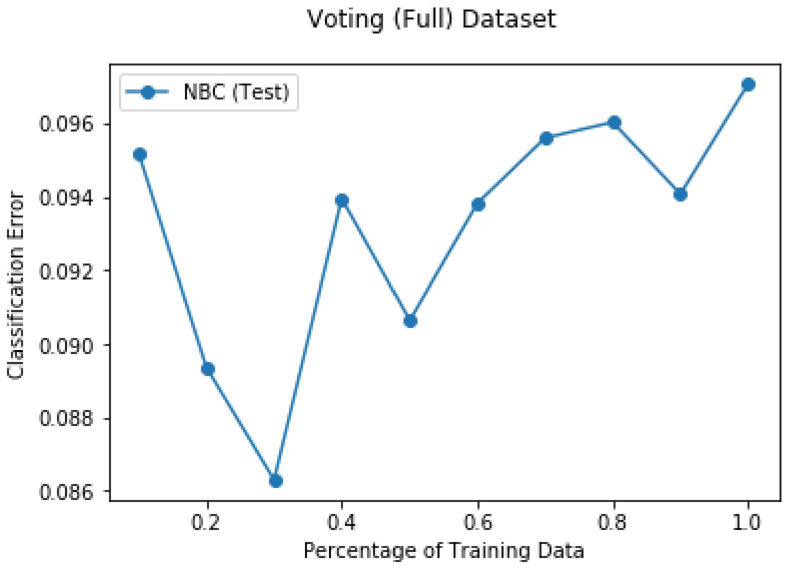
In [90]:

```
# Congressional Voting Records (Full)
XVoteFull, yVoteFull = cp.load(open('voting-full.pickle', 'rb'))
# Replace missing features with mean
for i in range(XVoteFull.shape[1]):
    feature = XVoteFull[:,i]
    numZeros = np.sum(feature == 0)
    numOnes = np.sum(feature == 1)
    mean = numOnes / (numZeros + numOnes)
    feature[feature == 2] = mean
    XVoteFull[:,i] = feature

featureTypes = ['b' for i in range(XVote.shape[1])]
nbc = NBC(feature_types=featureTypes, num_classes=2)
nbcVoteFullTrainCurve, nbcVoteFullTestCurve = \
    getLearningCurveNBC(XVoteFull, yVoteFull, nbc, 150)
nbcVoteFullTestCurve = np.array(nbcVoteFullTestCurve)
nbcVoteFullTestCurve = 1 - nbcVoteFullTestCurve

logReg = LogisticRegression(max_iter=1000, solver='lbfgs')
logRegVoteFullTrainCurve, logRegVoteFullTestCurve = \
    getLearningCurveLogReg(XVoteFull, yVoteFull, logReg, 150)
logRegVoteFullTestCurve = np.array(logRegVoteFullTestCurve)
logRegVoteFullTestCurve = 1 - logRegVoteFullTestCurve
```

In [91]:
```
dataPerc = np.linspace(0.1,1.0,10)
plt.figure()
plt.plot(dataPerc, logRegVoteFullTestCurve, '-o', label='LogReg (Test)')
plt.xlabel("Percentage of Training Data")
plt.ylabel("Classification Error")
plt.suptitle("Voting (Full) Dataset")
plt.legend()
plt.figure()
plt.plot(dataPerc, nbcVoteFullTestCurve, '-o', label='NBC (Test)')
plt.xlabel("Percentage of Training Data")
plt.ylabel("Classification Error")
plt.suptitle("Voting (Full) Dataset")
plt.legend()
```

Out[91]:
```
<matplotlib.legend.Legend at 0x2195f77a388>
```


Voting (Full) Dataset
LogReg (Test)
Classification Error
Percentage of Training Data


Voting (Full) Dataset
NBC (Test)
Classification Error
Percentage of Training Data