Machine Learning: Programming Exercise 4

Neural Networks Learning

In this exercise, you will implement the backpropagation algorithm for neural networks and apply it to the task of hand-written digit recognition.

Files needed for this exercise

- `ex4.mlx` - MATLAB Live Script that steps you through the exercise
- `ex4data1.mat` - Training set of hand-written digits
- `ex4weights.mat` - Neural network parameters for exercise 4
- `submit.m` - Submission script that sends your solutions to our servers
- `displayData.m` - Function to help visualize the dataset
- `fmincg.m` - Function minimization routine (similar to `fminunc`)
- `sigmoid.m` - Sigmoid function
- `computeNumericalGradient.m` - Numerically compute gradients
- `checkNNGradients.m` - Function to help check your gradients
- `debugInitializeWeights.m` - Function for initializing weights
- `predict.m` - Neural network prediction function
- *`sigmoidGradient.m` - Compute the gradient of the sigmoid function
- *`randInitializeWeights.m` - Randomly initialize weights
- *`nnCostFunction.m` - Neural network cost function

_ indicates files you will need to complete_

Confirm that your Current Folder is set correctly

Click into this section, then click the 'Run section' button above. This will execute the `dir` command below to list the files in your Current Folder. The output should contain all of the files listed above and the 'lib' folder. If it does not, right-click the 'ex4' folder and select 'Open' before proceding or see the instructions in `README.mlx` for more details.

```
dir
```

```
.                          debugInitializeWeights.m   ex4data1.mat          nnCostFunction.m
..                         displayData.m              ex4weights.mat        predict.m
checkNNGradients.m         ex4.mlx                    fmincg.m              randInitializeWeights.
computeNumericalGradient.m ex4_companion.mlx          lib                   sigmoid.m
```

Before you begin

The workflow for completing and submitting the programming exercises in MATLAB Online differs from the original course instructions. Before beginning this exercise, make sure you have read through the instructions in `README.mlx` which is included with the programming exercise files. `README` also contains solutions to the many common issues you may encounter while completing and submitting the exercises in MATLAB Online. Make sure you are following instructions in `README` and have checked for an existing solution before seeking help on the discussion forums.

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1. Neural Networks

In the previous exercise, you implemented feedforward propagation for neural networks and used it to predict handwritten digits with the weights we provided. In this exercise, you will implement the backpropagation algorithm to learn the parameters for the neural network.

1.1 Visualizing the data

The code below will load the data and display it on a 2-dimensional plot (Figure 1) by calling the function `displayData`. This is the same dataset that you used in the previous exercise. Run the code below to load the training data into the variables `X` and `y`.

Figure 1: Examples from the dataset

```
load('ex4data1.mat');
m = size(X, 1);

% Randomly select 100 data points to display
sel = randperm(size(X, 1));
sel = sel(1:100);

displayData(X(sel, :));
```

There are 5000 training examples in `ex4data1.mat`, where each training example is a 20 pixel by 20 pixel grayscale image of the digit. Each pixel is represented by a floating point number indicating the grayscale intensity at that location. The 20 by 20 grid of pixels is 'unrolled' into a 400-dimensional vector. Each of these training examples becomes a single row in our data matrix X. This gives us a 5000 by 400 matrix X where every row is a training example for a handwritten digit image.

$$X = \begin{bmatrix} -(x^{(1)})^T - \\ -(x^{(2)})^T - \\ \vdots \\ -(x^{(m)})^T - \end{bmatrix}$$

The second part of the training set is a 5000-dimensional vector y that contains labels for the training set. To make things more compatible with MATLAB indexing, where there is no zero index, we have mapped the digit zero to the value ten. Therefore, a '0' digit is labeled as '10', while the digits '1' to '9' are labeled as '1' to '9' in their natural order.

1.2 Model representation

Our neural network is shown in Figure 2. It has 3 layers- an input layer, a hidden layer and an output layer. Recall that our inputs are pixel values of digit images. Since the images are of size 20 x 20, this gives us 400 input layer units (not counting the extra bias unit which always outputs +1).
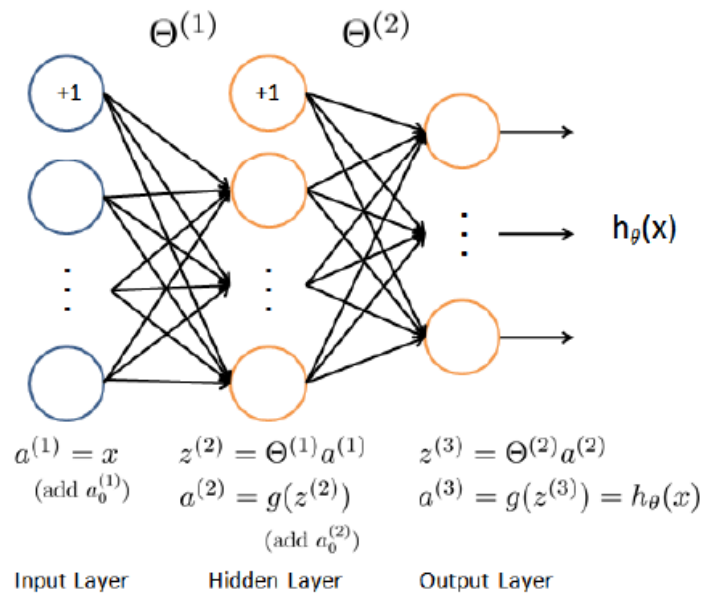
Figure 2: Neural network model.

