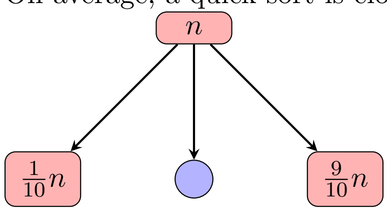
Algorithms

0.1 The Average Case

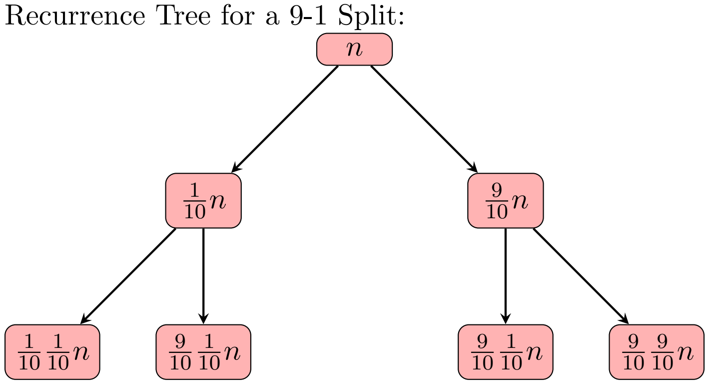
On average, a quick sort is close to its base case.



0.1.1 Recurrence

$$T(n) \leq T\left(\frac{9}{10}n\right) + T\left(\frac{1}{10}n\right) + C \cdot n$$

Recurrence Tree for a 9-1 Split:



The left will reach the bottom quickly, the right will take longer.
In general, any split that is a constant fraction will be in the same complexity category as its best case because it is not dependent on n.
The best split is when the pivor is in the middle.
The worst split is when nothing is on one side of the pivot.

0.1.2 Alternating Tree

Best Case Scenario:

$$\frac{n-1}{2} + \frac{n-1}{2} = n - 1 \in O(n)$$

When the splits rotate from worst to best and back, it is the same time complexity as the best case..

0.1.3 Randomized Quicksort

Choose a random item from the list, rather than taking the first item as a point. Do this by grabbing a random item and swapping it with the first item.

0.2 Statistics Recap

The **Sample Space** S is the set of all possible outcomes.

$$S = \{S_1, S_2, S_3, ..., S_k\}$$

An **Event** is a subset of a sample space, such as the event of "getting at least 1 heads" of the sample space "flipping two coins".
Probability is the likelihood of an event.
$P(Event) =$ The sum of basic outcomes in the event.
1. Probability is always between 0 and 1
2. For any event $E \in S$, $P(E) = \sum_{S_i \in E} P(S_i)$
3. $P(S) = 1,\ P(0) = 0$

0.2.1 Discrete Random Variables

Every basic outcome is matched to a real number.

$$X : S \to \mathbb{R}$$

$$s \in S : X(s) = x$$

Probability Density Function:

$$f(x) = P(X = x)$$

$$0 \leq P(X = x) \leq 1$$

$$\sum_{i=1}^{k} P(X = x_i) = 1$$

Example:
Define X to be the highest result of 2 dice rolled.
$P(X = 5)$ would be the chance that the highest roll is 5.
The set defined by $X = 5$:

$$\begin{bmatrix} 15 & 51 & 25 \\ 52 & 35 & 53 \\ 45 & 54 & 55 \end{bmatrix}$$

$$P(X = 5) = \frac{1}{36} \cdot 9 = \frac{1}{4}$$

The **Expected Value of X**, or $E[X]$, is

$$E[X] = \sum_{i=1}^{k} X_i \cdot P(X_i)$$

Example:
In a game where you flip 2 coins, getting 3 points for heads and losing 2 points for tails, X represents the payout from a single round.

$$E[X] = 6 \cdot P(H,H) + 1 \cdot P(H,T) + 1 \cdot P(T,H) + -4 \cdot P(T,T) = 1$$

0.2.2 Indicator Random Variable

A way to go between P and E.

Example:
Flipping a coin n times, what is the expected number of heads?
Let C be the random variable representing the number of heads.
$C_i = 0$ if tails, $C_i = 1$ for i from 1 to n

$$C = \sum_{i=1}^{n} C_i$$

$$P(C_i = 1) = \frac{1}{2},\ P(C_i = 0) = \frac{1}{2}$$

$$E\left[C_i\right] = 0 \cdot P(C_i = 0) + 1 \cdot P(C_i = 1) = \frac{1}{2}$$

$$E\left[C\right] = E\left[\sum_{i=1}^{k} C_i\right] = \frac{n}{2}$$