Exercice 1

Dans chaque cas une primitive F de la fonction f sera donnée en posant la constante $k = 0$ puis on calculera l'intégrale donnée.

1) $f(x) = 2x$ donc $F(x) = x^2 + k$. On pose $k = 0$, donc $F(x) = x^2$.

$$\begin{aligned}\int_{-1}^{2} f(x)dx &= [F(x)]_{-1}^{2} = F(2) - F(-1)\\ &= 2^2 - 1^2\\ &= 4 - 1\\ &= \boxed{3}\end{aligned}$$

2) $f(x) = -x + 7$ donc $F(x) = -\dfrac{x^2}{2} + 7x + k$. On pose $k = 0$, donc $F(x) = -\dfrac{x^2}{2} + 7x$.

$$\begin{aligned}\int_{1}^{5} f(x)dx &= [F(x)]_{1}^{5} = F(5) - F(1)\\ &= \left(-\frac{5^2}{2} + 7 \times 5\right) - \left(-\frac{1^2}{2} + 7 \times 7\right)\\ &= 22,5 - 6,5\\ &= \boxed{16}\end{aligned}$$

3) $f(x) = -x^2 + 2x + 4$ donc $F(x) = -\dfrac{x^3}{3} + x^2 + 4x + k$. On pose $k = 0$, donc $F(x) = -\dfrac{x^3}{3} + x^2 + 4x$.

$$\begin{aligned}\int_{0}^{2} f(x)dx &= [F(x)]_{0}^{2} = F(2) - F(0)\\ &= \left(-\frac{2^3}{3} + 2^2 + 4 \times 2\right) - \left(-\frac{0^3}{3} + 0^2 + 4 \times 0\right)\\ &= \frac{28}{3} - 0\\ &= \boxed{\frac{28}{3}}\\ &\approx 9,33\end{aligned}$$

4) $f(x) = x^3 - x + 2$ donc $F(x) = \dfrac{x^4}{4} - \dfrac{x^2}{2} + 2x + k$. On pose $k = 0$, donc $F(x) = \dfrac{x^4}{4} - \dfrac{x^2}{2} + 2x$.

$$\begin{aligned}\int_{-1}^{1} f(x)dx &= [F(x)]_{-1}^{1} = F(1) - F(-1)\\ &= \left(\frac{1^4}{4} - \frac{1^2}{2} + 2 \times 1\right) - \left(\frac{(-1)^4}{4} - \frac{(-1)^2}{2} + 2 \times (-1)\right)\\ &= \frac{7}{4} - \frac{-9}{4}\\ &= \boxed{4}\end{aligned}$$

5) $f(x) = \mathrm{e}^x$ donc $F(x) = \mathrm{e}^x + k$. On pose $k = 0$, donc $F(x) = \mathrm{e}^x$

$$\begin{aligned}\int_{-1}^{1} f(x)dx &= [F(x)]_{-1}^{1} = F(1) - F(-1) \\ &= \mathrm{e}^1 - \mathrm{e}^{-1} \\ &= \boxed{\mathrm{e} - \frac{1}{e} = \mathrm{e} - \mathrm{e}^{-1}} \\ &\approx 2,35\end{aligned}$$

6) $f(x) = \dfrac{x^2 - 1}{x^2} = 1 - \dfrac{1}{x^2}$ donc $F(x) = x + \dfrac{1}{x} + k$. On pose $k = 0$, donc $F(x) = x + \dfrac{1}{x}$.

$$\begin{aligned}\int_{1}^{10} f(x)dx &= [F(x)]_{1}^{10} = F(10) - F(1) \\ &= \left(10 + \frac{1}{10}\right) - \left(1 + \frac{1}{1}\right) \\ &= 10,1 - 2 \\ &= \boxed{8,1}\end{aligned}$$

7) $f(x) = 1 - \mathrm{e}^{-x}$ donc $F(x) = x + \mathrm{e}^{-x} + k$. On pose $k = 0$, donc $F(x) = x + \mathrm{e}^{-x}$.

$$\begin{aligned}\int_{0}^{\ln 2} f(x)dx &= [F(x)]_{0}^{\ln 2} = F(\ln 2) - F(0) \\ &= \left(\ln 2 + \mathrm{e}^{-\ln 2}\right) - \left(0 + \mathrm{e}^{-0}\right) \\ &= \ln 2 + \frac{1}{2} - 1 \\ &= \boxed{\ln 2 - \frac{1}{2}} \\ &\approx \boxed{0{,}19}\end{aligned}$$

8) $f(x) = x - \mathrm{e}^{0,5x-1}$ donc $F(x) = \dfrac{x^2}{2} - \dfrac{\mathrm{e}^{0,5x-1}}{0,5} = k = \dfrac{x^2}{2} - 2\mathrm{e}^{0,5x-1} = k$. On pose $k = 0$, donc $F(x) = \dfrac{x^2}{2} - 2\mathrm{e}^{0,5x-1}$.

$$\begin{aligned}\int_{2}^{4} f(x)dx &= [F(x)]_{2}^{4} = F(4) - F(2) \\ &= \left(\frac{4^2}{2} - 2\mathrm{e}^{0,5\times 4-1}\right) - \left(\frac{2^2}{2} - 2\mathrm{e}^{0,5\times 2-1}\right) \\ &= (8 - 2\mathrm{e}) - (2 - 2) \\ &= \boxed{8 - 2\mathrm{e}} \\ &\approx 2,56\end{aligned}$$