You have been provided with a set of network parameters $(\Theta^{(1)}, \Theta^{(2)})$ already trained by us. These are stored in `ex4weights.mat`. Run the code below to load them into `Theta1` and `Theta2`. The parameters have dimensions that are sized for a neural network with 25 units in the second layer and 10 output units (corresponding to the 10 digit classes).

```
% Load the weights into variables Theta1 and Theta2
load('ex4weights.mat');
```

1.3 Feedforward and cost function

Now you will implement the cost function and gradient for the neural network. First, complete the code in `nnCostFunction.m` to return the cost. Recall that the cost function for the neural network (without regularization) is

$$ J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \sum_{k=1}^{K} \left[-y_k^{(i)} \log((h_\theta(x^{(i)}))_k) - (1 - y_k^{(i)}) \log(1 - (h_\theta(x^{(i)}))_k) \right], $$

where $h_\theta(x^{(i)})$ is computed as shown in the Figure 2 and $K = 10$ is the total number of possible labels. Note that $h_\theta(x^{(i)})_k = a_k^{(3)}$ is the activation (output value) of the k-th output unit. Also, recall that whereas the original labels (in the variable y) were $1, 2, \ldots, 10$, for the purpose of training a neural network, we need to recode the labels as vectors containing only values 0 or 1, so that

$$y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \dots \text{ or } \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

For example, if $x^{(i)}$ is an image of the digit 5, then the corresponding $y^{(i)}$ (that you should use with the cost function) should be a 10-dimensional vector with $y_5 = 1$, and the other elements equal to 0. You should implement the feedforward computation that computes $h_\theta(x^{(i)})$ for every example i and sum the cost over all examples. Your code should also work for a dataset of any size, with any number of labels (you can assume that there are always at least $K \geq 3$ labels).

Implementation Note: The matrix X contains the examples in rows (i.e., X(i,:)' is the i-th training example $x^{(i)}$, expressed as a n x 1 vector.) When you complete the code in nnCostFunction.m, you will need to add the column of 1's to the X matrix. The parameters for each unit in the neural network is represented in Theta1 and Theta2 as one row. Specifically, the first row of Theta1 corresponds to the first hidden unit in the second layer. You can use a for loop over the examples to compute the cost. We suggest implementing the feedforward cost *without* regularization first so that it will be easier for you to debug. Later, you will get to implement the regularized cost.

Once you are done, run the code below to call your nnCostFunction using the loaded set of parameters for Theta1 and Theta2. You should see that the cost is about 0.287629.

```
input_layer_size  = 400;   % 20x20 Input Images of Digits
hidden_layer_size = 25;    % 25 hidden units
num_labels = 10;           % 10 labels, from 1 to 10 (note that we have mapped "0" to la

% Unroll parameters
nn_params = [Theta1(:) ; Theta2(:)];

% Weight regularization parameter (we set this to 0 here).
lambda = 0;

J = nnCostFunction(nn_params, input_layer_size, hidden_layer_size, num_labels, X, y, la
```

```
J = 0.2876
```

```
fprintf('Cost at parameters (loaded from ex4weights): %f', J);
```

```
Cost at parameters (loaded from ex4weights): 0.287629
```

6

1.4 Regularized cost function

The cost function for neural networks with regularization is given by

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \sum_{k=1}^{K} \left[-y_k^{(i)} \log((h_\theta(x^{(i)})_k)) - (1 - y_k^{(i)}) \log(1 - (h_\theta(x^{(i)}))_k) \right]$$

$$+ \frac{\lambda}{2m} \left[\sum_{j=1}^{25} \sum_{k=1}^{400} \left(\Theta_{j,k}^{(1)} \right)^2 + \sum_{j=1}^{10} \sum_{k=1}^{25} \left(\Theta_{j,k}^{(2)} \right)^2 \right]$$

You can assume that the neural network will only have 3 layers- an input layer, a hidden layer and an output layer. However, your code should work for any number of input units, hidden units and outputs units. While we have explicitly listed the indices above for $\Theta^{(1)}$ and $\Theta^{(2)}$ for clarity, do note that **your code should in general work with $\Theta^{(1)}$ and $\Theta^{(2)}$ of any size.**

Note that you should not be regularizing the terms that correspond to the bias. For the matrices Theta1 and Theta2, this corresponds to the first column of each matrix. You should now add regularization to your cost function. Notice that you can first compute the unregularized cost function J using your existing `nnCostFunction.m` and then later add the cost for the regularization terms. Once you are done, run the code below to call your `nnCostFunction` using the loaded set of parameters for `Theta1` and `Theta2`, and $\lambda = 1$. You should see that the cost is about 0.383770.

```
% Weight regularization parameter (we set this to 1 here).
lambda = 1;

J = nnCostFunction(nn_params, input_layer_size, hidden_layer_size, num_labels, X, y, la
```

```
J = 0.2876
```

```
fprintf('Cost at parameters (loaded from ex4weights): %f', J);
```

```
Cost at parameters (loaded from ex4weights): 0.383770
```

2. Backpropagation

In this part of the exercise, you will implement the backpropagation algorithm to compute the gradient for the neural network cost function. You will need to complete the `nnCostFunction.m` so that it returns an appropriate value for `grad`. Once you have computed the gradient, you will be able to train the neural network by minimizing the cost function $J(\theta)$ using an advanced optimizer such as `fmincg`.

