TO PASS 60% or higher

GRADE 100%

# **Splay Trees**

### TOTAL POINTS 3

1. What is going to happen if you forget to splay the last accessed vertex in the implementation of Find in case the key was not found in the splay tree?

The tree will work too slow on some sequences of operations.

O Some of the tree operations will work incorrectly after that.

 ${\it Correct!} \ {\it See this} \ \underline{{\it visualization}} \ {\it and try} \ to \ insert \ many \ elements \ in \ perfect \ order \ starting \ from \ an \ empty \ tree:$ insert 1, then 2, then 3, and so on. See how the tree grows unbalanced, it is just a chain! However, by now each operation took O(1) time, so it's ok. Now think what will happen if you look for element 0 in this tree. If you  $use the {\it visualization}, you {\it will} {\it see} {\it that} {\it you} {\it will} {\it have} {\it to} {\it go} {\it all} {\it the} {\it way} {\it down} {\it through} {\it the} {\it tree} {\it and} {\it then} {\it find} {\it out}$ in the end that you didn't find anything. The tree in the visualization then splays the lowest vertex, and the tree  $\frac{1}{2}$  $becomes \ more \ balanced. \ But \ let's \ suppose \ you \ for got \ to \ implement \ that - then \ the \ tree \ won't \ change \ after \ the$ call to Find. If you then try to find 0 again in the tree, you will have to go all the way down again! So, after inserting n elements in the tree in the perfect order, if you look for an element that is smaller than all the keys in the tree n times, then each of the last n operations will take O(n) time, so the tree no longer works in amortized  $O(\log n)$  time!

2. What will happen if you splay the node with the smallest key in a splay tree?

1/1 point

- The root of the new tree won't have left child.
- The root of the new tree will have both children.
- O The root of the new tree won't have children.
- The root of the new tree won't have right child.

### ✓ Correct

Correct! The node with the smallest key will become the root after splaying, and it cannot have a left child, because the key of the left child must be smaller than the key of its parent.

3. What will happen if you select a node N, splay its predecessor P (the node with the largest key smaller than the key of N), then splay the node N itself?

- igotimes N will be the root, P will be the left child of the root, P won't have a right child.
- O P will be the root.
- $\bigcirc$  N will be the root, P will be the right child of the root.
- $\bigcirc$  N will be the root, P will be the left child of the root, P won't have a left child.

## ✓ Correct

Correct! After the first splay, P will become the root. After the second splay, N will become the root, and Pwill become its child, and it will be on the left, because its key is smaller. P won't have a right child, because a right child of P must have key bigger than the key of P, and also it must have key smaller than the key of N(because it is now in the left subtree of N), but it can't happen, because P is the predecessor of N, so there are no keys between the key of  ${\cal P}$  and the key of  ${\cal N}.$