You will first implement the backpropagation algorithm to compute the gradients for the parameters for the (unregularized) neural network. After you have verified that your gradient computation for the unregularized case is correct, you will implement the gradient for the regularized neural network.

2.1 Sigmoid gradient

To help you get started with this part of the exercise, you will first implement the sigmoid gradient function. The gradient for the sigmoid function can be computed as

$$g'(z) = \frac{d}{dz} g(z) = g(z)(1 - g(z))$$

where

$$\text{sigmoid}(z) = g(z) = \frac{1}{1 + e^{-z}}$$

When you are done, try testing a few values by calling `sigmoidGradient(z)` below. For large values (both positive and negative) of `z`, the gradient should be close to 0. When `z = 0`, the gradient should be exactly 0.25. Your code should also work with vectors and matrices. For a matrix, your function should perform the sigmoid gradient function on every element.

```
% Call your sigmoidGradient function
sigmoidGradient(0)
```

```
ans = 0.2500
```

*You should now submit your solutions. Enter **submit** at the command prompt, then enter or confirm your login and token when prompted.*

2.2 Random initialization

When training neural networks, it is important to randomly initialize the parameters for symmetry breaking. One effective strategy for random initialization is to randomly select values for $\Theta^{(l)}$ uniformly in the range $[-\epsilon_{int}, \epsilon_{init}]$. You should use $\epsilon_{init} = 0.12^*$. This range of values ensures that the parameters are kept small and makes the learning more efficient.

Your job is to complete `randInitializeWeights.m` to initialize the weights for Θ; modify the file and fill in the following code:

```
% Randomly initialize the weights to small values
epsilon_init = 0.12;
W = rand(L out, 1 + L in) * 2 * epsilon_init - epsilon init;
```

When you are done, run the code below to call `randInitialWeights` and initialize the neural network parameters.

```
initial_Theta1 = randInitializeWeights(input_layer_size, hidden_layer_size);
initial_Theta2 = randInitializeWeights(hidden_layer_size, num_labels);

% Unroll parameters
```

```
initial_nn_params = [initial_Theta1(:) ; initial_Theta2(:)];
```

*One effective strategy for choosing ϵ_{init} is to base it on the number of units in the network. A good choice of ϵ_{init} is $\epsilon_{init} = \dfrac{\sqrt{6}}{\sqrt{L_{in} + L_{out}}}$, where $L_{in} = s_l$ and $L_{out} = s_l + 1$ are the number of units in the layers adjacent to $\Theta^{(l)}$.

You do not need to submit any code for this part of the exercise.

2.3 Backpropagation

Now, you will implement the backpropagation algorithm. Recall that the intuition behind the backpropagation algorithm is as follows. Given a training example $(x^{(t)}, y^{(t)})$, we will first run a 'forward pass' to compute all the activations throughout the network, including the output value of the hypothesis $h_\Theta(x)$. Then, for each node j in layer l, we would like to compute an 'error term' $\delta_j^{(l)}$ that measures how much that node was 'responsible' for any errors in our output.

For an output node, we can directly measure the difference between the network's activation and the true target value, and use that to define $\delta_j^{(3)}$ (since layer 3 is the output layer). For the hidden units, you will compute $\delta_j^{(l)}$ based on a weighted average of the error terms of the nodes in layer $(l + 1)$.
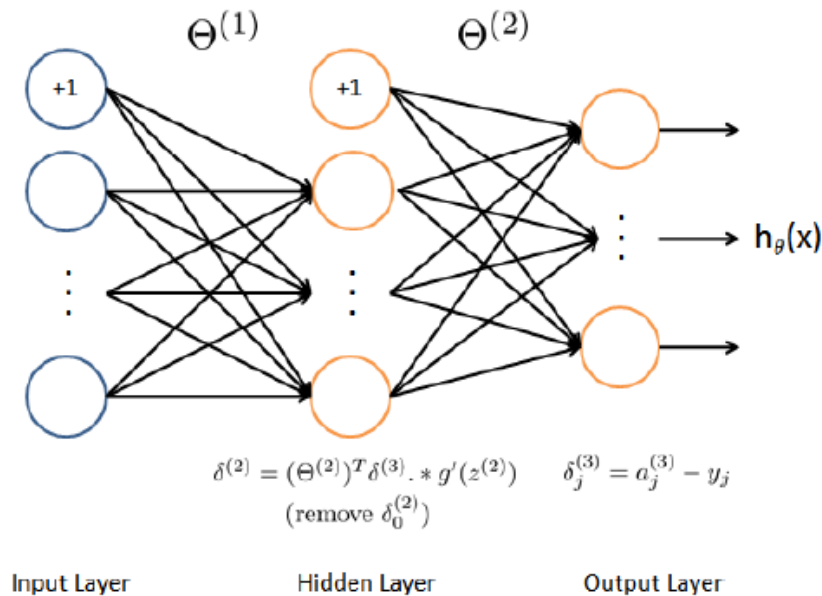
Figure 3: Backpropagation Updates.

In detail, here is the backpropagation algorithm (also depicted in Figure 3). You should implement steps 1 to 4 in a loop that processes one example at a time. Concretely, you should implement a `for` loop for `t = 1:m` and place steps 1-4 below inside the `for` loop, with the tth iteration performing the calculation on the tth training example $(x^{(t)}, y^{(t)})$. Step 5 will divide the accumulated gradients by m to obtain the gradients for the neural network cost function.

1. Set the input layer's values $(a^{(1)})$ to the t-th training example $x^{(t)}$. Perform a feedforward pass (Figure 2), computing the activations $(z^{(2)}, a^{(2)}, z^{(3)}, a^{(3)})$ for layers 2 and 3. Note that you need to add a $+1$ term to ensure that the vectors of activations for layers $a^{(1)}$ and $a^{(2)}$ also include the bias unit. In MATLAB, if `a_1` is a column vector, adding one corresponds to `a_1 = [1; a_1]`.

2. For each output unit k in layer 3 (the output layer), set $\delta_k^{(3)} = (a_k^{(3)} - y_k)$ where $y_k \in \{0, 1\}$ indicates whether the current training example belongs to class k $(y_k = 1)$, or if it belongs to a different class $(y_k = 0)$. You may find logical arrays helpful for this task (explained in the previous programming exercise).

3. For the hidden layer $l = 2$, set $\delta^{(2)} = (\Theta^{(2)})^T \delta^{(3)}. * g'(z^{(2)})$

4. Accumulate the gradient from this example using the following formula: $\Delta^{(l)} = \Delta^{(l)} + \delta^{(l+1)}(a^{(l)})^T$. Note that you should skip or remove $\delta_0^{(2)}$. In MATLAB, removing $\delta_0^{(2)}$ corresponds to `delta_2 = delta_2(2:end)`.

5. Obtain the (unregularized) gradient for the neural network cost function by dividing the accumulated gradients by $\frac{1}{m}$: $\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = D_{ij}^{(l)} = \frac{1}{m} \Delta_{ij}^{(l)}$

MATLAB Tip: You should implement the backpropagation algorithm only after you have successfully completed the feedforward and cost functions. While implementing the backpropagation algorithm, it is often useful to use the size function to print out the sizes of the variables you are working with if you run into dimension mismatch errors ("nonconformant arguments") errors.

After you have implemented the backpropagation algorithm, the code in the next section will run gradient checking on your implementation. The gradient check will allow you to increase your confidence that your code is computing the gradients correctly.

2.4 Gradient checking

In your neural network, you are minimizing the cost function $J(\Theta)$. To perform gradient checking on your parameters, you can imagine 'unrolling' the parameters $\Theta^{(1)}, \Theta^{(2)}$ into a long vector θ. By doing so, you can think of the cost function being $J(\theta)$ instead and use the following gradient checking procedure.

Suppose you have a function $f_i(\theta)$ that purportedly computes $\frac{\partial}{\partial \theta_i} J(\theta)$; you'd like to check if f_i is outputting correct derivative values.

$$\text{Let } \theta^{i+} = \theta + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \epsilon \\ \vdots \\ 0 \end{bmatrix} \text{ and } \theta^{i-} = \theta - \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \epsilon \\ \vdots \\ 0 \end{bmatrix}$$

So, $\theta^{(i+)}$ is the same as θ, except its i-th element has been incremented by ϵ. Similarly, $\theta^{(i-)}$ is the corresponding vector with the i-th element decreased by ϵ. You can now numerically verify $f_i(\theta)$'s correctness by checking, for each i, that:

$$f_i(\theta) \approx \frac{J(\theta^{(i+)}) - J(\theta^{(i-)})}{2\epsilon}$$

The degree to which these two values should approximate each other will depend on the details of J. But assuming $\epsilon = 10^{-4}$, you'll usually find that the left- and right-hand sides of the above will agree to at least 4 signicant digits (and often many more). We have implemented the function to compute the numerical gradient for you in `computeNumericalGradient.m`. While you are not required to modify the file, we highly encourage you to take a look at the code to understand how it works.

The code below will run the provided function `checkNNGradients.m` which will create a small neural network and dataset that will be used for checking your gradients. If your backpropagation implementation is correct, you should see a relative dierence that is less than 1e-9.

```
checkNNGradients;
```

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   -0.0067    -0.0067
   -0.0000    -0.0000
    0.0000     0.0000
   -0.0000    -0.0000
    0.0000     0.0000
   -0.0000    -0.0000
   -0.0002    -0.0002
    0.0002     0.0002
   -0.0003    -0.0003
    0.0003     0.0003
   -0.0004    -0.0004
   -0.0001    -0.0001
    0.0001     0.0001
```

```
  -0.0001    -0.0001
   0.0002     0.0002
  -0.0002    -0.0002
   0.3145     0.3145
   0.1111     0.1111
   0.0974     0.0974
   0.1641     0.1641
   0.0576     0.0576
   0.0505     0.0505
   0.1646     0.1646
   0.0578     0.0578
   0.0508     0.0508
   0.1583     0.1583
   0.0559     0.0559
   0.0492     0.0492
   0.1511     0.1511
   0.0537     0.0537
   0.0471     0.0471
   0.1496     0.1496
   0.0532     0.0532
   0.0466     0.0466


 The above two columns you get should be very similar.
 (Left-Your Numerical Gradient, Right-Analytical Gradient)

 If your backpropagation implementation is correct, then
 the relative difference will be small (less than 1e-9).

 Relative Difference: 2.54664e-11
```

Practical Tip: When performing gradient checking, it is much more efficient to use a small neural network with a relatively small number of input units and hidden units, thus having a relatively small number of parameters. Each dimension of θ requires two evaluations of the cost function and this can be expensive. In the function `checkNNGradients`, our code creates a small random model and dataset which is used with `computeNumericalGradient` for gradient checking. Furthermore, after you are condent that your gradient computations are correct, you should turn off gradient checking before running your learning algorithm.

Practical Tip: Gradient checking works for any function where you are computing the cost and the gradient. Concretely, you can use the same `computeNumericalGradient.m` function to check if your gradient implementations for the other exercises are correct too (e.g. logistic regression's cost function). Once your cost function passes the gradient check for the (unregularized) neural network cost function, you should submit the neural network gradient function (backpropagation).

> *You should now submit your solutions. Enter* **submit** *at the command*
> *prompt, then enter or confirm your login and token when prompted.*

2.5 Regularized neural networks

After you have successfully implemeted the backpropagation algorithm, you will add regularization to the gradient. To account for regularization, it turns out that you can add this as an additional term after computing the gradients using backpropagation. Specically, after you have computed $\Delta_{ij}^{(l)}$ using backpropagation, you should add regularization using

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = D_{ij}^{(l)} = \frac{1}{m} \Delta_{ij}^{(l)} \text{ for } j = 0,$$

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = D_{ij}^{(l)} = \frac{1}{m} \Delta_{ij}^{(l)} + \frac{\lambda}{m} \Theta_{ij}^{(l)} \text{ for } j \geq 1$$

Note that you should *not* be regularizing the first column of $\Theta^{(l)}$ which is used for the bias term. Furthermore, in the parameters $\Theta_{ij}^{(l)}$, i is indexed starting from 1, and j is indexed starting from 0. Thus,

$$\Theta^{(l)} = \begin{bmatrix} \Theta_{1,0}^{(l)} & \Theta_{1,1}^{(l)} & \cdots \\ \Theta_{2,0}^{(l)} & \Theta_{2,1}^{(l)} & \\ \vdots & & \ddots \end{bmatrix}$$

Somewhat confusingly, indexing in MATLAB starts from 1 (for both i and j), thus `Theta1(2, 1)` actually corresponds to $\Theta_{2,0}^{(l)}$ (i.e., the entry in the second row, first column of the matrix $\Theta^{(1)}$ shown above)

Now, modify your code that computes `grad` in `nnCostFunction` to account for regularization. After you are done, run the code below to run gradient checking on your implementation. If your code is correct, you should expect to see a relative difference that is less than 1e-9.

```
%  Check gradients by running checkNNGradients
lambda = 3;
checkNNGradients(lambda);
```

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J = 2.1010
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  -0.0084   -0.0084
   0.0076    0.0076
  -0.0067   -0.0067
  -0.0168   -0.0168
   0.0394    0.0394
   0.0593    0.0593
   0.0248    0.0248
  -0.0327   -0.0327
  -0.0602   -0.0602
  -0.0320   -0.0320
   0.0249    0.0249
   0.0598    0.0598
   0.0386    0.0386
  -0.0174   -0.0174
  -0.0576   -0.0576
  -0.0452   -0.0452
   0.0091    0.0091
```

```
     0.0546    0.0546
     0.3145    0.3145
     0.1111    0.1111
     0.0974    0.0974
     0.1187    0.1187
     0.0000    0.0000
     0.0337    0.0337
     0.2040    0.2040
     0.1171    0.1171
     0.0755    0.0755
     0.1257    0.1257
    -0.0041   -0.0041
     0.0170    0.0170
     0.1763    0.1763
     0.1131    0.1131
     0.0862    0.0862
     0.1323    0.1323
    -0.0045   -0.0045
     0.0015    0.0015

The above two columns you get should be very similar.
(Left-Your Numerical Gradient, Right-Analytical Gradient)

If your backpropagation implementation is correct, then
the relative difference will be small (less than 1e-9).

Relative Difference: 2.45988e-11
```

```matlab
% Also output the costFunction debugging value
% This value should be about 0.576051
debug_J  = nnCostFunction(nn_params, input_layer_size, hidden_layer_size, num_labels, )
```

```
J = 0.2876
```

```matlab
fprintf('Cost at (fixed) debugging parameters (w/ lambda = 3): %f', debug_J);
```

```
Cost at (fixed) debugging parameters (w/ lambda = 3): 0.576051
```

You should now submit your solutions. Enter **submit** *at the command*
prompt, then enter or confirm your login and token when prompted.

2.6 Learning parameters using `fmincg`

After you have successfully implemented the neural network cost function and gradient computation, run
the code below to use `fmincg` to learn a good set of parameters. After the training completes, the code will
report the training accuracy of your classifier by computing the percentage of examples it got correct. If your
implementation is correct, you should see a reported training accuracy of about 95.3% (this may vary by about
1% due to the random initialization). It is possible to get higher training accuracies by training the neural network
for more iterations.

```matlab
options = optimset('MaxIter', 50);
lambda = 1;

% Create "short hand" for the cost function to be minimized
costFunction = @(p) nnCostFunction(p, input_layer_size, hidden_layer_size, num_labels,

% Now, costFunction is a function that takes in only one argument (the
```

```
% neural network parameters)
[nn_params, ~] = fmincg(costFunction, initial_nn_params, options);
```

```
J = 7.0985
J = 6.2391
J = 4.8999
J = 3.2882
Iteration     1 | Cost: 3.293566e+00
J = 12.3730
J = 4.1593
J = 3.2533
J = 3.2286
Iteration     2 | Cost: 3.234026e+00
J = 3.2293
J = 3.1971
Iteration     3 | Cost: 3.202552e+00
J = 3.1302
J = 2.9752
J = 2.5937
J = 2.5524
Iteration     4 | Cost: 2.565109e+00
J = 2.2503
Iteration     5 | Cost: 2.278016e+00
J = 1.8906
Iteration     6 | Cost: 1.923296e+00
J = 2.2316
J = 1.7878
Iteration     7 | Cost: 1.820969e+00
J = 1.6361
Iteration     8 | Cost: 1.670169e+00
J = 1.5014
Iteration     9 | Cost: 1.536359e+00
J = 1.4148
Iteration    10 | Cost: 1.451160e+00
J = 1.3144
Iteration    11 | Cost: 1.354542e+00
J = 1.1923
Iteration    12 | Cost: 1.235328e+00
J = 1.1555
J = 1.1233
Iteration    13 | Cost: 1.167682e+00
J = 1.0341
Iteration    14 | Cost: 1.080305e+00
J = 1.0667
J = 1.0084
Iteration    15 | Cost: 1.054811e+00
J = 0.9748
Iteration    16 | Cost: 1.022122e+00
J = 0.9354
Iteration    17 | Cost: 9.835534e-01
J = 0.8996
Iteration    18 | Cost: 9.487658e-01
J = 0.8564
J = 0.8351
Iteration    19 | Cost: 8.877320e-01
J = 0.7696
Iteration    20 | Cost: 8.296503e-01
J = 0.7133
Iteration    21 | Cost: 7.757746e-01
J = 0.7464
J = 0.6900
Iteration    22 | Cost: 7.538789e-01
J = 0.6605
Iteration    23 | Cost: 7.259977e-01
J = 0.6500
```

```
J = 0.6456
Iteration     24 | Cost: 7.118843e-01
J = 0.6250
Iteration     25 | Cost: 6.928333e-01
J = 0.6070
Iteration     26 | Cost: 6.756728e-01
J = 0.5962
Iteration     27 | Cost: 6.660360e-01
J = 0.6100
J = 0.5906
Iteration     28 | Cost: 6.619006e-01
J = 0.5828
J = 0.5727
Iteration     29 | Cost: 6.446441e-01
J = 0.5702
J = 0.5655
Iteration     30 | Cost: 6.376277e-01
J = 0.5654
J = 0.5619
Iteration     31 | Cost: 6.342238e-01
J = 0.5553
J = 0.5468
Iteration     32 | Cost: 6.208050e-01
J = 0.5332
Iteration     33 | Cost: 6.086360e-01
J = 0.5155
J = 0.5067
Iteration     34 | Cost: 5.866748e-01
J = 0.4740
Iteration     35 | Cost: 5.617508e-01
J = 0.4454
Iteration     36 | Cost: 5.370093e-01
J = 0.4346
J = 0.4301
Iteration     37 | Cost: 5.241099e-01
J = 0.4195
J = 0.4187
Iteration     38 | Cost: 5.152348e-01
J = 0.4068
Iteration     39 | Cost: 5.059309e-01
J = 0.3958
Iteration     40 | Cost: 4.975483e-01
J = 0.3828
Iteration     41 | Cost: 4.887821e-01
J = 0.3710
Iteration     42 | Cost: 4.800555e-01
J = 0.3608
Iteration     43 | Cost: 4.733011e-01
J = 0.3473
Iteration     44 | Cost: 4.660747e-01
J = 0.3350
Iteration     45 | Cost: 4.561627e-01
J = 0.3275
Iteration     46 | Cost: 4.503039e-01
J = 0.3522
J = 0.3249
Iteration     47 | Cost: 4.486452e-01
J = 0.3222
J = 0.3205
Iteration     48 | Cost: 4.443667e-01
J = 0.3187
J = 0.3177
Iteration     49 | Cost: 4.417934e-01
J = 0.3155
Iteration     50 | Cost: 4.399828e-01
```

```
% Obtain Theta1 and Theta2 back from nn_params
Theta1 = reshape(nn_params(1:hidden_layer_size * (input_layer_size + 1)), hidden_layer_
Theta2 = reshape(nn_params((1 + (hidden_layer_size * (input_layer_size + 1))):end), num

pred = predict(Theta1, Theta2, X);
fprintf('\nTraining Set Accuracy: %f\n', mean(double(pred == y)) * 100);
```

```
Training Set Accuracy: 96.540000
```

3. Visualizing the hidden layer

One way to understand what your neural network is learning is to visualize what the representations captured by the hidden units. Informally, given a particular hidden unit, one way to visualize what it computes is to find an input x that will cause it to activate (that is, to have an activation value $(a_i^{(l)})$ close to 1). For the neural network you trained, notice that the ith row of $\Theta^{(1)}$ is a 401-dimensional vector that represents the parameter for the ith hidden unit. If we discard the bias term, we get a 400 dimensional vector that represents the weights from each input pixel to the hidden unit.

Thus, one way to visualize the 'representation' captured by the hidden unit is to reshape this 400 dimensional vector into a 20 x 20 image and display it.*
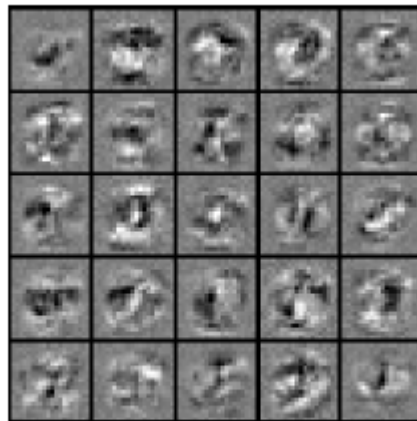

Figure 4: Visualization of Hidden Units.

The code below does this by using the `displayData` function and it will show you an image (similar to Figure 4) with 25 units, each corresponding to one hidden unit in the network. In your trained network, you should find that the hidden units corresponds roughly to detectors that look for strokes and other patterns in the input.

```
% Visualize Weights
displayData(Theta1(:, 2:end));
```

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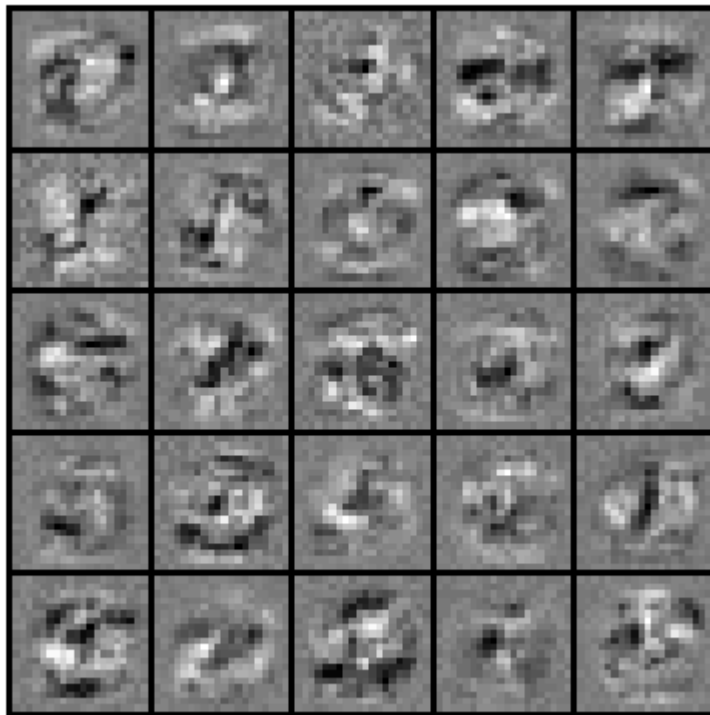
It turns out that this is equivalent to finding the input that gives the highest activation for the hidden unit, given a 'norm' constraint on the input (i.e., $\|x\|_2 \leq 1$).

3.1 Optional (ungraded) exercise

In this part of the exercise, you will get to try out different learning settings for the neural network to see how the performance of the neural network varies with the regularization parameter and number of training steps (the `MaxIter` option when using `fmincg`). Neural networks are very powerful models that can form highly complex decision boundaries. Without regularization, it is possible for a neural network to 'overfit' a training set so that it obtains close to 100% accuracy on the training set but does not do as well on new examples that it has not seen before. You can set the regularization λ to a smaller value and the `MaxIter` parameter to a higher number of iterations to see this for youself. You will also be able to see for yourself the changes in the visualizations of the hidden units when you change the learning parameters λ and `MaxIter`.

```
% Change lambda and MaxIter to see how it affects the result
lambda = 30;
MaxIter = 50;

options = optimset('MaxIter', MaxIter);

% Create "short hand" for the cost function to be minimized
costFunction = @(p) nnCostFunction(p,input_layer_size, hidden_layer_size, num_labels, X

% Now, costFunction is a function that takes in only one argument (the neural network p
```

```
[nn_params, ~] = fmincg(costFunction, initial_nn_params, options);
```

```
J = 7.0985
J = 6.2391
J = 4.8999
J = 3.2884
Iteration     1 | Cost: 3.304444e+00
J = 12.3654
J = 4.1564
J = 3.2532
J = 3.2287
Iteration     2 | Cost: 3.244833e+00
J = 3.2309
J = 3.1974
Iteration     3 | Cost: 3.213745e+00
J = 3.1314
J = 2.9784
J = 2.5978
J = 2.5544
Iteration     4 | Cost: 2.590832e+00
J = 2.2444
Iteration     5 | Cost: 2.324071e+00
J = 1.8953
Iteration     6 | Cost: 1.991264e+00
J = 2.1721
J = 1.7875
Iteration     7 | Cost: 1.886049e+00
J = 1.6280
Iteration     8 | Cost: 1.730996e+00
J = 1.4907
Iteration     9 | Cost: 1.596524e+00
J = 1.4138
Iteration    10 | Cost: 1.524138e+00
J = 1.3369
Iteration    11 | Cost: 1.461416e+00
J = 1.2072
Iteration    12 | Cost: 1.342640e+00
J = 1.1670
J = 1.1347
Iteration    13 | Cost: 1.271874e+00
J = 1.0391
Iteration    14 | Cost: 1.178731e+00
J = 1.0602
J = 1.0093
Iteration    15 | Cost: 1.149421e+00
J = 0.9658
Iteration    16 | Cost: 1.108002e+00
J = 0.9317
Iteration    17 | Cost: 1.075967e+00
J = 0.8841
Iteration    18 | Cost: 1.032507e+00
J = 0.8480
Iteration    19 | Cost: 9.983463e-01
J = 0.8237
Iteration    20 | Cost: 9.771485e-01
J = 0.8035
J = 0.8001
Iteration    21 | Cost: 9.592894e-01
J = 0.7688
J = 0.7444
Iteration    22 | Cost: 9.077883e-01
J = 0.7355
J = 0.7149
Iteration    23 | Cost: 8.816438e-01
J = 0.6869
```

```
Iteration    24 | Cost: 8.588840e-01
J = 0.6614
Iteration    25 | Cost: 8.408216e-01
J = 0.6341
Iteration    26 | Cost: 8.235590e-01
J = 0.5976
Iteration    27 | Cost: 7.969360e-01
J = 0.5688
Iteration    28 | Cost: 7.732745e-01
J = 0.5890
J = 0.5579
Iteration    29 | Cost: 7.650965e-01
J = 0.5476
Iteration    30 | Cost: 7.568414e-01
J = 0.5392
Iteration    31 | Cost: 7.508762e-01
J = 0.5259
Iteration    32 | Cost: 7.421697e-01
J = 0.5137
Iteration    33 | Cost: 7.313863e-01
J = 0.5152
J = 0.5098
Iteration    34 | Cost: 7.275229e-01
J = 0.5066
Iteration    35 | Cost: 7.244159e-01
J = 0.5027
Iteration    36 | Cost: 7.209570e-01
J = 0.4985
Iteration    37 | Cost: 7.171203e-01
J = 0.4938
Iteration    38 | Cost: 7.127795e-01
J = 0.4886
Iteration    39 | Cost: 7.081271e-01
J = 0.4848
Iteration    40 | Cost: 7.050025e-01
J = 0.4793
Iteration    41 | Cost: 7.012900e-01
J = 0.4723
J = 0.4673
Iteration    42 | Cost: 6.913258e-01
J = 0.4579
Iteration    43 | Cost: 6.850198e-01
J = 0.4489
J = 0.4486
Iteration    44 | Cost: 6.798318e-01
J = 0.4396
J = 0.4345
Iteration    45 | Cost: 6.678291e-01
J = 0.4503
J = 0.4303
Iteration    46 | Cost: 6.644661e-01
J = 0.4280
J = 0.4276
Iteration    47 | Cost: 6.624915e-01
J = 0.4244
Iteration    48 | Cost: 6.602657e-01
J = 0.4217
Iteration    49 | Cost: 6.584789e-01
J = 0.4175
Iteration    50 | Cost: 6.560140e-01
```

```
% Obtain Theta1 and Theta2 back from nn_params
Theta1 = reshape(nn_params(1:hidden_layer_size * (input_layer_size + 1)), hidden_layer_
```

```
Theta2 = reshape(nn_params((1 + (hidden_layer_size * (input_layer_size + 1))):end), num
```

```
pred = predict(Theta1, Theta2, X);
fprintf('\nTraining Set Accuracy: %f\n', mean(double(pred == y)) * 100);
```

```
Training Set Accuracy: 95.360000
```

```
% Visualize Weights
displayData(Theta1(:, 2:end));
```


You do not need to submit any solutions for this optional (ungraded) exercise.

Submission and Grading

After completing various parts of the assignment, be sure to use the submit function system to submit your solutions to our servers. The following is a breakdown of how each part of this exercise is scored.

Part	Submitted File	Points
Feedforward and Cost Function	nnCostFunction.m	30 points
Regularized Cost Function	nnCostFunction.m	15 points
Sigmoid Gradient	sigmoidGradient.m	5 points
Neural Net Gradient Function (Backpropagation)	nnCostFunction.m	40 points
Regularized Gradient	nnCostFunction.m	10 points
Total Points		100 points

You are allowed to submit your solutions multiple times, and we will take only the highest score into consideration.